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WAC OF Statistical Accounting & Management

ROLL NO.

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What we want to know?

We want to estimate the future demand for company's products in UK by forecasting overall sales of household goods in UK since we know that both are closely related. The estimation of demand is important so that the company can decide if it is worth entering new market or not.

Statistical tool to be used: Time series will be used because there is no other independent variable related to overall sales is available to run the regression analysis, except time. Time will act as proxy to cover other unknown variables.

Since seasonal patterns exist on initial look at data *(Figure 1, Page 9)*, using multiplicative or additive time series decomposition models would be suitable to first remove the effect of seasonality from data. Then regression analysis can be run on the trend estimate.

Experiment for getting average household sales

What

Average household Sales

Who

Basic EOI: 1 month

How

Pick a month in database and check household sales

Once the data has been obtained, models can be developed.

I. Multiplicative model

Seasonal effects are removed first. Time series is deseasonalized by dividing each observation by its corresponding seasonal index. If appropriate allowance can be made for seasonal, trend and cyclic effects, then probabilistic forecast might be made. Then the seasonal allowance is made in the end. The multiplicative model is

Y = T*C*S*I

Where

T = Trend estimate

C = Cyclical index

S = Seasonality index (Cyclical effect but for shorter period, less than a year)

I = Irregular effect

Since we are interested in estimating the forecast for only next year, cyclical effect will be ignored since it is long term.

Updated multiplicative model

Y = T*S*I   equation 1

1. Moving average

A simple plot of sales against time showed that data has recurring pattern in sales within each year, patten of sales visible in each month.

So, a **12-month moving average** will be calculated. Moving average cancels out the effect of irregularity and seasonality. Since the average would contain 12 months, the differences in monthly sales pattern would tend to cancel out and not reflect in our sales. Unexplained variations in each month would tend to balance out over 12 months.

The first average would lie between 6 and 7 months, because 12 is an even number. The calculation of moving average illustrated in Exhibit 2 (PAGE 10).

2. Central Moving Average

Since the moving average corresponds to midway between 6 and 7 months, and so on. Central moving average is calculated for our average to correspond to the exact month.

For instance: since the first moving average corresponds to the point between 6 and 7 months, and second moving average corresponds to point between 7 and 8 months, the average of these two moving averages is located at exact 7^{th} month observation.

The CMA calculations have been illustrated in exhibit 3 (Page 11).

As explained earlier how the central moving average smoothen out the effect of seasonal and irregular effects, CMA now represents only trend estimate and random variation.

3. Finding seasonal-irregular value

Since CMA = T in our equation 1, the effect of only seasonal-irregular factor can be calculated by dividing Y (that in this case is sales), by T. We are left with only seasonal-irregular effect.

$$\frac{Y}{T} = S * I$$

The reason for this exercise is by knowing the effect of seasonal-irregular effect, a permanent seasonal index can be derived. The calculation can be observed in exhibit 4 (Page 12).

4. Calculating Seasonal Index

To remove the irregular effect, the seasonal-irregular effect of each month is averaged out to obtain a value that shows only seasonal effect.

For instance, seasonal index of January would be calculated by averaging the seasonal-irregular effect values. Have a look at the exhibit 5 (PAGE 13) for calculations.

Months	Si	Interpretation
Jan	0.96	The average retail sales of household goods in Jan will be 4% lower than trend estimate due to seasonality and irregular effects
Feb	0.87	The average household sales in Feb will be 13% lower than trend estimate due to seasonality and irregular effects
Mar	1.10	The average household sales in March will be 10% higher than trend estimate due to seasonality and irregular effects
Apr	0.91	The average household sales in April will be 9% lower than trend estimate due to seasonality and irregular effects
May	0.89	The average household sales in May will be 11% lower than trend estimate due to seasonality and irregular effects

Jun	1.07	The average household sales in June will be 7% higher than trend estimate due to seasonality and irregular effects
Jul	0.88	The average household sales in July will be 12% lower than trend estimate due to seasonality and irregular effects
Aug	0.87	The average household sales in Aug will be 13% lower than trend estimate due to seasonality and irregular effects
Sep	1.10	The average household sales in Sep will be 10% higher than trend estimate due to seasonality and irregular effects
Oct	0.93	The average household sales in Oct will be 7% lower than trend estimate due to seasonality and irregular effects
Nov	1.00	The average household sales in Nov will be equal to trend estimate
Dec	1.44	The average household sales in Dec will be 44% higher than trend estimate due to seasonality and irregular effects

5. De-seasonalizing the Time Series

Seasonal effect is removed so that linear regression model is not affected by seasonal effects too. Time series is deseasonalized by dividing each observation by its corresponding seasonal index.

$$\frac{Y}{S} = Desaonalized\ sales$$

Exhibit 6 (Page 14) shows the calculations for deseasonalized sales.

