שם: שחר אשר
ת.ז. 209305408

מבוא להצפנה – תרגיל 4

.1

.א

```
In this capter we calculate the private key d using the extended
Euclidean algorithm.

i = 0, r = 33,        s = 0, t = 1
i = 1, r = 17, q = 1, s = 1, t = 0
i = 2, r = 16, q = 1, s = -1, t = 1
i = 3, r = 1, q = 16, s = 2, t = -1
-----------------
we got that 1 = 17*(2) + 33*(-1)
-----------------
So:
The value of s is 2
The value of t is -1
-----------------
Now we calculate:

C_a^s*C_b^t = m^(se_a)*m^(te_b) = m^(se_a + te_b) = m (mod 16157)

Calculate 11671^-1:
First we need to calculate the inverse of 11671: 11671^-1 = 11671^-1
(mod 16157)
Now we calculate it using the extended Euclidean algorithm:
i = 0, r = 16157,        s = 0, t = 1
i = 1, r = 11671, q = 1, s = 1, t = 0
i = 2, r = 4486, q = 2, s = -1, t = 1
i = 3, r = 2699, q = 1, s = 3, t = -2
i = 4, r = 1787, q = 1, s = -4, t = 3
i = 5, r = 912, q = 1, s = 7, t = -5
i = 6, r = 875, q = 1, s = -11, t = 8
i = 7, r = 37, q = 23, s = 18, t = -13
i = 8, r = 24, q = 1, s = -425, t = 307
i = 9, r = 13, q = 1, s = 443, t = -320
i = 10, r = 11, q = 1, s = -868, t = 627
i = 11, r = 2, q = 5, s = 1311, t = -947
i = 12, r = 1, q = 2, s = -7423, t = 5362
-----------------
we got that 1 = 11671*(-7423) + 16157*(5362)
-----------------
So:
The value of s is -7423
The value of t is 5362
-----------------
```

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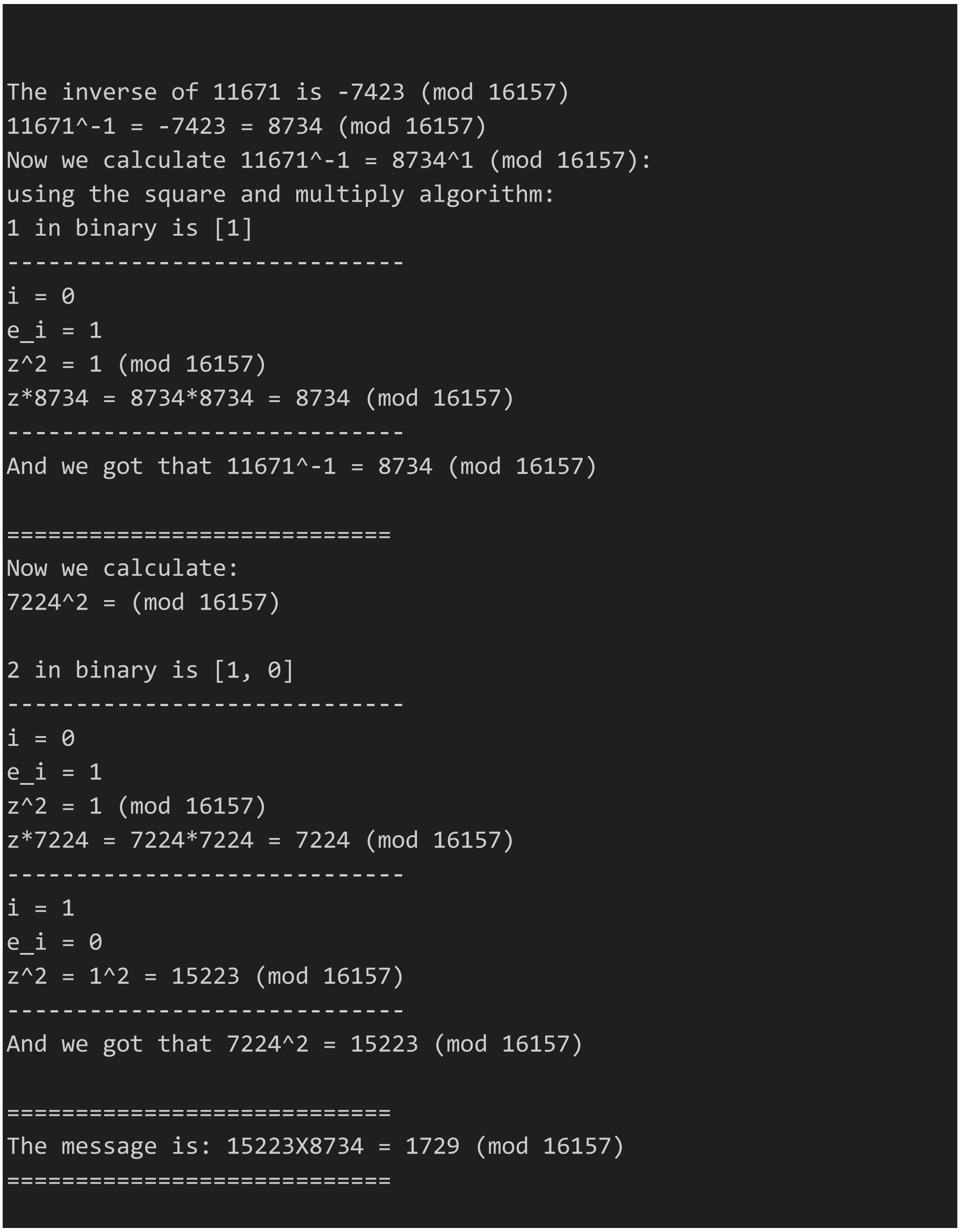

```
The inverse of 11671 is -7423 (mod 16157)
11671^-1 = -7423 = 8734 (mod 16157)
Now we calculate 11671^-1 = 8734^1 (mod 16157):
using the square and multiply algorithm:
1 in binary is [1]
-----------------------------
i = 0
e_i = 1
z^2 = 1 (mod 16157)
z*8734 = 8734*8734 = 8734 (mod 16157)
-----------------------------
And we got that 11671^-1 = 8734 (mod 16157)

============================
Now we calculate:
7224^2 = (mod 16157)

2 in binary is [1, 0]
-----------------------------
i = 0
e_i = 1
z^2 = 1 (mod 16157)
z*7224 = 7224*7224 = 7224 (mod 16157)
-----------------------------
i = 1
e_i = 0
z^2 = 1^2 = 15223 (mod 16157)
-----------------------------
And we got that 7224^2 = 15223 (mod 16157)

============================
The message is: 15223X8734 = 1729 (mod 16157)
============================
```

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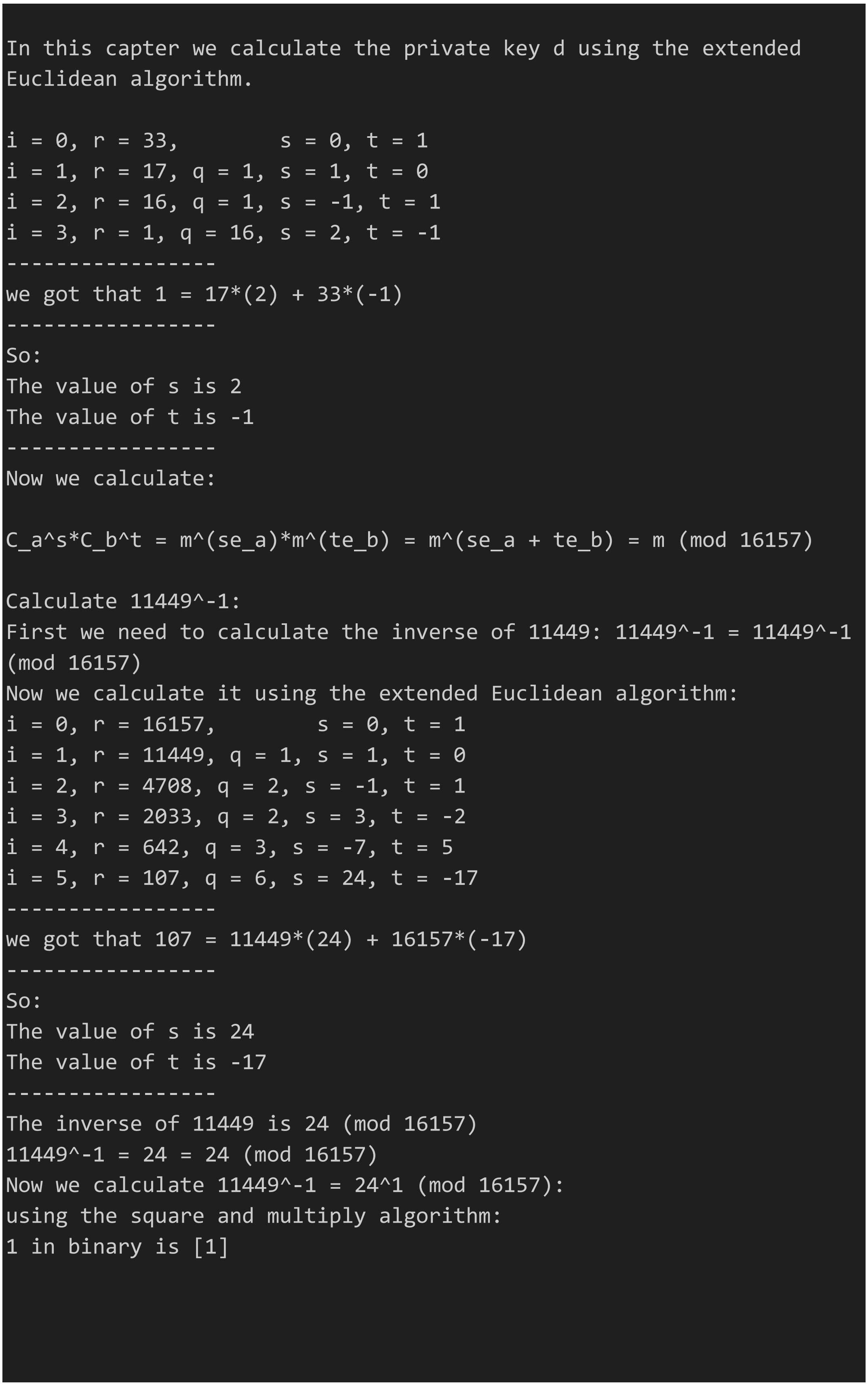
ב.

