9 MATH

Quarter 1

PIVOT 4A LEARNER'S MATERIAL

This module is a resource of information and guide in understanding the Most Essential Learning Competencies (MELCs). Understanding the target contents and skills can be further enriched thru the K to 12 Learning Materials and other supplementary materials such as worksheets/activity sheets provided by schools and/ or Schools Division Offices and thru other learning delivery modalities including radio-based and TV-based instruction (RB/TVI).

CLMD CALABARZON

Mathematics

Grade 9

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Mathematics Grade 9
PIVOT IV-A Learner's Material
Quarter 1 Module 1
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Guide in Using PIVOT Learner's Material

For the Parents/Guardian

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners in guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

As a facilitator, you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are

For the Learner

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will have to process the contents of the learning resource while being an active learner.

PARTS OF PIVOT LEARNER'S MATERIAL

	Parts of the LM	Description
Introduction	What I need to know	The teacher utilizes appropriate strategies in presenting the MELC and desired learning outcomes for the day or week, purpose of the lesson, core content and relevant samples. This allows teachers to maximize learners awareness of their own knowledge as regards content and skills required for the lesson
	What is new	
Development	What I know	The teacher presents activities, tasks , contents of value and interest to the learners. This shall expose the learners on what he/she knew, what he /she does not know and what she/he wanted to know and learn. Most of the activities and tasks must simply and directly revolved around the concepts to develop and master the skills or the MELC.
	What is in	
	What is it	
Engagement	What is more	The teacher allows the learners to be engaged in various tasks and opportunities in building their KSA's to meaningfully connect their learnings after doing the tasks in the D. This part exposes the learner to real life situations /tasks that shall ignite his/ her interests to meet the expectation, make their performance satisfactory or produce a product or performance which lead him/ her to understand fully the skills and concepts .
	What I can do	
	What else I can do	
Assimilation	What I have learned	The teacher brings the learners to a process where they shall demonstrate ideas, interpretation , mindset or values and create pieces of information that will form part of their knowledge in reflecting, relating or using it effectively in any situation or context. This part encourages learners in creating conceptual structures giving them the avenue to integrate new and old learnings.
	What I can achieve	

WEEK 1

Quadratic Equations

I

Lesson

After going through this lesson, you are expected to:

- Illustrates quadratic equations.
- Solves quadratic equations by:

(a) extracting square roots; (b) factoring; (c) completing the square; and (d) using the quadratic formula.

You learned about linear equation in one variable which is in the form of ax + b = 0, where a is not 0 otherwise the equation is constant. Quadratic equation is in the form of $ax^2 + bx + c = 0$ and a cannot be zero otherwise the equation will become linear equation.

Learning Task 1. Group the given equations into two based on observed common properties.

$n^2 - 3n + 10 = 0$	$8 - 3k = 12$	$2y - z = 9$	$2x^2 + 2x + 1 = 0$
$25b^2 - 16 = 0$	$3r + 2e = -6$	$5w + 5 = 0$	$f^2 - 3f + 2 = 0$
$d = 3e - 7$	$\frac{1}{3}m^2 + 2m = 4$	$10u - 5 = 8$	$a^2 = 225$

D

The standard form of quadratic equation is $\boldsymbol{ax^2 + bx + c = 0}$, *where a, b and c* are real numbers and a is not equal to zero.

Illustrative Examples:

1. $2x + 5 = 0$ is not a quadratic equation in one variable. It is a linear equation in one variable.
2. $2x^2 - x - 1 = 0$ is a quadratic equation in standard form with a = 2, b = -1, and c = -1.
3. $3x - 4 = 5x^2$ is a quadratic equation not in standard form, in this case we need to express it in its standard form to identify the values of a, b and c. To write it: $3x - 4 - 5x^2 = 5x^2 - 5x^2$ (Subtraction Property of Equality)

   $3x - 4 - 5x^2 = 0$ (Arrange the terms)

   $(-5x^2 + 3x - 4 = 0)\ {-1}$ (Obtain a > 0, by multiplying – 1to each term of the equation.)

   $5x^2 - 3x + 4 = 0$

   In this form, a = 5, b = -3 and c = 4.

4. $(2x + 3)(x - 1) = 0$ is also a quadratic equation but is not written in standard form.

Expanding :

$2x(x - 1) + 3(x - 1) = 0$

$2x^2 - 2x + 3x - 3 = 0$ (Distributive Property)

$2x^2 + x - 3 = 0$ (Combining similar terms)

In this form, $a = 2$, $b = 1$ and $c = -3$.

