WEEK 6

Quadratic Inequalities

Lesson

I

After going through this lesson, you are expected to:

- Illustrates quadratic inequalities.
- Solves quadratic inequalities.
- Solves problems involving quadratic inequalities.

Learning Task 1, Complete the table below by identifying whether the given is quadratic inequality or not. Put a check mark (√) in the column of your choice. Then answer the questions that follows.

Given	Quadratic Inequality	Not
1. $x^2 - x - 6$		
2. $3x^2 - 4x - 7 = 0$		
3. $(x - 4)(x - 2) > 0$		
4. $x^2 - 8x - 16 \le 0$		
5. $2x^2 - 5x - 10 > 0$		

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A quadratic inequality is mathematical statement that relates a quadratic expression as either less than or greater than another.

Illustrative Examples:

1. $3x^2 - 6x - 2$; this is not quadratic inequality in one variable because there is no inequality symbol indicted.
2. $x^2 - 2x - 5 = 0$; this is not quadratic inequality in one variable because symbol used is equal sign. It is quadratic equation in one variable.
3. $x - 2x - 8 \ge 0$; this is not quadratic inequality in one variable because the highest exponent is not 2.
4. $(x - 5)(x - 1) > 0$; this is quadratic inequality in one variable because the symbol used is an inequality symbol and the highest exponent is 2. (But take note this one is tricky because you need to multiply the factors to expressed to its lowest term then can identify the highest exponent. Thus,

$(x - 5)(x - 1) > 0$ using FOIL Method

$(x - 5x - x + 5) > 0$ Combine similar terms to simplify

Then, $\underline{\mathbf{x^2 - 6x + 5 > 0}}$ is the standard form of the given equation

A **quadratic inequality in one variable** is an inequality that contains polynomial whose highest exponent is 2. The general forms are the following, where a, b, and c are real numbers with $a > 0$ and $a \neq 0$.

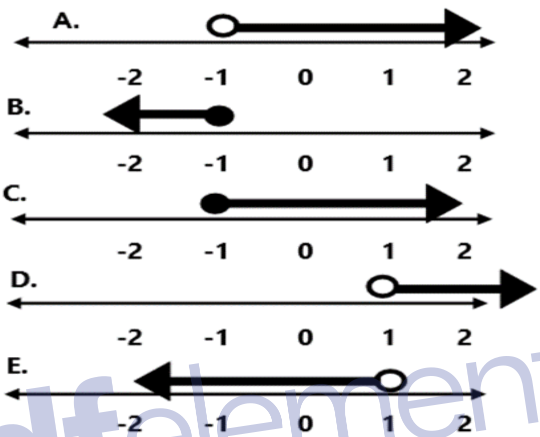
$$\mathbf{ax^2 + bx + c > 0} \qquad \mathbf{ax^2 + bx + c < 0}$$

$$\mathbf{ax^2 + bx + c \leq 0} \qquad \mathbf{ax^2 + bx + c \geq 0}$$

In solving quadratic inequalities, we need to find its solution set. The **solution set** of a quadratic inequality can be written as a set and can be illustrated through a number line.

To illustrate the solution set on the number line, we need to consider the inequality symbols used in given quadratic inequality.

1. $x > -1$
2. $y \leq -1$
3. $y \geq -1$
4. $x > 1$
5. $x < -1$



SYMBOLS	Circles to be used
<	○ Hollow circle (the critical points are not included)
>	
≤	● Solid circle (the critical points are included)
≥	

Illustrative Examples:

Find the solution set of $\mathbf{3x^2 - 3x - 18 \leq 0}$.

