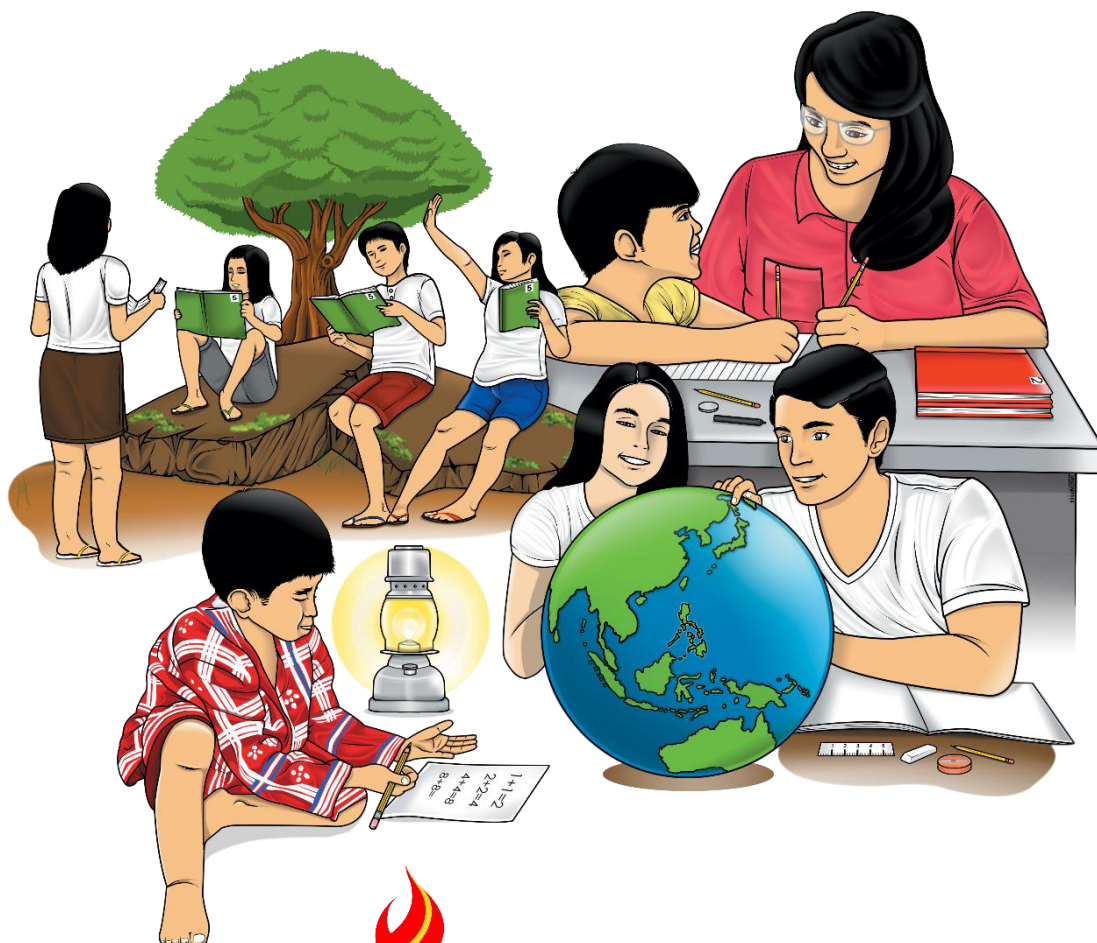


# Mathematics

## Quarter 1 - Module 3: Arithmetic Means and $n^{\text{th}}$ Term of an Arithmetic Sequence



**Mathematics – Grade 10**  
**Alternative Delivery Mode**

**Quarter I – Module 3: Arithmetic Means and nth Term of an Arithmetic Sequence**

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# 10

# Mathematics

## Quarter 1 - Module 3

### Arithmetic Means and $n^{\text{th}}$ Term of an Arithmetic Sequence

M10AL - Ib - c - 1

## Introductory Message

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This module deals with the third learning competency in our Mathematics 10 curriculum standards; hence mastery of the skills is significant to have a smooth progress in the next lesson. This learning material also designed to equip the students with essential knowledge about finding the  $n^{\text{th}}$  term and arithmetic means of an arithmetic sequence.



### **For the facilitator:**










Hi. As the facilitator of this module, kindly orient the learner on how to go about in reading and answering this learning material. Please be patient and encourage the learner to complete this module. By the way, do not forget to remind the learner to use separate sheets in answering all of the activities found in this module.

