

### מבוא להצפנה – תרגיל 3

.1

.א.

```
-----  
a = 2  
  
b0 = 2^11799 = 1014 mod 47197  
b1 = 1014^2 = 37059 mod 47197  
47197 is not a pseudoprime or a Strong pseudoprime to base 2  
////////////////////////////////
```

```
-----  
a = 3  
  
b0 = 3^11799 = 1 mod 47197  
47197 is a Strong pseudoprime to base 3  
////////////////////////////////
```

```
-----  
a = 4  
  
b0 = 4^11799 = 37059 mod 47197  
b1 = 37059^2 = 31175 mod 47197  
47197 is not a pseudoprime or a Strong pseudoprime to base 4  
////////////////////////////////
```

```
-----  
a = 5  
  
b0 = 5^11799 = 40004 mod 47197  
b1 = 40004^2 = 11337 mod 47197  
47197 is not a pseudoprime or a Strong pseudoprime to base 5  
////////////////////////////////
```

```
-----  
a = 6  
  
b0 = 6^11799 = 1014 mod 47197  
b1 = 1014^2 = 37059 mod 47197  
47197 is not a pseudoprime or a Strong pseudoprime to base 6  
////////////////////////////////
```

```
-----  
a = 7  
  
b0 = 7^11799 = 34445 mod 47197  
b1 = 34445^2 = 19839 mod 47197  
47197 is not a pseudoprime or a Strong pseudoprime to base 7  
/////////////////////////////////  
  
-----  
a = 8  
  
b0 = 8^11799 = 9014 mod 47197  
b1 = 9014^2 = 26159 mod 47197  
47197 is not a pseudoprime or a Strong pseudoprime to base 8  
/////////////////////////////////  
  
-----  
a = 9  
  
b0 = 9^11799 = 1 mod 47197  
47197 is a Strong pseudoprime to base 9  
/////////////////////////////////  
  
-----  
a = 10  
  
b0 = 10^11799 = 21833 mod 47197  
b1 = 21833^2 = 37386 mod 47197  
47197 is not a pseudoprime or a Strong pseudoprime to base 10  
/////////////////////////////////
```