6. Identifying the trend

Regression analysis is run on this deseasonalized data to estimate the trend (which we had observed in earlier figure 1 in the start too).

The estimated regression equation for trend will be

$$T_t = b_0 + b_1 t$$

Where T = linear trend forecast, b_0 is intercept, b_1 is slope of trend line and t is time period.

Time is independent variable. Its future data is available (we know the next month) and past data is available too. The regression run can be observed in exhibit 7 (Page 15). The equation comes out to be

T = 2437.184+2.32t

As we can see that F value of model is less than 0.05. So, model is significant. Furthermore, p value of independent variable t is less than 0.05, so it is significant too.

Interpretation: Every month, sales will be 2.32 times higher than past month sales, due to trend.

Since we are assuming that past is reasonably good indicator of future, trend projections for next year is developed, by simply putting the values for next periods starting from 181…

7. Developing a forecast

If assumed that past patterns will continue in the future, future values can be predicted by projecting individual parts and then combining the projections.

Trend component calculated in previous step is multiplied by seasonal index to get the final forecast. This is done by simply putting values in equation 1. **Exhibit 8 (PAGE 16) for reference.**

For instance, For Jan sale. The Trend estimate 2857 is multiplied by seasonal index 0.96 to get forecasted value of 2739.

Jan	2739.489
Feb	2477.492
Mar	3141.313
Apr	2605.621
May	2538.182
Jun	3055.695
Jul	2527.364
Aug	2510.396
Sep	3152.513
Oct	2688.323
Nov	2874.774
Dec	4139.07

II. Additive model

An additive model can also be used to handle both seasonality and trend. Like in multiplicative model, time series is decomposed in three components: trend T, seasonal S and irregular effects I.

An additive decomposition takes the following form

Y = T + S + I

The seasonality factor is catered by dummy variables for categories, which in our case are months in a year. So, we make N-1 dummy variables, which come out to be 11. Jan has been taken as base year.

The equation takes the following form

$$Y = b_0 + b_1 t + b_2\, FEB + b_3\, MARCH + b_4 APRIL + b_5\, MAY + b_6\, JUNE + b_7\, JULY + b_8\, AUG + b_9\, SEP + b_{10}\, OCT + b_{11}\, NOV + b_{12}\, \text{DEC}$$

Y= forecast of sales

Feb = 1 if time period corresponds to first month of year: 0 otherwise.

Similarly, for other dummy variables of other months.

t = time period, a month.

On running the regression, results can be observed in Exhibit 9 (Page 17). R square comes out to be 0.8 showing strong relationship between our independent variables and dependent variable. F value is less than 0.05, so model is significant.

B1 = 2.261, showing trend effect.

The seasonality for each month comes out to be:

Months	Seasonality	Interpretation
Jan	0	
Feb	-239.68	Sales in Feb will be 239.68 million pounds lower than sales in Jan, keeping other variables constant
Mar	369.873	Sales in March will be 369.873 million pounds higher than sales in Jan, keeping other variables constant
Apr	-131.155	Cannot be interpreted since its p value 0.08 is greater than significance level
May	-192.434	Sales in May will be 192.434 million pound lower than sales in Jan, keeping other variables constant
Jun	293.348	Sales in June will be 293.34 million pounds higher than sales in Jan, keeping other variables constant
Jul	-201.7	Sales in July will be 201.7 million pounds lower than sales in Jan, keeping other variables constant