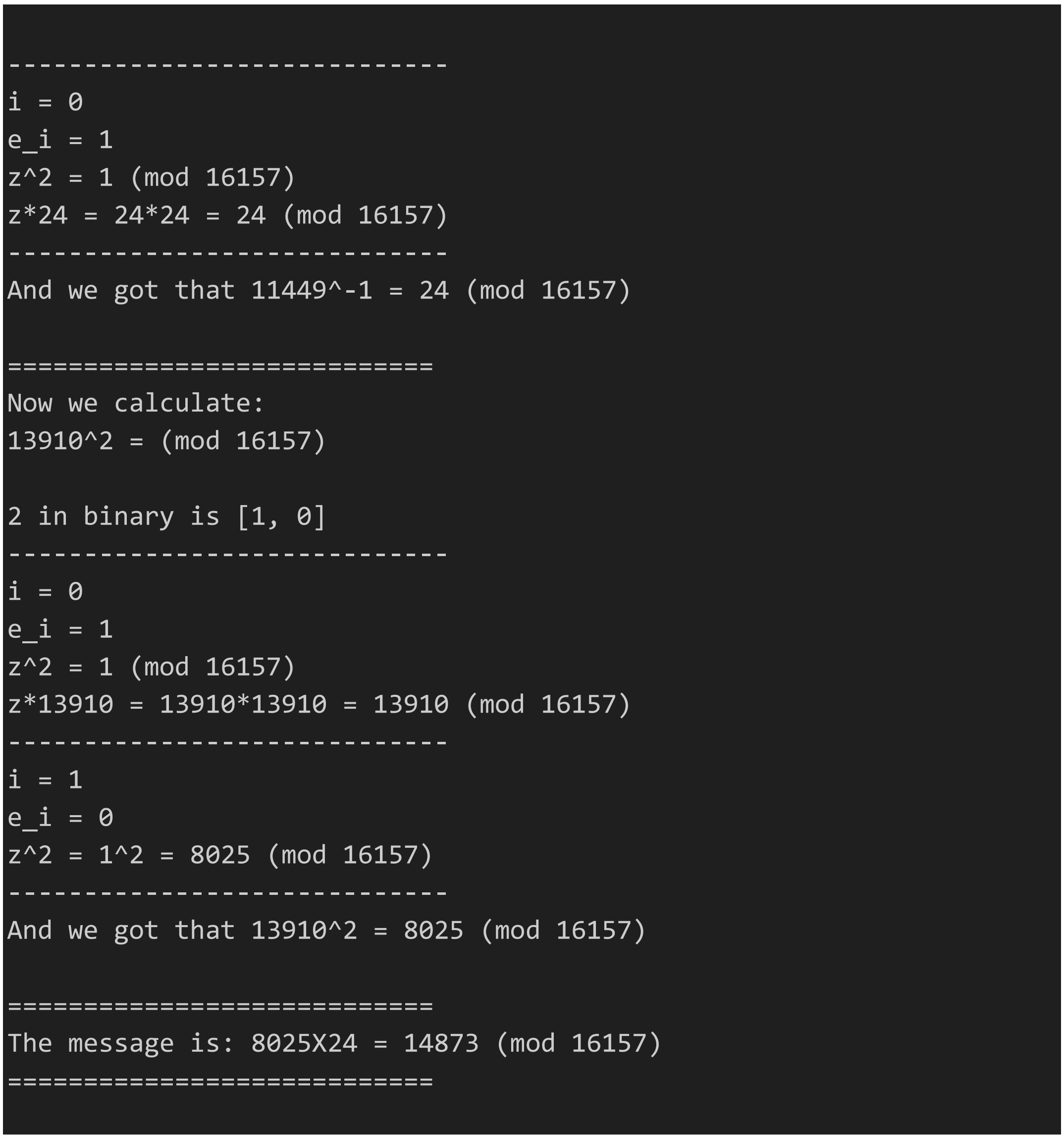


```
In this capter we calculate the private key d using the extended
Euclidean algorithm.

i = 0, r = 33,        s = 0, t = 1
i = 1, r = 17, q = 1, s = 1, t = 0
i = 2, r = 16, q = 1, s = -1, t = 1
i = 3, r = 1, q = 16, s = 2, t = -1
-----------------
we got that 1 = 17*(2) + 33*(-1)
-----------------
So:
The value of s is 2
The value of t is -1
-----------------
Now we calculate:

C_a^s*C_b^t = m^(se_a)*m^(te_b) = m^(se_a + te_b) = m (mod 16157)

Calculate 11449^-1:
First we need to calculate the inverse of 11449: 11449^-1 = 11449^-1
(mod 16157)
Now we calculate it using the extended Euclidean algorithm:
i = 0, r = 16157,        s = 0, t = 1
i = 1, r = 11449, q = 1, s = 1, t = 0
i = 2, r = 4708, q = 2, s = -1, t = 1
i = 3, r = 2033, q = 2, s = 3, t = -2
i = 4, r = 642, q = 3, s = -7, t = 5
i = 5, r = 107, q = 6, s = 24, t = -17
-----------------
we got that 107 = 11449*(24) + 16157*(-17)
-----------------
So:
The value of s is 24
The value of t is -17
-----------------
The inverse of 11449 is 24 (mod 16157)
11449^-1 = 24 = 24 (mod 16157)
Now we calculate 11449^-1 = 24^1 (mod 16157):
using the square and multiply algorithm:
1 in binary is [1]
```



```
-----------------------------
i = 0
e_i = 1
z^2 = 1 (mod 16157)
z*24 = 24*24 = 24 (mod 16157)
-----------------------------
And we got that 11449^-1 = 24 (mod 16157)

============================
Now we calculate:
13910^2 = (mod 16157)

2 in binary is [1, 0]
-----------------------------
i = 0
e_i = 1
z^2 = 1 (mod 16157)
z*13910 = 13910*13910 = 13910 (mod 16157)
-----------------------------
i = 1
e_i = 0
z^2 = 1^2 = 8025 (mod 16157)
-----------------------------
And we got that 13910^2 = 8025 (mod 16157)

============================
The message is: 8025X24 = 14873 (mod 16157)
============================
```

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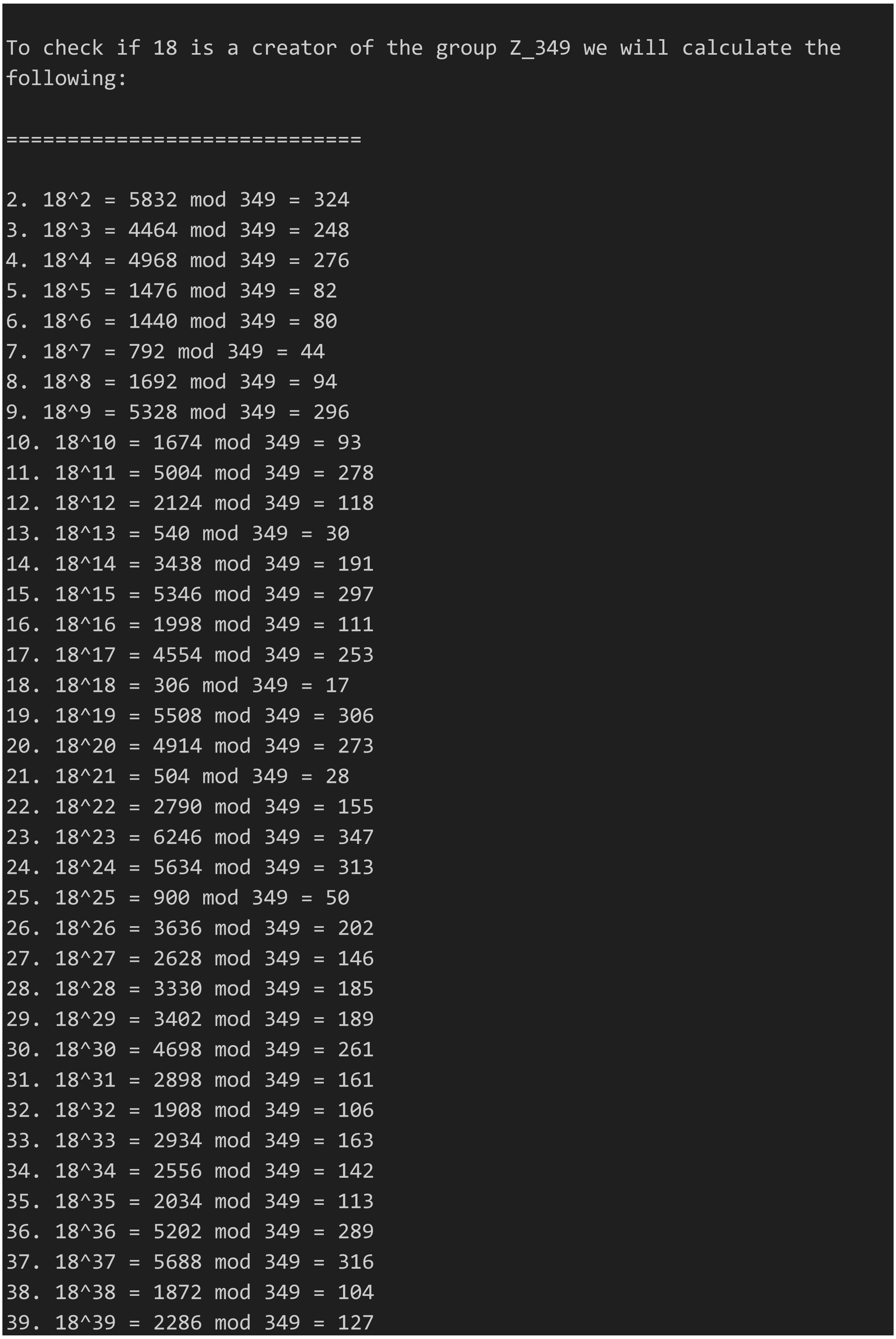
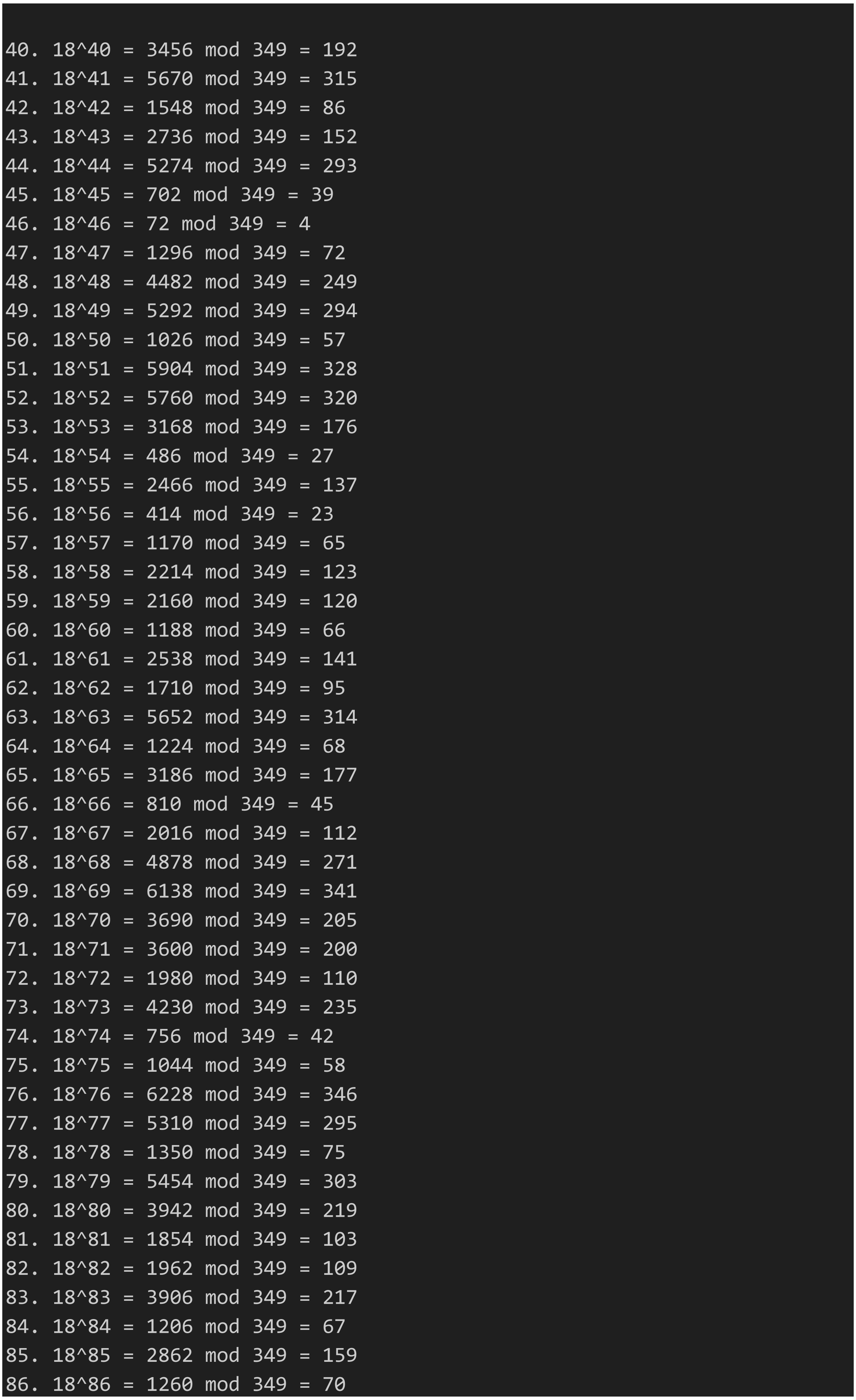
2.

