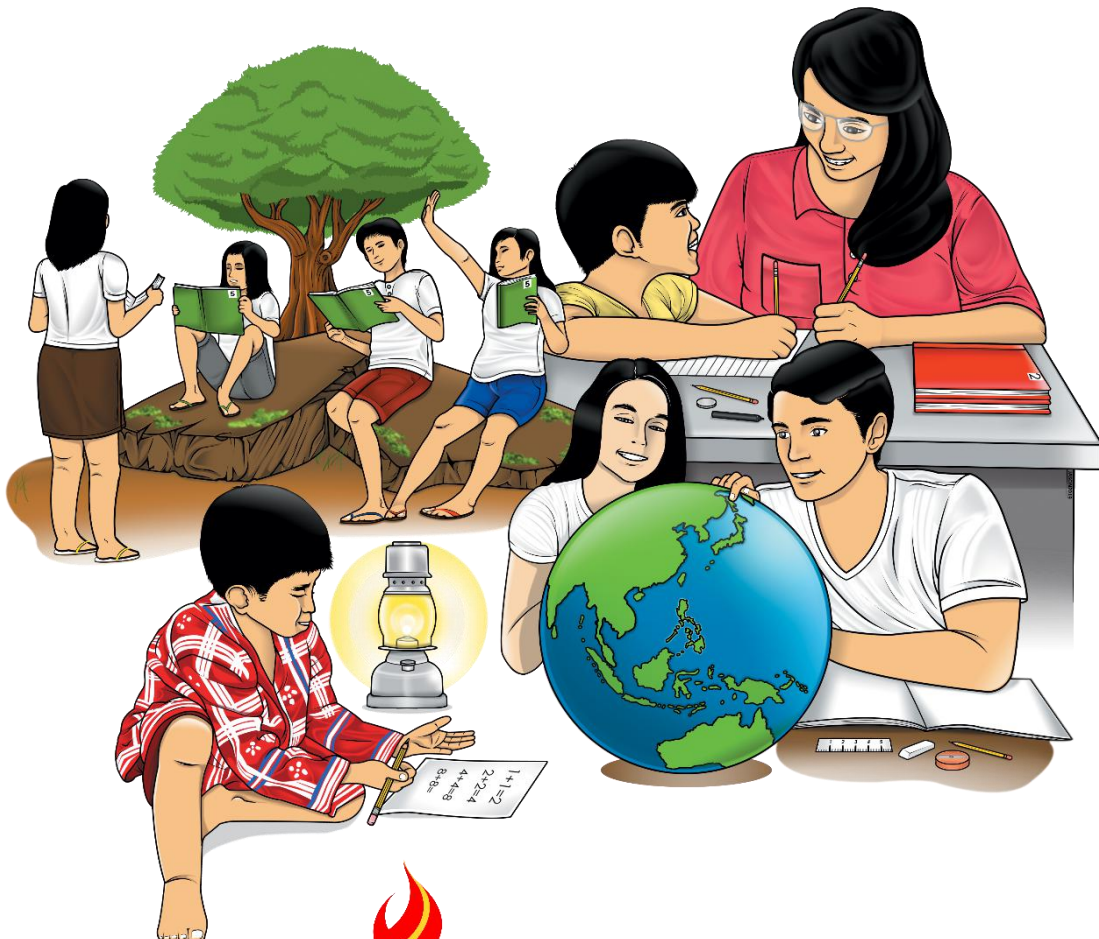**10**

Mathematics

Quarter 1 – Module 6:
Finding the nth Term of a Geometric Sequence and Geometric Means

Mathematics – Grade 10
Alternative Delivery Mode
Quarter 1 – Module 6: Finding the *nth* Term of a Geometric Sequence and
Geometric Means

DepED
DEPARTMENT OF EDUCATION

10

Mathematics

Quarter 1 – Module 6: Finding the *nth* Term of a Geometric Sequence and Geometric Means

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This is the sixth learning competency in our Mathematics 10 curriculum standards hence mastery of the skills is significant to have a smooth progress in the succeeding lessons.

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling

For the facilitator:

Being the facilitator of this module, please be patient in orienting the learner about the importance of this module. Kindly emphasize to the learner that this module is a big contribution. By the way, do not forget to remind the learner to use separate sheets in answering all of the activities found in this module.