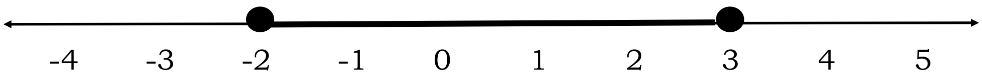
$3x^2 - 3x - 18 \leq 0$

$3x^2 - 3x - 18 = 0$		Transform the given inequality into equation
$(3x + 6)$	$(x - 3) = 0$	Factor the quadratic expression
$3x + 6 = 0$	$x - 3 = 0$	Equate each factor by zero
$3x + 6 = 0$	$x - 3 = 0$	Transpose the constants to the right side
$3x = -6$	$x = 3$	Solve for x.
$\mathbf{x = -2}$	$\mathbf{x = 3}$	

The values of x which are -2 and 3 are the **critical points** which can be used to obtain the following intervals: **(x ≤ -2) , (-2 ≤ x ≤ 3), and (x ≥ -3).**

Take note that -2 and 3 are included in the solution set because the symbol used is ≤ (less than or equal to).

Therefore, the solution set of the inequality is { **-2 ≤ x ≤ 3**}, and its graph is shown below.



To solve quadratic inequalities, the following procedures can be followed.

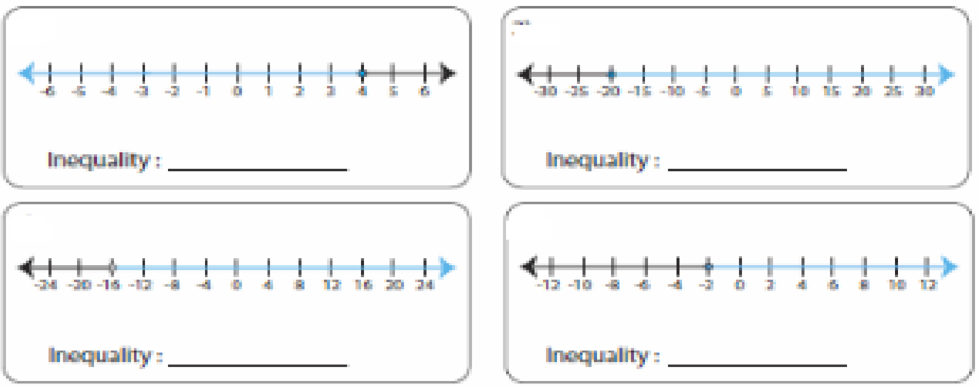
1. Transform the given quadratic inequality into quadratic equation. (Make sure to express into standard form.)
2. Solve for the roots (critical points). You may use the different methods in solving quadratic equations.
3. Use the roots (critical points) to obtain the intervals.
4. Choose a test point in each interval to determine the solution set.
5. Graph the solution set.
6. Check the obtained solution set in the original inequality

Learning Task 2

A. Graph the following on a number line

1. x > 2
2. x ≥ −3
3. x > 5
4. x < - 4
5. x ≤ 5

B. Write the inequality



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Learning Task 3. Tell whether the given statements below illustrate quadratic inequality in one variable or not. You may translate into mathematical symbols to justify your answers.

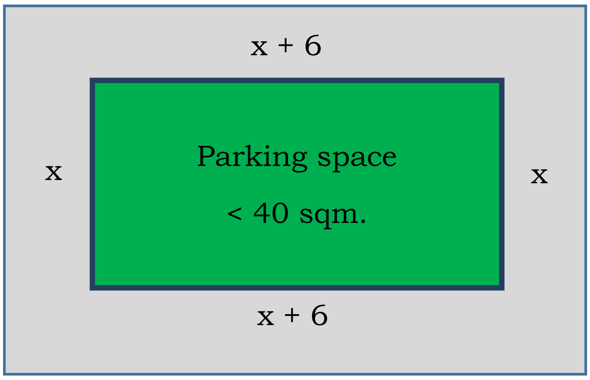
1. Four more than thrice number is greater than forty.
2. The length of the floor is 8 m longer than its width and the area is greater than 20 square meters.
3. The width of a rectangular plot is 5 m less than its length and its area is 84 m^2.

Steps in Solving Problems involving Inequalities.