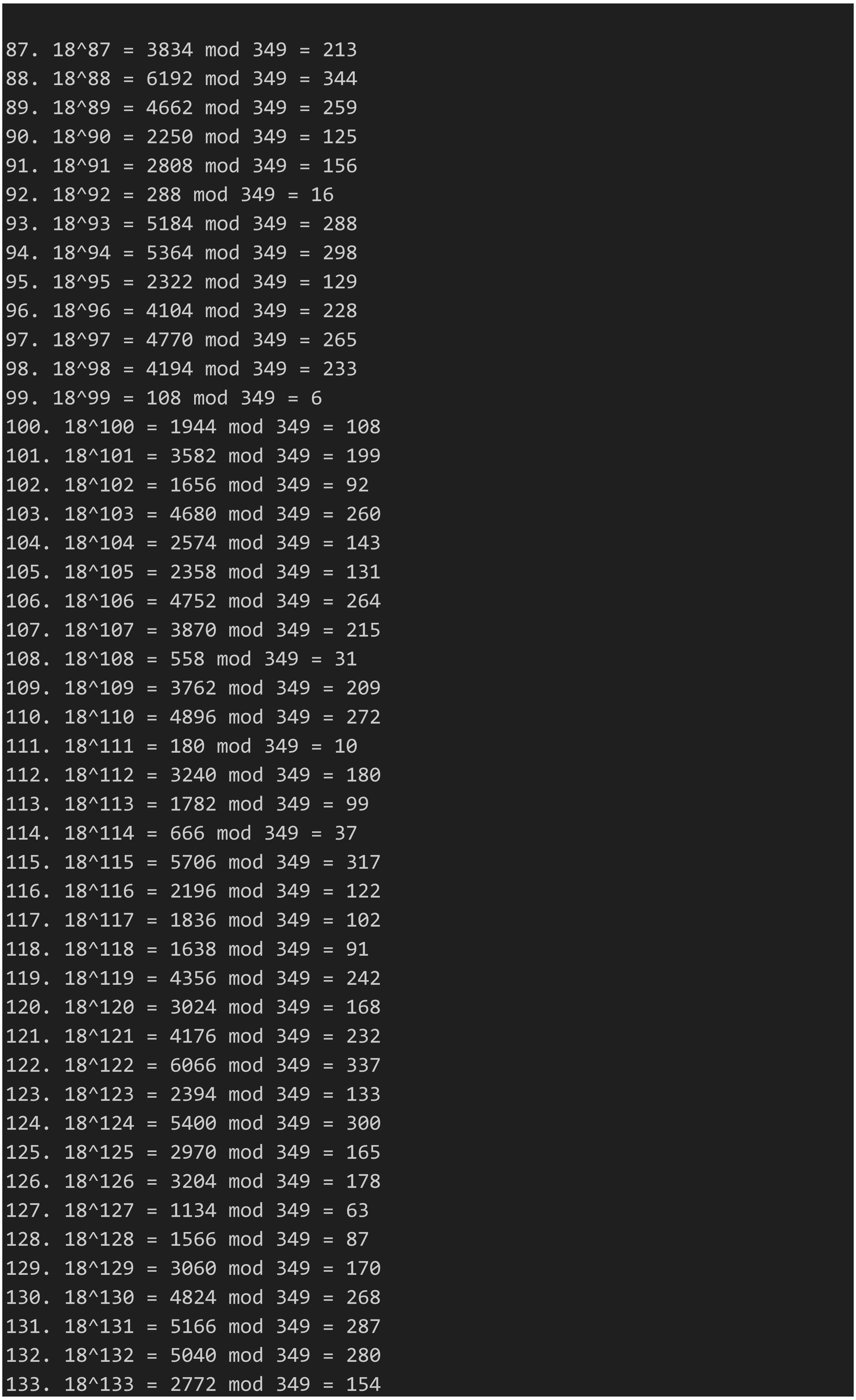
א.



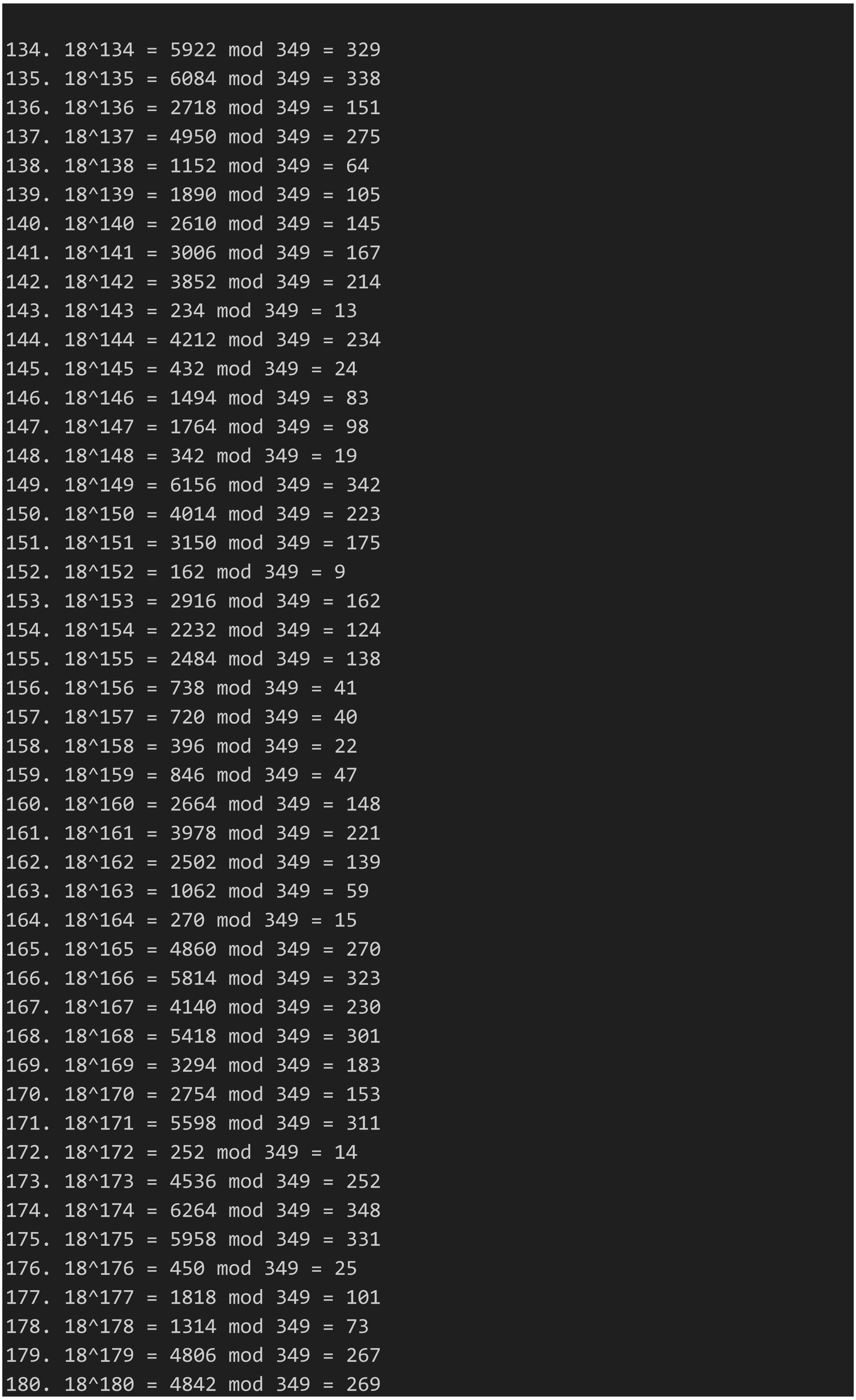
```
To check if 18 is a creator of the group Z_349 we will calculate the
following:

=============================

2. 18^2 = 5832 mod 349 = 324
3. 18^3 = 4464 mod 349 = 248
4. 18^4 = 4968 mod 349 = 276
5. 18^5 = 1476 mod 349 = 82
6. 18^6 = 1440 mod 349 = 80
7. 18^7 = 792 mod 349 = 44
8. 18^8 = 1692 mod 349 = 94
9. 18^9 = 5328 mod 349 = 296
10. 18^10 = 1674 mod 349 = 93
11. 18^11 = 5004 mod 349 = 278
12. 18^12 = 2124 mod 349 = 118
13. 18^13 = 540 mod 349 = 30
14. 18^14 = 3438 mod 349 = 191
15. 18^15 = 5346 mod 349 = 297
16. 18^16 = 1998 mod 349 = 111
17. 18^17 = 4554 mod 349 = 253
18. 18^18 = 306 mod 349 = 17
19. 18^19 = 5508 mod 349 = 306
20. 18^20 = 4914 mod 349 = 273
21. 18^21 = 504 mod 349 = 28
22. 18^22 = 2790 mod 349 = 155
23. 18^23 = 6246 mod 349 = 347
24. 18^24 = 5634 mod 349 = 313
25. 18^25 = 900 mod 349 = 50
26. 18^26 = 3636 mod 349 = 202
27. 18^27 = 2628 mod 349 = 146
28. 18^28 = 3330 mod 349 = 185
29. 18^29 = 3402 mod 349 = 189
30. 18^30 = 4698 mod 349 = 261
31. 18^31 = 2898 mod 349 = 161
32. 18^32 = 1908 mod 349 = 106
33. 18^33 = 2934 mod 349 = 163
34. 18^34 = 2556 mod 349 = 142
35. 18^35 = 2034 mod 349 = 113
36. 18^36 = 5202 mod 349 = 289
37. 18^37 = 5688 mod 349 = 316
38. 18^38 = 1872 mod 349 = 104
39. 18^39 = 2286 mod 349 = 127
```



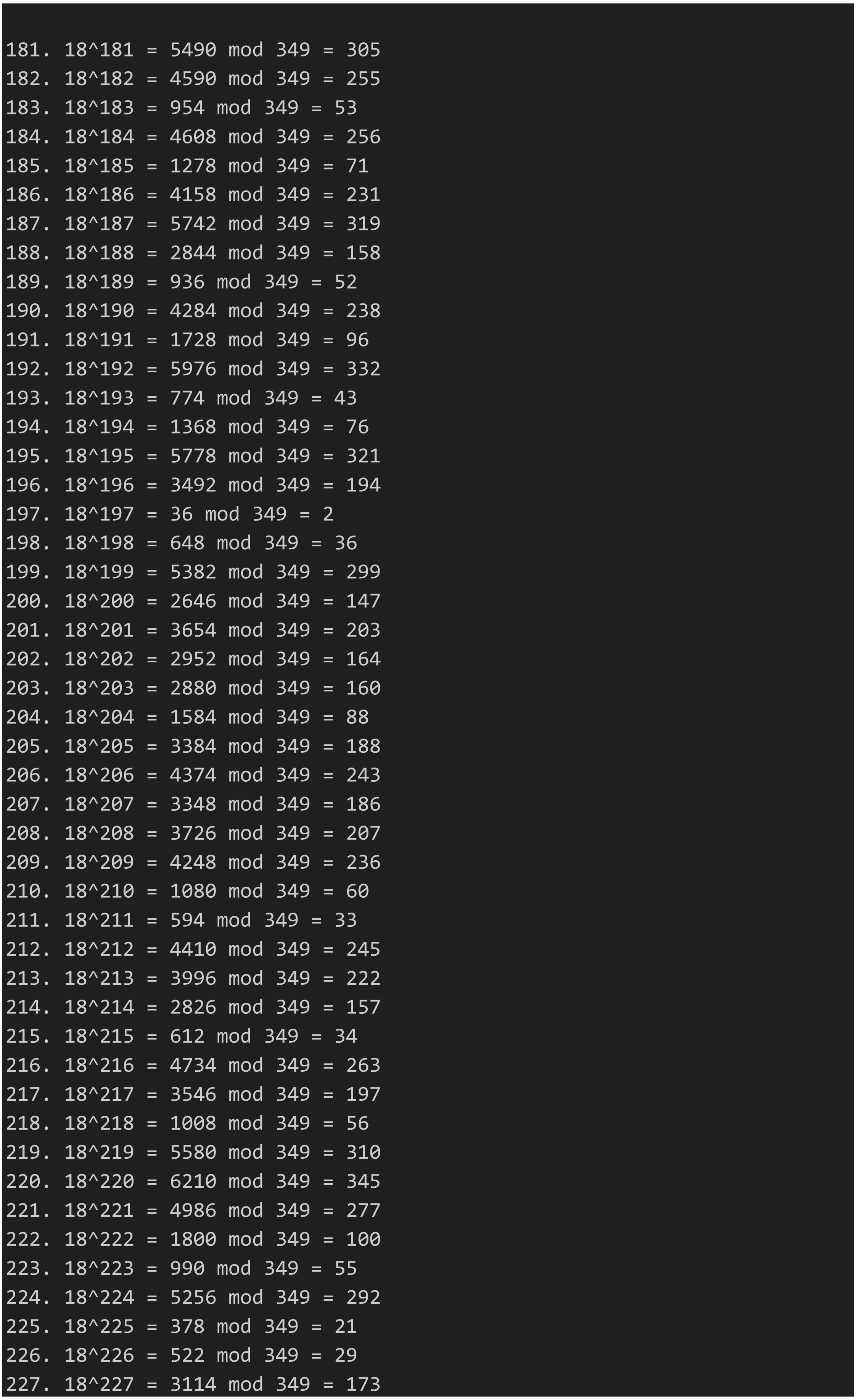
```
40. 18^40 = 3456 mod 349 = 192
41. 18^41 = 5670 mod 349 = 315
42. 18^42 = 1548 mod 349 = 86
43. 18^43 = 2736 mod 349 = 152
44. 18^44 = 5274 mod 349 = 293
45. 18^45 = 702 mod 349 = 39
46. 18^46 = 72 mod 349 = 4
47. 18^47 = 1296 mod 349 = 72
48. 18^48 = 4482 mod 349 = 249
49. 18^49 = 5292 mod 349 = 294
50. 18^50 = 1026 mod 349 = 57
51. 18^51 = 5904 mod 349 = 328
52. 18^52 = 5760 mod 349 = 320
53. 18^53 = 3168 mod 349 = 176
54. 18^54 = 486 mod 349 = 27
55. 18^55 = 2466 mod 349 = 137
56. 18^56 = 414 mod 349 = 23
57. 18^57 = 1170 mod 349 = 65
58. 18^58 = 2214 mod 349 = 123
59. 18^59 = 2160 mod 349 = 120
60. 18^60 = 1188 mod 349 = 66
61. 18^61 = 2538 mod 349 = 141
62. 18^62 = 1710 mod 349 = 95
63. 18^63 = 5652 mod 349 = 314
64. 18^64 = 1224 mod 349 = 68
65. 18^65 = 3186 mod 349 = 177
66. 18^66 = 810 mod 349 = 45
67. 18^67 = 2016 mod 349 = 112
68. 18^68 = 4878 mod 349 = 271
69. 18^69 = 6138 mod 349 = 341
70. 18^70 = 3690 mod 349 = 205
71. 18^71 = 3600 mod 349 = 200
72. 18^72 = 1980 mod 349 = 110
73. 18^73 = 4230 mod 349 = 235
74. 18^74 = 756 mod 349 = 42
75. 18^75 = 1044 mod 349 = 58
76. 18^76 = 6228 mod 349 = 346
77. 18^77 = 5310 mod 349 = 295
78. 18^78 = 1350 mod 349 = 75
79. 18^79 = 5454 mod 349 = 303
80. 18^80 = 3942 mod 349 = 219
81. 18^81 = 1854 mod 349 = 103
82. 18^82 = 1962 mod 349 = 109
83. 18^83 = 3906 mod 349 = 217
84. 18^84 = 1206 mod 349 = 67
85. 18^85 = 2862 mod 349 = 159
86. 18^86 = 1260 mod 349 = 70
```