A **quadratic equation** in one variable is a mathematical sentence of degree two that can be written in the following standard form: $\mathbf{ax^2 + bx + c = 0}$ where, a, b and c are real numbers $a \neq 0$. In this equation, ax^2 is the quadratic term (degree two), bx is the linear term (degree one) and c is the constant term (degree zero).

When $b = 0$, in the equation $ax^2 + bx + c = 0$, the result is a quadratic equation of the form $ax^2 + c = 0$. For example: $x^2 - 16 = 0$, $25x^2 - 81 = 0$ and $5x^2 = 500$. Furthermore, when $c = 0$, the quadratic equation is reduced to $ax^2 + bx = 0$. That is, in $x^2 + 3x = 0$ and $5x^2 - x = 0$, there is no constant term, $c = 0$.

In solving quadratic equation, we can apply the following methods:

A. Solving quadratic equations by extracting square roots.

Remember when it was mentioned that a quadratic equation of the form $ax^2 + bx + c = 0$ may have $b = 0$, such that, $ax^2 + c = 0$. In other words, since c could be any constant, then, $ax^2 = c$. And, if $a= 1$, the quadratic equation is further reduced to $x^2 = c$. Recall square roots.

In order to solve a quadratic equation by extracting square roots, the equation must be written in the form $x^2 = c$, before extracting the square roots of the left and right sides of the said equation so as to have the equation balanced. Inspect the given examples.

ILLUSTRATIVE EXAMPLES:

Solving for the values of the variable x by extracting square roots:

1. $x^2 = 9$ Quadratic equation in the form $x^2 = c$

$\sqrt{x^2} = \sqrt{9}$ Extracting the square roots of the left and right sides

$x = \pm 3$ Possible values that will satisfy the quadratic equation

To check for the solved values, substitute both +3 and -3 in the given equation. Also, always remember that a negative number does not have a square root.

2. $x^2 - 25 = 0$ — Quadratic equation in the form $x^2 + c = 0$

$x^2 = 25$ — b y Addition Property of Equality

$\sqrt{x^2} = \sqrt{25}$ — Extracting the square roots of the left and right sides

$x = \pm 5$ — Possible values that will satisfy the quadratic equation

3. $4x^2 = 49$ — Quadratic equation in the form $ax^2 = c$

$x^2 = \frac{49}{4}$ — by Multiplication Property of Equality multiplying both sides by $\frac{1}{4}$

$\sqrt{x^2} = \sqrt{\frac{49}{4}}$ — Extracting the square roots of the left and right sides

$X = \pm \frac{7}{2}$ — Possible values that will satisfy the quadratic equation

4. Find the roots of the equation $(x - 1)^2 = 0$.

Again, applying extracting square roots:

$\sqrt{(x-1)^2} = \sqrt{0}$ — Extracting square roots of the left and right sides

$x - 1 = 0$

$x = 1$ — by Addition Property of Equality

B. Solving quadratic equations by factoring

Only quadratic equation that is factorable can be solved by factoring. To solve such quadratic equation, the following procedure can be followed.

1. Transform the quadratic expression into standard form if necessary.
2. Factor the quadratic expression.
3. Apply zero product property by setting each factor of the quadratic expression equal to 0.
4. Solve the resulting equation.
5. Check the values of the variable obtained by substituting each in the original equation.

Zero Product Property. If a and b, are real numbers , then (a)(b) = 0, such that, a = 0 or b = 0 or a and b are both equal to zero.

So, if a given quadratic equation is in the form (a)(b) = 0, the Zero Product Property can be applied. To do this, the given quadratic equation must be written in the standard form $ax^2 + bx + c = 0$ before applying the factoring method.

ILLUSTRATIVE EXAMPLES:

Solve each equation by factoring.

1. $n^2 + 2n + 1 = 0$	Quadratic equation in standard form
$(n + 1)(n + 1) = 0$	Factoring the left side of the equation
$n + 1 = 0 \quad \vert \quad n + 1 = 0$	by Zero Product Property
$n = -1 \quad \vert \quad n = -1$	by Addition Property of Equality (APE)
2. $m^2 + 3m + 2 = 0$	Quadratic equation in standard form
$(m + 2)(m + 1) = 0$	Factoring the left side of the equation
$m + 2 = 0 \quad \vert \quad m + 1 = 0$	by Zero Product Property
$m = -2 \quad \vert \quad m = -1$	by Addition Property of Equality

C. Solving quadratic equations by completing the square.

If the first two methods in solving quadratic equations cannot be used to solve such quadratic equations, then, you must need another method to solve the said equations. This other way of solving quadratic equations is actually referred to as the mother of all methods in solving any quadratic equation – the completing the square method.