Aug	-222.608	Sales in Aug will be 239.68 million pounds lower than sales in Jan, keeping other variables constant
Sep	371.178	Sales in September will be 371.178 million pounds higher than sales in Jan, keeping other variables constant
Oct	-58.443	Cannot be interpreted since its p value 0.445 is greater than 0.05 significance level
Nov	103.016	Cannot be interpreted since its p value 0.179 is greater than 0.05 significance level
Dec	1256.346	Sales in Feb will be 1256.34 million pounds higher than sales in Jan, keeping other variables constant

Putting the seasonality in the equation earlier, the equation becomes

Sales forecast Y = 2330.95 + 2.261t - 239.68*Feb + 369.873*March - 131.155*April - 192.434*May + 293.348*June - 201.7*July - 222.608*Aug + 371.178*Sept - 58.443*Oct + 103.016*Nov + 1256.346*Dec

Putting in 1 for the month we want the forecast, the next year forecast follows, as observed in exhibit 10 (Page 18).

Months	Sales forecast		Months	Sales forecast
Jan	2740.192		**Jul**	2552.058
Feb	2502.773		**Aug**	2533.411
Mar	3114.587		**Sep**	3129.458
Apr	2615.82		**Oct**	2702.098
May	2556.802		**Nov**	2865.818
Jun	3044.845		**Dec**	4021.409

III. Which method is better?

Additive model is better because. To compare we used Mean squared error. It is calculated by calculating the forecast error. Then squaring the forecast error by summing and dividing it by number of values. MSE for multiplicative model is 41688.28. (exhibit 11, Page 19) MSE for multiplicative model comes out to be 40970.93. (exhibit 12, Page 20)

The possible reason for additive model giving better result is that seasonal fluctuations in earlier periods are about the same as the fluctuations in later periods.

IV. Develop a logistic regression model to predict chances of the month to be December

What: Predict probability of month to be December based on amount of monthly household sales

Dependent variable is YES if month is December and No if some other month. So, we convert into binary 1 for December and 0 for other months.

Modeling

Logit g = b0 +b1*x

Where logit g is natural log of odds in favor of y =1

Where x is sales

Solving equation, probability comes to be

$P(y = 1) = \frac{e^g}{1+e^g}$

Running the logistic regression, exhibit 13 (Page 21), the chi-squared p value is less than 0.05, showing model is significant. The p value for sales is not significant at 0.05 significance level, but significant at 0.85 level.

$$P(y = 1) = \frac{e^{-105.46+0.031x}}{1 + e^{-105.46+0.031x}}$$

Odd ratio

The odd ratio comes out to be 1.032. For a million-pound increase in overall household sales, the estimated odds of the month being Friday is 1.032 times higher.

Amount of retail sale for fixed probability of month being December

- By hit and trial, the amount of sales on 50% p comes out to be 3402. **Interpretation:** For a month having 3402 million pounds of sales, the probability of it being December is 50%
- By hit and trial, the amount of sales on 90% p comes out to be 3473. **Interpretation**: For a month having 3473 million pounds of sales, the probability of it being December is 90%