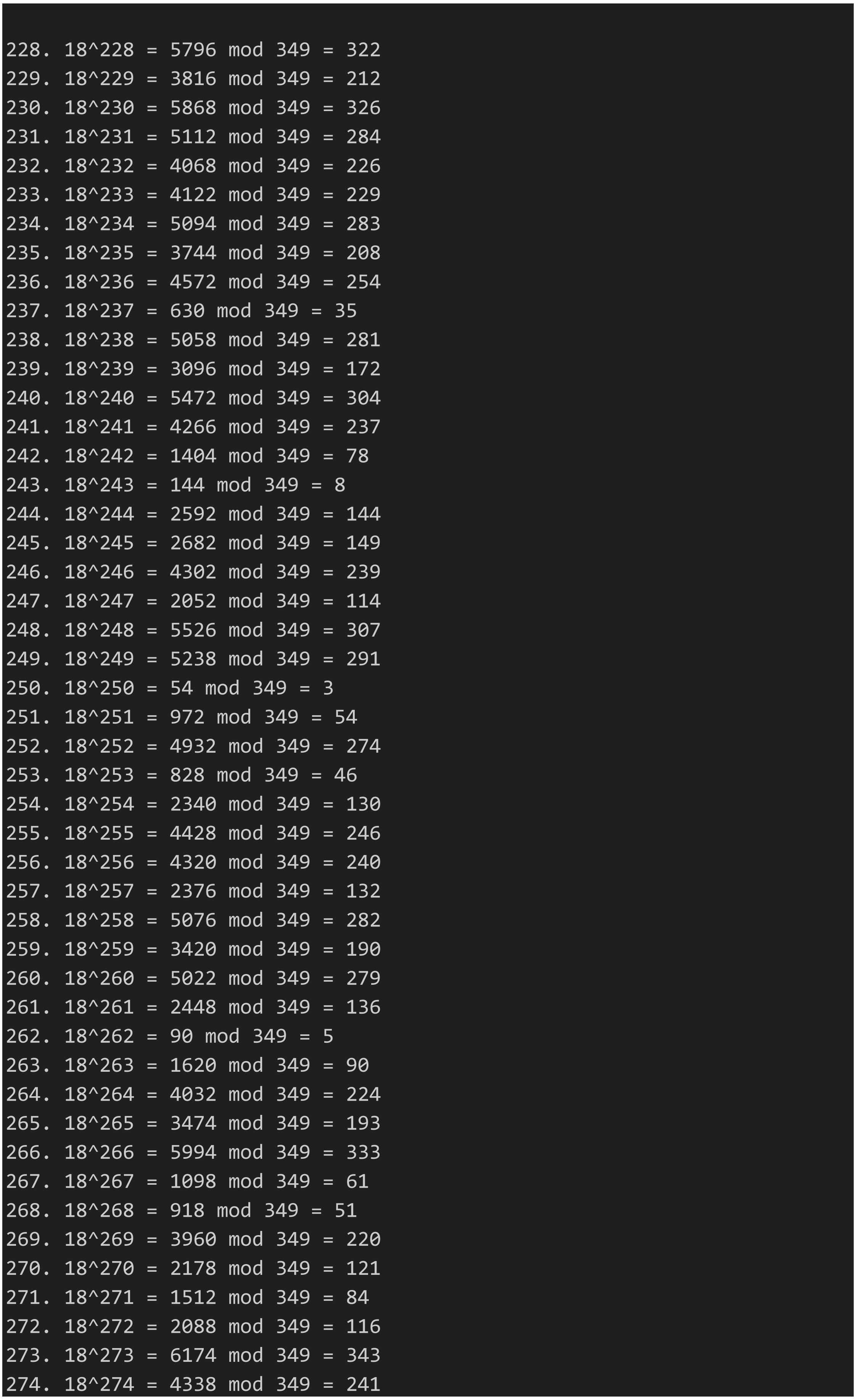
```
87. 18^87 = 3834 mod 349 = 213
88. 18^88 = 6192 mod 349 = 344
89. 18^89 = 4662 mod 349 = 259
90. 18^90 = 2250 mod 349 = 125
91. 18^91 = 2808 mod 349 = 156
92. 18^92 = 288 mod 349 = 16
93. 18^93 = 5184 mod 349 = 288
94. 18^94 = 5364 mod 349 = 298
95. 18^95 = 2322 mod 349 = 129
96. 18^96 = 4104 mod 349 = 228
97. 18^97 = 4770 mod 349 = 265
98. 18^98 = 4194 mod 349 = 233
99. 18^99 = 108 mod 349 = 6
100. 18^100 = 1944 mod 349 = 108
101. 18^101 = 3582 mod 349 = 199
102. 18^102 = 1656 mod 349 = 92
103. 18^103 = 4680 mod 349 = 260
104. 18^104 = 2574 mod 349 = 143
105. 18^105 = 2358 mod 349 = 131
106. 18^106 = 4752 mod 349 = 264
107. 18^107 = 3870 mod 349 = 215
108. 18^108 = 558 mod 349 = 31
109. 18^109 = 3762 mod 349 = 209
110. 18^110 = 4896 mod 349 = 272
111. 18^111 = 180 mod 349 = 10
112. 18^112 = 3240 mod 349 = 180
113. 18^113 = 1782 mod 349 = 99
114. 18^114 = 666 mod 349 = 37
115. 18^115 = 5706 mod 349 = 317
116. 18^116 = 2196 mod 349 = 122
117. 18^117 = 1836 mod 349 = 102
118. 18^118 = 1638 mod 349 = 91
119. 18^119 = 4356 mod 349 = 242
120. 18^120 = 3024 mod 349 = 168
121. 18^121 = 4176 mod 349 = 232
122. 18^122 = 6066 mod 349 = 337
123. 18^123 = 2394 mod 349 = 133
124. 18^124 = 5400 mod 349 = 300
125. 18^125 = 2970 mod 349 = 165
126. 18^126 = 3204 mod 349 = 178
127. 18^127 = 1134 mod 349 = 63
128. 18^128 = 1566 mod 349 = 87
129. 18^129 = 3060 mod 349 = 170
130. 18^130 = 4824 mod 349 = 268
131. 18^131 = 5166 mod 349 = 287
132. 18^132 = 5040 mod 349 = 280
133. 18^133 = 2772 mod 349 = 154
```



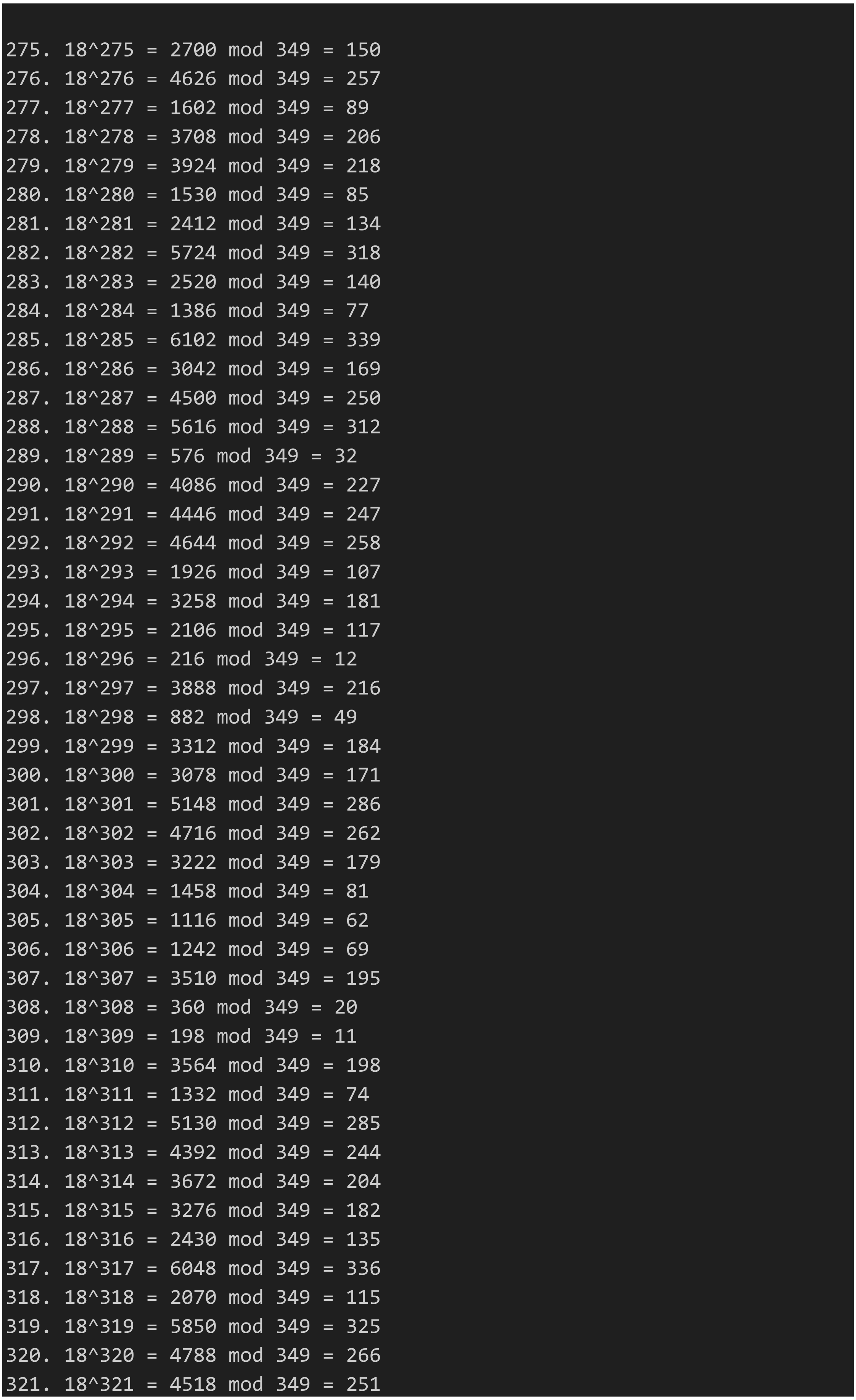
```
134. 18^134 = 5922 mod 349 = 329
135. 18^135 = 6084 mod 349 = 338
136. 18^136 = 2718 mod 349 = 151
137. 18^137 = 4950 mod 349 = 275
138. 18^138 = 1152 mod 349 = 64
139. 18^139 = 1890 mod 349 = 105
140. 18^140 = 2610 mod 349 = 145
141. 18^141 = 3006 mod 349 = 167
142. 18^142 = 3852 mod 349 = 214
143. 18^143 = 234 mod 349 = 13
144. 18^144 = 4212 mod 349 = 234
145. 18^145 = 432 mod 349 = 24
146. 18^146 = 1494 mod 349 = 83
147. 18^147 = 1764 mod 349 = 98
148. 18^148 = 342 mod 349 = 19
149. 18^149 = 6156 mod 349 = 342
150. 18^150 = 4014 mod 349 = 223
151. 18^151 = 3150 mod 349 = 175
152. 18^152 = 162 mod 349 = 9
153. 18^153 = 2916 mod 349 = 162
154. 18^154 = 2232 mod 349 = 124
155. 18^155 = 2484 mod 349 = 138
156. 18^156 = 738 mod 349 = 41
157. 18^157 = 720 mod 349 = 40
158. 18^158 = 396 mod 349 = 22
159. 18^159 = 846 mod 349 = 47
160. 18^160 = 2664 mod 349 = 148
161. 18^161 = 3978 mod 349 = 221
162. 18^162 = 2502 mod 349 = 139
163. 18^163 = 1062 mod 349 = 59
164. 18^164 = 270 mod 349 = 15
165. 18^165 = 4860 mod 349 = 270
166. 18^166 = 5814 mod 349 = 323
167. 18^167 = 4140 mod 349 = 230
168. 18^168 = 5418 mod 349 = 301
169. 18^169 = 3294 mod 349 = 183
170. 18^170 = 2754 mod 349 = 153
171. 18^171 = 5598 mod 349 = 311
172. 18^172 = 252 mod 349 = 14
173. 18^173 = 4536 mod 349 = 252
174. 18^174 = 6264 mod 349 = 348
175. 18^175 = 5958 mod 349 = 331
176. 18^176 = 450 mod 349 = 25
177. 18^177 = 1818 mod 349 = 101
178. 18^178 = 1314 mod 349 = 73
179. 18^179 = 4806 mod 349 = 267
180. 18^180 = 4842 mod 349 = 269
```



