

DSU

Disjoint Set Union

— Priyansh

N people in a village.

1 2 3 4 5 6

(1, 2)

(3, 4)

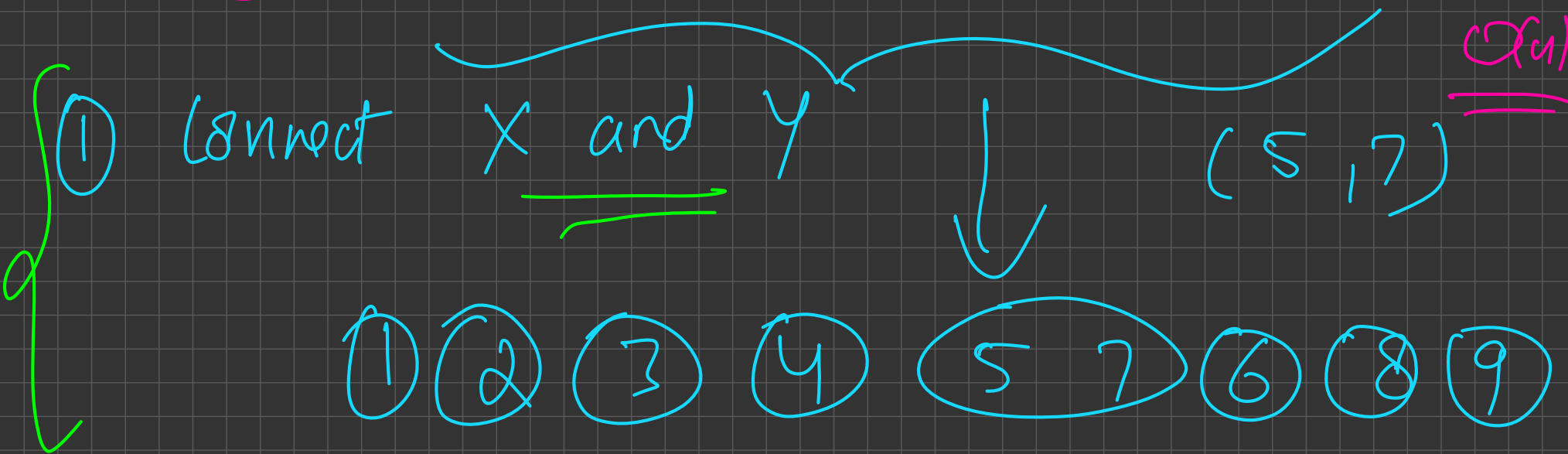
(1, 3)