ב.

```
-----  
a = 2  
n = 47197, k = 2, r = 11799  
  
b0 = 2^11799 = 1014 mod 47197  
b1 = 1014^2 = 37059 mod 47197  
/////////////////////////////////  
47197 is composite  
gcd(47197, 37059) = 1  
/////////////////////////////////  
  
-----  
a = 3  
n = 47197, k = 2, r = 11799  
  
b0 = 3^11799 = 1 mod 47197  
/////////////////////////////////  
47197 is probably prime  
/////////////////////////////////  
  
-----  
a = 4  
n = 47197, k = 2, r = 11799  
  
b0 = 4^11799 = 37059 mod 47197  
b1 = 37059^2 = 31175 mod 47197  
/////////////////////////////////  
47197 is composite  
gcd(47197, 31175) = 109  
and we found that the composite is 47197 = 109 * 433  
/////////////////////////////////
```

```
-----  
x = 218  
x^2 - 47197 = 327  
sqrt(x^2 - 47197) = 18.083141320025124
```

```
-----  
x = 219  
x^2 - 47197 = 764  
sqrt(x^2 - 47197) = 27.640549922170507
```

```
-----  
x = 220  
x^2 - 47197 = 1203  
sqrt(x^2 - 47197) = 34.68429039204925
```

```
-----  
x = 221  
x^2 - 47197 = 1644  
sqrt(x^2 - 47197) = 40.54626986542659
```

```
-----  
x = 222  
x^2 - 47197 = 2087  
sqrt(x^2 - 47197) = 45.68369512200168
```

```
-----  
x = 223  
x^2 - 47197 = 2532  
sqrt(x^2 - 47197) = 50.3189825016365
```

```
-----  
x = 224  
x^2 - 47197 = 2979  
sqrt(x^2 - 47197) = 54.58021619598075
```

```
-----  
x = 225  
x^2 - 47197 = 3428  
sqrt(x^2 - 47197) = 58.54912467321779
```

```
-----  
x = 226  
x^2 - 47197 = 3879  
sqrt(x^2 - 47197) = 62.281618476080084
```

```
-----  
x = 227  
x^2 - 47197 = 4332  
sqrt(x^2 - 47197) = 65.81793068761733
```

```
-----  
x = 228  
x^2 - 47197 = 4787  
sqrt(x^2 - 47197) = 69.18814927427962
```

```
-----  
x = 229  
x^2 - 47197 = 5244  
sqrt(x^2 - 47197) = 72.41546796092669
```

```
-----  
x = 230  
x^2 - 47197 = 5703  
sqrt(x^2 - 47197) = 75.5182097245426
```

```
-----  
x = 231  
x^2 - 47197 = 6164  
sqrt(x^2 - 47197) = 78.51114570556209
```

```
-----  
x = 232  
x^2 - 47197 = 6627  
sqrt(x^2 - 47197) = 81.40638795573723
```

```
-----  
x = 233  
x^2 - 47197 = 7092  
sqrt(x^2 - 47197) = 84.2140130857092
```

```
-----  
x = 234  
x^2 - 47197 = 7559  
sqrt(x^2 - 47197) = 86.94250974063263
```

```
-----  
x = 235  
x^2 - 47197 = 8028  
sqrt(x^2 - 47197) = 89.59910713840847
```

```
-----  
x = 236  
x^2 - 47197 = 8499  
sqrt(x^2 - 47197) = 92.19002115196633  
  
-----  
x = 237  
x^2 - 47197 = 8972  
sqrt(x^2 - 47197) = 94.72064188971694  
  
-----  
x = 238  
x^2 - 47197 = 9447  
sqrt(x^2 - 47197) = 97.19567891629751  
  
-----  
x = 239  
x^2 - 47197 = 9924  
sqrt(x^2 - 47197) = 99.61927524329818  
  
-----  
x = 240  
x^2 - 47197 = 10403  
sqrt(x^2 - 47197) = 101.99509792141973  
  
-----  
x = 241  
x^2 - 47197 = 10884  
sqrt(x^2 - 47197) = 104.32641084595981  
  
-----  
x = 242  
x^2 - 47197 = 11367  
sqrt(x^2 - 47197) = 106.61613386350116  
  
-----  
x = 243  
x^2 - 47197 = 11852  
sqrt(x^2 - 47197) = 108.86689120205463  
  
-----  
x = 244  
x^2 - 47197 = 12339  
sqrt(x^2 - 47197) = 111.0810514894417
```

```
-----  
x = 245  
x^2 - 47197 = 12828  
sqrt(x^2 - 47197) = 113.26076107814215
```

```
-----  
x = 246  
x^2 - 47197 = 13319  
sqrt(x^2 - 47197) = 115.40797199500561
```

```
-----  
x = 247  
x^2 - 47197 = 13812  
sqrt(x^2 - 47197) = 117.52446553803169
```

```
-----  
x = 248  
x^2 - 47197 = 14307  
sqrt(x^2 - 47197) = 119.61187232043481
```

```
-----  
x = 249  
x^2 - 47197 = 14804  
sqrt(x^2 - 47197) = 121.67168939404104
```

```
-----  
x = 250  
x^2 - 47197 = 15303  
sqrt(x^2 - 47197) = 123.70529495538985
```

```
-----  
x = 251  
x^2 - 47197 = 15804  
sqrt(x^2 - 47197) = 125.71396103854178
```

```
-----  
x = 252  
x^2 - 47197 = 16307  
sqrt(x^2 - 47197) = 127.69886452118516
```

```
-----  
x = 253  
x^2 - 47197 = 16812  
sqrt(x^2 - 47197) = 129.66109670984585
```

```
-----  
x = 254  
x^2 - 47197 = 17319  
sqrt(x^2 - 47197) = 131.6016717219048  
  
-----  
x = 255  
x^2 - 47197 = 17828  
sqrt(x^2 - 47197) = 133.52153384379613  
  
-----  
x = 256  
x^2 - 47197 = 18339  
sqrt(x^2 - 47197) = 135.42156401400774  
  
-----  
x = 257  
x^2 - 47197 = 18852  
sqrt(x^2 - 47197) = 137.30258555467918  
  
-----  
x = 258  
x^2 - 47197 = 19367  
sqrt(x^2 - 47197) = 139.16536925542934  
  
-----  
x = 259  
x^2 - 47197 = 19884  
sqrt(x^2 - 47197) = 141.01063789657857  
  
-----  
x = 260  
x^2 - 47197 = 20403  
sqrt(x^2 - 47197) = 142.83907028540895  
  
-----  
x = 261  
x^2 - 47197 = 20924  
sqrt(x^2 - 47197) = 144.65130486794789  
  
-----  
x = 262  
x^2 - 47197 = 21447  
sqrt(x^2 - 47197) = 146.4479429695071
```



```
-----  
x = 263  
x^2 - 47197 = 21972  
sqrt(x^2 - 47197) = 148.22955170950223
```

```
-----  
x = 264  
x^2 - 47197 = 22499  
sqrt(x^2 - 47197) = 149.9966666296288
```

```
-----  
x = 265  
x^2 - 47197 = 23028  
sqrt(x^2 - 47197) = 151.74979406905302
```

```
-----  
x = 266  
x^2 - 47197 = 23559  
sqrt(x^2 - 47197) = 153.48941331570722
```

```
-----  
x = 267  
x^2 - 47197 = 24092  
sqrt(x^2 - 47197) = 155.21597855890997
```

```
-----  
x = 268  
x^2 - 47197 = 24627  
sqrt(x^2 - 47197) = 156.9299206652447
```

```
-----  
x = 269  
x^2 - 47197 = 25164  
sqrt(x^2 - 47197) = 158.63164879682742
```

```
-----  
x = 270  
x^2 - 47197 = 25703  
sqrt(x^2 - 47197) = 160.3215518886965
```

```
-----  
x = 271  
x^2 - 47197 = 26244  
sqrt(x^2 - 47197) = 162.0
```

-----  
The factors are:  $x-y$  and  $x+y$ , where  $x$  and  $y$  are the values from the table above, and  $n$  is the number to be factored.