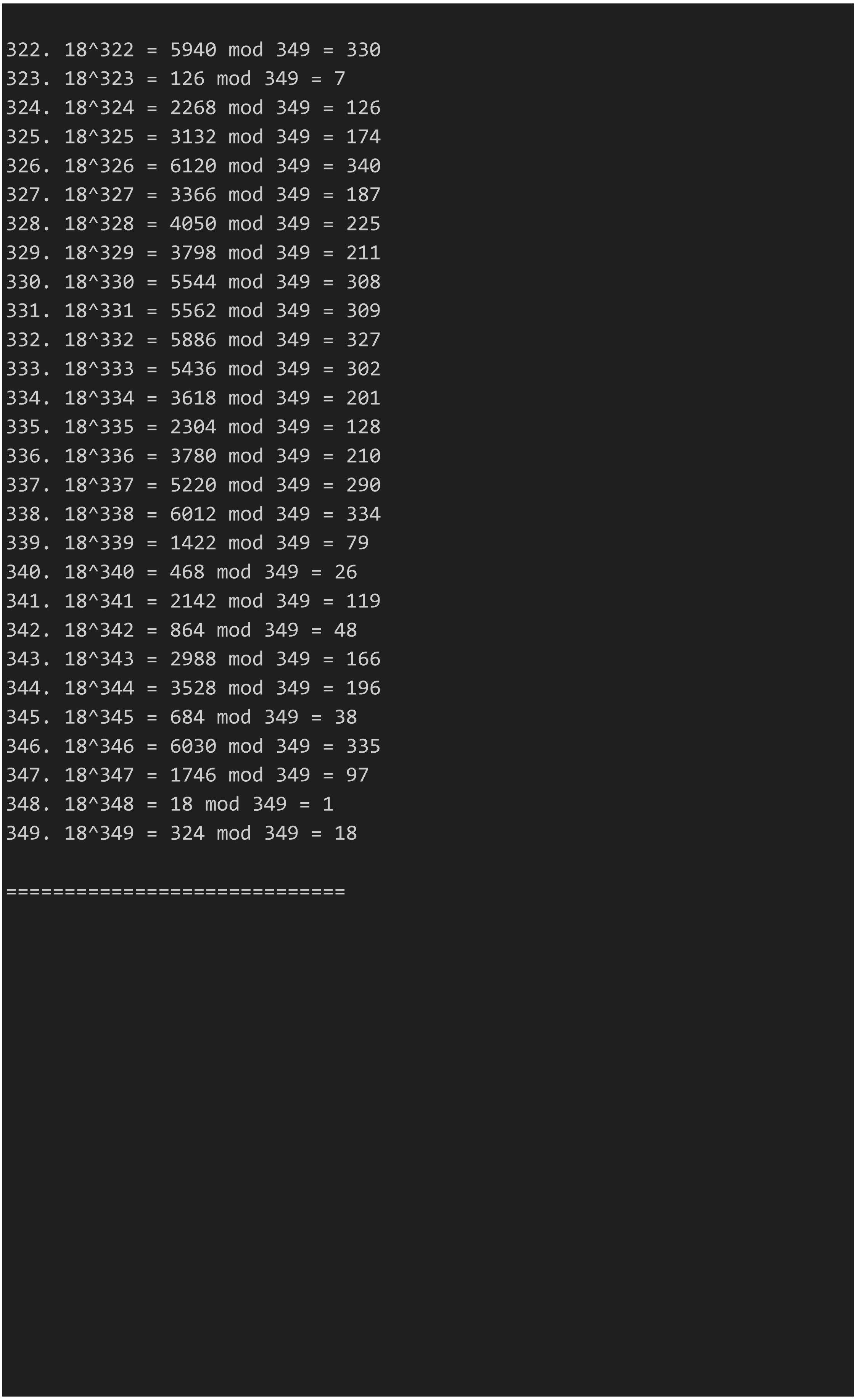
```
181. 18^181 = 5490 mod 349 = 305
182. 18^182 = 4590 mod 349 = 255
183. 18^183 = 954 mod 349 = 53
184. 18^184 = 4608 mod 349 = 256
185. 18^185 = 1278 mod 349 = 71
186. 18^186 = 4158 mod 349 = 231
187. 18^187 = 5742 mod 349 = 319
188. 18^188 = 2844 mod 349 = 158
189. 18^189 = 936 mod 349 = 52
190. 18^190 = 4284 mod 349 = 238
191. 18^191 = 1728 mod 349 = 96
192. 18^192 = 5976 mod 349 = 332
193. 18^193 = 774 mod 349 = 43
194. 18^194 = 1368 mod 349 = 76
195. 18^195 = 5778 mod 349 = 321
196. 18^196 = 3492 mod 349 = 194
197. 18^197 = 36 mod 349 = 2
198. 18^198 = 648 mod 349 = 36
199. 18^199 = 5382 mod 349 = 299
200. 18^200 = 2646 mod 349 = 147
201. 18^201 = 3654 mod 349 = 203
202. 18^202 = 2952 mod 349 = 164
203. 18^203 = 2880 mod 349 = 160
204. 18^204 = 1584 mod 349 = 88
205. 18^205 = 3384 mod 349 = 188
206. 18^206 = 4374 mod 349 = 243
207. 18^207 = 3348 mod 349 = 186
208. 18^208 = 3726 mod 349 = 207
209. 18^209 = 4248 mod 349 = 236
210. 18^210 = 1080 mod 349 = 60
211. 18^211 = 594 mod 349 = 33
212. 18^212 = 4410 mod 349 = 245
213. 18^213 = 3996 mod 349 = 222
214. 18^214 = 2826 mod 349 = 157
215. 18^215 = 612 mod 349 = 34
216. 18^216 = 4734 mod 349 = 263
217. 18^217 = 3546 mod 349 = 197
218. 18^218 = 1008 mod 349 = 56
219. 18^219 = 5580 mod 349 = 310
220. 18^220 = 6210 mod 349 = 345
221. 18^221 = 4986 mod 349 = 277
222. 18^222 = 1800 mod 349 = 100
223. 18^223 = 990 mod 349 = 55
224. 18^224 = 5256 mod 349 = 292
225. 18^225 = 378 mod 349 = 21
226. 18^226 = 522 mod 349 = 29
227. 18^227 = 3114 mod 349 = 173
```



```
228. 18^228 = 5796 mod 349 = 322
229. 18^229 = 3816 mod 349 = 212
230. 18^230 = 5868 mod 349 = 326
231. 18^231 = 5112 mod 349 = 284
232. 18^232 = 4068 mod 349 = 226
233. 18^233 = 4122 mod 349 = 229
234. 18^234 = 5094 mod 349 = 283
235. 18^235 = 3744 mod 349 = 208
236. 18^236 = 4572 mod 349 = 254
237. 18^237 = 630 mod 349 = 35
238. 18^238 = 5058 mod 349 = 281
239. 18^239 = 3096 mod 349 = 172
240. 18^240 = 5472 mod 349 = 304
241. 18^241 = 4266 mod 349 = 237
242. 18^242 = 1404 mod 349 = 78
243. 18^243 = 144 mod 349 = 8
244. 18^244 = 2592 mod 349 = 144
245. 18^245 = 2682 mod 349 = 149
246. 18^246 = 4302 mod 349 = 239
247. 18^247 = 2052 mod 349 = 114
248. 18^248 = 5526 mod 349 = 307
249. 18^249 = 5238 mod 349 = 291
250. 18^250 = 54 mod 349 = 3
251. 18^251 = 972 mod 349 = 54
252. 18^252 = 4932 mod 349 = 274
253. 18^253 = 828 mod 349 = 46
254. 18^254 = 2340 mod 349 = 130
255. 18^255 = 4428 mod 349 = 246
256. 18^256 = 4320 mod 349 = 240
257. 18^257 = 2376 mod 349 = 132
258. 18^258 = 5076 mod 349 = 282
259. 18^259 = 3420 mod 349 = 190
260. 18^260 = 5022 mod 349 = 279
261. 18^261 = 2448 mod 349 = 136
262. 18^262 = 90 mod 349 = 5
263. 18^263 = 1620 mod 349 = 90
264. 18^264 = 4032 mod 349 = 224
265. 18^265 = 3474 mod 349 = 193
266. 18^266 = 5994 mod 349 = 333
267. 18^267 = 1098 mod 349 = 61
268. 18^268 = 918 mod 349 = 51
269. 18^269 = 3960 mod 349 = 220
270. 18^270 = 2178 mod 349 = 121
271. 18^271 = 1512 mod 349 = 84
272. 18^272 = 2088 mod 349 = 116
273. 18^273 = 6174 mod 349 = 343
274. 18^274 = 4338 mod 349 = 241
```