$N \leq 10^5$

① connect x, y

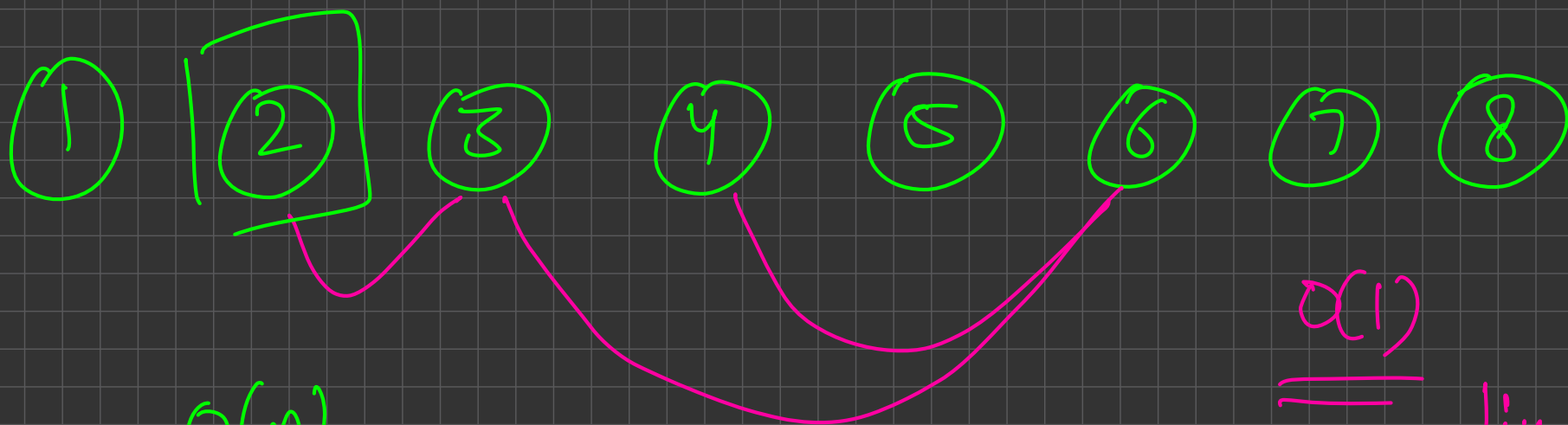
② Number of people in x 's team

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨



② Give us the no. of people in X's team

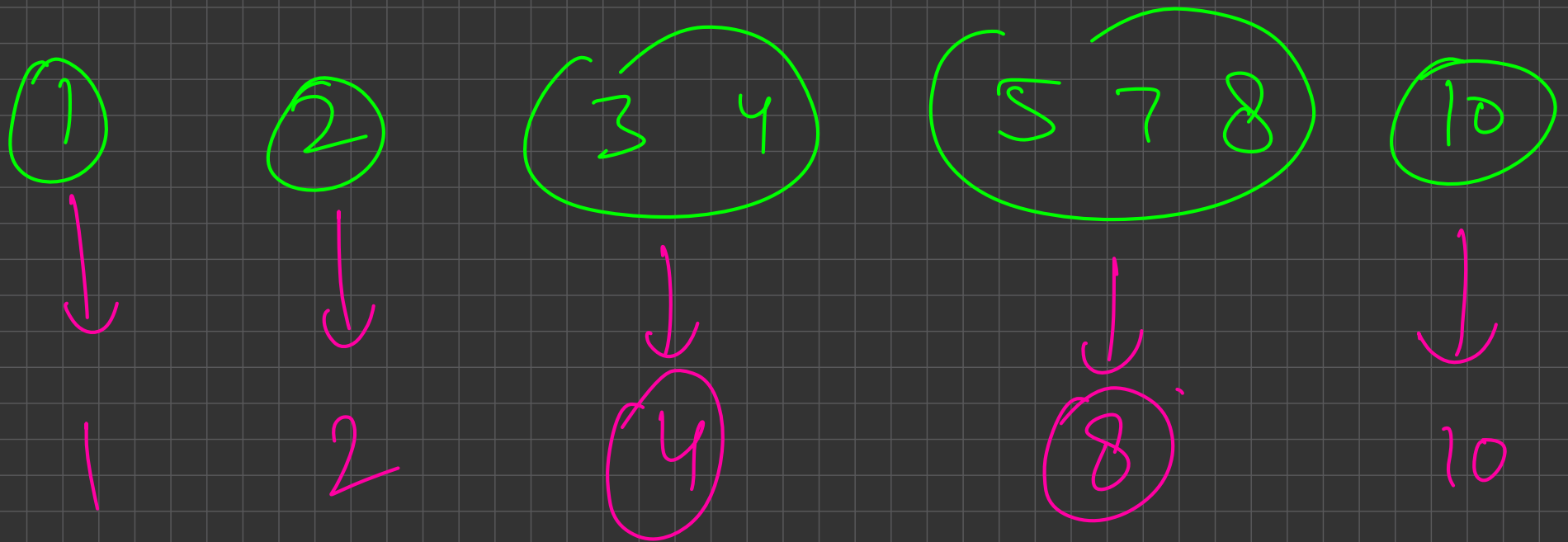
$N \leq 10^5$ $Q \leq 10^5$



$O(n)$

$O(1)$ time

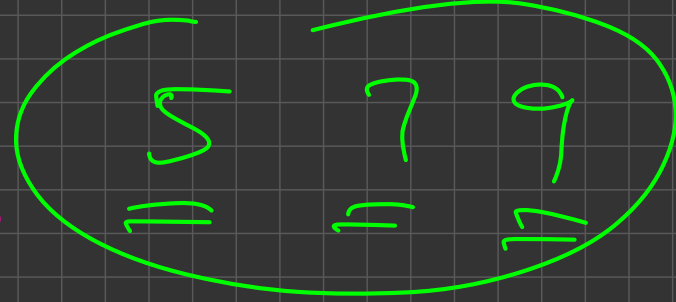
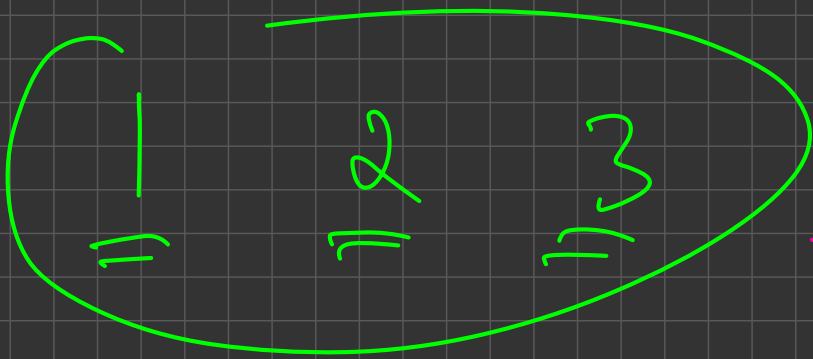
if we imagine
this as a graph



connect set of 3 with set of 8

|| || leader[3] || || || leader[8]

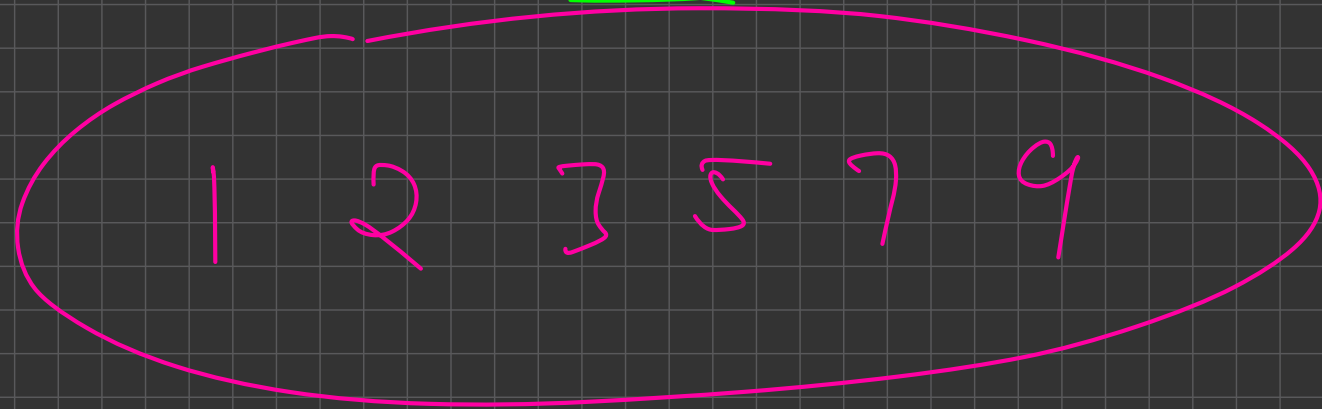
now leader $\rightarrow \max(l_1, l_2)$ $O(1)$
team[11] \rightarrow team[4] + team[8] $O(1)$



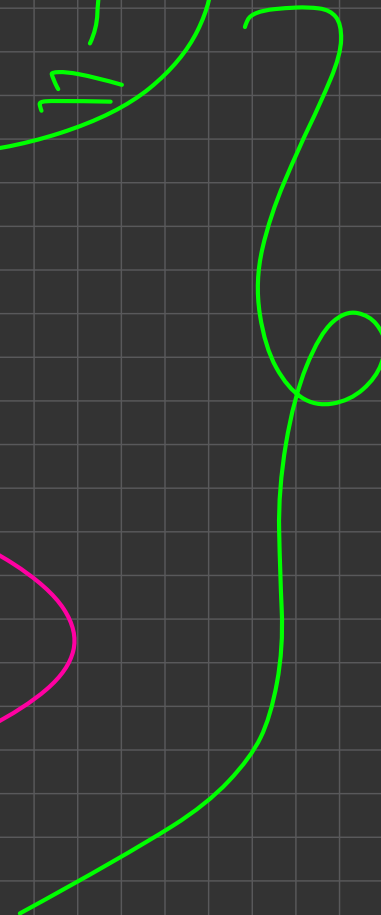
3

9

~~2~~

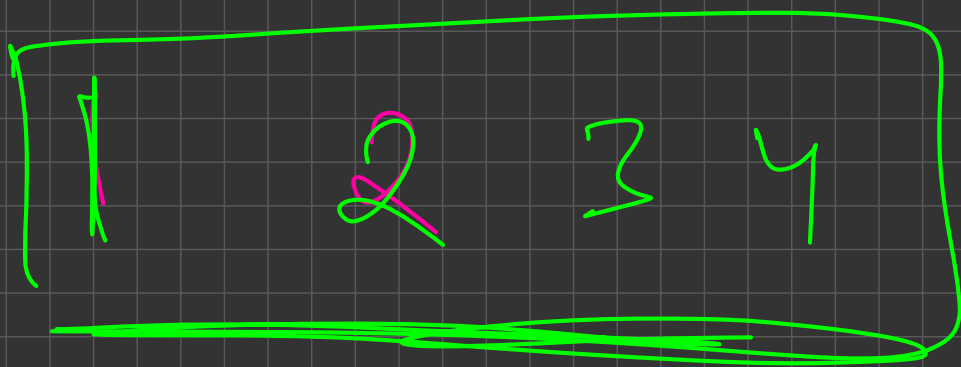


9

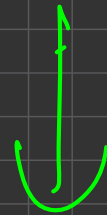
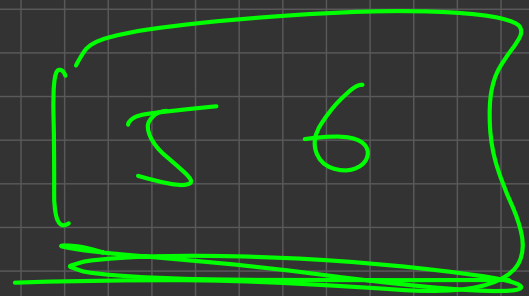