For the learner:

Hello learner. I hope you are ready to progress in your Grade 10 Mathematics by accomplishing this learning module. This is designed to provide you with interactive tasks to further develop the desired learning competencies prescribed in the K-12 Curriculum of the Department of Education. In this module, you are expected to determine the terms of geometric sequence and means.

Here is a guide on the parts of the learning modules which you need to understand as you progress in reading and analyzing its content.

ICON	LABEL	DETAIL
	What I need to know	This will give you an idea of the skills or competencies you are expected to learn in the module.
	What I know	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
	What's In	This is a brief drill or review to help you link the current lesson with the previous one.

	What's New	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a
	What Is It	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
	What's More	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
	What I have Learned	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
	What I Can Do	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
	Assessment	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
	Additional Activities	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned
	Answer Key	This contains answers to all activities in the module.

At the end of this module you will also find:

References   This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that though this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module aims to provide the material necessary to introduce the mathematical concept of geometric sequences to Grade – 10 students. This module will discuss the procedures in finding the nth term of geometric sequences and identifying geometric means. It also includes interesting activities which will help learners understand well the derivation of formulas for nth term of geometric sequences and geometric means.

After going through this module, the learner should be able to:

a. familiarize the formulas in finding terms of geometric sequence,
b. find the nth term of a geometric sequence, and
c. determine the geometric mean/s of a geometric sequence.
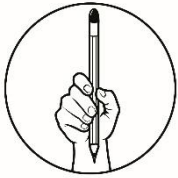
What I Know

A.   Multiple Choice. Read and analyze the following items and determine the letter of the correct answer from the given choices. Write your answer on a separate sheet of paper.

_____ 1. What expression is same as $(a \cdot a \cdot a)(b \cdot b)$?
A. $(4a)(2b)$ C. $a^4\, b^2$
B. $a^2\, b^2$ D. 8ab

_____ 2. What is the common ratio of the sequence 36, 12, 4, …?
A. 3   C. $\frac{1}{3}$
B. – 3   D. $-\frac{1}{3}$

_____ 3. The first term of a geometric sequence is 5, and the second term is 10, what is the sixth term?
A. 160   C. 320
B. 80   D. 640

_____ 4. The common ratio of a geometric sequence is $\frac{1}{2}$, and the fourth term is $\frac{1}{4}$. What is the third term of the sequence?
A. 1   C. $\frac{1}{4}$
B. $\frac{1}{2}$ D. $\frac{1}{8}$

_____ 5. The common ratio of a geometric sequence is $\frac{3}{2}$. Find the first three terms if the fifth term is 1.

A. $\frac{16}{81}, \frac{8}{27}, \frac{4}{9}$

B. $\frac{8}{27}, \frac{4}{9}, \frac{2}{3}$

C. $\frac{32}{243}, \frac{16}{81}, \frac{8}{27}$

D. $2, 1, \frac{1}{2}$

For items 6 – 10, find the specified term of the following geometric sequences.

_____ 6.) 3, 6, 12, … find a_7

_____ 7.) 10, 5, $\frac{5}{2}$, … find a_6

_____ 8.) 1, 3, 9, … find a_9

_____ 9.) 2, − 1 , $\frac{1}{2}$, … find a_{10}

_____ 10.) 100, 20, 4, … find a_8

For items 11 – 15, insert geometric means in each geometric sequence.

11.) 3, ____, 8

12.) $\frac{3}{2}$, _____, $\frac{3}{8}$

13.) 7, _____, _____, 56

14.) 36, _____, _____, _____, $\frac{9}{64}$

15.) − 1, _____, − 9 , _____, − 81

Lesson 1	**Finding the *nth* term of a Geometric Sequence**

What's In

In the previous modules, you learned about geometric sequences and how to find the next terms of geometric sequences.

Look at the sequences below.

5, 10, 20,…

36, 12, 4,…

If you are asked to find the next three terms of the geometric sequences, what method are you going to use?

Basically, you are going to multiply the common ratio to obtain the next terms as taught in the previous modules.

The sequence 5, 10, 20… is a geometric sequence with a common ratio of 2.

Therefore, the next three terms are 40, 80 and 160.