The completing the square method also includes the use of extracting square roots after the completing of square part. You may use a scientific calculator in writing the approximate value/s of the answer/s if they are irrational number values.

Completing the square includes the following steps:

1. Divide both sides of the equation by "a "then simplify.
2. Write the equation such that the terms with variables are on the left side of the equation and the constant term is on the right side.
3. Add the square of one-half of the coefficient of "x" on both sides of the resulting equation. The left side of the equation becomes a perfect square trinomial.
4. Express the perfect square trinomial on the left side of the equation as a square of a binomial.
5. Solve the resulting quadratic equation by extracting the square root.
6. Solve the resulting linear equations.
7. Check the solutions obtained against the original equation.

ILLUSTRATIVE EXAMPLES:

Express the following as a squared binomial by completing the square.

1. $x^2 +$ _____ $+ 9$ — Incomplete perfect square trinomial (the quadratic/ first and constant/last terms are perfect squares)

$x^2 + \underline{\ 6x\ } + 9$ — The middle or linear term is found by doubling the product of the square roots of the first and last terms

$2\bullet\sqrt{x^2}\bullet\sqrt{9}$

$(x + 3)(x + 3)$ — Writing as product of the same binomial or as a

$(x + 3)^2$ — Squared binomial

2. $4e^2 -$ _____ $+ 25$ — Incomplete perfect square trinomial (the quadratic/ first and constant/last terms are perfect square)

$4e^2 – \underline{20e\ } + 25$ — The middle or linear term is found by doubling the product of the square roots of the first and last terms

$2\bullet\sqrt{4e^2}\bullet\sqrt{25}$

$(2e – 5)(2e – 5)$ — Writing as product of the same binomial or as

$(2e – 5)^2$ — Squared binomial

NOTE: The sign of the middle (missing) term will also be the sign of the operation between the two terms in the squared binomial.

What do you do when the quadratic trinomial to be completed does not seem to be incomplete – it consists of three terms but is not a perfect square trinomial?

3. Express $x^2 + 2x + 4$ as a squared binomial by completing the square

$x^2 + 2x + 4$ — Quadratic but not a perfect square trinomial

$x^2 + 2x +$ ____ $+ 4 -$ ____ — Terms to be added must sum up to zero

$x^2 + 2x + \underline{\ 1\ } + 4 - \underline{\ 1\ }$ — The added term is the square of half the numerical coefficient of the middle term

$x^2 + 2x + 1 + 3$ — Combining constants

$(x + 1)^2 + 3$ — Writing the perfect square trinomial as a squared binomial

D. Solving quadratic equations using the quadratic formula.

For any given quadratic equation (in one variable) in the standard form $ax^2 + bx + c = 0$, all you need to do is substitute the corresponding values of the numerical coefficients a, b and c from the standard form of the quadratic equation in the formula;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ILLUSTRATIVE EXAMPLES:

1. $x^2 + 6x + 5 = 0$, the values of $a = 1$, $b = 6$ and $c = 5$.

   Then, using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$x = \frac{-6 \pm \sqrt{16}}{2}$$

$$x = \frac{-6 \pm 4}{2}$$

$x = -1$ or -5 (Check the solutions/roots by substituting these to the original quadratic equation)

Note: Quadratic equation has at most two zeros or roots.

Learning Task 2

A. Complete the table below

Given	**Standard Form**	**Values of**		
		a	**b**	**c**
1. $2x – 3x^2 = 5$				
2. $4 – x^2 = 5x$				
3. $(2x + 5)(x – 4) = 0$				
4. $2x(x – 1) = 6$				
5. $(x + 1)(x + 4) = 8$				

B. Solve the quadratic equation using appropriate method.

1. $x^2 - 81 = 0$ 2. $x^2 + 5x + 6 = 0$ 3. $2x^2 — 4x + 3 = 0$

E

Learning Task 3 . Solve for the variable of the following quadratic equations

A. by extracting square roots.

1. $\underline{x^2 = 169}$
2. $9b^2 = 25$
3. $(3y - 1)^2 = 0$
4. $(x - 2)^2 = 16$
5. $2(t - 3)^2 - 72 = 0$

B. by factoring

1. $x^2 + 7x = 0$
2. $m^2 + 8m = -16$
3. $x^2 + 5x - 14 = 0$
4. $2y^2 + 8y - 10 = 0$

C. by completing the square.

1. $x^2 + 5x + 6 = 0$
2. $x^2 + 2x = 8$
3. $2x^2 + 2x = 24$

D. using quadratic formula.

1. $x^2 + 5x = 14$
2. $2x^2 + 8x - 10 = 0$
3. $2x^2 + 3x = 27$

A

Learning Task 3. Using the concept map below explain what you have learned in this module.