S_1 S_2
= =

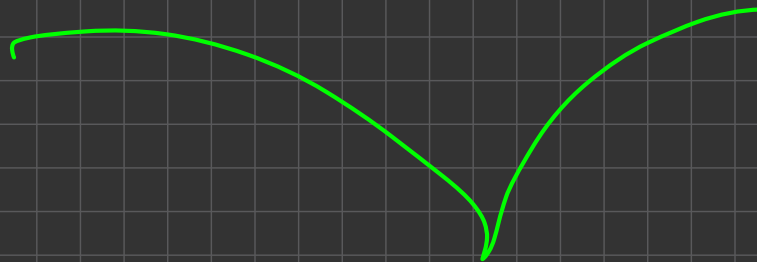
for element in S_1
 leader[element] = new-leader



2



6



2

Q

queries

$\leq n$

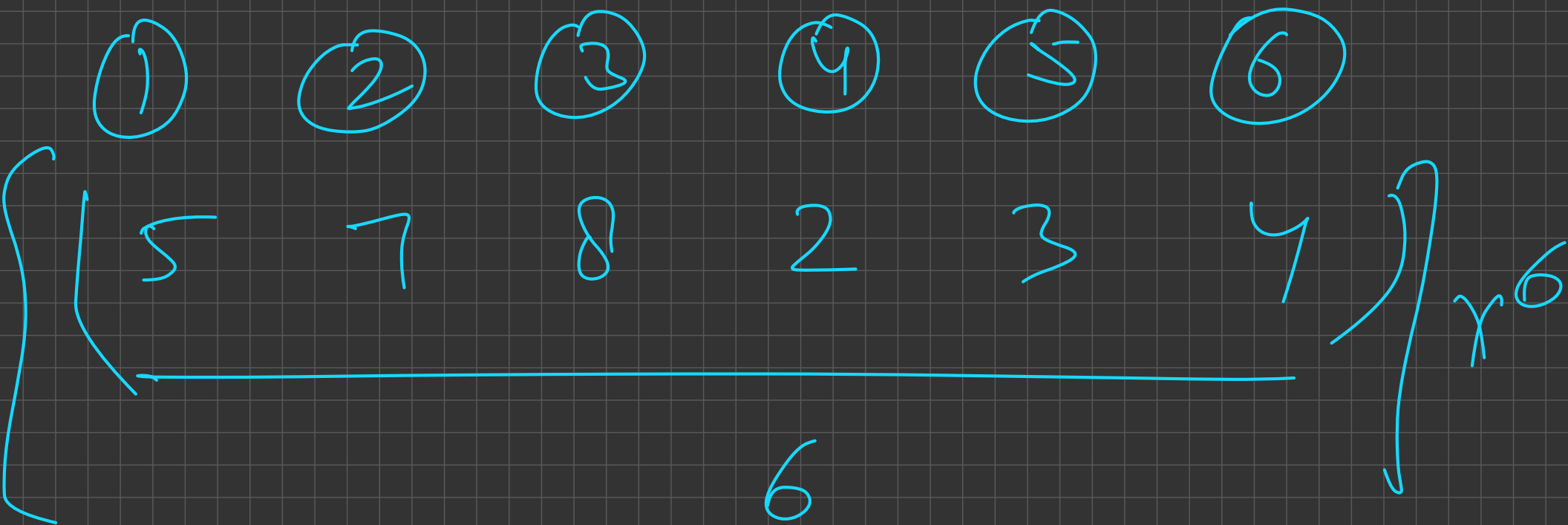
set

$(N - 1)$

total

updates

total updates



Average updates for a single element

[Average updates of 1 element] $\times N$

$$\begin{aligned} &= T.C \\ &= \end{aligned}$$

x, y, a, s, c

d

$x \rightarrow 1$

x

1, 2, 3, 4, 5

① \rightarrow 1 \rightarrow ≥ 1

② \rightarrow ≥ 2 \rightarrow ≥ 2

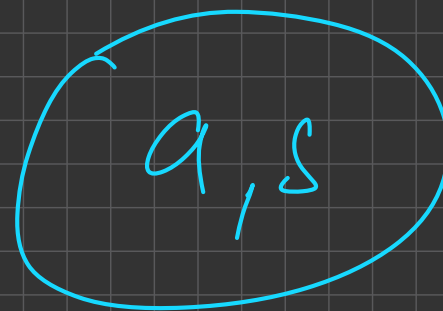
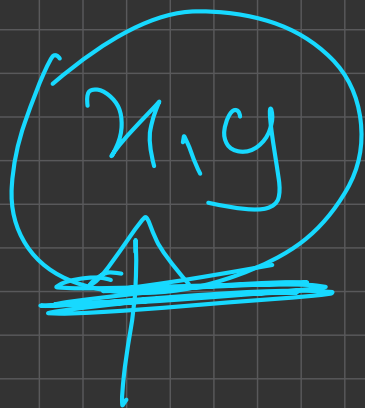
③ \rightarrow ≥ 4 \rightarrow ≥ 4



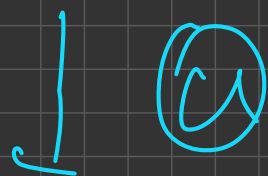
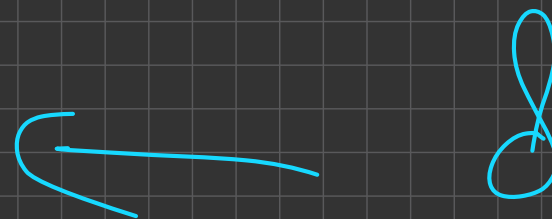
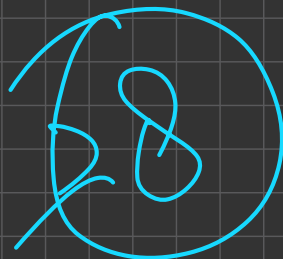
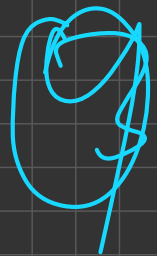
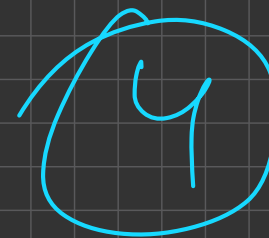
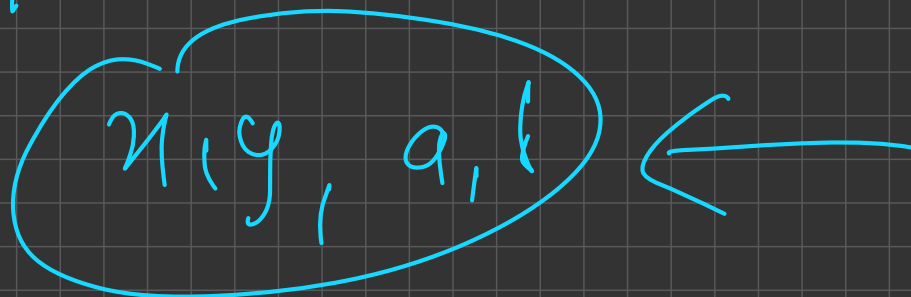
$\leq \log n$ times