### **For the learner:**

Hello learner. I hope you are ready to progress in your Grade 10 Mathematics by accomplishing this learning module. This is designed to provide you with interactive tasks to further develop the desired learning competencies on determining arithmetic means and  $n^{\text{th}}$  term of an arithmetic sequence. This module is especially crafted for you to be able to cope with the current lessons taken by your classmates. Please read completely the written texts and follow the instructions carefully so that you will be able to get the most of this learning material. We hope that you will enjoy learning.

Here is a guide on the parts of the learning modules which you need to understand as you progress in reading and analyzing its content.

ICON	LABEL	DETAIL
	<b>What I need to know</b>	This will give you an idea of the skills or competencies you are expected to learn in the module.
	<b>What I know</b>	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.

	<b>What's In</b>	This is a brief drill or review to help you link the current lesson with the previous one.
	<b>What's New</b>	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
	<b>What Is It</b>	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
	<b>What's More</b>	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
	<b>What I have Learned</b>	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
	<b>What I Can Do</b>	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
	<b>Assessment</b>	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
	<b>Additional Activities</b>	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
	<b>Answer Key</b>	This contains answers to all activities in the module.

At the end of this module you will also find:

**References**

This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

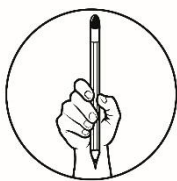


## What I Need to Know

This module was designed and written with you in mind. This will help you determine arithmetic means and  $n^{th}$  term of an arithmetic sequence. The scope of this module will be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course but how you read and answer this module is dependent on your ability.

After going through this module, you are expected to be able to demonstrate knowledge and skill related to sequence and apply these in solving problems. Specifically, you should be able to:

1. write a formula for the  $n^{th}$  term of an arithmetic sequence,
2. use the formula to find the  $n^{th}$  term or unknown term of an arithmetic sequence,
3. define arithmetic means, and
4. determine arithmetic means of a sequence.



## What I Know

Find out how much you already know about the topics in this module. Choose the letter of your answer from the given options. Take note of the items that you are not able to answer correctly and find the right answer as you go through this module. Write your answer on a separate sheet of paper.

- Which term of the arithmetic sequence 7, 10, 13, 16, . . . is 304?  
a. 99<sup>th</sup> term      b. 100<sup>th</sup> term      c. 111<sup>th</sup> term      d. 102<sup>th</sup> term
- Find the  $n^{\text{th}}$  term of the arithmetic sequence given the following conditions:  
 $a_1=2$        $d=3$        $n=9$   
a. 26      b. 27      c. 28      d. 29
- Which term of the arithmetic sequence 2, 6, 10,..... is 102?  
a. 20<sup>th</sup> term      b. 25<sup>th</sup> term      c. 30<sup>th</sup> term      d. 35<sup>th</sup> term
- If three arithmetic means are inserted between -15 and 9, find the first of these arithmetic means.  
a. 3      b. -3      c. -6      d. -9
- Find the 21<sup>st</sup> term of the arithmetic sequence 6, 9, 12, 15,...  
a. 61      b. 60      c. 62      d. 66
- If three arithmetic means are inserted between 8 and 16, find the second arithmetic mean.  
a. 10      b. 12      c. 14      d. 16
- Which term of the arithmetic sequence  $3, \frac{7}{3}, \frac{5}{3}, \dots, \dots$  is -27?  
a. 9<sup>th</sup> term      b. 20<sup>th</sup> term      c. 41<sup>th</sup> term      d. 46<sup>th</sup> term
- What is the arithmetic mean between 10 and 24?  
a. 18.5      b. 19      c. 16      d. 17



9. What is the 10<sup>th</sup> term of the following arithmetic sequence: -5, -1, 3, 7, 11,...?
- a. 31                      b. 19                      c. 27                      d. 22
10. Insert two arithmetic means between  $\sqrt{2}$  and  $4\sqrt{2}$ . Which of the following is the first arithmetic mean?
- a.  $\sqrt{2}$                       b.  $2\sqrt{2}$                       c.  $3\sqrt{2}$                       d.  $4\sqrt{2}$
11. If  $a_1 = -4$  and  $a_{25} = -100$ . Find  $a_{100}$ ?
- a. -104                      b. -150                      c. -316                      d. -400
12. If  $a_3 = 8$  and  $a_{16} = 47$  and  $a_k$  is the  $k^{\text{th}}$  term of the sequence and  $a_k = 212$ , then what is the value of  $k$ ?
- a. 61                      b. 71                      c. 81                      d. 91
13. Insert 2 arithmetic means between 3 and 30.
- a. 12, 14                      b. 10, 11                      c. 14, 22                      d. 12, 30
14. After one second, a rocket is 30 ft above the ground. After another second, it is 85 feet above the ground. Then after another second, it is already 140 feet above the ground. If it continues to rise at this rate, how many feet above the ground will the rocket be after 16 seconds?
- a. 780 ft                      b. 830 ft                      c. 855 ft                      d. 910 ft
15. 2. An object is dropped from a plane and falls 32 feet during the first second. For each succeeding second, it falls 40 feet more than the distance covered in the preceding second. How far has it fallen after 11 seconds?
- a. 118 feet                      b. 120 feet                      c. 115 feet                      d. 112feet

# Lesson 1

## Finding the $n^{\text{th}}$ Term of an Arithmetic Sequence



### *What's In*

In the previous module, we define arithmetic sequence and give the next term by adding a constant number.

