WEEK 8

Graphing Quadratic Functions and Analyzing the Effects on its Graph

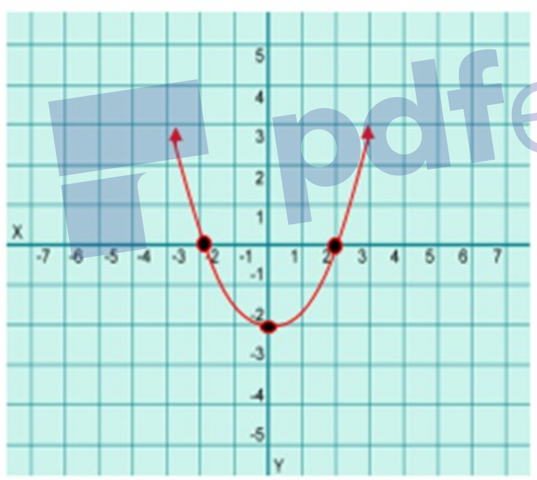
I

After going through this lesson, you are expected to:

- Graphs a quadratic function: (a) domain; (b) range; (c) intercepts; (d) axis of symmetry; (e) vertex; (f) direction of the opening of the parabola.
- Analyzes the effects of changing the values of a, h and k in the equation y = a(x – h)2 + k of a quadratic function on its graph..

In this time, you will learn how to graph quadratic functions and analyze the effects of changing the values of a, h and k in the equation $y = a(x - h)^2 + k$.

Learning Task 1. Complete the table below using the given graph.



Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x – intercept	
y – intercept	

D

Learning Task 2. Sketch the graph of the following. Give the domain and range of the following quadratic function defined by the given equation. Determine the direction where the parabola opens, its vertex, its axis of symmetry, and its x- and y- intercepts.

1. $y = x^2 + 8x + 4$
2. $y = x^2 + 4x + 5$
3. $y = -3x^2 + 12x - 7$
4. $y = 2x^2 - 4x - 6$

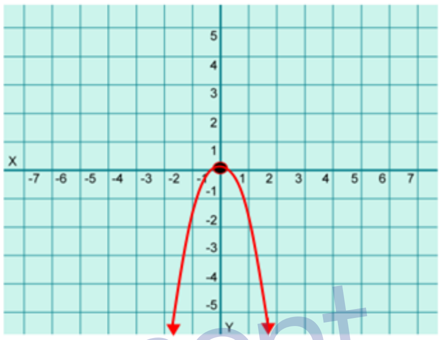
Take note that we have two kinds of parabola, either it opens upward or downward, the given illustration is an example of upward parabola. Study the given illustrative examples as follows to know how to construct parabola and determine its domain, range, intercepts, axis of symmetry, vertex, and its

Study the following.

ILLUSTRATIVE EXAMPLES:

1. Given: $y = -2x^2$

Domain	**Set of all real numbers**
Range	**$\{y/y \leq 0\}$**
Opening of the parabola	**Downward**
Vertex	**(0,0)**
Axis of Symmetry	**$x = 0$**
x - intercept	**$y = -2x^2$** → **$(0 = -2x^2)$** **$x = \{0\}$**
y – intercept	**$y = -2x^2$** → **$y = -2(0)^2$** **$y = \{0\}$**

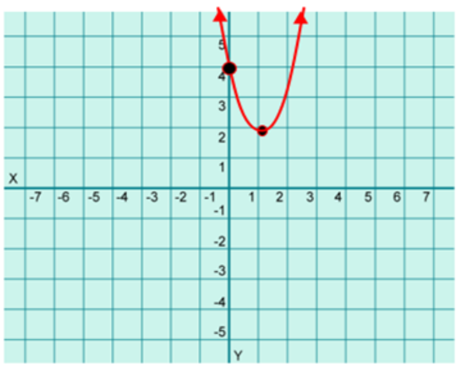


2. Given: $y = 2x^2 – 4x + 4$

Transform into $y = a(x – h)^2 + k$ (Vertex Form).

$y = 2x^2 – 4x + 4 \quad \rightarrow \quad y = 2(x – 1)^2 + 2$

Domain	**Set of all real numbers**
Range	**$\{y/y \geq 2\}$**
Opening of the parabola	**Upward**
Vertex	**(1, 2)**
Axis of Symmetry	**$x = 1$**
x - intercept	**y** $= 2x^2 – 4x+4$ → **0** $= 2x^2 – 4x+4$ **no x- intercept**
y - intercept	**y**$=2x^2 – 4x+4$ → **y**$=2(0)^2 – 4(0)+4$ **$y = \{4\}$**



Note: To find the vertex in standard form, we can also use the formula for (h,k): $\mathbf{h} = \frac{-b}{2a}$; $\mathbf{k} = \frac{4ac - b^2}{4a}$

In the function $y = 2x^2 - 4x + 4$; a = 2; b= -4; c = 4. Substitute the values to the formula and solve then simplify.

Thus, $\mathbf{h} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$;

$$\mathbf{k} = \frac{4(2)(4) - (-4)^2}{4(2)} = \frac{32 - 16}{8} = \frac{16}{8} = 2 \text{ ; } \mathbf{(1, 2) - Vertex}$$

The **domain** of quadratic function is the set of real numbers. In the form $y = a(x - h)^2 + k$, if a > 0, then the **range** of the quadratic function is $\{y / y \geq k\}$; if a < 0, then the range of the quadratic function is $\{y / y \leq k\}$.

The value of "a" indicates the **opening of the parabola**, if the value of "a" is positive (+) then the parabola opens upward and if the value of "a" negative (-) it opens downward.