①



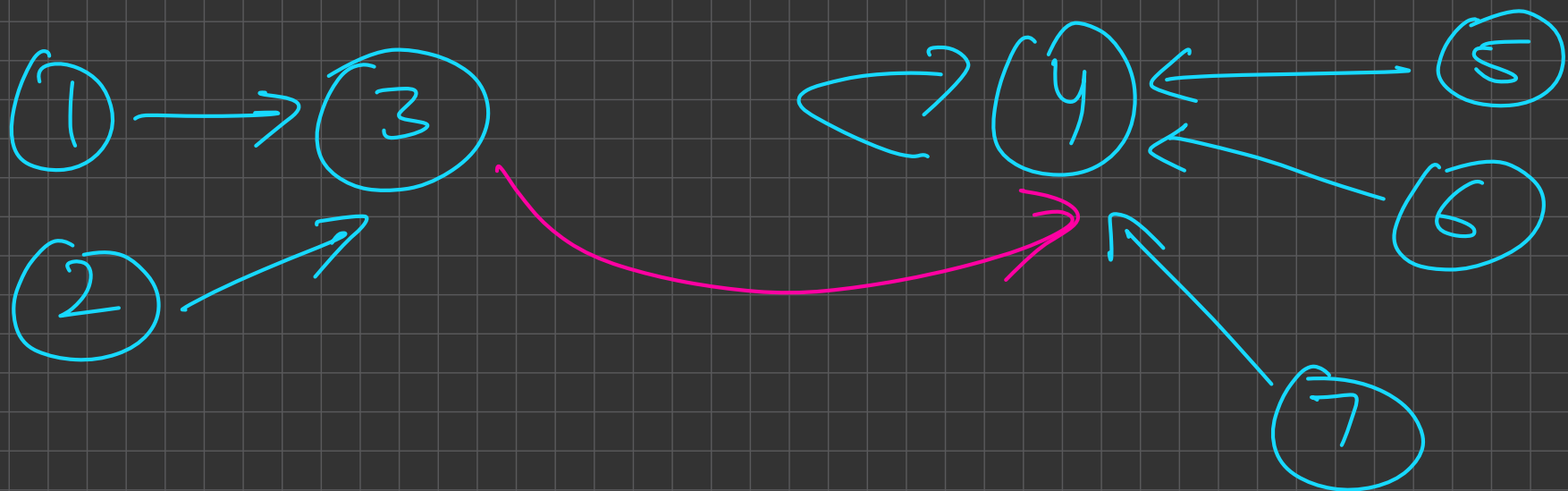
②

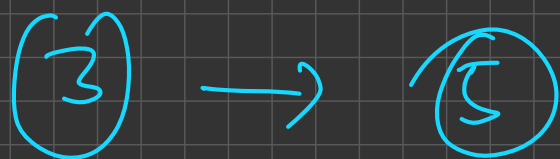
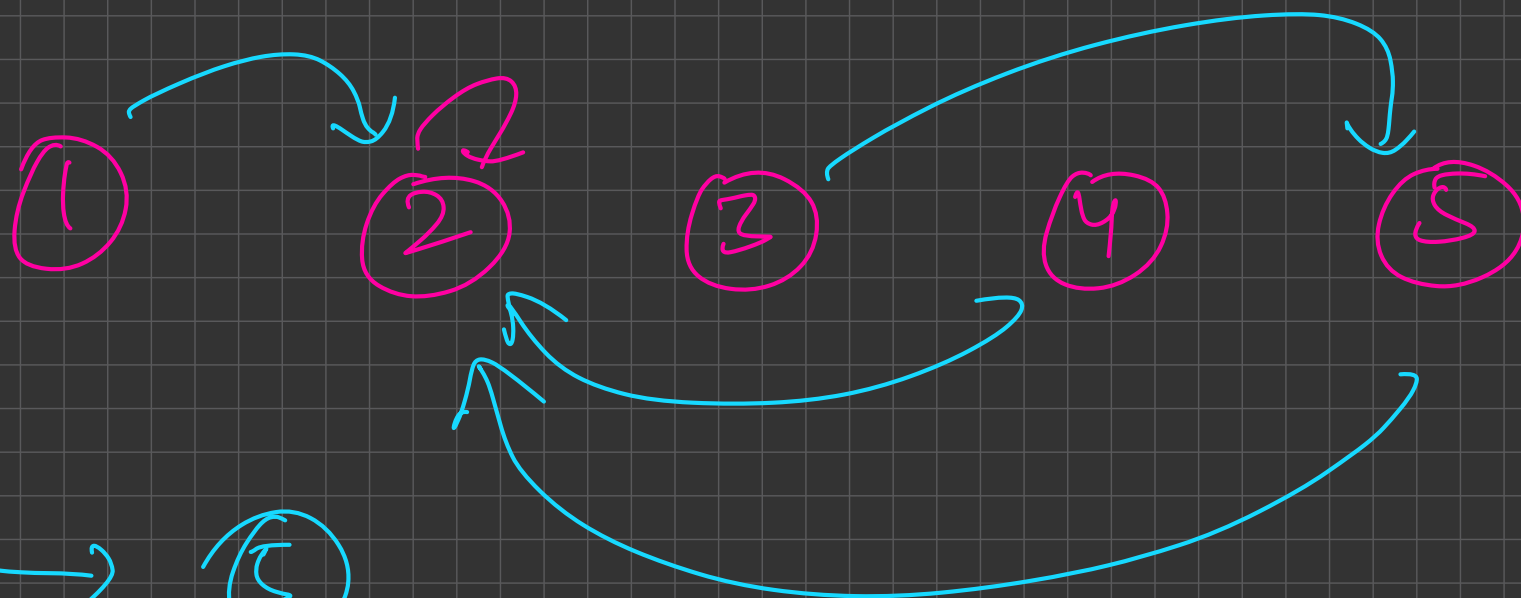


