WEEKS 4-5

Solving Quadratic Equations and Ra... Algebraic Equations

Lesson

I

After going through this lesson, you are expected to:

- Solve equations transformable to quadratic equations (including rational algebraic equations).
- Solve problems involving quadratic equations and rational algebraic equations.

So far, what you have solved are quadratic equations in standard form or one that can be written in standard form. What if the quadratic equation involves rational expressions? Can one also use the learned methods to solve such equations?

Since, the quadratic equations this time involves rational expressions, you must recall the concept on finding the least common denominator of rational expressions.

Learning Task 1. Find the least common denominators of the following expressions:

1. $\frac{1}{2}+\frac{3}{4}$
2. $\frac{1}{x}+\frac{x}{2}$
3. $\frac{2}{x}-\frac{x+1}{5}$
4. $\frac{2}{x-3}+\frac{x}{2}$

What about the rational expressions in each equation, what should be the corresponding least common denominator?

1. $\frac{1}{x}+\frac{x}{2}=\frac{11}{6}$
2. $\frac{2}{x}-\frac{x+1}{5}=-\frac{4}{5}$
3. $\frac{2}{x-3}+\frac{x}{2}=-\frac{1}{2}$

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In order to simplify a rational equation, it has to be transformed into an equation without the denominators. This is made possible by using the least common denominator of all rational expressions in the equation containing such expressions.

ILLUSTRATIVE EXAMPLES:

1. Solve the rational equation $\frac{x}{3}+\frac{1}{x}=\frac{19}{12}$ by transforming it into a quadratic equation. The least common denominator (LCD) of the left rational parts in the equation is (3)(x). The LCD is generally found by multiplying all the denominators – as long as they are relatively prime (no other factor will divide the expressions except 1).

Then, $\frac{x^2+3}{3x}=\frac{19}{12}$	by Addition (using the LCD on the left side)
$12(x^2 + 3) = (3x)(19)$	by Proportionality
$12x^2 + 36 = 57x$	Distributive Property
$12x^2 – 57x + 36 = 0$	by APE
$4x^2 – 19x + 12 = 0$	by MPE
$(4x – 3)\ (x – 4) = 0$	by factoring method
$4x – 3 = 0 \mid x – 4 = 0$	Zero Product Property
$4x = 3 \mid x = 4$	by APE
$x = \frac{3}{4}$	by MPE

To transform rational equations into quadratic equations, the following procedures can be followed:

1. Multiply both sides of the equation by the LCD (Least Common Denominator).
2. Write the resulting quadratic equation in standard form.

Then, you can solve the resulting quadratic equation using any methods of solving quadratic equation. And don't forget to check the obtained values by substituting in the original equation.

So far, how did you find solving some of the problem applications regarding the different concepts on quadratic equations? Let us now have a deeper understanding of some other problem conditions involving solving quadratic and rational equations.

ILLUSTRATIVE EXAMPLES:

1. Odette can finish washing a certain number of dishes in 15 minutes less than it takes Pam. If they work together, they can do the dishes in 10 minutes.

Let w = (1) whole work to be done

$\frac{1}{t}$ = part of work done by Pam in a minute

$\frac{1}{t-15}$ = part of work done by Odette in a minute

$\frac{1}{10}$ = part of work done together by both girls in a minute

Equation: $\frac{1}{t} + \frac{1}{t-15} = \frac{1}{10}$

Find the LCD, Then add: $\frac{1}{t} + \frac{1}{t-15} = \frac{1}{10} \rightarrow \frac{(t-15)+t}{t(1-15)} = \frac{1}{10}$

$$\frac{2t-15}{t^2-15t} = \frac{1}{10} \rightarrow 10(2t-15) = t^2-15$$

$$20t - 150 = t^2 - 5 \rightarrow t^2 - 20t + 145 = 0$$

You may solve the equation by any method applicable.

Learning Task 2.

A. Translate the following verbal sentences to mathematical sentence. Then express into quadratic equations in terms of "x".

Given	Quadratic Equations
1. The length of a wooden frame is 1 foot longer than its width and its area is equal to 12 ft^2.	
2. The length of the floor is 8 m longer than its width and there is 20 square meters.	
3. The length of a plywood is 0.9 m more than its width and its area is 0.36 m^2.	
4. The area of rectangle whose length is six less than twice its width is thirty-six.	
5. The width of a rectangular plot is 5 m less than its length and its area is 84 m^2.	

B. Solve for x:

$$\frac{1}{x} + \frac{1}{x+5} = \frac{1}{10}$$

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Learning Task 3

A. Solve the following equations transformable to quadratic equations.

1. $\frac{1}{x}+\frac{x}{2}=\frac{11}{6}$	2. $\frac{2}{x}-\frac{x+1}{5}=-\frac{4}{5}$	3. $\frac{2}{x-3}+\frac{x}{2}=-\frac{1}{2}$

B. Solve

1. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

2. If the difference between a number and its reciprocal is 24/5, find the number.

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Learning Task 4

Solve the following problems. Show the step by step process. And write a brief explanation of what you do.

1. The width of a bedroom floor is 5 m less than its length and its area is 84 m^2. Find its dimensions.

2. A certain pipe can fill up a tank 2 hours faster than another pipe. It takes 3 and hours for both pipes to fill up the same tank. In how many hours would the first pipe fill up the tank?

3. A picture frame has its length 8 cm longer than its width. It has an inner 1-cm boundary such that a maximum 660 cm^2-picture may fit into it. Find the dimensions of this frame.

4. After how many seconds is needed for a thrown ball to reach a 14-m distance if its distance is given by the relation d = 10 + 40t – $5t^2$? (t=time in seconds).