The same is through with the sequence 36, 12, 4, …with a common ratio of $\frac{1}{3}$. Therefore, the next three terms are $\frac{4}{3}$, $\frac{4}{9}$, and $\frac{4}{27}$.

> **Essential Question:**
>   What if I am going to ask for the 10th term, 20th term and 100th respectively? Are you going to use the same method?

Using the same method (multiplying the common ratio) to identify the 10th term, 20th term and 100th is such a waste of time. Don't worry! This module will solve this problem.

What's New

THE RIGHT TERM

Use any method to find the *nth* term of the given geometric sequences.

1.   What is the 7th term of the sequence 5, 10, 20, …? _____

Since you obtained already the common ratio of this sequence which is 2 and the next three terms are 40, 80, and 160, you can easily identify the 7th term. By multiplying 160 by 2, you will obtain the 7th term which is 320.

Now, what is the 10th term? 20th term?

Here is another representation:

First term: 5
Second term: 5 x 2 = 10
Third term: 10 x 2 = 20
Fourth term: 20 x 2 = 40
Fifth term: 40 x 2 = 80
Sixth term: 80 x 2 = 160
Seventh term: 160 x 2 = 320
Eighth term: 320 x 2 = 640
Ninth term: 640 x 2 = 1 280
Tenth term: 1 280 x 2 = 2 560

This method seems to be time – consuming and impractical, so, let us try to derive a shorter and a more accurate formula.

Therefore, the 10th term is 2 560.

What is It

Using the geometric sequence: 5, 10, 20, … you are asked to find for the 10th term.

Let us now derive a formula which may help us find an unknown term of a geometric sequence.

We obtained the 10th term of the sequence by using the representation.

can be written as

1st term (a_1)	5	5
2nd term(a_2)	5 x 2 = 10	5 x 2
3rd Term(a_3)	10 x 2 = 20	5 x 2 x 2
4th Term(a_4)	20 x 2 = 40	5 x 2 x 2 x 2
5th Term(a_5)	40 x 2 = 80	5 x 2 x 2 x 2 x 2
6th Term(a_6)	80 x 2 = 160	5 x 2 x 2 x 2 x 2 x 2
7th Term(a_7)	160 x 2 = 320	5 x 2 x 2 x 2 x 2 x 2 x 2
8th Term(a_8)	320 x 2 = 640	5 x 2 x 2 x 2 x 2 x 2 x 2 x 2
9th Term(a_9)	640 x 2 = 1 28	5 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2
10th Term(a_{10})	1 280 x 2 = 2 560	5 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

Looking at the illustration, we can derive a pattern that is being followed by the sequence. Let us consider the first five terms.

(a_1) = 5	5×2^0
(a_2) = 5 x 2 = 10	5×2^1
(a_3) = 10 x 2 = 20	5×2^2
(a_4) = 20 x 2 = 40	5×2^3
(a_5) = 40 x 2 = 80	5×2^4

First Term of the sequence denoted by $\mathbf{a_1}$

Common Ratio denoted by $\mathbf{r}$
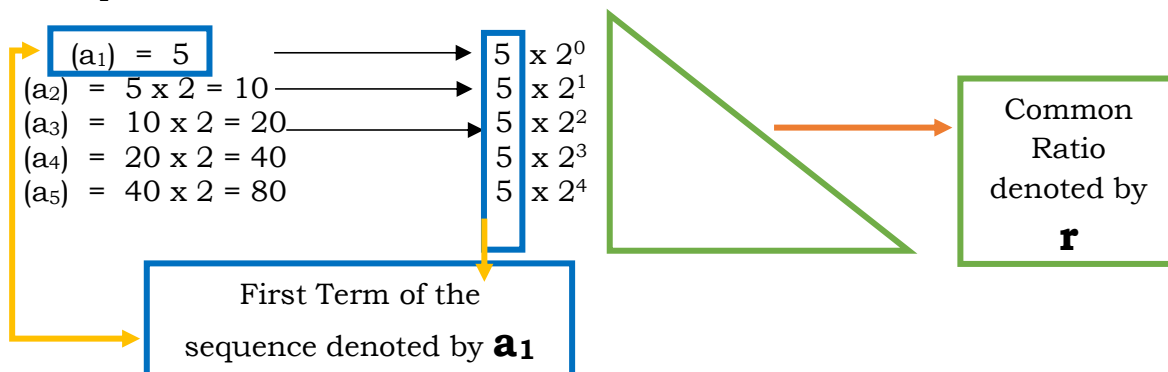
Let's try to **change the numbers to variables**.