The table below indicates the different forms of quadratic functions and its properties.

FORM	VERTEX	AXIS OF SYMMETRY
$y = ax^2$	(0,0)	x = 0
$y = ax^2 + k$	(0,k)	x = 0
$y = a(x-h)^2 + k$	(h,k)	x = h
$y = ax^2 + bx + c$	(h,k)	x = h

The **vertex** of the parabola is the point (h,k). It is the minimum point of the parabola if a >0 and maximum point of the parabola if a < 0.

The **axis of symmetry** of the parabola is the vertical line **x = h** and also divides the parabola into two equal parts.

The **x-intercepts** is determined by setting y = 0, and then solving for x. On the other hand, the **y – intercept** is determined by setting x = 0, and solve for y. The function $y = ax^2 + bx + c$ has two distinct x – intercepts if $b^2 - 4ac > 0$; only one x – intercept if $b^2 - 4ac = 0$; and no x – intercepts if $b^2 - 4ac < 0$.

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Learning Task 3. Complete the tables below and graph on the same coordinate plane. Then analyze the relationship between the graphs.

1. Given: $y = -2x^2$

Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y – intercept	

2. Given: $y = -x^2 + 4$

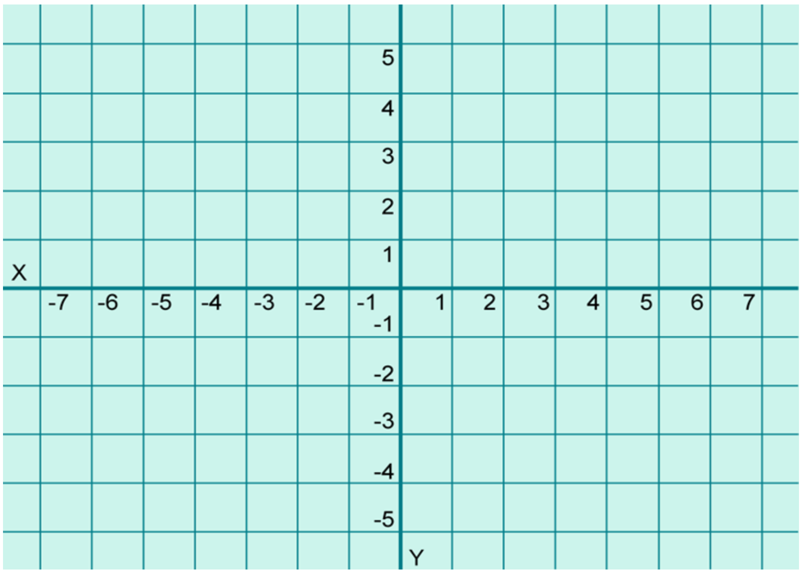
Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y – intercept	

3. Given: $y = (x + 1)^2$

Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y – intercept	

4. Given: $y = 2x^2 - 4x + 4$

Domain	
Range	
Opening of the parabola	
Vertex	
Axis of Symmetry	
x - intercept	
y – intercept	



B. Show the following effects of the changing values of a, h and k in the equation $y = a(x - h)^2 + k$ of a quadratic function by formulating your own quadratic functions and graphing it.

- The parabola opens upward if $a > 0$ (positive) and opens downward if $a < 0$ (negative).
- The graph of $y = ax^2$ narrows if the value of "a" becomes larger and widens when the value of "a" is smaller. Its vertex is always located at the origin (0, 0) and the axis of symmetry is $x = 0$.
- The graph of $y = ax^2 + k$ is obtained by shifting $y = ax^2$, k units upward if $k > 0$ (positive) and $/k/$ units downward if $k < 0$(negative). Its vertex is located at the point of (0, k) and an axis of symmetry of $x = 0$.
- The graph of $y = a(x - h)^2$ is obtained by shifting $y = ax^2$, h units to the right if $h > 0$(positive) and $/h/$ units to the left if $h < 0$(negative). Its vertex is located at the point of (h, k) and an axis of symmetry of $x = h$.
- The graph of $y = a(x - h)^2 + k$ is obtained by shifting $y = ax^2$, h units to the right if $h > 0$(positive) $/h/$ units to the left if $h < 0$(negative).; and k units upward if $k > 0$ (positive) and $/k/$ units downward if $k < 0$(negative). Its vertex is located at the point of (h, k) and an axis of symmetry of $x = h$.

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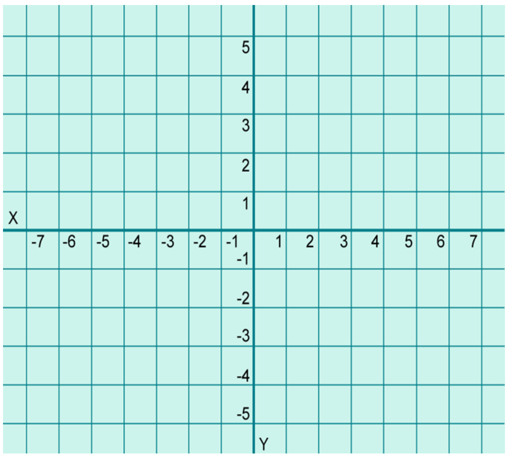
Learning Task 4. Illustrate the graphs of the following quadratic functions then analyze the effects from each other.

1. $y = 3x^2$
2. $y = 3x^2 - 3$
3. $y = 3x^2 + 3$
4. $y = 3(x - 3)^2 + 1$
5. $y = 3(x + 3)^2 - 1$



Explanation:

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