Appendix

Figure 1 Plot of average household sales over the years

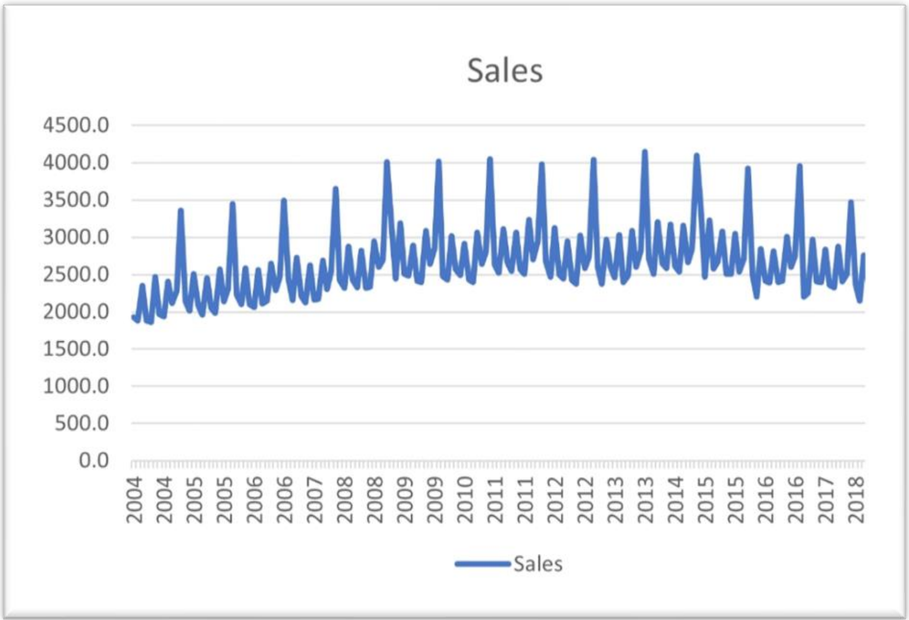

Figure 2 Moving average

r #	Year	Month	Sales	MA	CMA
1	2004	**Jan**	1931.3		
2	2004	**Feb**	1884.0		
3	2004	**Mar**	2351.0		
4	2004	**Apr**	1881.6		
5	2004	**May**	1862.0		
6	2004	**Jun**	2466.1	=AVERAGE(D2:D13)	
7	2004	**Jul**	1972.2	**AVERAGE**(number1, [	
8	2004	**Aug**	1934.3	2232.4	2227.1
9	2004	**Sep**	2408.2	2245.7	2239.0
10	2004	**Oct**	2117.9	2263.0	2254.3
11	2004	**Nov**	2281.2	2271.5	2267.2
12	2004	**Dec**	3360.9	2270.5	2271.0
13	2005	**Jan**	2141.1	2277.4	2273.9
14	2005	**Feb**	2013.0	2281.8	2279.6
15	2005	**Mar**	2509.5	2295.3	2288.5
16	2005	**Apr**	2089.1	2297.5	2296.4
17	2005	**May**	1964.1	2299.2	2298.4
18	2005	**Jun**	2453.9	2306.5	2302.9
19	2005	**Jul**	2055.8	2314.0	2310.2
20	2005	**Aug**	1987.1	2321.6	2317.8
21	2005	**Sep**	2569.4	2328.0	2324.8
22	2005	**Oct**	2144.6	2328.8	2328.4
23	2005	**Nov**	2302.1	2337.1	2332.9
24	2005	**Dec**	3448.4	2346.0	2341.6
25	2006	**Jan**	2230.3	2350.6	2348.3
26	2006	**Feb**	2104.3	2364.0	2357.3
27	2006	**Mar**	2586.7	2370.5	2367.2

Figure 3 Central Moving Average

Sr #	Year	Month	Sales	MA	CMA
1	2004	**Jan**	1931.3		
2	2004	**Feb**	1884.0		
3	2004	**Mar**	2351.0		
4	2004	**Apr**	1881.6		
5	2004	**May**	1862.0		
6	2004	**Jun**	2466.1	2204.2	
7	2004	**Jul**	1972.2	2221.7	=AVERAGE(E7:E8)
8	2004	**Aug**	1934.3	2232.4	**AVERAGE**(number1
9	2004	**Sep**	2408.2	2245.7	2239.0
10	2004	**Oct**	2117.9	2263.0	2254.3
11	2004	**Nov**	2281.2	2271.5	2267.2
12	2004	**Dec**	3360.9	2270.5	2271.0
13	2005	**Jan**	2141.1	2277.4	2273.9
14	2005	**Feb**	2013.0	2281.8	2279.6
15	2005	**Mar**	2509.5	2295.3	2288.5
16	2005	**Apr**	2089.1	2297.5	2296.4
17	2005	**May**	1964.1	2299.2	2298.4
18	2005	**Jun**	2453.9	2306.5	2302.9
19	2005	**Jul**	2055.8	2314.0	2310.2
20	2005	**Aug**	1987.1	2321.6	2317.8
21	2005	**Sep**	2569.4	2328.0	2324.8
22	2005	**Oct**	2144.6	2328.8	2328.4
23	2005	**Nov**	2302.1	2337.1	2332.9
24	2005	**Dec**	3448.4	2346.0	2341.6
25	2006	**Jan**	2230.3	2350.6	2348.3
26	2006	**Feb**	2104.3	2364.0	2357.3
27	2006	**Mar**	2586.7	2370.5	2367.2
28	2006	**Apr**	2098.6	2382.3	2376.4