$$a_1 = a_1 \cdot 1 = a_1 \cdot r^0$$

$$a_2 = a_1 \cdot r = a_1 \cdot r^1$$

$$a_3 = a_1 \cdot r \cdot r = a_1 \cdot r^2$$

$$a_4 = a_1 \cdot r \cdot r \cdot r = a_1 \cdot r^3$$

$$\boldsymbol{a_5 \;=\; a_1 \cdot r \cdot r \cdot r \cdot r \;=\; a_1 \cdot r^4}$$

Recall: In the laws of exponents, $a \cdot a \cdot a$ can be written as a^3, and any non-zero number raised to zero (0) is always equal to 1. Hence, $a^0 = 1$.

What can you observe from the exponent of $\boldsymbol{r}$ and the subscript of $\boldsymbol{a}$?

The exponent of $\boldsymbol{r}$ is less than one of the subscript of $\boldsymbol{a}$. The subscript of $\boldsymbol{a}$ refers to the placement of the term which is denoted by $\boldsymbol{n.}$ Therefore, using these variables, we derived the formula
$$a_n \;=\; a_1 r^{n-1}$$

GENERAL IDEA. The formula in finding the *nth* term of a geometric sequence is
$$a_n \;=\; a_1 r^{n-1}$$
Wherein:
a_n = *nth term*
a_1 = *first term*
r = *common ratio*
n = *number of terms*

Using the sequence: 5, 10, 20, ..., we identified the 7th term and the 10th term, which are **320** and **2,560** respectively. Now, let us use the derived formula to check if we are at the right track.

For the 7th term (a_7)

Given: $a_1 = 5, r = 2, n = 7$

$$a_n = a_1 r^{n-1}$$
$$a_7 = (5)(2^{7-1})$$
$$a_7 = (5)(2^6)$$
$$a_7 = (5)(64)$$
$$a_7 = 320 ✓$$

For the 10th term (a_{10})

Given: $a_1 = 5, r = 2, n = 10$

$$a_n = a_1 r^{n-1}$$
$$a_{10} = (5)(2^{10-1})$$
$$a_{10} = (5)(2^9)$$
$$a_{10} = (5)(512)$$
$$a_{10} = 2,560 ✓$$
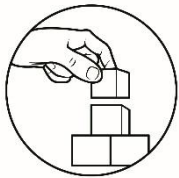
What's More

ACTIVITY 1. THE nth TIME

Find the specified term of the given geometric sequence.

1. $\frac{1}{2}$, 2, 8,...; find a_6

To solve for the unknown term, identify first the given values. The first term is ½, and the unknown term is the sixth term, so, the value of n is equal to 6. To find common ratio:

$$r = \frac{a_n}{a_{n-1}} \qquad \frac{a_2}{a_1} \to \frac{2}{\frac{1}{2}} = 4$$

The given values will then be substituted to the formula in finding the nth term of a geometric sequence.

Given: $a_1 = \frac{1}{2}$; $r = 4$; $n = 6$ Unknown = a_6

$$a_n = a_1 r^{n-1}$$

$$a_6 = \left(\frac{1}{2}\right)(4)^{6-1}$$

$$a_6 = \left(\frac{1}{2}\right)(4)^5$$

$$a_6 = \left(\frac{1}{2}\right)(1\,024)$$

$$a_6 = 512$$

ASSESSMENT 1. FIND OUT!

Find the specified term of the given geometric sequences. Show your complete solutions.

1.) $3, 6, 12, \dots$ a_7
2.) $18, 6, 2, \dots$ a_9
3.) $1, \frac{1}{2}, \frac{1}{4}, \dots$ a_7

ACTIVITY 2. FIND ME!

1.   What is the seventh term of a geometric sequence whose fourth term is 128 and common ratio equal to 4?

To begin with the problem, you must have to analyze carefully what does it ask for. The problem is asking for the 7th term but the first term was not given.

