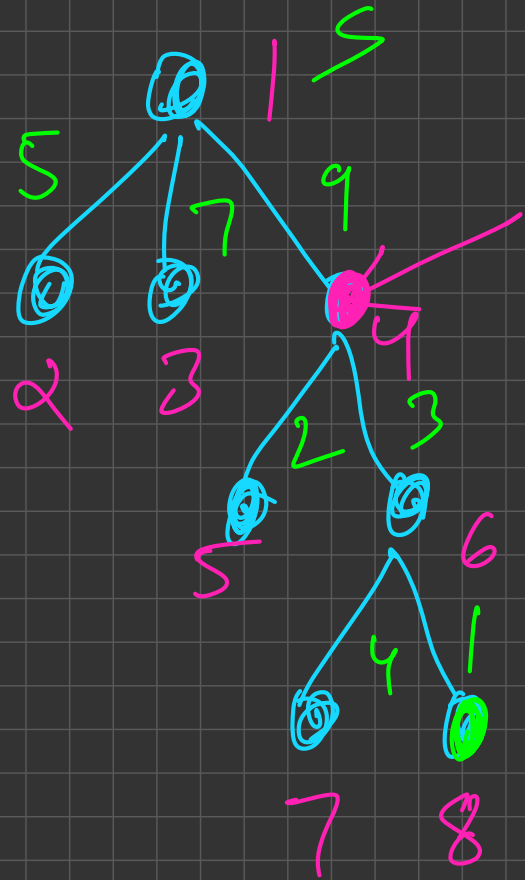


Df on DAG

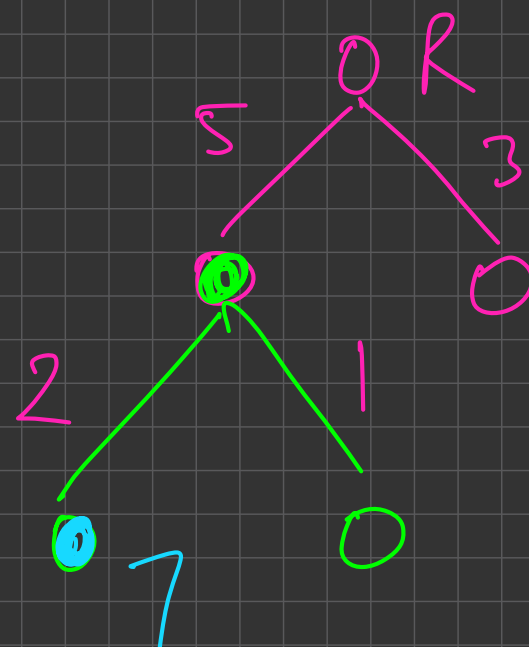
# Graph Theory

DP on Directed Graphs

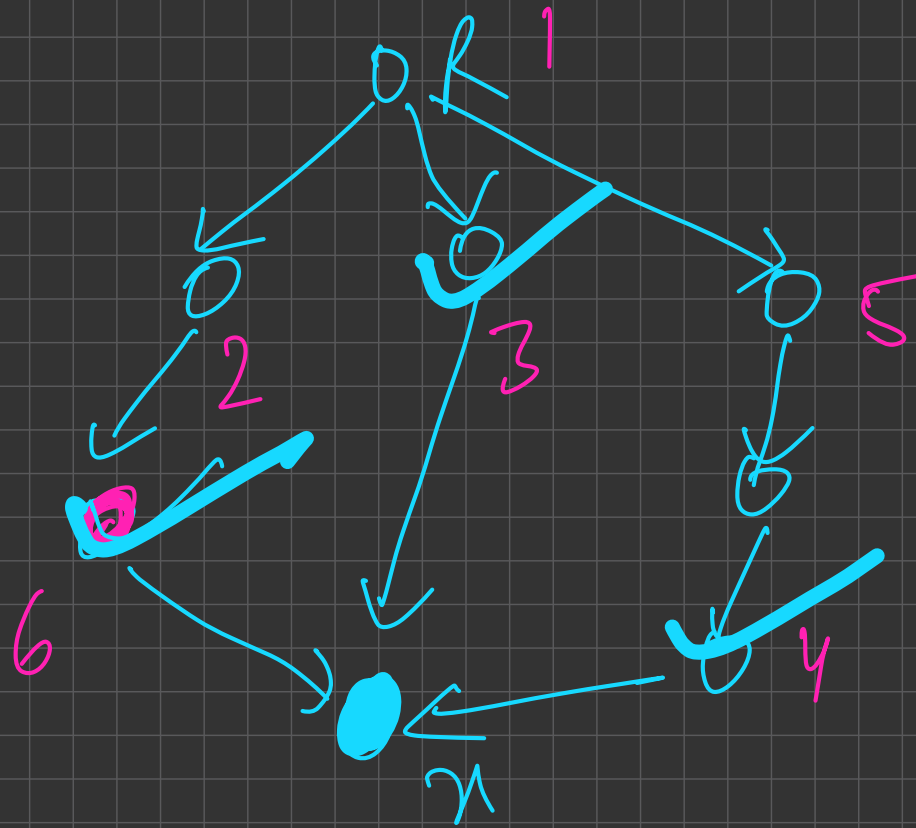


DP on Trees