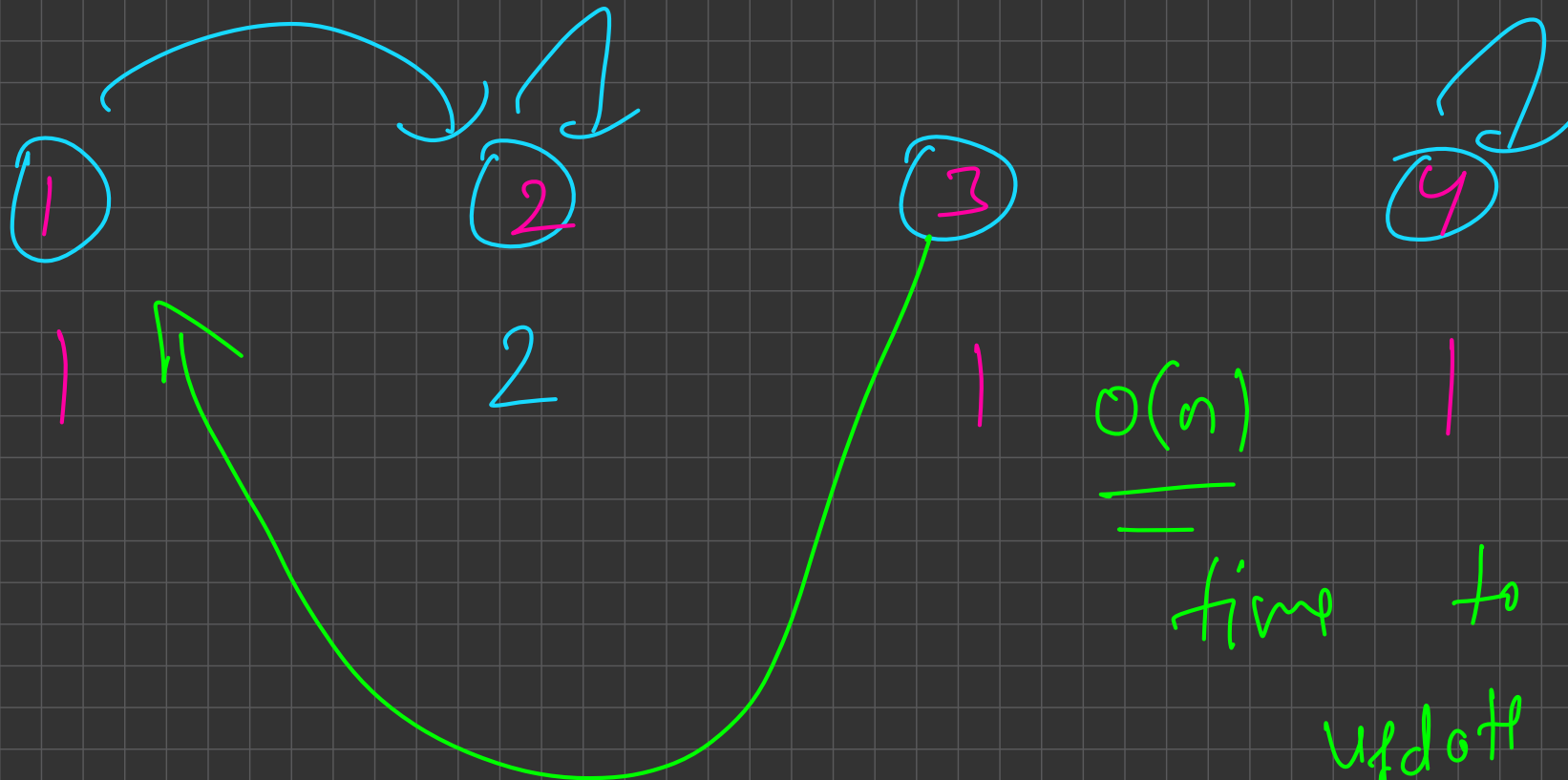
$$1 \leq \log n \cdot n$$

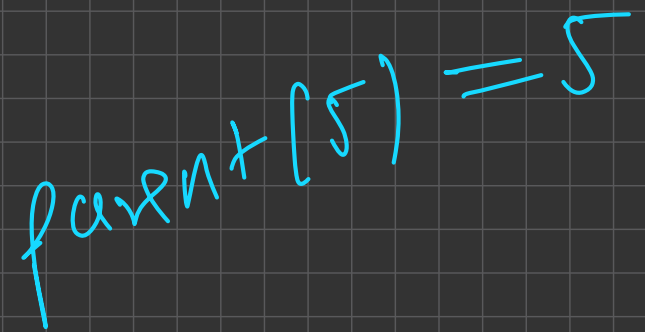
} size trick
small to large
merge trick

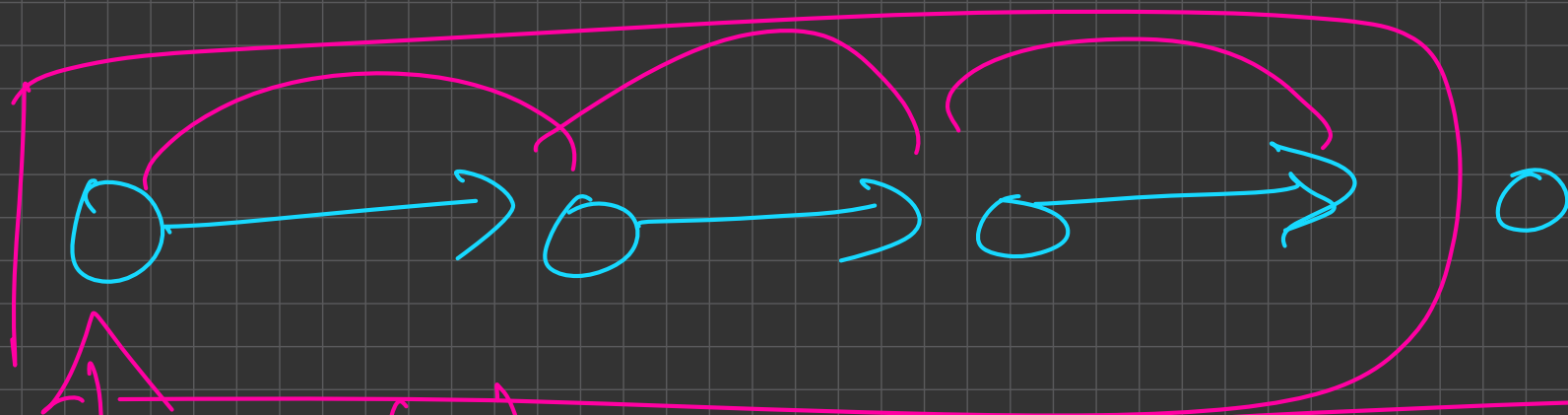