For example: Find the next three terms of the arithmetic sequence: 3, 8, 13, 18, ...

Solution:

- The terms are  $a_1 = 3$ ,  $a_2 = 8$ ,  $a_3 = 13$ , and  $a_4 = 18$ . So, we will be finding  $a_5$ ,  $a_6$ , and  $a_7$ .
- The common difference ( $d$ ) in the sequence is 5.
- To get the next three terms, add 5 to each of the preceding term.

$$\begin{aligned}\text{Thus: } a_5 &= a_4 + 5 = 18 + 5 = 23 \\ a_6 &= a_5 + 5 = 23 + 5 = 28 \\ a_7 &= a_6 + 5 = 28 + 5 = 33\end{aligned}$$

What about if the problem is to find the 100<sup>th</sup> term or the 250<sup>th</sup> term? Can you find the terms? Using the process that is illustrated above will take much of your time and effort. There is a short cut in doing this and that is one of the focus of this module.



## What's New

Before we find other higher terms of a sequence, let us first find lower terms. In the arithmetic sequence: 3, 8, 13, 18,...; what is the 15<sup>th</sup> term?

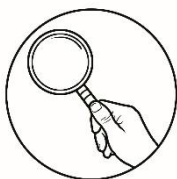
Solution:

- a. By adding the common difference to each of the preceding terms, we get the following values.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$a_n$	3	8	13	18	23	28	33	38	43	48	53	58	63	68	73

- b. Thus, the 15<sup>th</sup> term is 73.

However, using this procedure to get any higher  $n^{\text{th}}$  term would be tedious. Thus, a formula is necessary to find any  $n^{\text{th}}$  term.



## What is It

Let us investigate on how to determine the  $n^{\text{th}}$  term of a sequence. In the table:

$$\begin{array}{ll}
 a_1 = 3 & = 3 \\
 a_2 = 3 + 5 & = 8 \\
 a_3 = 3 + 5 + 5 & = 13 \\
 a_4 = 3 + 5 + 5 + 5 & = 18 \\
 . & . \\
 . & . \\
 . & . \\
 a_{13} = 3 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 & = 63 \\
 a_{14} = 3 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 & = 68 \\
 a_{15} = 3 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 & = 73
 \end{array}$$

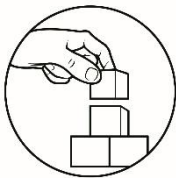
These terms can be written in the following manner as a short cut.

$$\begin{array}{rcl}
 a_1 & = & 3 \\
 a_2 & = & 3 + 5 (1) \\
 a_3 & = & 3 + 5 (2) \\
 a_4 & = & 3 + 5 (3) \\
 & \cdot & \\
 & \cdot & \\
 & \cdot & \\
 a_{13} & = & 3 + 5 (12) \\
 a_{14} & = & 3 + 5 (13) \\
 a_{15} & = & 3 + 5 (14)
 \end{array}
 \begin{array}{rcl}
 & = & 3 \\
 & = & 8 \\
 & = & 13 \\
 & = & 18 \\
 & \cdot & \\
 & \cdot & \\
 & \cdot & \\
 & = & 63 \\
 & = & 68 \\
 & = & 73
 \end{array}$$

Thus, if we find for the 16<sup>th</sup> term of the arithmetic sequence, then  $a_{16} = 3 + 5 (15) = 78$ . We can conclude that using the pattern observed the  $n^{\text{th}}$  term of the sequence is  $a_n = a_1 + d (n-1)$ , where  $a_n$  is the term that corresponds to  $n^{\text{th}}$  position,  $a_1$  is the first term, and  $d$  is the common difference.

The  $n^{\text{th}}$  term of an arithmetic sequence with first term  $a_1$  and common difference  $d$  is given by:

$$a_n = a_1 + d (n-1)$$



## What's More

Let us apply this formula in solving the following:

- A. Find the 21<sup>st</sup> term of the arithmetic sequence: 6, 9, 12, 15,...