$$\text{Ans}[4] = f(\text{Ans}[5], \text{Ans}[6])$$



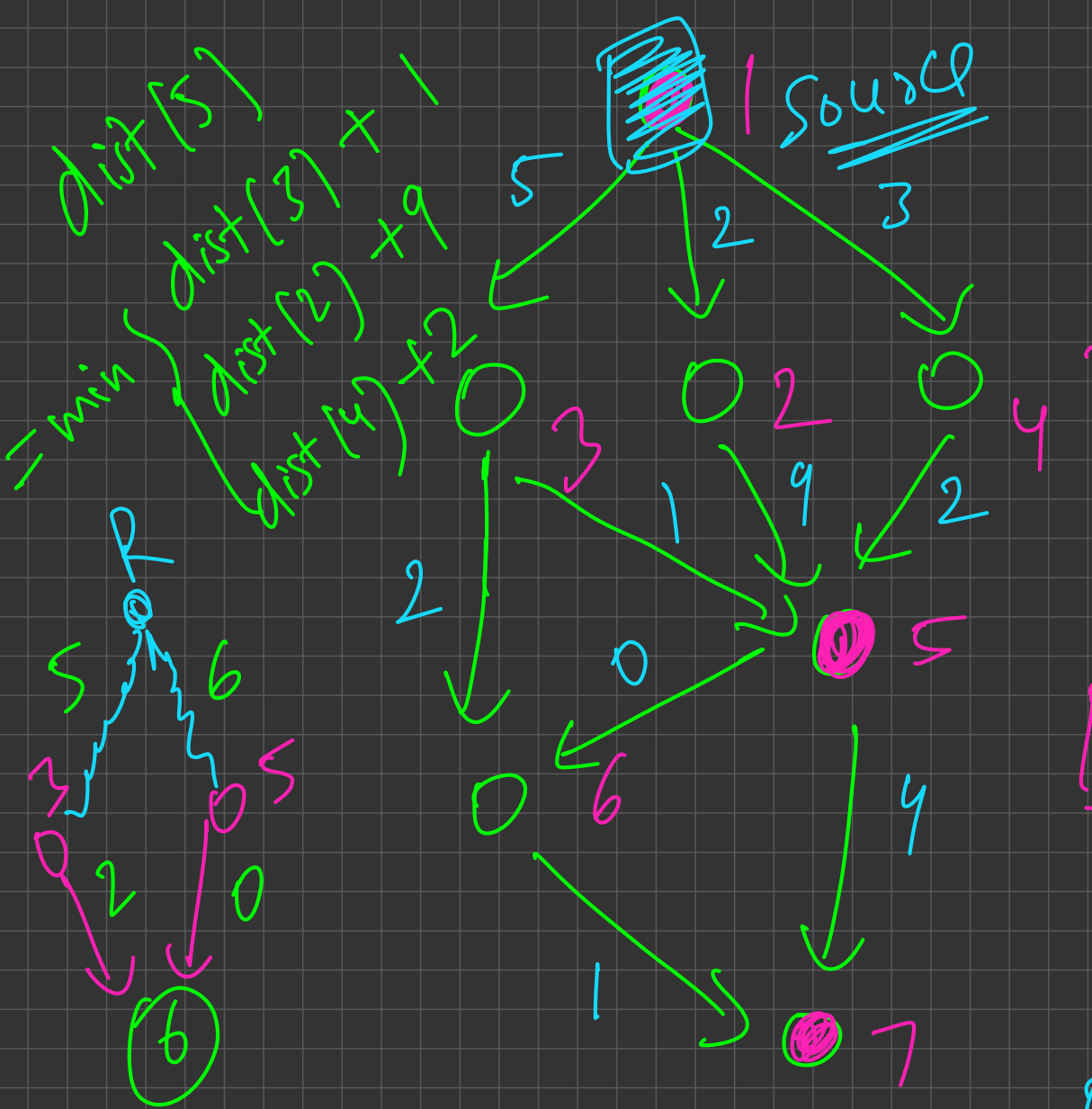
$$D =$$



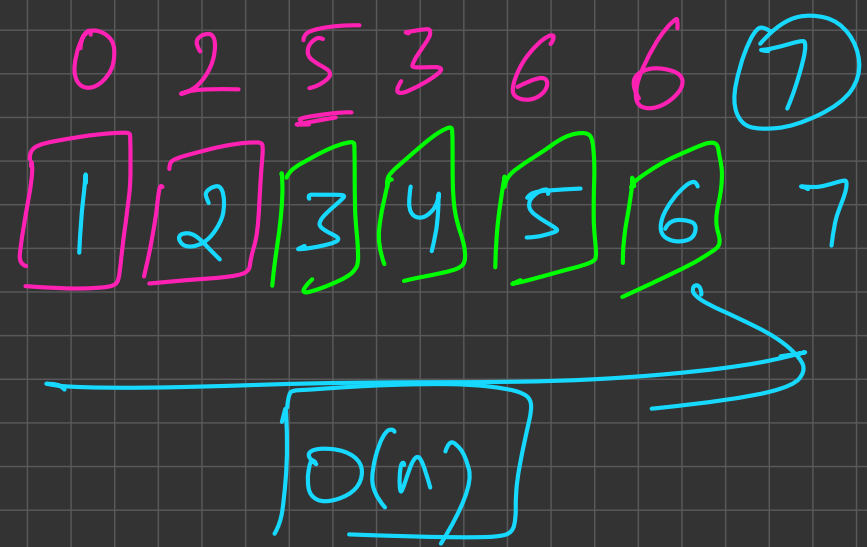
DAG

$\text{dist}[6]$ ,  $\text{dist}[3]$ ,  $\text{dist}[4]$

$$\text{dist}(x) = \min \begin{cases} \text{dist}[6] + \text{edge}(6 \text{ to } x) \\ \text{dist}[3] + \text{edge}(3 \text{ to } x) \\ \text{dist}[4] + \text{edge}(4 \text{ to } x) \end{cases}$$