$$\underline{n \log n}$$





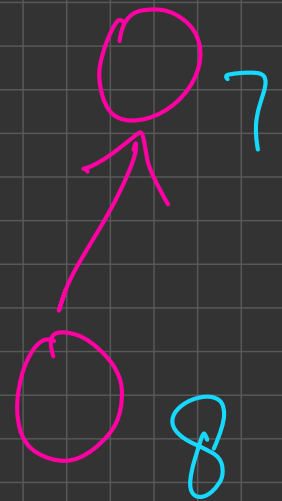
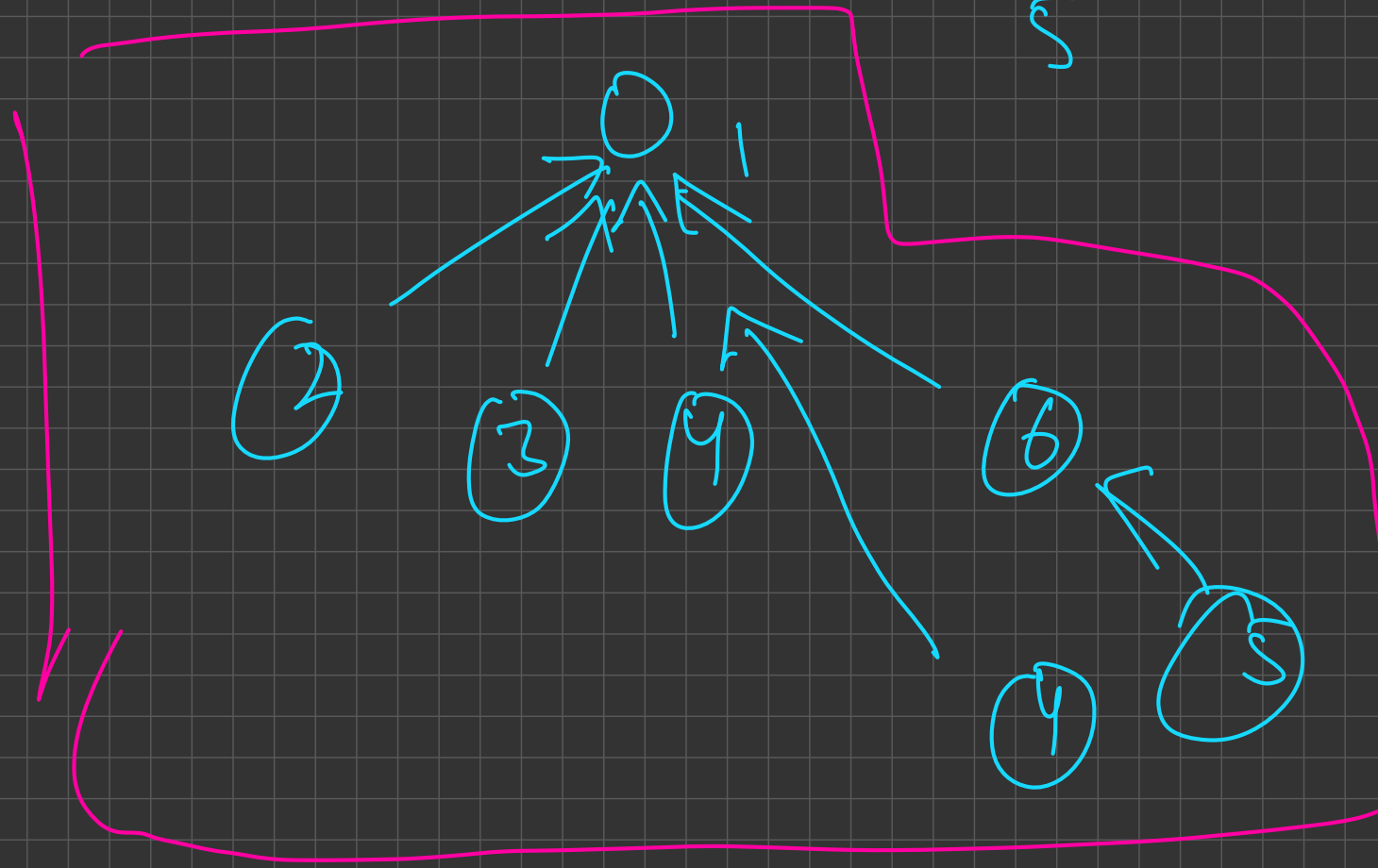
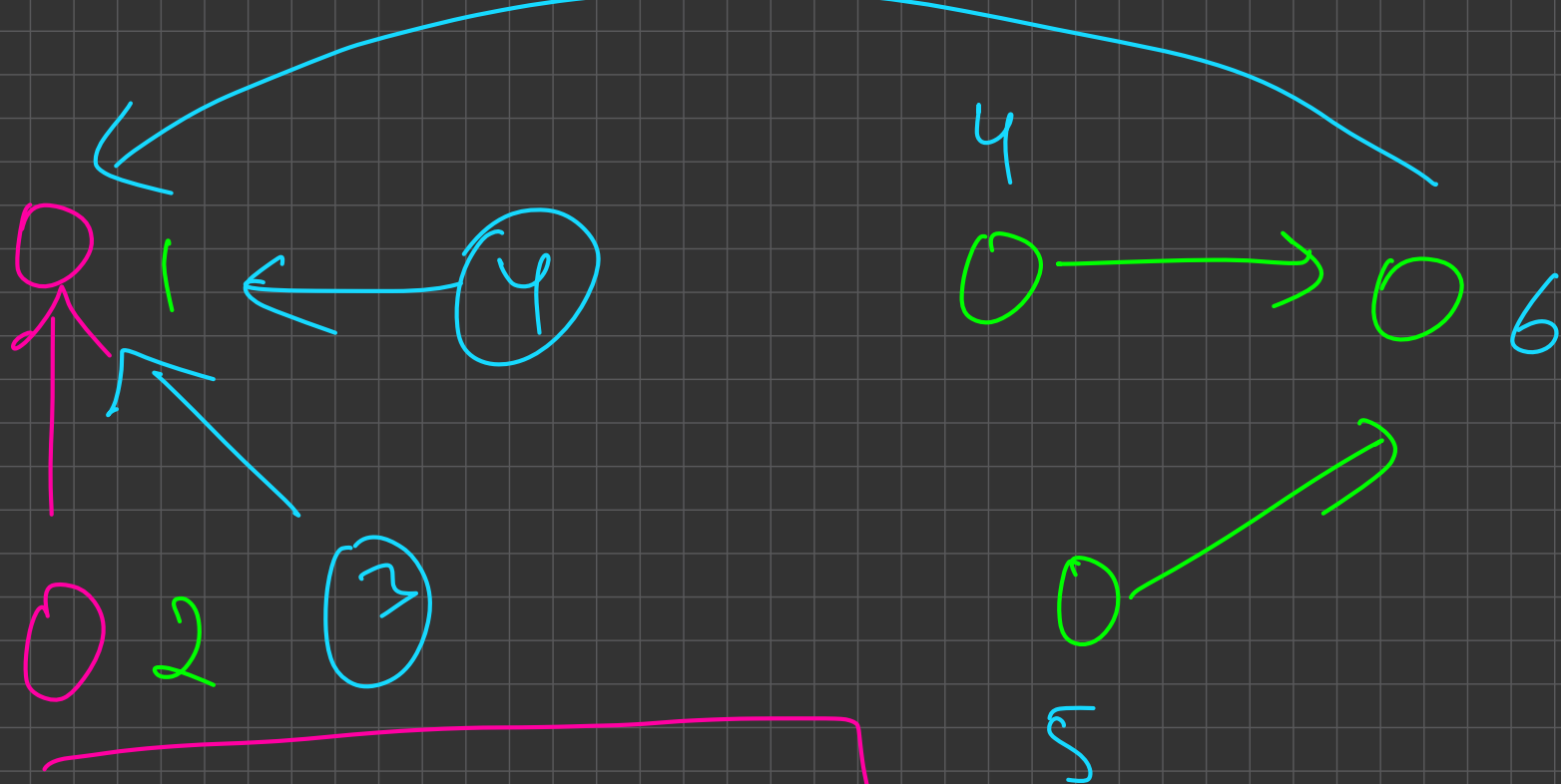