Figure 4 Seasonal Irregular effect

Sr #	Year	Month	Sales	MA	CMA	Seasonal-irregular
1	2004	**Jan**	1931.3			
2	2004	**Feb**	1884.0			
3	2004	**Mar**	2351.0			
4	2004	**Apr**	1881.6			
5	2004	**May**	1862.0			
6	2004	**Jun**	2466.1	2204.2		
7	2004	**Jul**	1972.2	2221.7	2212.9	=D8/F8
8	2004	**Aug**	1934.3	2232.4	2227.1	0.868523
9	2004	**Sep**	2408.2	2245.7	2239.0	1.075539
10	2004	**Oct**	2117.9	2263.0	2254.3	0.939504
11	2004	**Nov**	2281.2	2271.5	2267.2	1.006159
12	2004	**Dec**	3360.9	2270.5	2271.0	1.479941
13	2005	**Jan**	2141.1	2277.4	2273.9	0.941573
14	2005	**Feb**	2013.0	2281.8	2279.6	0.883051
15	2005	**Mar**	2509.5	2295.3	2288.5	1.096567
16	2005	**Apr**	2089.1	2297.5	2296.4	0.909759
17	2005	**May**	1964.1	2299.2	2298.4	0.854578
18	2005	**Jun**	2453.9	2306.5	2302.9	1.065588
19	2005	**Jul**	2055.8	2314.0	2310.2	0.889886
20	2005	**Aug**	1987.1	2321.6	2317.8	0.857329
21	2005	**Sep**	2569.4	2328.0	2324.8	1.105212
22	2005	**Oct**	2144.6	2328.8	2328.4	0.921082
23	2005	**Nov**	2302.1	2337.1	2332.9	0.986788
24	2005	**Dec**	3448.4	2346.0	2341.6	1.472664
25	2006	**Jan**	2230.3	2350.6	2348.3	0.949726
26	2006	**Feb**	2104.3	2364.0	2357.3	0.89267
27	2006	**Mar**	2586.7	2370.5	2367.2	1.092728
28	2006	**Apr**	2098.6	2382.3	2376.4	0.883111

Figure 5 Seasonal indexes

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Si
Jan		0.941573	0.949726	0.988871	0.943643	1.122689	0.891915	0.928421	0.94206	0.939599	0.941782	1.156176	0.924251	0.822941	0.930033	=AVERAGE(E218:R218)
Feb		0.883051	0.89267	0.876412	0.899209	0.871126	0.874937	0.887856	0.874366	0.856621	0.870042	0.840487	0.819046	0.83943	0.844772	AVERAGE(number1, [number2], ...
Mar		1.096567	1.092728	1.108585	1.105949	1.133672	1.08717	1.089566	1.112604	1.068805	1.110107	1.101192	1.059467	1.113622	1.087652	**1.097692**
Apr		0.909759	0.883111	0.898339	0.926083	0.887724	0.926528	0.930003	0.895174	0.935093	0.914377	0.882678	0.901437	0.905787	0.940598	**0.909764**
May		0.854578	0.863924	0.861353	0.882524	0.877218	0.898353	0.885203	0.878912	0.884031	0.891469	0.917904	0.891277	0.910222	0.900028	**0.8855**
Jun		1.065588	1.0679	1.059102	1.061218	1.017383	1.049952	1.06567	1.062335	1.085697	1.095274	1.063243	1.045758	1.089115	1.084326	**1.065183**
Jul	0.891188	0.889886	0.876691	0.873027	0.858078	0.858096	0.872912	0.893417	0.872796	0.856395	0.894129	0.880272	0.89551	0.91181		**0.880301**
Aug	0.868523	0.857329	0.888062	0.871499	0.851523	0.861341	0.860888	0.873384	0.857191	0.883505	0.860551	0.894571	0.904461	0.898752		**0.873684**
Sep	1.075539	1.105212	1.091067	1.075298	1.069185	1.112878	1.095374	1.126573	1.096739	1.09207	1.069141	1.098488	1.125162	1.115097		**1.096273**
Oct	0.939504	0.921082	0.938232	0.916123	0.936985	0.953662	0.940792	0.942705	0.938239	0.915278	0.904349	0.924487	0.971134	0.934816		**0.934099**
Nov	1.006159	0.986788	1.010885	1.002986	0.967151	1.029809	0.998093	1.031081	0.990407	0.990571	0.965492	0.991027	1.025771	0.976899		**0.99808**
Dec	1.479941	1.472664	1.425156	1.431162	1.433657	1.446573	1.434574	1.397179	1.461223	1.450119	1.389765	1.447977	1.474568	1.357603		**1.435869**