First, identify the given values and the unknown variable/s.
Given: Fourth Term (a_4) = 128
Common Ratio (r) = 4

There are two unknowns, we have a_1 and a_7.

Let's have the formula $a_n = a_1 r^{n-1}$

There are two unknowns in the problem and to solve a_7, we need to solve a_1 first.

Since the given term is the fourth term, which is 128, we can use it to solve for the value of a_1. **Substitute the value of a_4, which is 128, n which is 4, and r, which is 4 in the formula $a_n = a_1 r^{n-1}$.**

$$a_n = a_1 r^{n-1}$$

$$128 = a_1(4)^{4-1}$$

$$128 = a_1(4)^3$$

$$\frac{128}{64} = \frac{64a_1}{64}$$

$$a_1 = 2$$

Afterwards, identify the unknown term which is a_7.

$$a_n = a_1 r^{n-1}$$

$$a_7 = (2)(4)^{7-1}$$
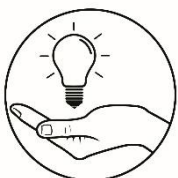
$$a_7 = (2)(4)^6$$

$$a_7 = 2(4\,096)$$

$$a_7 = 8{,}192$$

ASSESSMENT 2. FIND OUT!

Answer the problem completely.

1. The third and eighth term of a geometric sequence are 8 and $\frac{1}{4}$ respectively. What is the first term and the common ratio of the sequence?
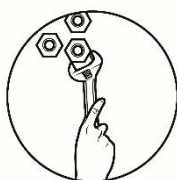
What I have Learned

FILL IN THE TABLE

Directions. Find the specified term of the geometric sequences given the first term and the common ratio.

Given	a_5	a_7	a_9
1. $a_1 = 3$ $r = 3$			
2. $a_1 = 2$ $r = 2$			
3. $a_1 = -\dfrac{1}{2}$ $r = -6$			

What I Can Do

Solve for what is asked in the problem. Show your solutions.

Warren deposited his P12, 000. 00 in a local bank in 2019.

Note: $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

A - final amount

P – initial principal balance

r - interest rate

n - number of times interest applied per time period

t – number of time periods elapsed

1. If the bank offers 5% interest rate compounded annually, how much will be in his bank account in 2028?

2. Suppose he will withdraw the principal in 2028 and will leave the interest in his account, how much will the interest earn after 5 years?

Geometric Means

What's In

In the previous lesson in this module, you were able to derive the formula in finding the nth term of geometric sequences.

To review what you have already learned on finding the nth term of a geometric sequence, accomplish the following activity.

WHAT'S THAT TERM?

Find the specified term of the following geometric sequence.

_____ 1. $3, 9, 27, \ldots$ **6th term**

_____ 2. $1, 2, 4, \ldots$ **7th term**

_____ 3. $12, \underline{\boldsymbol{a_2}}, 3$

_____ 4. $2, \boldsymbol{a_2}, \underline{\boldsymbol{a_3}}, 54$

Were you able to answer items 3 and 4? How did you come up with your answers?

Using the formula in finding the nth term of a geometric sequence to supply the unknown term in Items 3 and 4, will really let you experience a difficulty.

Through this module, let us discover a shorter way to identify the unknown term/s in between terms of geometric sequences.

What's New

Look at the problem below, can you determine what must be placed on the blank to form a geometric sequence?

$$12, \underline{\quad}, 3$$

You might use the formula in finding the nth term of geometric sequence to supply the middle term.