Solution:

- From the sequence,  $a_1 = 6$ ,  $d = 3$ , and  $n = 21$ .
- Using the formula, substitute these values.
 
$$a_{21} = 6 + 3 (21 - 1)$$

$$a_{21} = 6 + 3 (20)$$

$$a_{21} = 6 + 60$$

$$a_{21} = 66$$
- Thus, the 21<sup>st</sup> term is 66.

- B. In the arithmetic sequence: 7, 10, 13, 16, . . .; find  $n$  if  $a_n = 304$ .

Solution:

- From the sequence,  $a_1 = 7$ ,  $d = 3$ , and  $a_n = 304$ .
- Using the formula, substitute these values.  

$$a_n = a_1 + d(n-1)$$

$$304 = 7 + 3(n-1)$$

$$304 = 7 + 3n - 3$$

$$304 = 4 + 3n$$

$$300 = 3n$$

$$n = 100$$
- Thus, 304 is the 100<sup>th</sup> term of the sequence.

- C. The 3<sup>rd</sup> term of an arithmetic sequence is 8 and the 16<sup>th</sup> term is 47. Find  $d$ ,  $a_1$  and the 71<sup>st</sup> term.

Solution:

- From the sequence,  $a_3 = 8$  and  $a_{16} = 47$
- These imply that:  

$$a_3 = a_1 + d(3-1)$$

$$8 = a_1 + d(3-1)$$

$$8 = a_1 + 2d \quad \text{Eq. 1}$$

$$a_{16} = a_1 + d(16-1)$$

$$47 = a_1 + d(16-1)$$

$$47 = a_1 + 15d \quad \text{Eq. 2}$$
- Using Eq. 1 and Eq. 2, solve for  $a_1$  and  $d$ . By subtracting Eq. 2 by Eq. 1, then:

$$\begin{array}{r} 47 = a_1 + 15d \\ - (8 = a_1 + 2d) \\ \hline 39 = 13d \\ d = 3 \end{array}$$

To solve for  $a_1$ , substitute  $d = 3$  to either Eq. 1 or Eq. 2. Using Eq. 1:

$$\begin{array}{l} 8 = a_1 + 2(3) \\ 8 = a_1 + 6 \\ a_1 = 2 \end{array}$$

Thus, the  $n^{\text{th}}$  term of the arithmetic sequence is  $a_n = 2 + 3(n-1)$

- Using  $a_n = 2 + 3(n-1)$ , we can solve for the 71<sup>st</sup> term.

$$\begin{array}{l} a_{71} = 2 + 3(71-1) \\ a_{71} = 2 + 3(70) \\ a_{71} = 2 + 210 \\ a_{71} = 212 \end{array}$$

**Alternative Solution:** Another way to solve d is to use the difference

$$\text{formula: } d = \frac{a_n - a_k}{n - k}$$

Given :  $a_k = a_3 = 8$ ;  $k = 3$  and  $a_n = a_{16} = 47$ ;  $n = 16$

$$\text{Thus, } d = \frac{a_n - a_k}{n - k} = \frac{47 - 8}{16 - 3} = \frac{39}{13} = 3$$

- D. After one second, a rocket is 30 ft above the ground. After another second, it is 85 feet above the ground. Then after another second, it is already 140 feet above the ground. If it continues to rise at this rate, how many feet above the ground will the rocket be after 16 seconds?

Solution:

- a. From the problem we let the given be

$$\begin{aligned} a_1 &= 30 \\ a_2 &= 85 \\ a_3 &= 140 \end{aligned}$$

- b. Find first d by substituting the given value of  $a_1$  and  $a_2$  in the formula then simplify.

$$\begin{aligned} a_n &= a_1 + d(n-1) \\ 85 &= 30 + d(2-1) \\ 55 &= d \end{aligned}$$

- c. To find  $a_{16}$ , the unknown in the problem substitute the obtained value of d and the given value of  $a_1$  in the formula then simplify.

$$\begin{aligned} a_{16} &= a_1 + d(16-1) \\ &= 30 + 55(16-1) \\ &= 855 \end{aligned}$$

- d. Thus, the rocket will be 855 ft above the ground after 16 seconds.

## Assessment:

- A. Find the specified  $n^{\text{th}}$  term of each arithmetic sequence.

