Characterizing and Describing the Roots of Quadratic Equations

WEEKS 2-3

I

Lesson

After going through this lesson, you are expected to:

- Characterize the roots of a quadratic equation using the discriminant.
- Describe the relationship between the coefficients and the roots of a quadratic equation.

Roots of quadratic equation can be imaginary number, equal or two distinct roots. It can be determined by the value of $b^2 - 4ac$.

Learning Task 1. Complete the table given below and find the relations among constants a, b and c in the quadratic equation standard form.

Equation	a	b	c	$b^2 - 4ac$	Roots	
					x_1	x_2
$x^2 + 4x + 3 = 0$						
$x^2 - 5x + 4 = 0$						
$x^2 - 49 = 0$						
$4x^2 - 25 = 0$						
$2x^2 + 7x + 3 = 0$						

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The expression $b^2 - 4ac$, is the quadratic equation's discriminant. The discriminant determines the nature of the roots of a quadratic equation.

If $b^2 - 4ac = 0$, then the roots are real and equal

$b^2 - 4ac > 0$, then the roots are real and unequal

$b^2 - 4ac < 0$, then the roots are not real

Illustrative Examples:

1. Find the nature of roots of the equation $x^2 + 4x + 3 = 0$.

   The values of a, b and c in the equation are 1, 4 and 3, respectively.

   Evaluating $b^2 - 4ac$,

$$\begin{aligned} b^2 - 4ac &= (4)^2 - 4(1)(3) \\ &= 16 - 12 \\ &= 4 \end{aligned}$$

   Since $b^2 - 4ac > 0$, then the equation has two real and unequal roots.

To check, solve for the roots of the equation.

Checking,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 12}}{2}$$

$$x = \frac{-4 \pm 2}{2}$$

x = -1 or -3 (rational and unequal)

When the value of the determinant $b^2 - 4ac$ is greater than zero and is not a perfect square, then the roots are irrational numbers and unequal.

2. Determine the nature of roots of the equation $x^2 - 6x + 9 = 0$.

The values of a, b and c in the equation are 1, -6 and 9, respectively.

Evaluating $b^2 - 4ac$,

$$b^2 - 4ac = (-6)^2 - 4(1)(9)$$
$$= 36 - 36$$
$$= 0$$

Since $b^2 - 4ac = 0$, then the roots of quadratic equation are real and equal.

Check: $x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0 \longrightarrow x - 3 = 0 \rightarrow x = 3$

3. What kind of roots does the equation $x^2 + 2x + 3 = 0$ have?

The values of a, b and c in the equation are 1, 2 and 3, respectively.

Evaluating $b^2 - 4ac$,

$$b^2 - 4ac = (2)^2 - 4(1)(3)$$
$$= 4 - 12$$
$$= -8$$

Since b - 4ac < -8, the n the roots are not real.

The sum and product of the roots of a given quadratic equation (in one variable) as you have noticed in the activity table have relations among the constants a, b and c of said equation in standard form. This is given by the actual sum and product of the roots in using the Quadratic Formula to solve a given quadratic equation: given that the roots are:

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Sum of the Roots $= x_1 + x_2$	Product of the Roots $= x_1 \bullet X2$
$= \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$	$= \frac{-b+\sqrt{b^2-4ac}}{2a} \bullet \frac{-b-\sqrt{b^2-4ac}}{2a}$
$= \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$	$= \frac{(-b)^2-(\sqrt{b^2-4ac})^2}{4a^2}$
$= \frac{-b-b}{2a}$	$= \frac{b^2-b^2+4ac}{4a^2}$
$= \frac{-2b}{2a}$	$= \frac{4ac}{4a^2}$
$= -\frac{b}{a}$	$= \frac{c}{a}$
$x_1 + x_2 = -\frac{b}{a}$	$x_1 \bullet x_2 = \frac{c}{a}$

So, the sum of the roots of a given quadratic equation is the ratio $-\frac{b}{a}$, while the product of its roots is the ratio $\frac{c}{a}$. In general, all you need to do is write the quadratic equation in standard form and identify the values of the constants a, b and c. In finding the sum and product of the roots of the equations:

a. $x^2 + 5x + 4 = 0$, the values of a = 1, b = 5 and c = 4, respectively.

Substituting values in the ratio $-\frac{b}{a}$ for

Sum of roots, $x_1 + x_2 = -\frac{b}{a} = -\frac{5}{1} = -5$, while for

Product of roots, $x_1 \bullet x_2 = \frac{c}{a} = \frac{4}{1} = 4$.

Checking:

Using factoring, $(x + 4)(x + 1) = 0$ — Factoring the quadratic trinomial part

$x + 4 = 0$; $x + 1 = 0$ — Zero Product Property

$x = -4$; $x = -1$ — by APE

$x_1 + x_2 = -4 - 1 = -5$ — Sum of roots

$x_1 \bullet x_2 = (-4)(-1) = 5$ — Product of roots

b. $2x^2 - 5x + 3 = 0$, the values of $a = 2$, $b = -5$ and $c = 3$, respectively.

Substituting values in the ratio $-\frac{b}{a}$ for

Sum of roots, $x_1 + x_2 = -\frac{b}{a} = -\frac{5}{2}$, while for

Product of roots, $x_1 \bullet x_2 = \frac{c}{a} = \frac{3}{2}$.

Checking:

Using factoring, $(2x - 3)(x - 1) = 0$ — Factoring the quadratic trinomial part

$2x - 3 = 0$; $x - 1 = 0$ — Zero Product Property

$2x = 3$; $x = 1$ — by APE

$x = \frac{3}{2}$ — by MPE

Learning Task 2. Complete the table

Equation	a	b	c	Discriminant	Nature of the Roots
$x^2 - 6x - 27 = 0$					
$x^2 - 25 = 0$					
$x^2 + 10x + 25 = 0$					
$2x^2 - 5x + 3 = 0$					

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Learning Task 3. Do the following:

A. Characterize the roots of the following quadratic equations using the discriminant.

1. $x^2 + 4x + 3 = 0$
2. $x^2 - 5x + 4 = 0$
3. $x^2 + 7 = 0$
4. $4x^2 - 4x + 1 = 0$
5. $2x^2 + 6x + 3 = 0$

B. Complete the table.

Equation	a	b	c	Roots		$x_1 + x_2$	$x_1 \cdot x_2$
				x_1	x_2		
$x^2 + 5x + 4 = 0$							
$x^2 - 6x - 27 = 0$							
$x^2 - 25 = 0$							
$x^2 + 10x + 25 = 0$							
$2x^2 - 5x + 3 = 0$							

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Learning Task 4. Solve the problem by applying the sum and product of roots of quadratic equations.

The perimeter of a rectangular metal plate is 36 dm and its area is 80 dm^2. Find its dimensions. (Relate the measures to the sum and product of a quadratic equation.)

The perimeter of a rectangle is twice the sum of its length and width while its area is the product of its length and width. Such that,

Perimeter = $2(L + w)$ and Area = $L \cdot w$