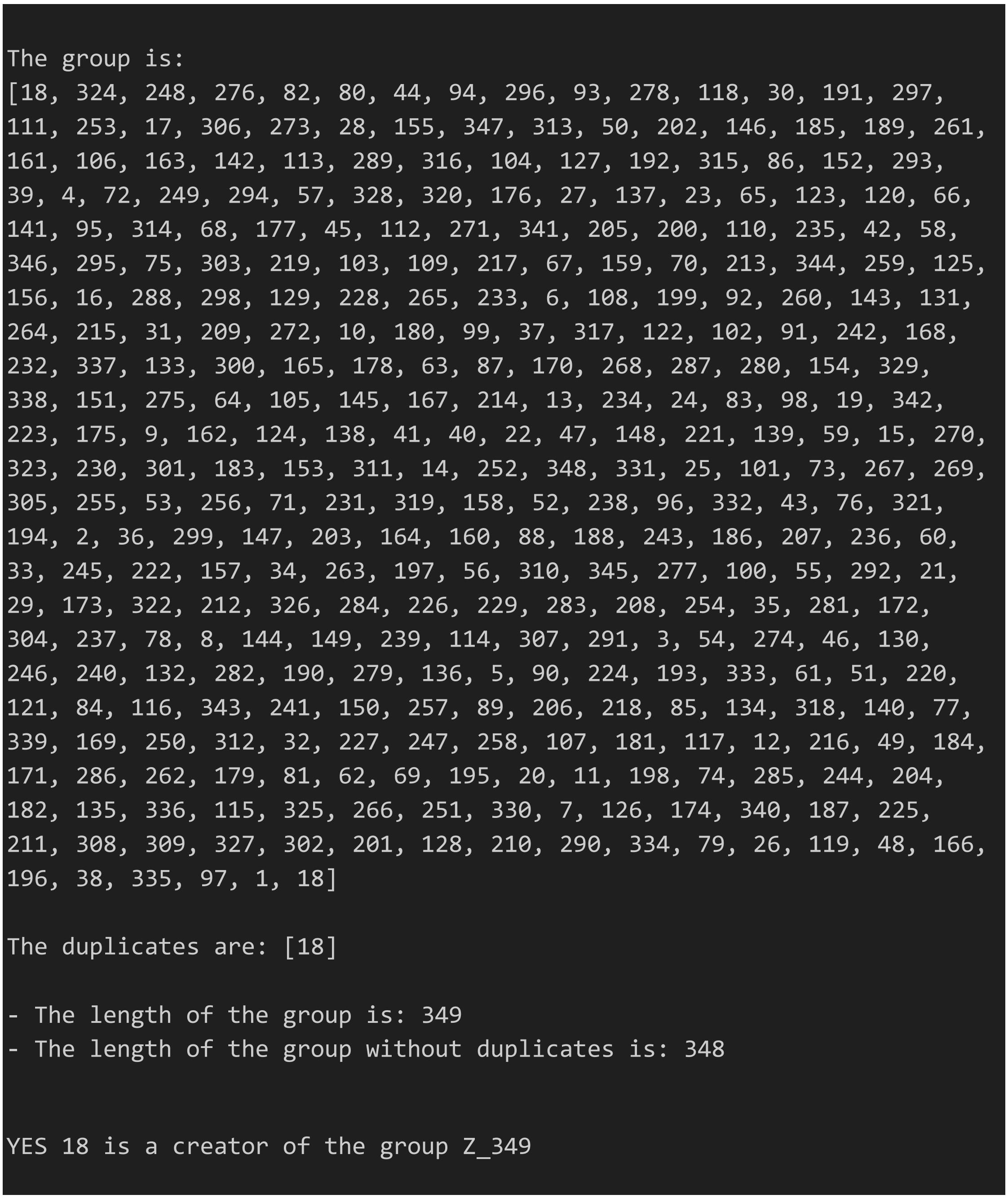


```
275. 18^275 = 2700 mod 349 = 150
276. 18^276 = 4626 mod 349 = 257
277. 18^277 = 1602 mod 349 = 89
278. 18^278 = 3708 mod 349 = 206
279. 18^279 = 3924 mod 349 = 218
280. 18^280 = 1530 mod 349 = 85
281. 18^281 = 2412 mod 349 = 134
282. 18^282 = 5724 mod 349 = 318
283. 18^283 = 2520 mod 349 = 140
284. 18^284 = 1386 mod 349 = 77
285. 18^285 = 6102 mod 349 = 339
286. 18^286 = 3042 mod 349 = 169
287. 18^287 = 4500 mod 349 = 250
288. 18^288 = 5616 mod 349 = 312
289. 18^289 = 576 mod 349 = 32
290. 18^290 = 4086 mod 349 = 227
291. 18^291 = 4446 mod 349 = 247
292. 18^292 = 4644 mod 349 = 258
293. 18^293 = 1926 mod 349 = 107
294. 18^294 = 3258 mod 349 = 181
295. 18^295 = 2106 mod 349 = 117
296. 18^296 = 216 mod 349 = 12
297. 18^297 = 3888 mod 349 = 216
298. 18^298 = 882 mod 349 = 49
299. 18^299 = 3312 mod 349 = 184
300. 18^300 = 3078 mod 349 = 171
301. 18^301 = 5148 mod 349 = 286
302. 18^302 = 4716 mod 349 = 262
303. 18^303 = 3222 mod 349 = 179
304. 18^304 = 1458 mod 349 = 81
305. 18^305 = 1116 mod 349 = 62
306. 18^306 = 1242 mod 349 = 69
307. 18^307 = 3510 mod 349 = 195
308. 18^308 = 360 mod 349 = 20
309. 18^309 = 198 mod 349 = 11
310. 18^310 = 3564 mod 349 = 198
311. 18^311 = 1332 mod 349 = 74
312. 18^312 = 5130 mod 349 = 285
313. 18^313 = 4392 mod 349 = 244
314. 18^314 = 3672 mod 349 = 204
315. 18^315 = 3276 mod 349 = 182
316. 18^316 = 2430 mod 349 = 135
317. 18^317 = 6048 mod 349 = 336
318. 18^318 = 2070 mod 349 = 115
319. 18^319 = 5850 mod 349 = 325
320. 18^320 = 4788 mod 349 = 266
321. 18^321 = 4518 mod 349 = 251
```



```
322. 18^322 = 5940 mod 349 = 330
323. 18^323 = 126 mod 349 = 7
324. 18^324 = 2268 mod 349 = 126
325. 18^325 = 3132 mod 349 = 174
326. 18^326 = 6120 mod 349 = 340
327. 18^327 = 3366 mod 349 = 187
328. 18^328 = 4050 mod 349 = 225
329. 18^329 = 3798 mod 349 = 211
330. 18^330 = 5544 mod 349 = 308
331. 18^331 = 5562 mod 349 = 309
332. 18^332 = 5886 mod 349 = 327
333. 18^333 = 5436 mod 349 = 302
334. 18^334 = 3618 mod 349 = 201
335. 18^335 = 2304 mod 349 = 128
336. 18^336 = 3780 mod 349 = 210
337. 18^337 = 5220 mod 349 = 290
338. 18^338 = 6012 mod 349 = 334
339. 18^339 = 1422 mod 349 = 79
340. 18^340 = 468 mod 349 = 26
341. 18^341 = 2142 mod 349 = 119
342. 18^342 = 864 mod 349 = 48
343. 18^343 = 2988 mod 349 = 166
344. 18^344 = 3528 mod 349 = 196
345. 18^345 = 684 mod 349 = 38
346. 18^346 = 6030 mod 349 = 335
347. 18^347 = 1746 mod 349 = 97
348. 18^348 = 18 mod 349 = 1
349. 18^349 = 324 mod 349 = 18

=============================
```

שם: שחר אשר
ת.ז. 209305408



```
The group is:
[18, 324, 248, 276, 82, 80, 44, 94, 296, 93, 278, 118, 30, 191, 297,
111, 253, 17, 306, 273, 28, 155, 347, 313, 50, 202, 146, 185, 189, 261,
161, 106, 163, 142, 113, 289, 316, 104, 127, 192, 315, 86, 152, 293,
39, 4, 72, 249, 294, 57, 328, 320, 176, 27, 137, 23, 65, 123, 120, 66,
141, 95, 314, 68, 177, 45, 112, 271, 341, 205, 200, 110, 235, 42, 58,
346, 295, 75, 303, 219, 103, 109, 217, 67, 159, 70, 213, 344, 259, 125,
156, 16, 288, 298, 129, 228, 265, 233, 6, 108, 199, 92, 260, 143, 131,
264, 215, 31, 209, 272, 10, 180, 99, 37, 317, 122, 102, 91, 242, 168,
232, 337, 133, 300, 165, 178, 63, 87, 170, 268, 287, 280, 154, 329,
338, 151, 275, 64, 105, 145, 167, 214, 13, 234, 24, 83, 98, 19, 342,
223, 175, 9, 162, 124, 138, 41, 40, 22, 47, 148, 221, 139, 59, 15, 270,
323, 230, 301, 183, 153, 311, 14, 252, 348, 331, 25, 101, 73, 267, 269,
305, 255, 53, 256, 71, 231, 319, 158, 52, 238, 96, 332, 43, 76, 321,
194, 2, 36, 299, 147, 203, 164, 160, 88, 188, 243, 186, 207, 236, 60,
33, 245, 222, 157, 34, 263, 197, 56, 310, 345, 277, 100, 55, 292, 21,
29, 173, 322, 212, 326, 284, 226, 229, 283, 208, 254, 35, 281, 172,
304, 237, 78, 8, 144, 149, 239, 114, 307, 291, 3, 54, 274, 46, 130,
246, 240, 132, 282, 190, 279, 136, 5, 90, 224, 193, 333, 61, 51, 220,
121, 84, 116, 343, 241, 150, 257, 89, 206, 218, 85, 134, 318, 140, 77,
339, 169, 250, 312, 32, 227, 247, 258, 107, 181, 117, 12, 216, 49, 184,
171, 286, 262, 179, 81, 62, 69, 195, 20, 11, 198, 74, 285, 244, 204,
182, 135, 336, 115, 325, 266, 251, 330, 7, 126, 174, 340, 187, 225,
211, 308, 309, 327, 302, 201, 128, 210, 290, 334, 79, 26, 119, 48, 166,
196, 38, 335, 97, 1, 18]

The duplicates are: [18]

- The length of the group is: 349
- The length of the group without duplicates is: 348


YES 18 is a creator of the group Z_349
```

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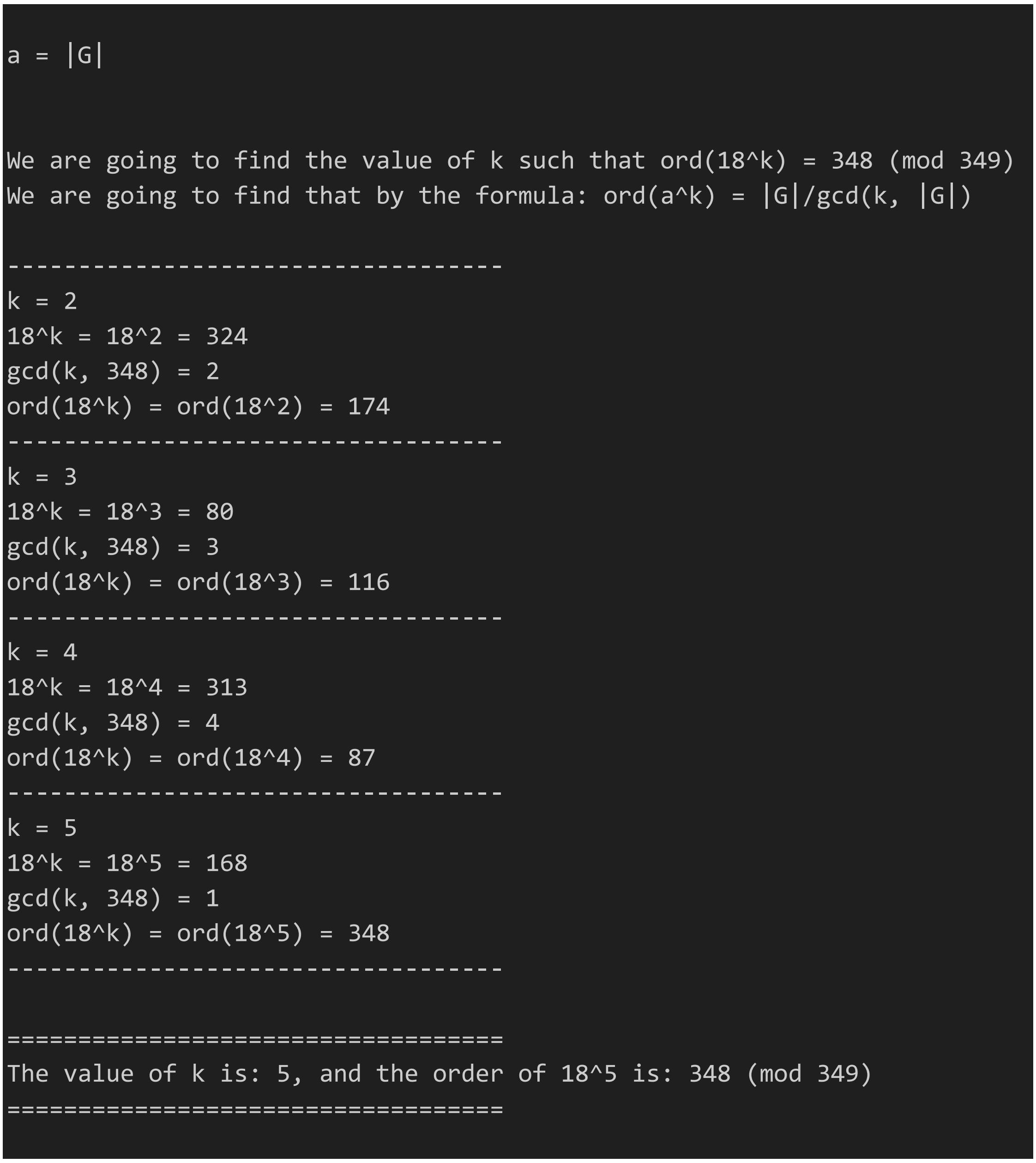
ב.