\_\_\_\_\_ 1. 2, 5, 8, ...; 9<sup>th</sup> term

\_\_\_\_\_ 2. 3, 5, 7, ...; 20<sup>th</sup> term

\_\_\_\_\_ 3.  $1, \frac{1}{2}, 0, \dots$ ; 16<sup>th</sup> term

\_\_\_\_\_ 4. 5, 11, 17, ...; 9<sup>th</sup> term

\_\_\_\_\_ 5. 26, 22, 18, ...; 40<sup>th</sup> term

\_\_\_\_\_ 6. 103<sup>rd</sup> term of the arithmetic sequence if  $a_1 = -5$  and  $d = -4$

\_\_\_\_\_ 7. 19<sup>th</sup> term of the arithmetic sequence if  $a_1 = 25$  and  $d = -2$

\_\_\_\_\_ 8. 25<sup>th</sup> term of the arithmetic sequence if  $a_1 = \frac{1}{2}$  and  $d = -\frac{3}{8}$ .

B. Solve what is asked.

\_\_\_\_\_ 1. In the sequence 2, 6, 10, ...; find n if the nth term is 102.

\_\_\_\_\_ 2. In the sequence  $3, \frac{7}{3}, \frac{5}{3}, \dots$ ; find n if the nth term is -27.

\_\_\_\_\_ 3. Find the 15<sup>th</sup> term of the sequence if  $a_8 = 5$  and  $a_{21} = -60$

\_\_\_\_\_ 4. Find 5<sup>th</sup> term of the sequence if  $a_{15} = 29$  and  $a_{27} = 47$

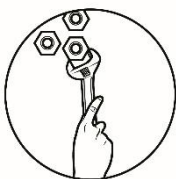
\_\_\_\_\_ 5. If  $a_1 = -4$ ,  $a_{25} = -100$ , what is the value of  $a_{100}$ ?



## What I Have Learned

Let us see if you understood our lesson by answering the following questions.

1. State the general formula of finding the  $n^{\text{th}}$  term of an arithmetic sequence.
2. Given an arithmetic sequence, how do we find the common difference?
3. Given two different  $n^{\text{th}}$  terms of an arithmetic sequence, how do we find for the common difference?



## What I Can Do

A. Give what is asked:

\_\_\_\_\_ 1. The 10<sup>th</sup> term of the arithmetic sequence if  $a_1 = -15$  and  $d = 6$

\_\_\_\_\_ 2. The 39<sup>th</sup> term of the arithmetic sequence if  $a_1 = 40$  and  $d = \frac{1}{2}$

B. Find the specified term of each arithmetic sequence.

\_\_\_\_\_ 1. 1.4, 4.5, 7.6, ...; The 41<sup>st</sup> term

\_\_\_\_\_ 2. 9, 18, 27,...; the 23<sup>rd</sup> term

\_\_\_\_\_ 3. 14, 6, -2,...; 27<sup>th</sup> term

\_\_\_\_\_ 4. 3, 3.25, 3.5,...; 16<sup>th</sup> term

\_\_\_\_\_ 5. 1, 4, 7,...; 28<sup>th</sup> term

C. Find the specified term.

\_\_\_ 1. In the sequence: 0.12, 0.17, 0.22, ...; find n if the nth term is 67?

\_\_\_ 2. In the sequence: 10, 7, 4, ...; what term has a value of -296?

\_\_\_ 3. In the sequence: 2, 6, 10, 14, ...; what n corresponds to  $a_n = 286$ ?

\_\_\_ 4. Find 1<sup>st</sup> term of the sequence if  $a_5 = 26$  and  $a_{12} = 47$

\_\_\_ 5. If  $a_{24} = 85$ , and  $a_{28} = 100$ , what is  $a_1$ ?

## Lesson 2

# Computing Arithmetic Means



## What's In

In the previous lesson, you learned how to determine the  $n^{\text{th}}$  term of an arithmetic sequence.

For example: In the sequence: 10, 15, 20, 25,...; what term has a value of 385?

Solution:

a. Using the formula,  $a_n = a_1 + d(n - 1)$ :

$$385 = 10 + 5(n - 1)$$

$$385 = 10 + 5n - 5$$

$$385 = 5n + 5$$

$$5n = 385 - 5$$

$$5n = 380$$

$$n = 76$$



- b. Thus, 385 is the 76<sup>th</sup> term of the given sequence.

The next lesson intends to discuss with you how to compute arithmetic means.



## What's New

The focus of this module has something to do with finding the arithmetic means.

For example: In the sequence: 4, 8, 12, 16, 20, 24; find its arithmetic means.

Solution:

- The arithmetic mean is a term between the first term and the last term.
- Thus, 8, 12, 16, and 20 are the arithmetic means of the sequence because these terms are between 4 and 24, which are the first and last term, respectively.



## What is It

The first and last terms of a finite arithmetic sequences are called **arithmetic extremes**, and the terms in between are called **arithmetic means**. In the sequence 4, 8, 12, 16, 20, 24; the terms 4 and 24 are the arithmetic extremes, while 8, 12, 16, and 20 are the arithmetic means. Also, 8 is the arithmetic mean of the arithmetic extremes, 4 and 12.