1. Read and understand the problem.
2. Identify the given conditions and the unknown.
3. Represent the unknown using variable.
4. Devise an equation or inequality that corresponds to the given conditions and the unknown in the problem.
5. Solve for the unknown,
6. Check your answer.

Solve the problem:

Mrs. Reyna is planning to fence her vacant lot for a parking space. The desired length of the lot is 6 meters longer than its width. What will be the possible dimensions of the rectangular parking space if it should be less than 40sqm? (Hint: length x width = Area of rectangle)



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Learning Task 4. Solve the problem below. (Show the step by step process)

The length of the floor is 32 m longer than its width and there is greater than 200 square meters. You will cover the floor completely with tiles. What will be the possible dimension of the floor?

Modeling, Representing and Transforming Quadratic Functions

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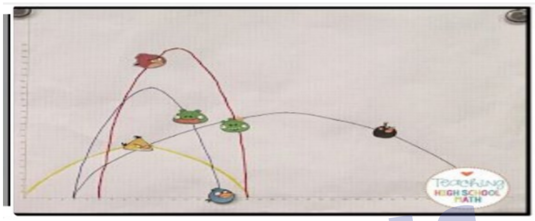
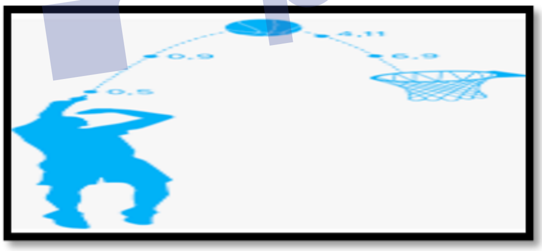
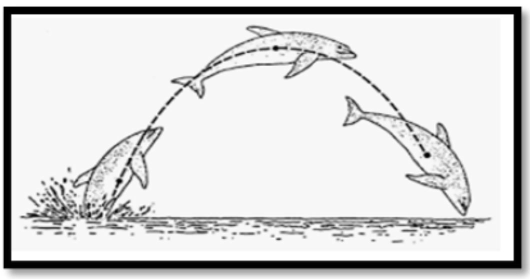
After going through this lesson, you are expected to:

- Models real-life situations using quadratic functions.
- Represents a quadratic function using: (a) table of values; (b) graph; and (c) equation.
- Transforms the quadratic function defined by

  $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$.

Learning Task 1. Tell whether each picture models a quadratic function or not. Justify your answer.

GIVEN	Answer	WHY?
 Source: Google search//www.pinterest.ph		
 Source: Google search//ww.coolmath.com		
 Source: Google search//ww.thinglink.com		

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The concepts of quadratic function is very useful in our life if you know further about it. You can solve different problems that involves quadratic function in real-life situations such as building structures, computing the maximum height or minimum point of an object my reach, analyzing the movement of an object, and etc.

Here are some examples of situations that models quadratic functions in real-life situations.

1. Targets an object in upward direction
2. Throwing an object downward
3. Shooting ball vertically upward
4. Minimum point submarine to submerge
5. Launching rocket to its maximum point

Study the illustrative examples below to know more about how quadratic function.

Illustrative Examples:

1: Is $y = f(x) = x^2 + 6x + 8$ a quadratic function or not?

Solution: $f(x) = x^2 + 6x + 8$ is a quadratic function since its highest degree is 2 and all numerical coefficients are real numbers.

2: Is $y = 7x + 12$ a quadratic function or not?

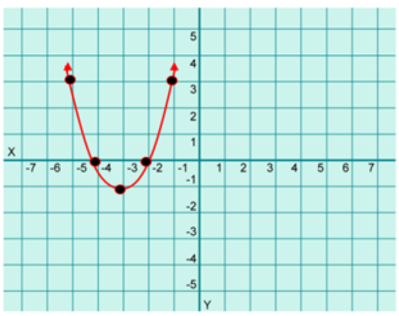
Solution: $y = 7x + 12$ is a not quadratic function since its highest degree is 1.