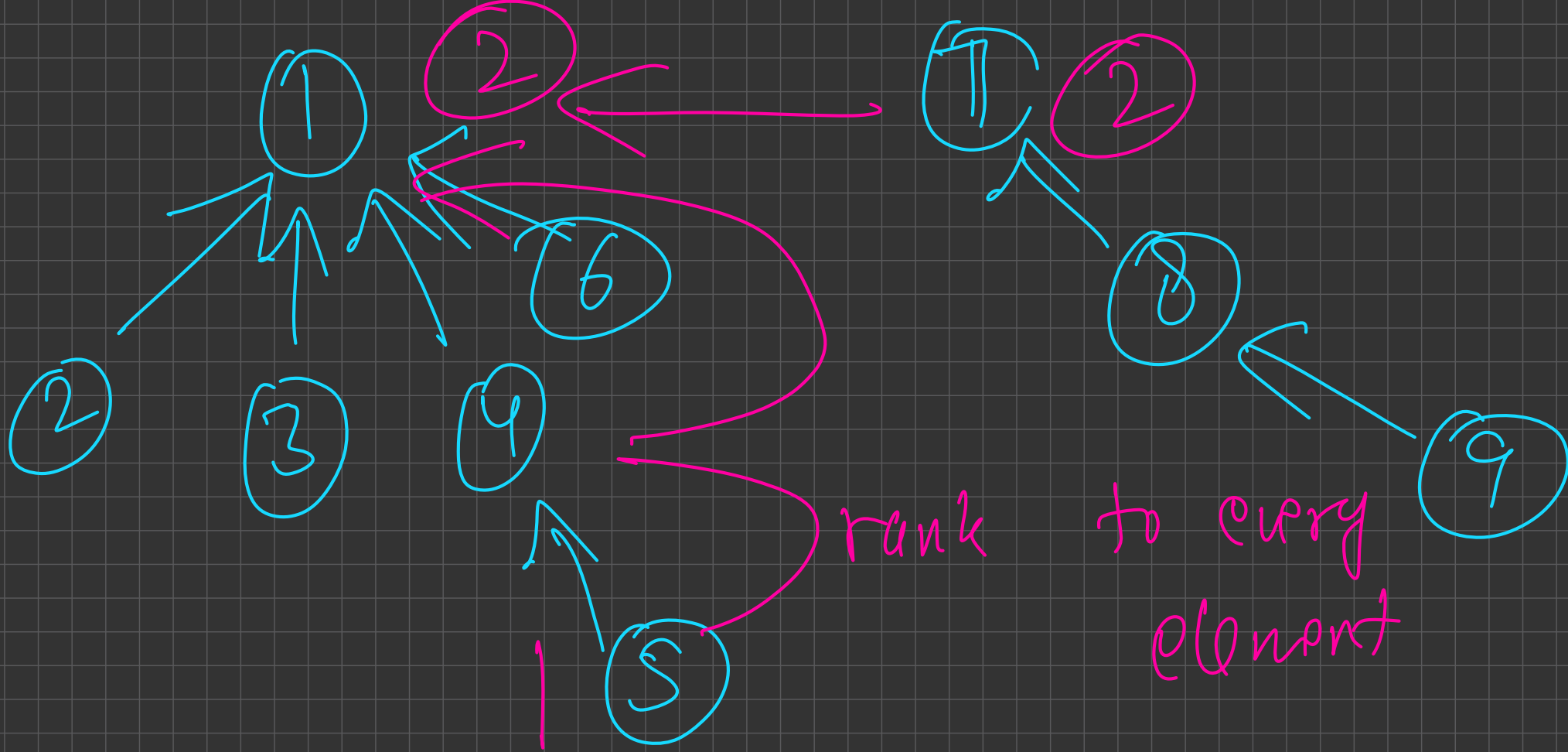


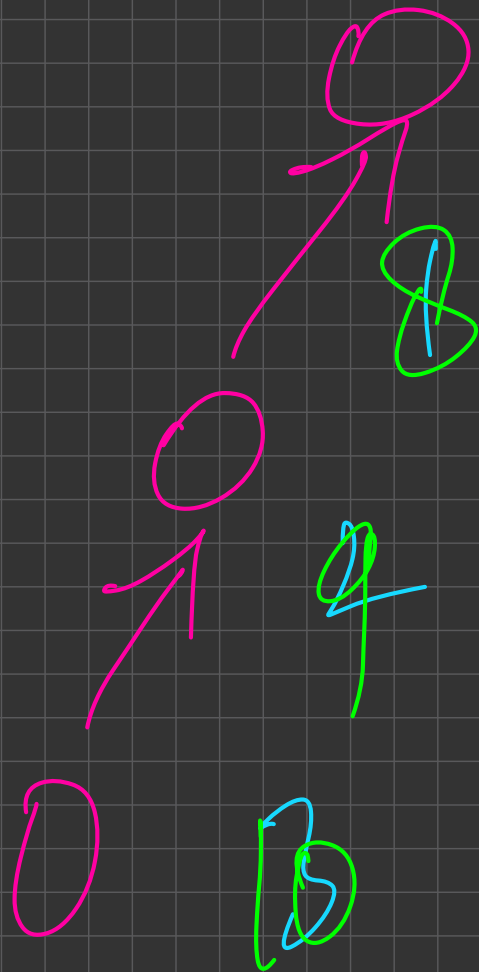


$O(n)$

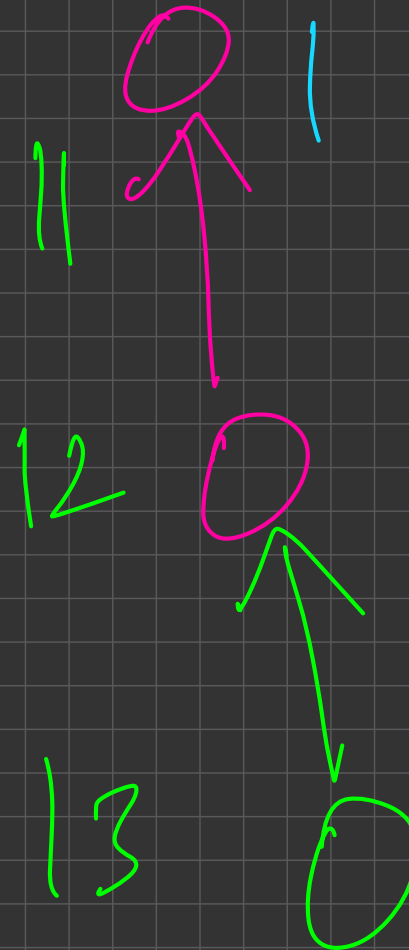
$O(1)$ time







2



1

$\text{rank}[a] == \text{rank}[s]$

$\text{rank}[a]++$

Size trick \rightarrow small to large
work

✓ Path Compression ①

✓ Rank ②

$\log^*(n)$

inverse

ackermann

function