The arithmetic mean between two numbers is sometimes called the average of two numbers. If more than one arithmetic means will be inserted between two arithmetic extremes, the formula for  $d$ , which is  $d = \frac{a_n - a_k}{n - k}$ , can be used.

The formula for  $d$  can be used to find the arithmetic means if more than one arithmetic means will be inserted between two arithmetic extremes.

$$d = \frac{a_n - a_k}{n - k}$$

**Let's Try!**

- A. What is the arithmetic mean between 10 and 24?

Solution

- a. Using the average formula, get the arithmetic mean of 10 and 24.  
b. Thus,  $\frac{10+24}{2} = 17$  is the arithmetic mean.

**Activity 1:** Using the example above, solve for the arithmetic mean of each of the pairs of arithmetic extremes.

- B. Insert three arithmetic means between 8 and 16.

Solution:

- a. If three arithmetic means will be inserted between 8 and 16, then  $a_1 = 8$  and  $a_5 = 16$ .  
b. Using the formula for  $d$ , compute for the common difference.

$$d = \frac{a_n - a_k}{n - k} = \frac{a_5 - a_1}{5 - 1} = \frac{16 - 8}{5 - 1} = \frac{8}{4} = 2$$

- c. The arithmetic means are  $a_2$ ,  $a_3$ , and  $a_4$ .  
 $a_2 = a_1 + d = 8 + 2 = 10$   
 $a_3 = a_2 + d = 10 + 2 = 12$   
 $a_4 = a_3 + d = 12 + 2 = 14$   
d. Thus, the three arithmetic means between the arithmetic extremes, 8 and 16, are 10, 12, and 14.

- C. Insert two arithmetic means between  $\sqrt{2}$  and  $4\sqrt{2}$

Solution:

- a. If two arithmetic means will be inserted between  $\sqrt{2}$  and  $4\sqrt{2}$ , then  $a_1 = \sqrt{2}$  and  $a_4 = 4\sqrt{2}$ .  
b. Using the formula for  $d$ , compute for the common difference.

$$d = \frac{a_n - a_k}{n - k} = \frac{a_4 - a_1}{4 - 1} = \frac{4\sqrt{2} - \sqrt{2}}{4 - 1} = \frac{3\sqrt{2}}{3} = \sqrt{2}$$

- c. The arithmetic means are  $a_2$  and  $a_3$

$$a_2 = a_1 + d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

$$a_3 = a_2 + d = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$$

- d. Thus, the two arithmetic means between  $\sqrt{2}$  and  $4\sqrt{2}$  are  $2\sqrt{2}$  and  $3\sqrt{2}$ .

- D. Find the missing terms of the arithmetic sequence: \_\_\_\_\_, 6, \_\_\_\_\_, \_\_\_\_\_, 30.

Solution:

- a. The arrangement of the terms tells that  $a_2 = 6$  and  $a_5 = 30$ . We are supposed to find for  $a_1$ ,  $a_3$ , and  $a_4$ .
- b. To find for the unknown, determine the common difference ( $d$ ).

$$d = \frac{a_n - a_k}{n - k} = \frac{a_5 - a_2}{5 - 2} = \frac{30 - 6}{5 - 2} = \frac{24}{3} = 8$$

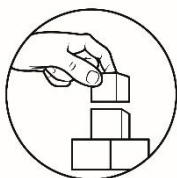
- c. Thus, the value of  $a_2$ ,  $a_3$ , and  $a_4$  are:

$$a_1 = a_2 - d = 6 - 8 = -2$$

$$a_3 = a_2 + d = 6 + 8 = 14$$

$$a_4 = a_3 + d = 14 + 8 = 22$$

**Activity 2:** Find the missing terms of the following sequence



## What's More

### Let's Do It!

- A. What is the arithmetic mean between the two given arithmetic extremes?

\_\_\_\_\_ 1. 5 and 19

\_\_\_\_\_ 2.  $3x^2 + 8$  and  $x^2 - 6$

- \_\_\_\_\_ 3. -2 and 58
- \_\_\_\_\_ 4.  $2x + 3y$  and  $x - 5y$
- \_\_\_\_\_ 5. 13.8 and 15.6

B. Insert the specified number of arithmetic means between the two given arithmetic extremes.

- \_\_\_\_\_ 1. Three arithmetic means between 2 and 22.
- \_\_\_\_\_ 2. Four arithmetic means between 8 and 23.
- \_\_\_\_\_ 3. Two arithmetic means between 41 and 95.
- \_\_\_\_\_ 4. Two arithmetic means between -5 and 1.
- \_\_\_\_\_ 5. Two arithmetic means between 97 and 172.