3: Is $f(x) = (x-2)\ (x+3)\ x^2$ a quadratic function or not?

Solution: $y = (x-2)\ (x+3)\ x^2$ is a not quadratic function because if you will expand the right side, its highest degree will be 4.

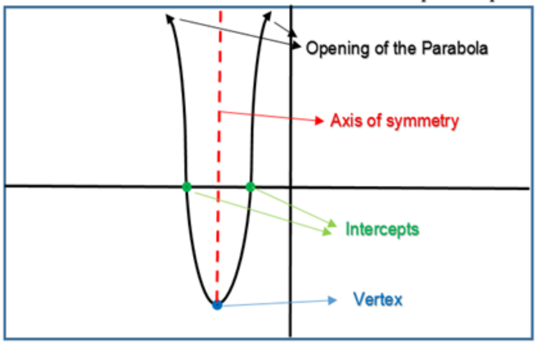
4. Consider the graph of the quadratic function $f(x) = x^2 + 6x + 8$.

x	-5	-4	-3	-2	-1
y= f(x)	3	0	-1	0	3



Observe the trend/ characteristics of the graph for you to determine how quadratic func-

The graph of a quadratic function is a **Parabola**. It has different properties including *vertex, axis of symmetry, opening of the parabola,* and *the intercepts.* Consider the illustration on the right to



A **quadratic function** is a second degree polynomial represented as $\mathbf{f(x) = ax^2 + bx + c}$ or $\mathbf{y = ax^2 + bx + c}$, $\mathbf{a \neq 0}$ where **a**, **b**, and **c** are real numbers.

The graph of a quadratic function is a **Parabola**. It has different properties including *vertex, axis of symmetry, opening of the parabola,* and *the intercepts.*

Study the process on how to transform quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$.

ILLUTRATIVE EXAMPLES:

1. Transform $y = x^2 - 6x - 6$ into $y = a(x - h)^2 + k$.

$y = x^2 - 6x - 6$

$y = (x^2 - 6x) - 6$ Group together the terms containing x.

$y = (x^2 - 6x + 9) - 6 - 9$ Make the expression in parenthesis a perfect square trinomial by adding the value of $(\frac{-b}{2})^2$ and subtracting the same value to the constant term, since a = 1.

$\boxed{(\frac{-6}{2})^2 = 9}$

$y = (x^2 - 6x + 9) - 15$ Simplify.

$\mathbf{y = (x - 3)^2 - 15}$ Express the perfect square trinomial into square of binomial

Learning Task 2. Which of the following represents a quadratic function?

1. $f(x) = 8x + 5$ ____________________

2. $f(x) = x^2 - 2x + 7$ ____________________

3.

x	-5	-4	-3	-2	-1
y= f(x)	3	0	-1	0	3

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4.

x	1	2	3	4	5
y= f(x)	0	1	2	3	4

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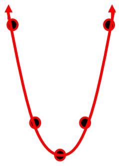
5.  ______________________________

6. ______________________________

B. Quadratic function defined by $\mathbf{y = ax^2 + bx + c}$ can be transformed into the form $\mathbf{y = a(x - h)^2 + k}$ (called the Vertex Form), by using Completing the Square Method. The following steps can be followed.

1. Group together the terms containing x.
2. Factor out a. If a = 1, proceed to step 3.
3. Complete the expression in parenthesis to make it perfect square trinomial by adding the value of $(\quad)^2$ and subtracting the value $a(\quad)^2$ to the constant term.
4. Simplify and express perfect square trinomial as the square of binomial.

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Learning Task 3. Transform the quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$.

1. $y = x^2 - 6x - 3$

2. $y = 5x^2 - 20x - 5$

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Learning Task 4 Suppose that you will put a square carpet on a floor, you need a carpet with an area of $(x^2 - 8x + k)$ square meters. What will be the value of "k" so that it will make it a perfect square?