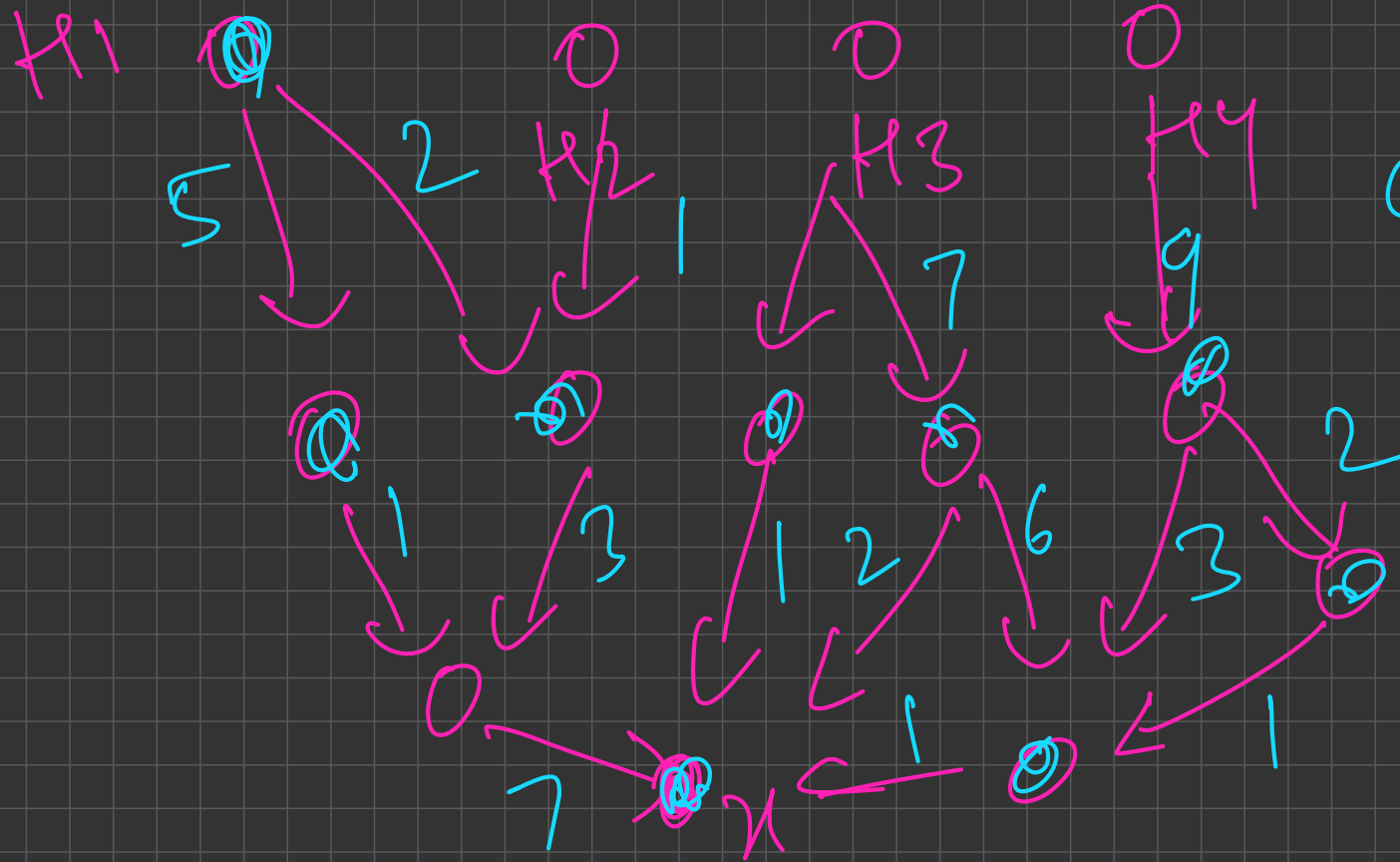
$O((n+m) \log n)$   
 $O(n+m)$



$O(n+m) + O(n+m)$

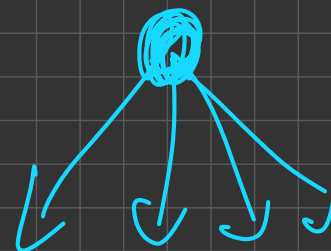
$\text{dist}[5] = 0$

$\text{dist}[n] \geq 0$   
 $\text{dist}[n] = 0$



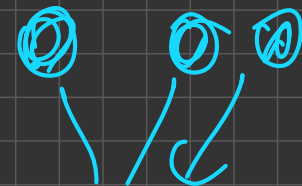
distances  $\Rightarrow$   
DAG

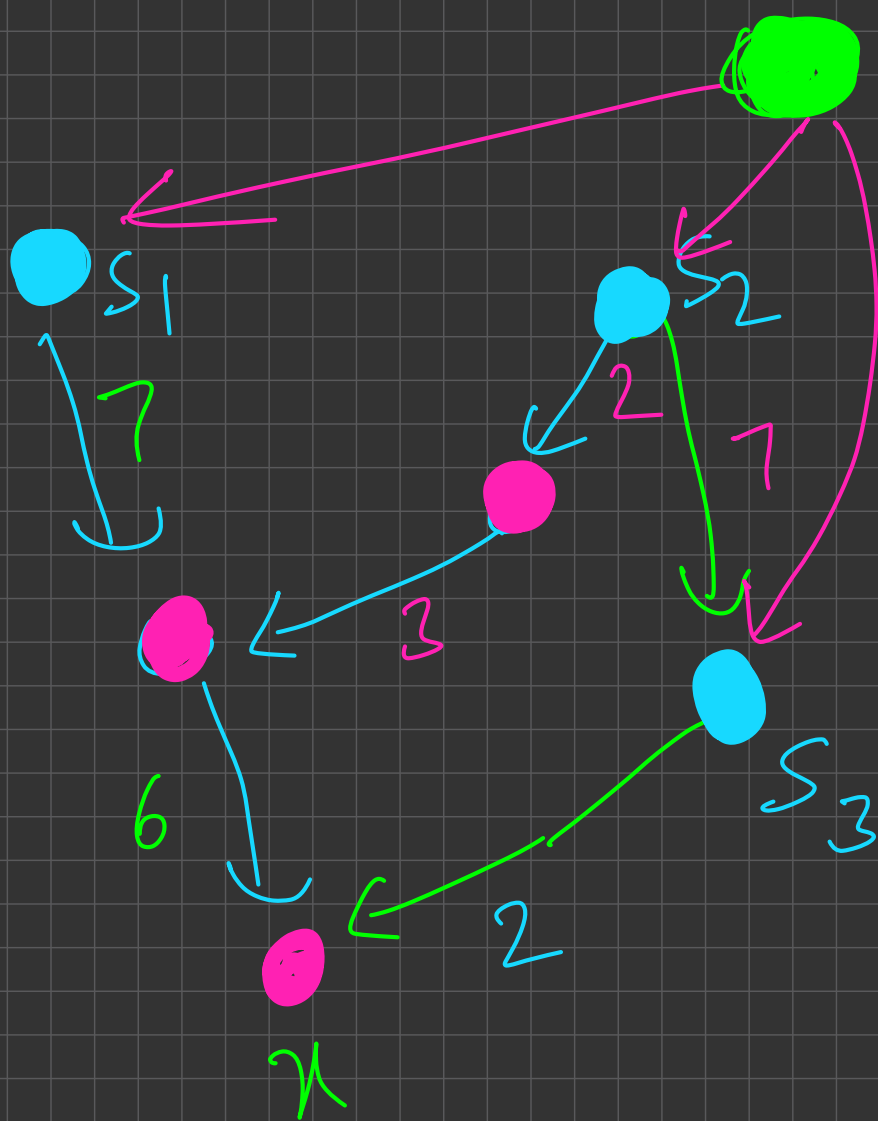
multi-source BFS



$$n \leq 10^5$$

$$k \leq n$$










## Problem 1

Shortest Path from Source to  
Destination in  $O(n+m)$  in DAG





# Problem Part 1 - DAG

A game has  $n$  rooms and  $m$  tunnels between them. Each room has a certain number of coins. What is the maximum number of coins you can collect while moving through the tunnels when you can freely choose your starting and ending room?

Input:

DAG

The first input line has two integers  $n$  and  $m$ : the number of rooms and tunnels. The rooms are numbered  $1, 2, \dots, n$ .