C. Find the missing terms of each arithmetic sequence:

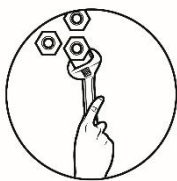
1. 7, \_\_\_\_\_, 9
2. 1, \_\_\_\_\_,  $\frac{1}{5}$
3. 8, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, 20
4. -4, \_\_\_\_\_, \_\_\_\_\_, 5
5. -5, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, 3



## What I Have Learned

Answer the following questions:

1. How do we find the arithmetic mean of two arithmetic extremes?
2. When two or more arithmetic means are inserted between two arithmetic extremes, how are they computed?
3. Do infinite sequences have arithmetic means? Why?



## What I Can Do

A. What is the arithmetic mean between the given arithmetic extremes?

\_\_\_\_\_ 1. 19 and 7

\_\_\_\_\_ 2.  $\frac{3}{11}$  and  $\frac{7}{11}$

\_\_\_\_\_ 3.  $15x$  and  $23x$

\_\_\_\_\_ 4.  $9\sqrt{3}$  and  $11\sqrt{3}$

\_\_\_\_\_ 5.  $6 - 7\sqrt{7}$  and  $2 + 3\sqrt{7}$

B. Insert the specified number of arithmetic means between the given arithmetic extremes.

\_\_\_\_\_ 1. Three arithmetic means between 18 and 92.

\_\_\_\_\_ 2. Three arithmetic means between -14 and 6.

\_\_\_\_\_ 3. Four arithmetic means between 24 and -8.

\_\_\_\_\_ 4. Five arithmetic means between 6 and -18.

\_\_\_\_\_ 5. Two arithmetic means between  $2\sqrt{5}$  and  $14\sqrt{5}$ .

C. Find the missing terms of each arithmetic sequence:

1.  $x$ , \_\_\_\_\_,  $9x$

2. 31, \_\_\_\_\_, \_\_\_\_\_, 85

3. \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, 17, 23, 29

4.  $x$ , \_\_\_\_\_,  $11x$

5. \_\_\_\_\_, 6, \_\_\_\_\_, \_\_\_\_\_, 30

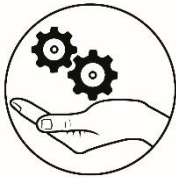


## Assessment

Choose the letter of your answer from the given options. Write your answer on a separate paper.

1. Which term of the arithmetic sequence 5, 9, 13, 17, ... is 409?  
a. 99<sup>th</sup> term                      b. 100<sup>th</sup> term                      c. 111<sup>th</sup> term                      d. 102<sup>th</sup> term
2. Find the  $n^{\text{th}}$  term of the arithmetic sequence given the following conditions:  
 $a_1=5$                        $d=5$                        $n=25$   
a. 25<sup>th</sup> term=115   b. 25<sup>th</sup> term=125   c. 25<sup>th</sup> term=120   d. 25<sup>th</sup> term=130
3. Which term of the arithmetic sequence 5, 9, 13, 17,..... is 401?  
a. 99<sup>th</sup> term                      b. 100<sup>th</sup> term                      c. 111<sup>th</sup> term                      d. 112<sup>th</sup> term
4. If three arithmetic means are inserted between -15 and 9, find the first of these arithmetic means.  
a. 3                                      b. -3                                      c. -6                                      d. -9
5. Find the 20<sup>th</sup> term of the arithmetic sequence 5, 9, 13, 17, 21,...  
a. 81                                      b. 80                                      c. 82                                      d. 87
6. If three arithmetic means are inserted between 11 and 39, find the second arithmetic mean.  
a. 18                                      b. 25                                      c. 32                                      d. 46
7. Which term of the arithmetic sequence 4, 1, -2, -5, ... is -29?  
a. 9<sup>th</sup> term                                      b. 10<sup>th</sup> term                                      c. 11<sup>th</sup> term                                      d. 12<sup>th</sup> term
8. What is the arithmetic mean between 15 and 40?  
a. 28.5                                      b. 29                                      c. 26                                      d. 27.5
9. What is the 8<sup>th</sup> term of the following arithmetic sequence: -5, -1, 3, 7, 11,...?  
a. 23                                      b. 19                                      c. 27                                      d. 22
10. Which of the following is an arithmetic mean between  $2-\sqrt{3}$  and  $4-\sqrt{3}$ ?  
a.  $3-\sqrt{3}$                                       b.  $3-2\sqrt{3}$                                       c.  $3+\sqrt{3}$                                       d.  $3+2\sqrt{3}$
11. If  $a_1 = -3$  and  $a_5 = 5$ . Find  $a_{10}$ ?  
a. 14                                      b. 15                                      c. 16                                      d. 17