$x = 271, y = 162.0, n = 47197$

$109.0 \times 433.0 = (271-162.0)(271+162.0) = 271^2 - 162.0^2 = 47197$

3.

$433 - 1 = 432 = 2^4 \times 3^3$  ו-  $109 - 1 = 108 = 2^3 \times 3^3$ .  
שיטת  $p - 1$  של פולארד מבוססת על העובדה כי  $a^{B!} - 1$  אם  $p - 1 | B!$ .  
אז, אם  $q - 1$  אינו מחלק את  $B!$ , יש סיכויים טובים ש-  $\gcd(a^{B!} - 1 \bmod n, n)$ .  
אבל במקרה הזה, עבור  $p - 1$  ו-  $q - 1$ , הגורם הראשוני הגדול ביותר הוא 3.  
ולכן, רק עבור  $B = 3$  נקבל ש-  $p - 1 | B!$ . ומכאן שגם  $q - 1 | B!$  ואז  
 $\gcd(a^{B!} - 1 \bmod n, n) = pq = n$  ולא נצליח לפרק את  $n$ .

.4

B = 2		$2^2! \bmod 168163 = 4$		$\gcd(2^2! - 1 \bmod 168163) = 1$
B = 3		$2^3! \bmod 168163 = 64$		$\gcd(2^3! - 1 \bmod 168163) = 1$
B = 4		$2^4! \bmod 168163 = 129079$		$\gcd(2^4! - 1 \bmod 168163) = 1$
B = 5		$2^5! \bmod 168163 = 66131$		$\gcd(2^5! - 1 \bmod 168163) = 1$
B = 6		$2^6! \bmod 168163 = 2423$		$\gcd(2^6! - 1 \bmod 168163) = 1$
B = 7		$2^7! \bmod 168163 = 13818$		$\gcd(2^7! - 1 \bmod 168163) = 337$

-----  
337 is a prime factor of 168163.  
And  $168163 = 337 \times 499$   
-----

.5

נבחר:  $B = 47$

$$217^2 \bmod 47197$$

$$217^2 = 47089 = 7^2 \times 31^2 = \bmod 47197$$

$$227^2 = 4332 = 2^2 \times 3 \times 19^2 = \bmod 47197$$

$$232^2 = 6627 = 3 \times 47^2 = \bmod 47197$$

$$(217 * 227 * 232)^2 = 7^2 \times 31^2 \times 2^2 \times 3^2 \times 19^2 \times 47^2 = \bmod 47197$$

$$(6414)^2 = (7 \times 31 \times 2 \times 3 \times 19 \times 47)^2 = \bmod 47197$$

$$(6414)^2 = 29958^2 = \bmod 47197$$

$$\gcd(47197, 6414 - 29958) =$$

$$\gcd(47197, 23653) = 109$$

$$47197 = 109 \times 433. \Leftarrow$$

6.

א.

פונקציית ההצפנה היא:

$$y = e(x) = x^e \bmod n$$

נתון:

$$m = 2024, \\ (n, e) = (47197, 17)$$

נציב:

$$2024^{17} \bmod 47197$$

נחשב:

i	e <sub>i</sub>	S <sub>k</sub>	r <sub>k</sub>
1	1	$1 \bmod 47197$	$2024 \times 1 \bmod 47197 =$ $= 2024 \bmod 47197$
2	0	$2024^2 \bmod 47197 =$ $= 37634 \bmod 47197$	
3	0	$37634^2 \bmod 47197 =$ $= 30380 \bmod 47197$	
4	0	$30380^2 \bmod 47197 =$ $= 7065 \bmod 47197$	
5	1	$7065^2 \bmod 47197 =$ $= 26996 \bmod 47197$	$2024 \times 26996 \bmod 47197 =$ $= 32975 \bmod 47197$

ולכן,

$$2024^{17} \bmod 47197 = \\ = 32975 \bmod 47197$$

ב.

נחשב את המפתח הפרטי על ידי האלגוריתם האוקלידי המורחב.  
נתונים:

$$n = 47197$$

$$e = 17$$

ראשית, נחשב את  $\varphi(n)$

$$\begin{aligned}\varphi(n) &= (109^1 - 109^0)(433^1 - 433^0) = (109 - 1)(433 - 1) = . \\ &= 108 \times 432 = 46656\end{aligned}$$

נחשב כעת את  $d = e^{-1} = 17^{-1} \bmod 46656$

i	$r_i$	$q_i$	$S_i$	$t_i$
0	$a = 46656$		0	1
1	$b = 17$	2744	1	0
2	8	2	-2744	1
3	1	8	5489	-2
4	0			

מצאנו בעזרת האלגוריתם האוקלידי המורחב כי:

$$(5489 \times 17) + (-2 \times 46656) = 1$$

$$5489 \times 17 = 1 \bmod 47197 \quad \Leftarrow$$

$$5489 = 17^{-1} \bmod 47197 \quad \Leftarrow$$

המפתח הפרטי הוא:

$$\underline{d = 5489}$$

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ג.

7.