

מבוא להצפנה – תרגיל 4

.1

.א.

In this chapter we calculate the private key d using the extended Euclidean algorithm.

```
i = 0, r = 33,      s = 0, t = 1
i = 1, r = 17, q = 1, s = 1, t = 0
i = 2, r = 16, q = 1, s = -1, t = 1
i = 3, r = 1, q = 16, s = 2, t = -1
```

we got that $1 = 17 \cdot (2) + 33 \cdot (-1)$

So:

The value of s is 2

The value of t is -1

Now we calculate:

$C_a^s \cdot C_b^t = m^{(se_a)} \cdot m^{(te_b)} = m^{(se_a + te_b)} = m \pmod{16157}$

Calculate 11671^{-1} :

First we need to calculate the inverse of 11671: $11671^{-1} = 11671^{-1} \pmod{16157}$

Now we calculate it using the extended Euclidean algorithm:

```
i = 0, r = 16157,      s = 0, t = 1
i = 1, r = 11671, q = 1, s = 1, t = 0
i = 2, r = 4486, q = 2, s = -1, t = 1
i = 3, r = 2699, q = 1, s = 3, t = -2
i = 4, r = 1787, q = 1, s = -4, t = 3
i = 5, r = 912, q = 1, s = 7, t = -5
i = 6, r = 875, q = 1, s = -11, t = 8
i = 7, r = 37, q = 23, s = 18, t = -13
i = 8, r = 24, q = 1, s = -425, t = 307
i = 9, r = 13, q = 1, s = 443, t = -320
i = 10, r = 11, q = 1, s = -868, t = 627
i = 11, r = 2, q = 5, s = 1311, t = -947
i = 12, r = 1, q = 2, s = -7423, t = 5362
```

we got that $1 = 11671 \cdot (-7423) + 16157 \cdot (5362)$

So:

The value of s is -7423

The value of t is 5362

```
The inverse of 11671 is -7423 (mod 16157)
11671-1 = -7423 = 8734 (mod 16157)
Now we calculate 11671-1 = 87341 (mod 16157):
using the square and multiply algorithm:
1 in binary is [1]
-----
i = 0
e_i = 1
z2 = 1 (mod 16157)
z*8734 = 8734*8734 = 8734 (mod 16157)
-----
And we got that 11671-1 = 8734 (mod 16157)

=====
Now we calculate:
72242 = (mod 16157)

2 in binary is [1, 0]
-----
i = 0
e_i = 1
z2 = 1 (mod 16157)
z*7224 = 7224*7224 = 7224 (mod 16157)
-----
i = 1
e_i = 0
z2 = 12 = 15223 (mod 16157)
-----
And we got that 72242 = 15223 (mod 16157)

=====
The message is: 15223X8734 = 1729 (mod 16157)
=====
```

ב.

In this chapter we calculate the private key d using the extended Euclidean algorithm.

```
i = 0, r = 33,      s = 0, t = 1
i = 1, r = 17, q = 1, s = 1, t = 0
i = 2, r = 16, q = 1, s = -1, t = 1
i = 3, r = 1, q = 16, s = 2, t = -1
```

we got that $1 = 17 \cdot (2) + 33 \cdot (-1)$

So:

The value of s is 2

The value of t is -1

Now we calculate:

$$C_a^s \cdot C_b^t = m^{(se_a)} \cdot m^{(te_b)} = m^{(se_a + te_b)} = m \pmod{16157}$$

Calculate 11449^{-1} :

First we need to calculate the inverse of 11449: $11449^{-1} = 11449^{-1} \pmod{16157}$

Now we calculate it using the extended Euclidean algorithm:

```
i = 0, r = 16157,      s = 0, t = 1
i = 1, r = 11449, q = 1, s = 1, t = 0
i = 2, r = 4708, q = 2, s = -1, t = 1
i = 3, r = 2033, q = 2, s = 3, t = -2
i = 4, r = 642, q = 3, s = -7, t = 5
i = 5, r = 107, q = 6, s = 24, t = -17
```

we got that $107 = 11449 \cdot (24) + 16157 \cdot (-17)$

So:

The value of s is 24

The value of t is -17

The inverse of 11449 is 24 $\pmod{16157}$

$$11449^{-1} = 24 = 24 \pmod{16157}$$

Now we calculate $11449^{-1} = 24^1 \pmod{16157}$:

using the square and multiply algorithm:

1 in binary is [1]

$i = 0$

$e_i = 1$

$$z^2 = 1 \pmod{16157}$$

$$z^{*24} = 24^{*24} = 24 \pmod{16157}$$

```
-----  
And we got that  $11449^{-1} = 24 \pmod{16157}$ 
```

```
=====
```

Now we calculate:

$13910^2 \pmod{16157}$

2 in binary is [1, 0]

```
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```

$i = 0$

$e_i = 1$

$z^2 = 1 \pmod{16157}$

$z \cdot 13910 = 13910 \cdot 13910 = 13910 \pmod{16157}$

```
-----
```

$i = 1$

$e_i = 0$

$z^2 = 1^2 = 8025 \pmod{16157}$

```
-----
```

And we got that $13910^2 = 8025 \pmod{16157}$

```
=====
```

The message is: $8025 \cdot 24 = 14873 \pmod{16157}$

```
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```