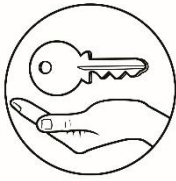
12. If  $a_3 = 11$  and  $a_5 = 7$  and  $a_k$  is the  $k^{\text{th}}$  term of the sequence and  $a_k = -9$ , then what is the value of  $k$ ?
- a. 11                      b. 12                      c. 13                      d. 14
13. Insert 3 arithmetic means between 8 and 16.
- b. 10, 12, 14              b. 9, 10, 11              c. 9, 11, 13              d. 12, 15, 16.
14. After one second, a rocket is 40 ft above the ground. After another second, it is 95 feet above the ground. Then after another second, it is already 150 feet above the ground. If it continues to rise at this rate, how many feet above the ground will the rocket be after 16 seconds?
- a. 780 ft                      b. 830 ft                      c. 855 ft                      d. 865 ft
15. Jose is the track and field representative of the Municipal NHS for the provincial meet. He begins training by running 5 miles during the first week, 6.5 miles during the second week, and 8 miles on the third week. If his training pattern continues, how far will he run on the tenth week?
- a. 18.5 miles              b. 20 miles              c. 21.5 miles              d. 23 miles



## Additional Activity

Solve the following word problems correctly. Give what is asked.

1. You have accepted a job with a salary of P27,000.00 a month during the first year. At the end of each year, you receive a P1,500.00 raise. What is your monthly salary during the first six years?
2. An object is dropped from a plane and falls 32 feet during the first second. For each succeeding second, it falls 40 feet more than the distance covered in the preceding second. How far has it fallen after 11 seconds?



# **Answer Key**

**What I Know:**

- B
- A
- B
- C
- D
- B
- D
- D
- A
- B
- D
- B
- C
- C
- D

**Lesson I: What's More**

- A.
- 26
- 41
- $-\frac{2}{13}$
- 53
- 130
- 413
- 13
- $-\frac{2}{17}$  or -8.5
1. 26<sup>th</sup>
- 121<sup>st</sup>
- 45
- $\frac{1}{2}$
- 392

**Lesson 1: What I Can Do**

1. 39
- 59
1. 125.4
- 207
- 194
- 6.75
- 82
1. 32<sup>nd</sup>
- 103<sup>rd</sup>
- 72<sup>nd</sup>
- 14
- 5/4 or -1.25

**Lesson 1: What I Have Learned**

- by using the general formula  $a_n = a_1 + (n-1)d$
- by manipulating the general formula or by substitution to the general formula.



**Additional Activity:**

- 34,500.00- expected salary after six years.
- After 11 seconds, the object is dropped, 112 feet.

**Assessment:**

- D
- B
- B
- D
- A
- B
- D
- D
- A
- A
- B
- C
- A
- D
- A

**Lesson 2: What I Have Learned.**

*Depends on students respond.*

**Lesson 2:What I Can Do**

A.1. 13

2. 5/11

3. 19x

4.  $10\sqrt{3}$

5.  $4-2\sqrt{7}$

B. 1. 36.5, 55, 73.5

2. -9, -4, 1

3. 17.6, 11.2, 4.8, 1.6

4. 2, -2, -6, -10, -14

5.  $6\sqrt{5}$ ,  $10\sqrt{5}$

C. 1. 5x

2. 49, 67

3. -1, 5, 11

4. 6x

5. -2, 14, 22

**Lesson 2:What's More:**

A.

1. 12

2.  $2x^2 + 1$

3. 28

4.  $\frac{2}{3}x - y$

5. 14, 75, 122, 147

C.

1. 8

2.  $\frac{5}{3}$

3. 11, 14, 17

4. 1, 2

5.  $-\frac{5}{17}, -\frac{5}{9}, -\frac{1}{5}, -\frac{5}{7}$

**What is It**

A. 1. 65.5

2. 152.5

3. 10

4. 92.5

5. 66

B. 1. 15, 21, 27, 33, 39, 45

2. 1, 7, 13, 19, 25

3. -3, 4, 11, 18, 25

4. 0, 9, 18, 27, 36

5. 16, 20, 24, 28, 32

## ***References***

Callanta, Melvin M., et al. 2015. *Mathematics Learner's Module*. Pasig City.

Nivera, Gladys C. and Lapinid, Minie Rose C. 2015, *Grade 10 Mathematics: Patterns and Practicalities*. Makati City, Don Bosco Press.

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