Figure 6 Deseasonalizing sales

Sr #	Year	Month	Sales	MA	CMA	Seasonal-ir	Desonalized sales
1	2004	**Jan**	1931.3				=D2/$218
2	2004	**Feb**	1884.0				2174.417
3	2004	**Mar**	2351.0				2141.73
4	2004	**Apr**	1881.6				2068.194
5	2004	**May**	1862.0				2102.803
6	2004	**Jun**	2466.1	2204.2			2315.146
7	2004	**Jul**	1972.2	2221.7	2212.9	0.891188	2240.317
8	2004	**Aug**	1934.3	2232.4	2227.1	0.868523	2213.907
9	2004	**Sep**	2408.2	2245.7	2239.0	1.075539	2196.702
10	2004	**Oct**	2117.9	2263.0	2254.3	0.939504	2267.348
11	2004	**Nov**	2281.2	2271.5	2267.2	1.006159	2285.56
12	2004	**Dec**	3360.9	2270.5	2271.0	1.479941	2340.66
13	2005	**Jan**	2141.1	2277.4	2273.9	0.941573	2233.002
14	2005	**Feb**	2013.0	2281.8	2279.6	0.883051	2323.355
15	2005	**Mar**	2509.5	2295.3	2288.5	1.096567	2286.197
16	2005	**Apr**	2089.1	2297.5	2296.4	0.909759	2296.359
17	2005	**May**	1964.1	2299.2	2298.4	0.854578	2218.1
18	2005	**Jun**	2453.9	2306.5	2302.9	1.065588	2303.751
19	2005	**Jul**	2055.8	2314.0	2310.2	0.889886	2335.393
20	2005	**Aug**	1987.1	2321.6	2317.8	0.857329	2274.368
21	2005	**Sep**	2569.4	2328.0	2324.8	1.105212	2343.732
22	2005	**Oct**	2144.6	2328.8	2328.4	0.921082	2295.94
23	2005	**Nov**	2302.1	2337.1	2332.9	0.986788	2306.554
24	2005	**Dec**	3448.4	2346.0	2341.6	1.472664	2401.577
25	2006	**Jan**	2230.3	2350.6	2348.3	0.949726	2326.022
26	2006	**Feb**	2104.3	2364.0	2357.3	0.89267	2428.704
27	2006	**Mar**	2586.7	2370.5	2367.2	1.092728	2356.53
28	2006	**Apr**	2098.6	2382.3	2376.4	0.883111	2306.75

Figure 7 Regression analysis

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.514529
R Square	0.26474
Adjusted R Square	0.260609
Standard Error	202.1088
Observations	180

ANOVA

	df	*SS*	*MS*	*F*	*ignificance F*
Regression	1	2617992	2617992	64.09113	1.48E-13
Residual	178	7270937	40847.96		
Total	179	9888929			

	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	2437.184	30.25457	80.55588	5.5E-142	2377.48	2496.888	2377.48	2496.888
Sr #	2.320987	0.289917	8.005694	1.48E-13	1.74887	2.893104	1.74887	2.893104

Figure 8 Developing final forecast

180	2018	**Dec**	3437.2
181	2019	**Jan**	=(2437.184+2.32*Sheet1!A182)*S218
182	2019	**Feb**	2477.492
183	2019	**Mar**	3141.313
184	2019	**Apr**	2605.621
185	2019	**May**	2538.182
186	2019	**Jun**	3055.695
187	2019	**Jul**	2527.364
188	2019	**Aug**	2510.396
189	2019	**Sep**	3152.513
190	2019	**Oct**	2688.323
191	2019	**Nov**	2874.774
192	2019	**Dec**	4139.07

Figure 9 Additive Time series Regression

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.905[a]	.819	.806	208.9728

a. Predictors: (Constant), Dec, Sr #, Jun, July, May, Aug, Apr, Sep, Mar, Oct, Feb, Nov