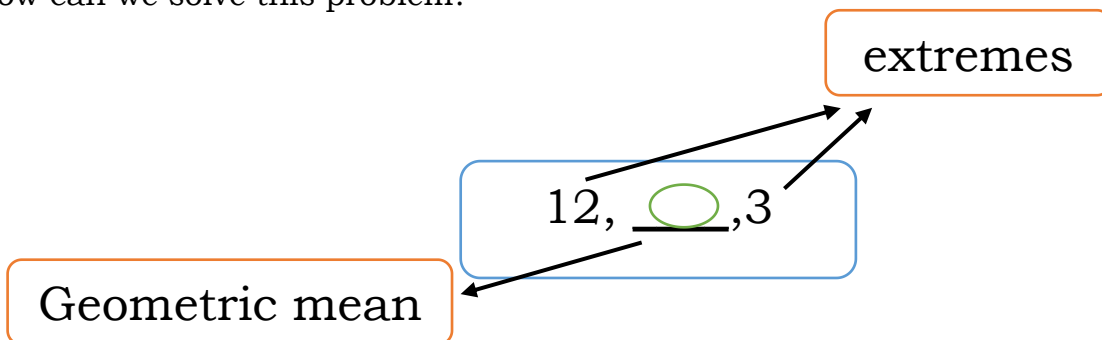**We're stuck in here!**

$$a_n = a_1 r^{n-1}$$

$$a_2 = 12r^{2-1}$$

$$a_2 = 12r$$

Let's discover how to answer his problem.

What is It

How can we solve this problem?

extremes

$$12, \underline{\bigcirc}, 3$$

Geometric mean

The given terms are the first and last terms. These terms are called the **extremes**, and the term/s in between the extremes are called **geometric mean/s**. In the geometric sequence, 2, 4, 8, 16, the numbers 4 and 8 are the geometric means of the extremes, 2 and 16.

Let us derive the formula:

The **common ratio** refers to the ratio of two consecutive terms,

$$r = \frac{a_n}{a_{n-1}} \; same \; through \; as \; \frac{a_3}{a_2} \; and \; \frac{a_2}{a_1}, therefore \; we \; can \; equate \; \frac{a_3}{a_2} = \frac{a_2}{a_1},$$

This can be our basis in finding geometric means.

$$\frac{a_3}{a_2} = \frac{a_2}{a_1},$$

Let's go back to the problem, 12, ____, 3. The first term is 12 and the last term is 3. Now, let us substitute.

$$\frac{a_3}{a_2} = \frac{a_2}{a_1},$$

$$\frac{3}{a_2} = \frac{a_2}{12},$$

$$(a_2)^2 = 36$$
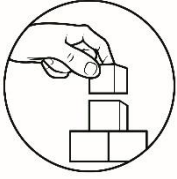
$$\sqrt{a_2} = \sqrt{36}$$

$$\boxed{a_2 = 6} \quad \checkmark$$

To insert terms, let us identify first the common ratio by using the formula:

$$r = \sqrt[n-k]{\frac{a_n}{a_k}}$$

Where: n = number of terms
k = 1, since it is the first term placement
r = common ratio

ACTIVITY 1. **MEET ME HALFWAY**

Find the missing term/s in the following geometric sequences.

1) 3, _____ , 45

To insert a geometric mean, let us first identify the extremes and number of terms. By substitution,

$$\frac{a_3}{a_2} = \frac{a_2}{a_1},$$
$$\frac{45}{a_2} = \frac{a_2}{3}$$
$$(a_2)^2 = 135$$
$$\sqrt{a_2} = \sqrt{135} \text{ or } 3\sqrt{15}$$

Therefore, $a_2 = 3\sqrt{15}$

2) 2, _____, _____ , 250

There are two geometric means needed in this problem. Let us identify first the extremes and the number of terms. The extremes are 2 and 250, and there are 4 terms in the sequence.

$$r = \sqrt[n-k]{\frac{a_n}{a_k}}$$

$$r = \sqrt[4-1]{\frac{a_4}{a_1}}$$

$$r = \sqrt[3]{\frac{250}{2}}$$

$$r = \sqrt[3]{125}$$

$$\boxed{r = 5}$$

Therefore, the sequence is 2, <u>10</u>, <u>50</u>, 250...

ASSESSMENT 1. HALFWAY

Find the geometric mean of the given extremes.

Given	Geometric Mean
1. 3 and 8	
2. 100 and 25	
3. $\frac{1}{2}$ and $\frac{1}{8}$	
4. 3 and $\frac{1}{3}$	
5. x and x^7	

ASSESSMENT 2. IN BETWEEN

Insert geometric means in each geometric sequence.

1.) 2, _____, _____, 686
2.) _____, 24, _____, _____, $\frac{3}{64}$
3.) _____, _____, _____, 4, 8
4.) _____, $\frac{1}{4}$, $\frac{1}{2}$, _____
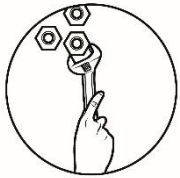5.) 81, _____, _____, _____, _____, $\frac{1}{3}$

What I have Learned

SHADE THAT TERM OF MINE

The table contains terms of a geometric sequence. Identify whether the terms in the right columns are geometric means or term of the sequence. Use any shade to color the geometric means or terms.