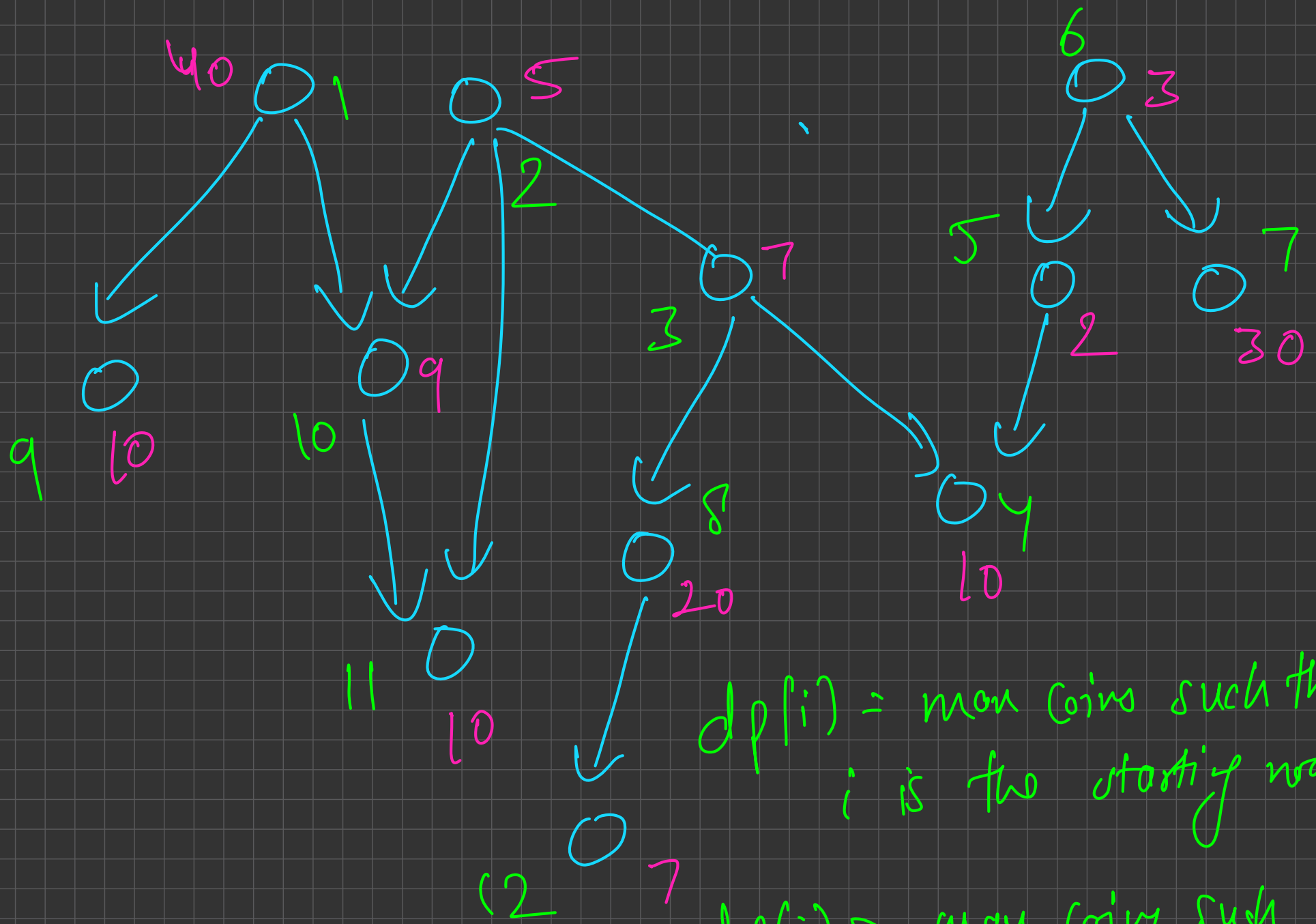
Then, there are  $n$  integers  $k_1, k_2, \dots, k_n$ : the number of coins in each room.

Finally, there are  $m$  lines describing the tunnels. Each line has two integers  $a$  and  $b$ : there is a tunnel from room  $a$  to room  $b$ . Each tunnel is a one-way tunnel.

Output:

Print one integer: the maximum number of coins you can

collect. Problem Source: [CSES-1686](#)



$dp(i)$  = max coins such that  $i$  is the starting node

$dp(i)$  = max coins such that  $i$  is the ending node

$dp[i] = \text{max coins you can pick s.t.}$

$\sum i$  is the starting node.

$$dp[i] = \text{coins}[i] + \max \{ dp[\text{child}] \}$$

B.C.

$dp[i] = \text{coins}[i]$  for  $i$   
not having any children

P-S

mean of  $dpln$



## Problem Part 2 - Normal Graph

A game has  $n$  rooms and  $m$  tunnels between them. Each room has a certain number of coins. What is the maximum number of coins you can collect while moving through the tunnels when you can freely choose your starting and ending room?