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	33074043.00	12	2756170.250	63.114	.000[b]
	Residual	7292825.974	167	43669.617		
	Total	40366868.97	179			

a. Dependent Variable: Sales

b. Predictors: (Constant), Dec, Sr #, Jun, July, May, Aug, Apr, Sep, Mar, Oct, Feb, Nov

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2330.951	59.694		39.048	.000
	Sr #	2.261	.300	.248	7.525	.000
	Feb	-239.608	76.307	-.140	-3.140	.002
	Mar	369.873	76.308	.216	4.847	.000
	Apr	-131.155	76.311	-.077	-1.719	.088
	May	-192.434	76.316	-.112	-2.522	.013
	Jun	293.348	76.321	.171	3.844	.000
	July	-201.699	76.327	-.118	-2.643	.009
	Aug	-222.608	76.335	-.130	-2.916	.004
	Sep	371.178	76.344	.217	4.862	.000
	Oct	-58.443	76.354	-.034	-.765	.445
	Nov	103.016	76.365	.060	1.349	.179
	Dec	1256.346	76.378	.733	16.449	.000

a. Dependent Variable: Sales

Figure 10 Sales forecast additive model

S#	Coeffecients			Sales forecast
181	0		Jan	=2330.951 +2.261*J2+K2
182	-239.68		Feb	2502.773
183	369.873		Mar	3114.587
184	-131.155		Apr	2615.82
185	-192.434		May	2556.802
186	293.348		Jun	3044.845
187	-201.7		Jul	2552.058
188	-222.608		Aug	2533.411
189	371.178		Sep	3129.458
190	-58.443		Oct	2702.098
191	103.016		Nov	2865.818
192	1256.346		Dec	4021.409

Figure 11 MSE multiplicative

X	Y	Z	AA
2815.344	2478.349	-114.9	13205.87401727
2817.664	2461.749	-129.8	16846.48104646
2819.984	3091.473	-212.8	45304.09679002
2822.304	2636.312	-232.3	53964.74118164
2824.624	2819.201	-312.7	97792.34598415
2826.944	4059.121	-588.0	345698.91987541
2829.264	2712.795	-341.7	116763.89439256
2831.584	2453.37	-306.7	94058.40452484
2833.904	3110.754	-354.6	125737.37706313
2836.224	2580.293	-197.7	39077.56346367
2838.544	2513.53	-234.0	54776.17529572
2840.864	3026.04	-283.1	80130.36803230
2843.184	2502.856	-221.5	49078.36775351
2845.504	2486.072	-272.0	73966.79291384
2847.824	3121.993	-295.7	87433.06902418
2850.144	2662.317	-229.1	52478.30951299
2852.464	2846.987	-379.3	143861.83300215
2854.784	4099.095	-661.9	438163.57396099
			7420514.16435395
			=AA182/178

Figure 12 MSE additive

AG	AH	AI
2497.745	-134.3	18039.979282740
2479.097	-147.1	21650.665164723
3075.144	-196.5	38619.685917966
2647.783	-243.8	59425.626562639
2811.503	-305.0	93037.316307636
3967.093	-495.9	245950.978696802
2713.008	-341.9	116909.464198468
2475.661	-329.0	108227.748285620
3087.403	-331.2	109722.541161845
2588.636	-206.0	42445.389999384
2529.617	-250.1	62564.846811716
3017.66	-274.7	75455.986037801
2524.873	-243.6	59318.224554089
2506.225	-292.1	85334.672798580
3102.272	-276.0	76159.584404468
2674.911	-241.7	58406.834626004
2838.632	-370.9	137593.167416381
3994.222	-557.1	310322.294388324
		7292825.989230190
		=AI182/178

Figure 13 Logistic regression

Omnibus Tests of Model Coefficients

		Chi-square	df	Sig.
Step 1	Step	98.514	1	.000
	Block	98.514	1	.000
	Model	98.514	1	.000

Model Summary

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	4.747[a]	.421	.966

a. Estimation terminated at iteration number 13 because parameter estimates changed by less than .001.

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	Sales	.031	.020	2.490	1	.115	1.032
	Constant	-105.467	67.292	2.456	1	.117	.000

a. Variable(s) entered on step 1: Sales.