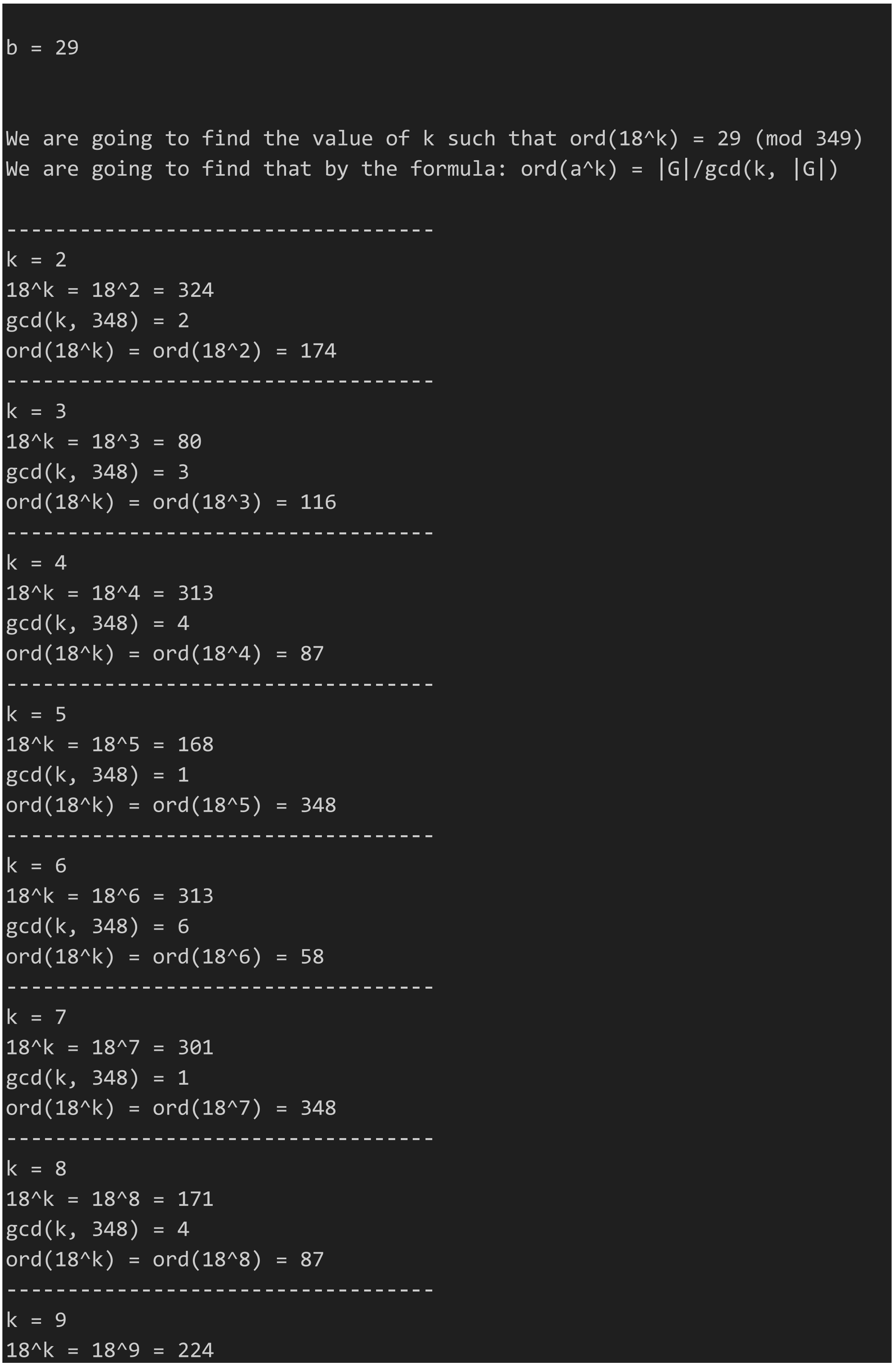


```
a = |G|


We are going to find the value of k such that ord(18^k) = 348 (mod 349)
We are going to find that by the formula: ord(a^k) = |G|/gcd(k, |G|)

-----------------------------------
k = 2
18^k = 18^2 = 324
gcd(k, 348) = 2
ord(18^k) = ord(18^2) = 174
-----------------------------------
k = 3
18^k = 18^3 = 80
gcd(k, 348) = 3
ord(18^k) = ord(18^3) = 116
-----------------------------------
k = 4
18^k = 18^4 = 313
gcd(k, 348) = 4
ord(18^k) = ord(18^4) = 87
-----------------------------------
k = 5
18^k = 18^5 = 168
gcd(k, 348) = 1
ord(18^k) = ord(18^5) = 348
-----------------------------------


===================================
The value of k is: 5, and the order of 18^5 is: 348 (mod 349)
===================================
```



```
b = 29


We are going to find the value of k such that ord(18^k) = 29 (mod 349)
We are going to find that by the formula: ord(a^k) = |G|/gcd(k, |G|)

-----------------------------------
k = 2
18^k = 18^2 = 324
gcd(k, 348) = 2
ord(18^k) = ord(18^2) = 174
-----------------------------------
k = 3
18^k = 18^3 = 80
gcd(k, 348) = 3
ord(18^k) = ord(18^3) = 116
-----------------------------------
k = 4
18^k = 18^4 = 313
gcd(k, 348) = 4
ord(18^k) = ord(18^4) = 87
-----------------------------------
k = 5
18^k = 18^5 = 168
gcd(k, 348) = 1
ord(18^k) = ord(18^5) = 348
-----------------------------------
k = 6
18^k = 18^6 = 313
gcd(k, 348) = 6
ord(18^k) = ord(18^6) = 58
-----------------------------------
k = 7
18^k = 18^7 = 301
gcd(k, 348) = 1
ord(18^k) = ord(18^7) = 348
-----------------------------------
k = 8
18^k = 18^8 = 171
gcd(k, 348) = 4
ord(18^k) = ord(18^8) = 87
-----------------------------------
k = 9
18^k = 18^9 = 224
```

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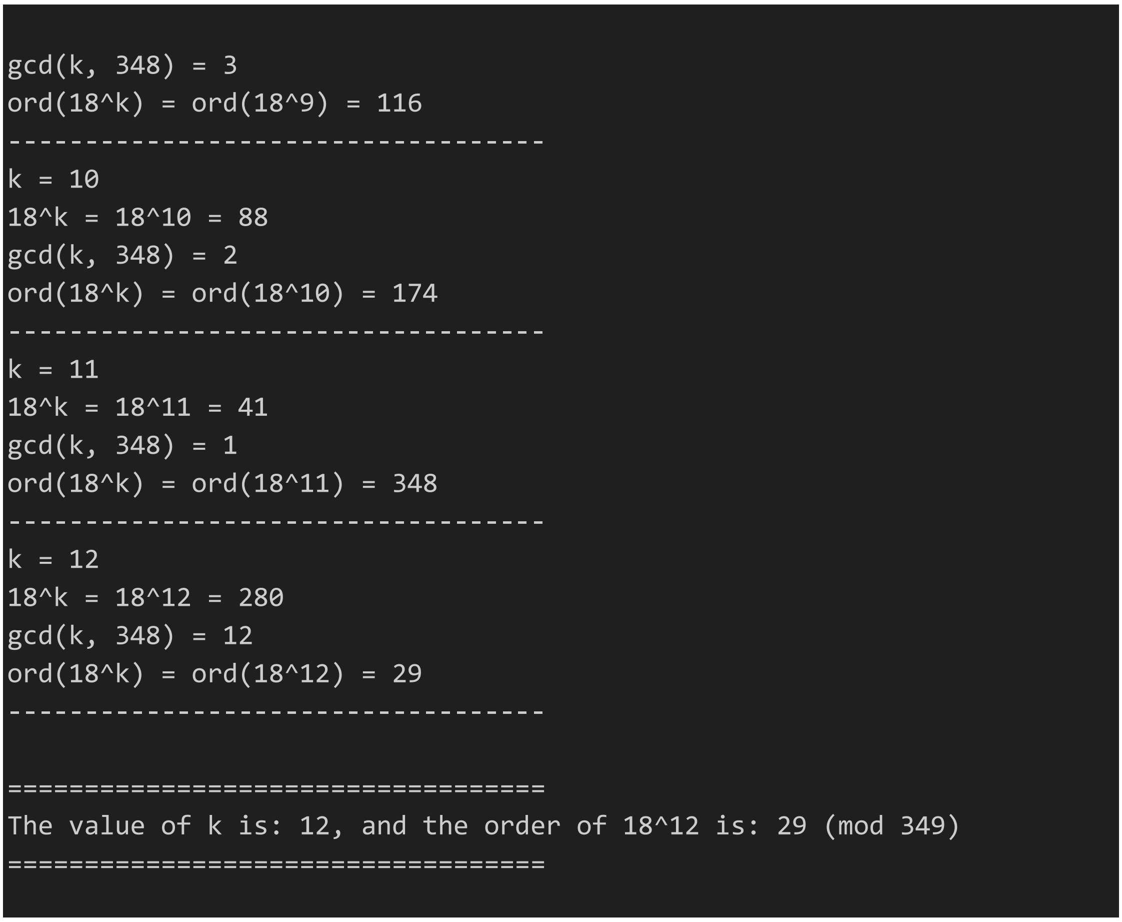

```
gcd(k, 348) = 3
ord(18^k) = ord(18^9) = 116
-----------------------------------
k = 10
18^k = 18^10 = 88
gcd(k, 348) = 2
ord(18^k) = ord(18^10) = 174
-----------------------------------
k = 11
18^k = 18^11 = 41
gcd(k, 348) = 1
ord(18^k) = ord(18^11) = 348
-----------------------------------
k = 12
18^k = 18^12 = 280
gcd(k, 348) = 12
ord(18^k) = ord(18^12) = 29
-----------------------------------

===================================
The value of k is: 12, and the order of 18^12 is: 29 (mod 349)
===================================
```

ג.

נחשב את $L_{18}(3)$, $L_{18}(11)$, $L_{18}(7)$.

$$\begin{cases} 18^{54} = 27 = 3^3 \; mod \; 349 \\ 18^{211} = 33 = 3 \times 11 \; mod \; 349 \\ 18^{284} = 77 = 7 \times 11 \; mod \; 349 \end{cases}$$

$$\Rightarrow \begin{cases} 54 = 3L_{18}(3) \; mod \; 348 \\ 211 = L_{18}(3) + L_{18}(11) \; mod \; 348 \\ 284 = L_{18}(7) + L_{18}(11) \; mod \; 348 \end{cases}$$

$$L_{18}(3) \colon 18 = L_{18}(3) \; mod \; 116$$

$$18 \; mod \; 116$$

$$18 + 116 = 134 \; mod \; 116$$

$$134 + 116 = 250 \; mod \; 116$$

$$L_{18}(3) = 18, 134, 250 \; mod \; 348$$

נבדוק איזה ערך ייתן את $L_{18}(3)$:

$$18^{18} = 17 \; mod \; 348$$

$$18^{134} = 329 \; mod \; 348$$

$$18^{250} = 3 \; mod \; 348$$

לכן, $L_{18}(3) = 250$

$$\Rightarrow \begin{cases} 250 = L_{18}(3) \; mod \; 348 \\ 211 = L_{18}(3) + L_{18}(11) \; mod \; 348 \\ 284 = L_{18}(7) + L_{18}(11) \; mod \; 348 \end{cases}$$

$$\Rightarrow \begin{cases} 250 = L_{18}(3) \; mod \; 348 \\ 309 = L_{18}(11) \; mod \; 348 \\ 284 = L_{18}(7) + L_{18}(11) \; mod \; 348 \end{cases}$$

$$\Rightarrow \begin{cases} 250 = L_{18}(3) \; mod \; 348 \\ 309 = L_{18}(11) \; mod \; 348 \\ 323 = L_{18}(7) \; mod \; 348 \end{cases}$$

לסיכום, $L_{18}(3) = 250$, $L_{18}(11) = 309$, $L_{18}(7) = 323$.

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ד.