Input:

The first input line has two integers  $n$  and  $m$ : the number of rooms and tunnels. The rooms are numbered  $1, 2, \dots, n$ .

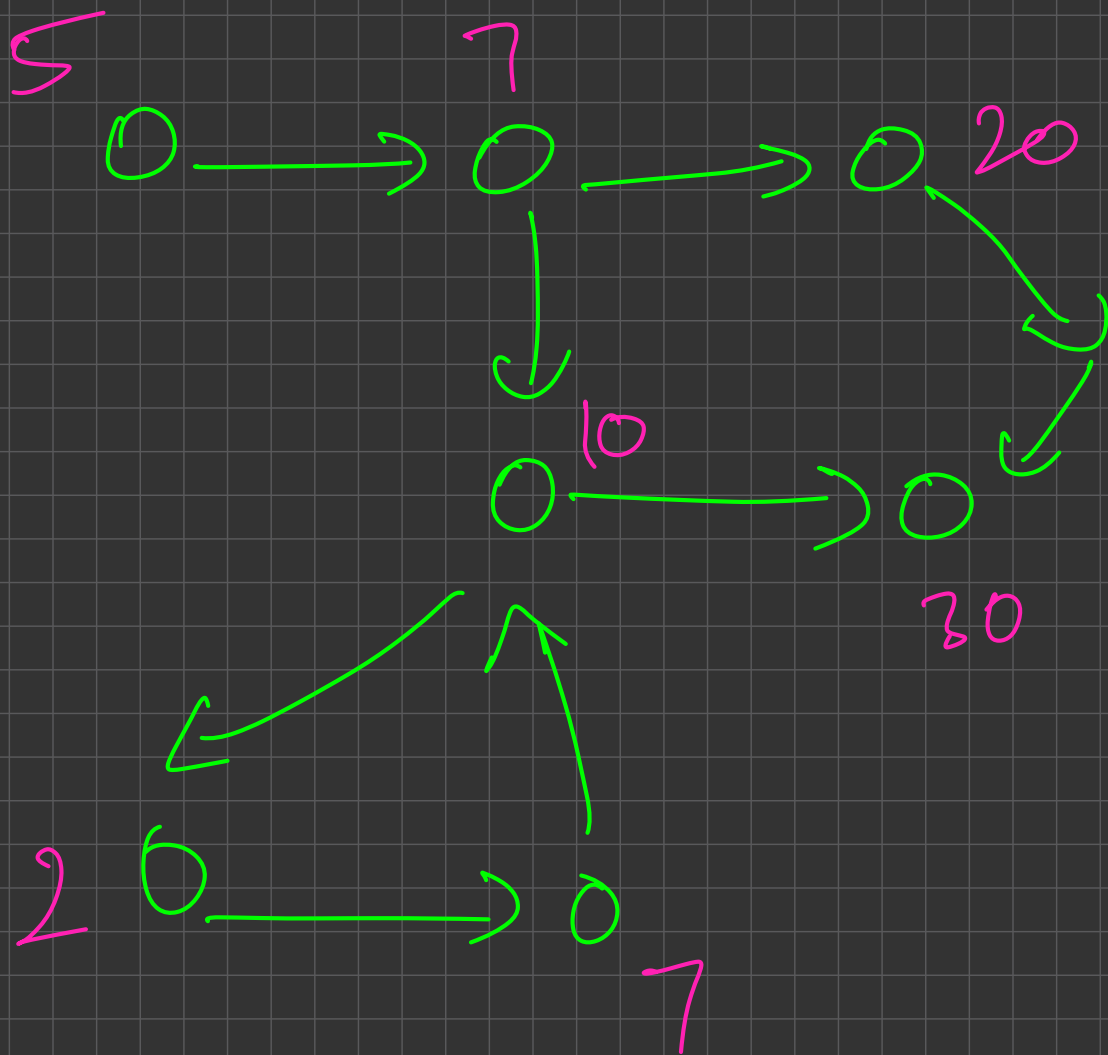
Then, there are  $n$  integers  $k_1, k_2, \dots, k_n$ : the number of coins in each room.