Geometric Sequence	Terms			
1.) 1, ___, ___, ___, 81	30	9	27	3
2.) ___, 3, ___, ___, $\frac{1}{9}$	1	6	9	$\frac{1}{3}$
3.) 24, ___, ___, ___, $\frac{3}{2}$	1	2	3	4
4.) 2, ___, $\frac{1}{2}$, ___, $\frac{1}{8}$	1	4	$\frac{1}{4}$	2
5.) ___, $\frac{1}{2}$, 1	2	4	$\frac{1}{4}$	$\frac{1}{3}$

What I Can Do

SALARY SCHEME

Angelie is working for a year as a clerk in a certain department. Her employer promised her that she will be having a salary increase every after 3 months. Assume that her basic salary is P10,000.00 and there is a constant rate of increase on her salary every after 3 months.

Your task is to construct/write a salary scheme for Angelie that determines her salary in the first up to the last quarter of the year.

Cycle of Salary Increase	1	2	3	4
Salary Received				

Assessment

A. Multiple Choice. Read and analyze the following items and determine the letter of the correct answer from the given choices. Write your answer on a separate sheet of paper.

_____ 1. What expression is same as $(a \cdot a \cdot a \cdot a \cdot a)(b \cdot b \cdot b)$?

  A. 15ab   C. a^3 b
  B. a^5 b^3 D. $5a^3b$

_____ 2. What is the common ratio of the sequence $\frac{1}{7}$, 1, 7, ...?

  A. 7   C. $\frac{1}{7}$

  B. -7 D. $-\frac{1}{7}$

_____ 3. The first term of a geometric sequence is 2, and the second term is 10, what is the fifth term?

  A. 250   C. 500
  B. 100   D. 750

_____ 4. The common ratio of a geometric sequence is $\frac{1}{4}$, and the fourth term is $\frac{1}{2}$. What is the third term of the sequence?

  A. 1   C. $\frac{1}{4}$

  B. $\frac{1}{2}$ D. $\frac{1}{8}$

_____ 5. Find the 12th term of the geometric sequence 1, 2, 4,...
  A. 512   C. 2 048
  B. 1 024   D. 1 112

For items 6 – 10, find the specified term of the following geometric
sequences.

_____ 6.) 6, 12, 24, ...   find a_7
_____ 7.) 20, 10, 5, ...   find a_6
_____ 8.) 7, - 14, 28, ...   find a_8
_____ 9.) $a_1 = 2$, r = 3   find a_{10}
_____ 10.) $a_1 = 0$, r = $\frac{1}{2}$ find a_8

For items 11 – 15, insert geometric means in each geometric sequence.

11.) 3, ____, 27
12.) $\frac{1}{2}$, _____, $\frac{1}{8}$
13.) $\frac{5}{2}$, _____, _____, $\frac{625}{16}$
14.) 3, ____, ____, ____, 3
15.) – 1, ____, – 125 , ____, – 3,125

Answer Keys

What I Know

1. C
2. C
3. A
4. B
5. A
6. 192
7. $\frac{5}{16}$
8. 6 561
9. $-\frac{1}{256}$
10. $\frac{4}{3\sqrt{125}}$
11. $2\sqrt{6}$
12. $\frac{3}{4}$
13. 14, 28
14. 9, $\frac{9}{9}$, $4\sqrt{16}$
15. 3, 27

What I have Learned

1. 243, 218, 19,683
2. 32, 128, 512
3. − 648, 162, − 40.5

ASSESSMENT

1. B
2. A
3. D
4. D
5. C
6. 384
7. $\frac{5}{8}$
8. − 896
9. 39,366
10. 0
11. 9
12. $\frac{1}{4}$
13. 25, $\frac{125}{8}$
14. − 3, 3, − 3
15. 5, 625

18

REFERENCES

www.VectorStock.com/15088776

www.yumpu.com/en/document/read/35995428

Cabral, J., Bernabe, J., & Valencia, E. (2004). *New Trend in Math Series, Algebra II.* Quezon City: Vibal Publishing House Incorporated.