נחשב את $L_{18}(100)$.

$$100 \times 18^3 = 21 = 3 \times 7 \; mod \; 349$$

$$\Longrightarrow L_{18}(100) + 3 \equiv L_{18}(3) + L_{18}(7) \; mod \; 348$$

$$\Longrightarrow L_{18}(100) + 3 \equiv 250 + 323 \; mod \; 348$$

$$\Longrightarrow L_{18}(100) + 3 \equiv 225 \; mod \; 348$$

$$\Longrightarrow L_{18}(100) \equiv 222 \; mod \; 348$$

$L_{18}(100) \equiv 222 \;\; \Longleftarrow$

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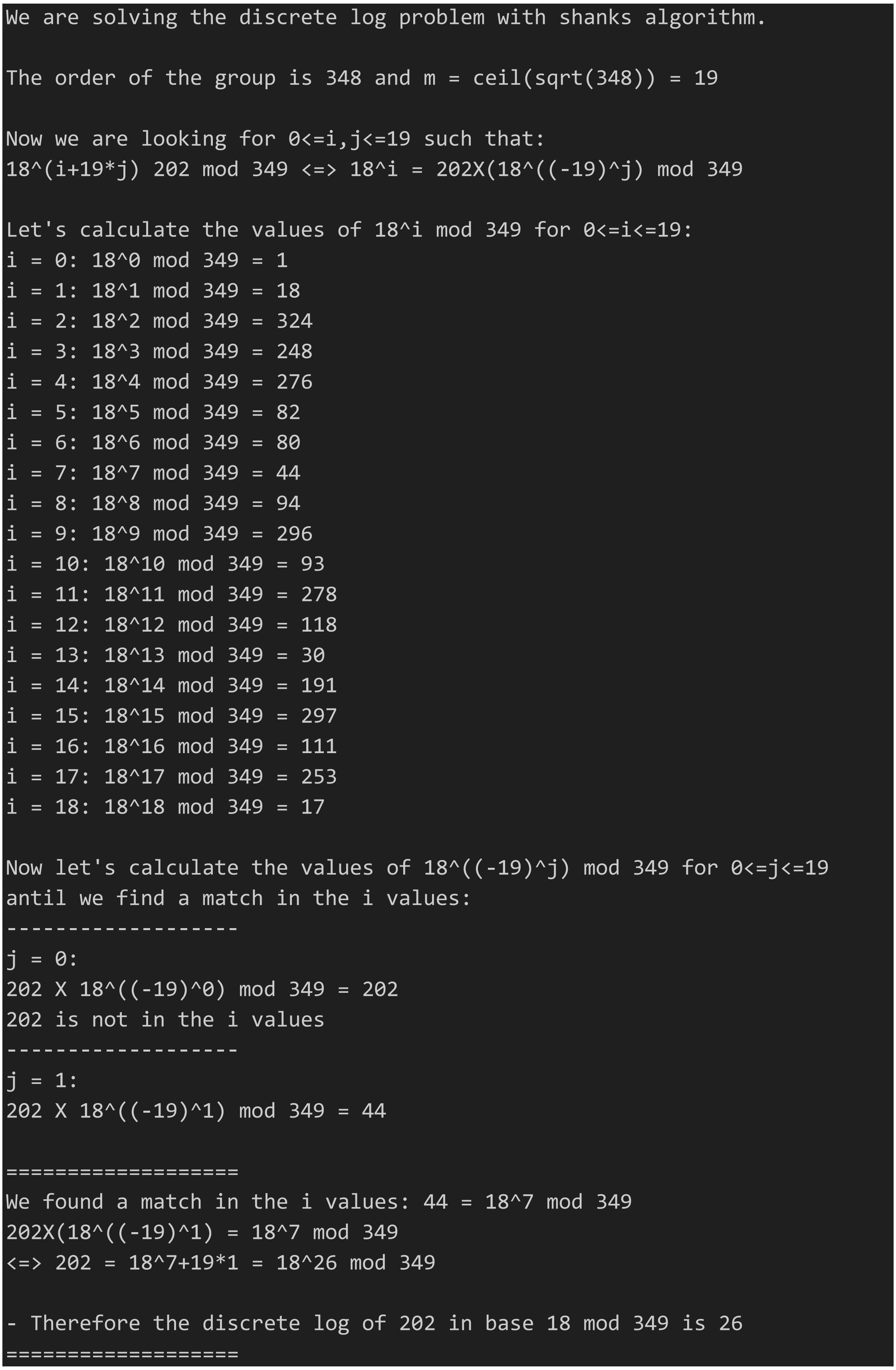
3.



```
We are solving the discrete log problem with shanks algorithm.

The order of the group is 348 and m = ceil(sqrt(348)) = 19

Now we are looking for 0<=i,j<=19 such that:
18^(i+19*j) 202 mod 349 <=> 18^i = 202X(18^((-19)^j) mod 349

Let's calculate the values of 18^i mod 349 for 0<=i<=19:
i = 0: 18^0 mod 349 = 1
i = 1: 18^1 mod 349 = 18
i = 2: 18^2 mod 349 = 324
i = 3: 18^3 mod 349 = 248
i = 4: 18^4 mod 349 = 276
i = 5: 18^5 mod 349 = 82
i = 6: 18^6 mod 349 = 80
i = 7: 18^7 mod 349 = 44
i = 8: 18^8 mod 349 = 94
i = 9: 18^9 mod 349 = 296
i = 10: 18^10 mod 349 = 93
i = 11: 18^11 mod 349 = 278
i = 12: 18^12 mod 349 = 118
i = 13: 18^13 mod 349 = 30
i = 14: 18^14 mod 349 = 191
i = 15: 18^15 mod 349 = 297
i = 16: 18^16 mod 349 = 111
i = 17: 18^17 mod 349 = 253
i = 18: 18^18 mod 349 = 17

Now let's calculate the values of 18^((-19)^j) mod 349 for 0<=j<=19
antil we find a match in the i values:
-------------------
j = 0:
202 X 18^((-19)^0) mod 349 = 202
202 is not in the i values
-------------------
j = 1:
202 X 18^((-19)^1) mod 349 = 44

===================
We found a match in the i values: 44 = 18^7 mod 349
202X(18^((-19)^1) = 18^7 mod 349
<=> 202 = 18^7+19*1 = 18^26 mod 349

- Therefore the discrete log of 202 in base 18 mod 349 is 26
===================
```

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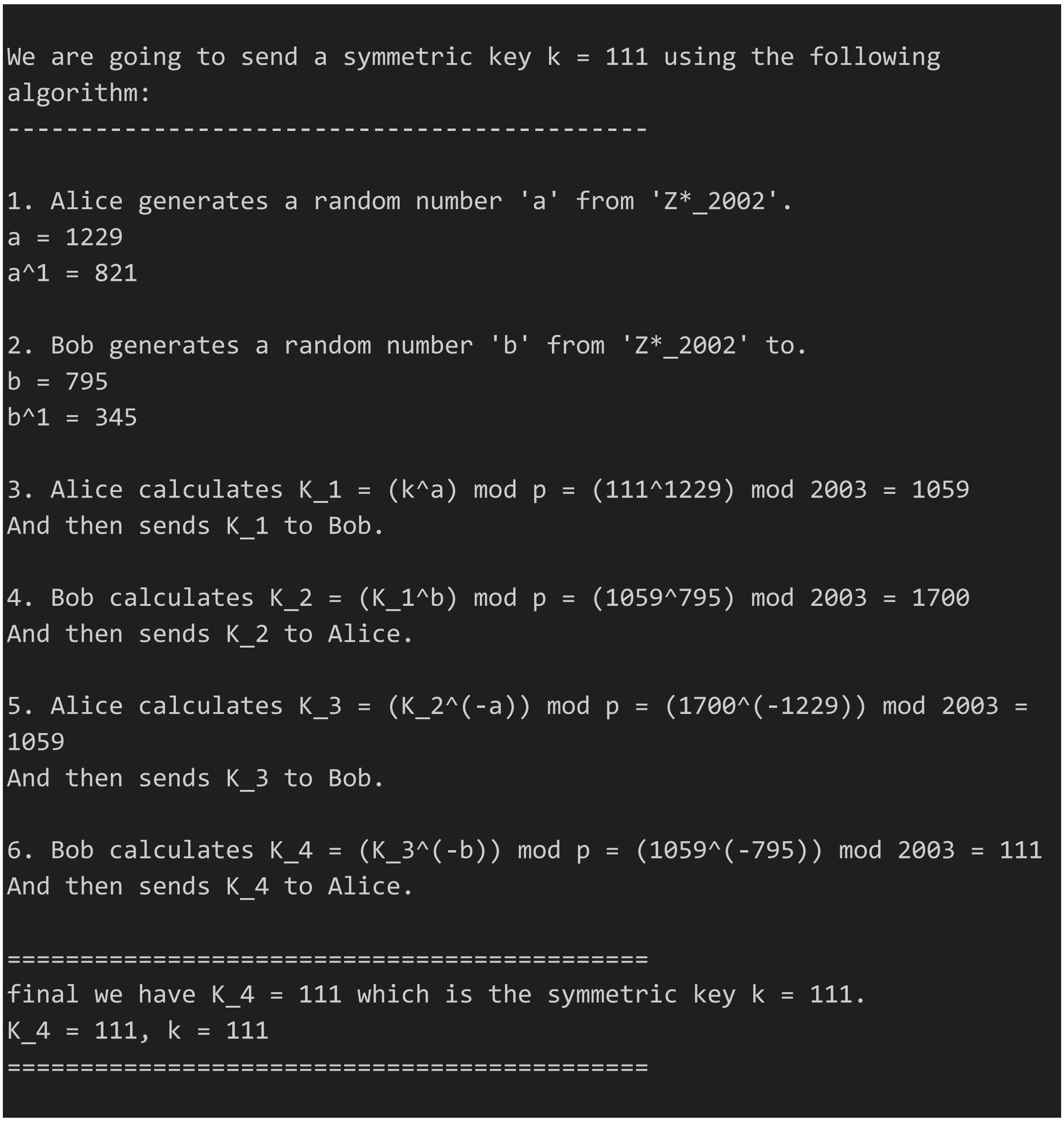
4.

א.



```
We are going to send a symmetric key k = 111 using the following
algorithm:
--------------------------------------------

1. Alice generates a random number 'a' from 'Z*_2002'.
a = 1229
a^1 = 821

2. Bob generates a random number 'b' from 'Z*_2002' to.
b = 795
b^1 = 345

3. Alice calculates K_1 = (k^a) mod p = (111^1229) mod 2003 = 1059
And then sends K_1 to Bob.

4. Bob calculates K_2 = (K_1^b) mod p = (1059^795) mod 2003 = 1700
And then sends K_2 to Alice.

5. Alice calculates K_3 = (K_2^(-a)) mod p = (1700^(-1229)) mod 2003 =
1059
And then sends K_3 to Bob.

6. Bob calculates K_4 = (K_3^(-b)) mod p = (1059^(-795)) mod 2003 = 111
And then sends K_4 to Alice.

============================================
final we have K_4 = 111 which is the symmetric key k = 111.
K_4 = 111, k = 111
============================================
```

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ב.

נציג מתקפה מסוג "man in the middle" עבור הפרוטוקול הזה, שהתוצאה של המתקפה היא שאליס חושבת שהיא שולחת את K לבוב אבל בסוף ההתקפה התוקף מלורי מקבל את K ובוב מקבל בסוף מפתח K' שנקבע על ידי מלורי.

ההתקפה:
אליס שולחת לבוב את $K_1 = K^a \ mod \ p$.

מלורי שנמצאת באמצע בוחרת $C \in \mathbb{Z}^*{}_{p-1}$ הופכי, ומוסיפה ללא ידיעת אליס ובוב את $K_1{}' = K_1{}^c = K^{ac} \ mod \ p$ ושולחת את $K_1{}'$ לבוב, ללא ידיעת אליס ובוב.

בוב מחשב את $K_2{}' = (K_1{}')^b = K^{abc}$ למרות שהוא ואליס חושבים שהוא מחשב את: $K_2 = K_1{}^b = K^{ab}$.

לאחר מכן אליס מחשבת את: $K_3{}' = (K_2{}')^{-a} = K^{bc}$.
ובוב מחשב את: $K' = K_4{}' = (K_3{}')^{-b} = K^c$.

כעת לבוב יש את: $K' = K^c$.

מלורי מחשבת כעת את: $K = K'^{-c} = (K_4{}')^{-c} = K$.

ולסיכום: לבוב יש את בסוף האלגוריתם את: $K' = K^c$.
ולמלורי יש בסוף האלגוריתם את: K.

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5.

בהצפנת אל גמאל בוחרים $1 < k < p - 1$ אקראי.
הצפנה של הודעה x היא $(\alpha^k \; mod \; p, x\beta^k \; mod \; p)$.
בשתי ההודעות המוצפנות של בוב יש את אותו רכיב ראשון, לכן אנו יודעים כי בוב השתמש באותו רכיב k עבור שתי ההודעות.
נסמן ב- x_1, x_2 את שתי ההודעות לפי הנתון, $x_1 = 222 \; mod \; 349$.

לכן,

$$97 = 222 \times \beta^k \; mod \; 349$$
$$\Longrightarrow \beta^k = 97 \times 222^{-1} \; mod \; 349$$

לפי הנתון:

$$114 = x_2\beta^k = x_2 \times 97 \times 222^{-1} \; mod \; 349$$

ולכן,

$$x_2 = 114 \times 222 \times 97^{-1} \; mod \; 349$$
$$\Longrightarrow x_2 = 114 \times 222 \times 18 \; mod \; 349$$

$$\Longrightarrow x_2 = 99 \; mod \; 349$$

<u>לסיכום</u>: הפענוח של ההודעה השנייה היא –

$$x_2 = 99 \; mod \; 349$$