Finally, there are  $m$  lines describing the tunnels. Each line has two integers  $a$  and  $b$ : there is a tunnel from room  $a$  to room  $b$ . Each tunnel is a one-way tunnel.

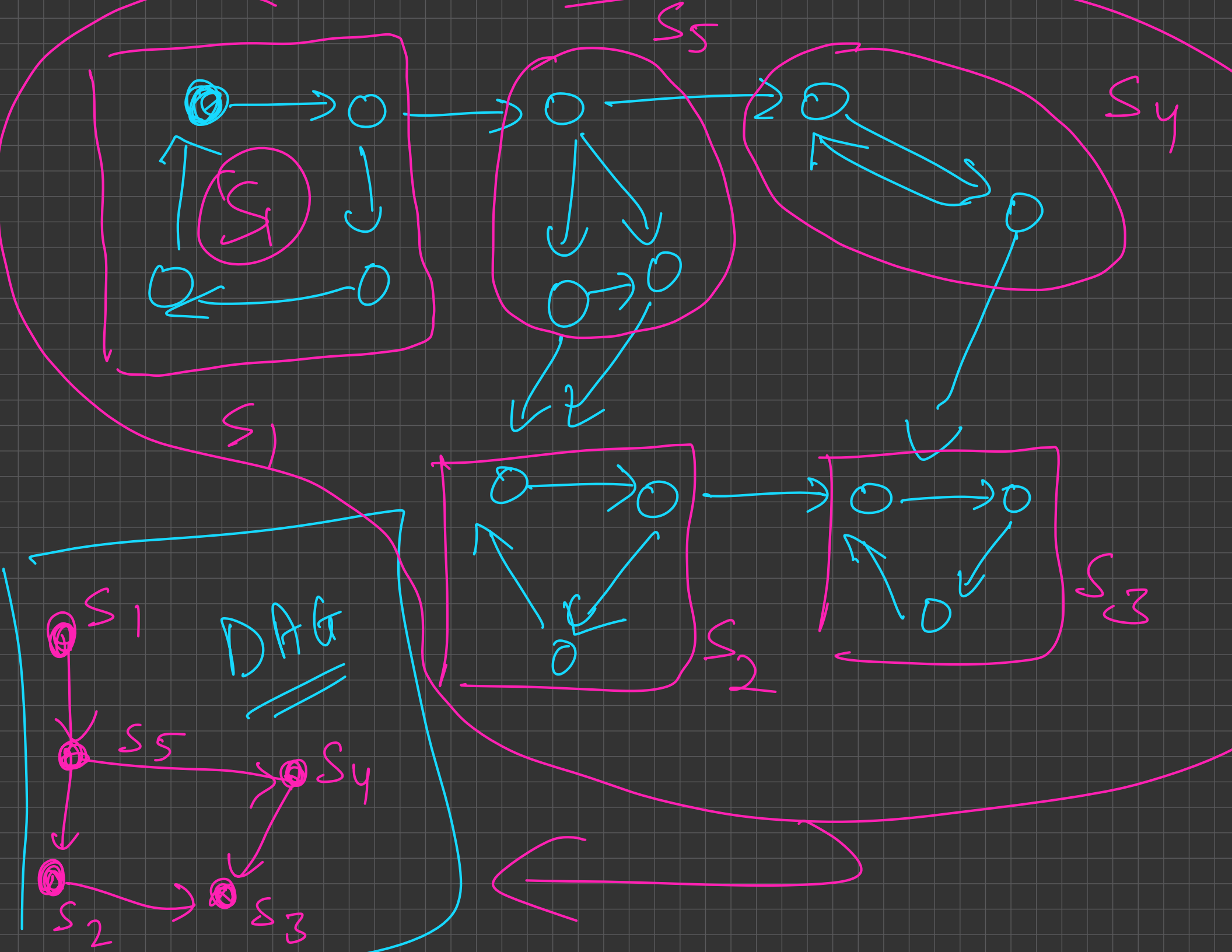
Output:

Print one integer: the maximum number of coins you can

collect. Problem Source: [CSES-1686](#)



# coins  $\geq 0$





Second best minimum spanning tree