No of time you

have to take the log of a
number to make it 1

$$\log_2 64 \rightarrow \log_2 6 \rightarrow \log_2 2 \rightarrow \textcircled{1}$$

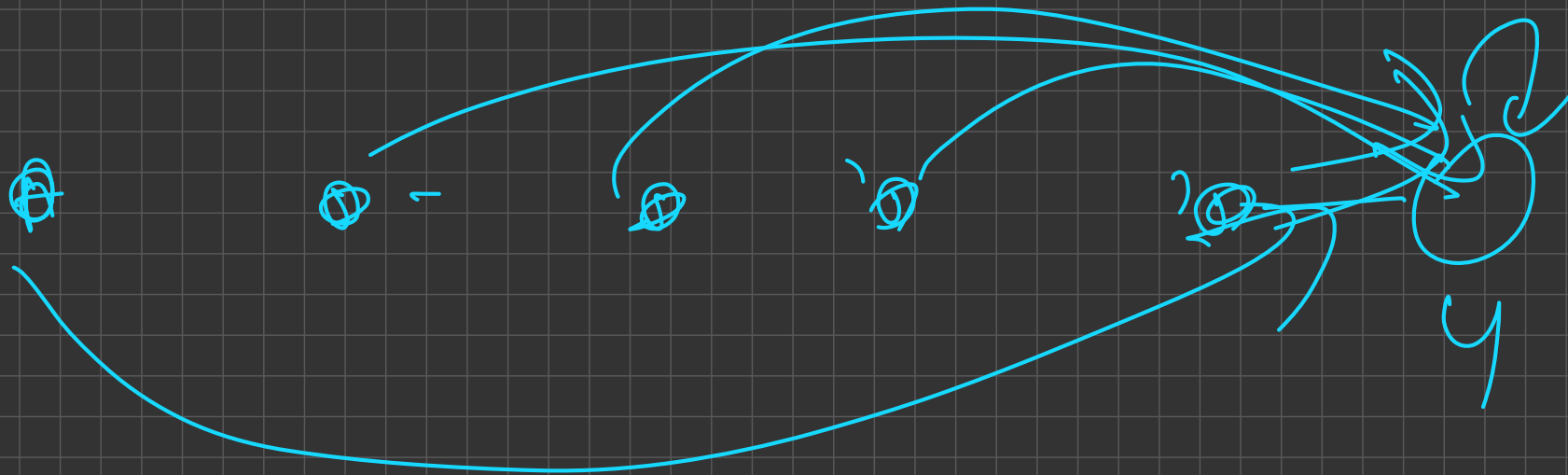
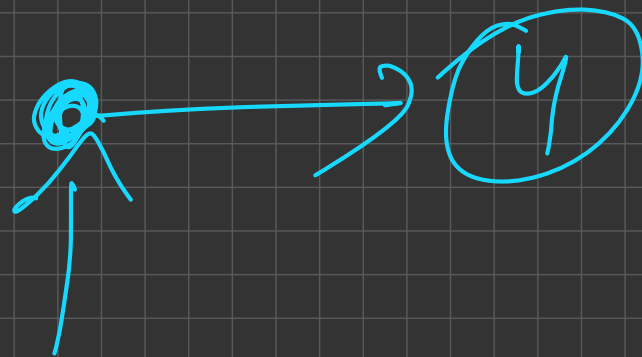
$$\textcircled{3}$$

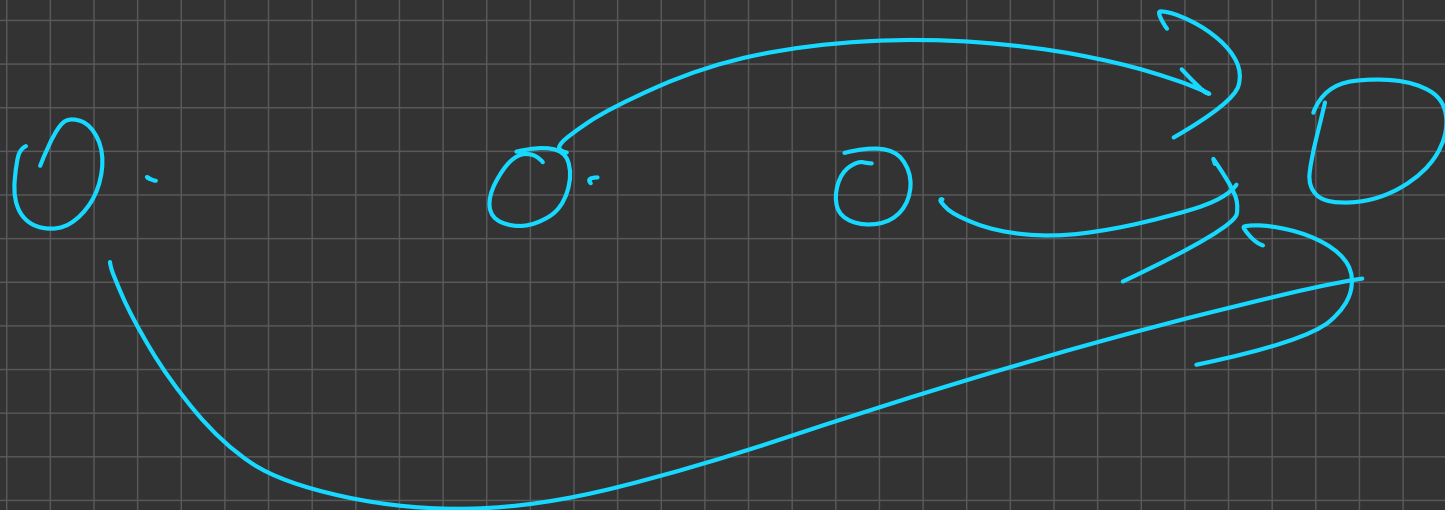
$$\log_2^*(64) = 3$$

$$\log_2(64) = 6$$

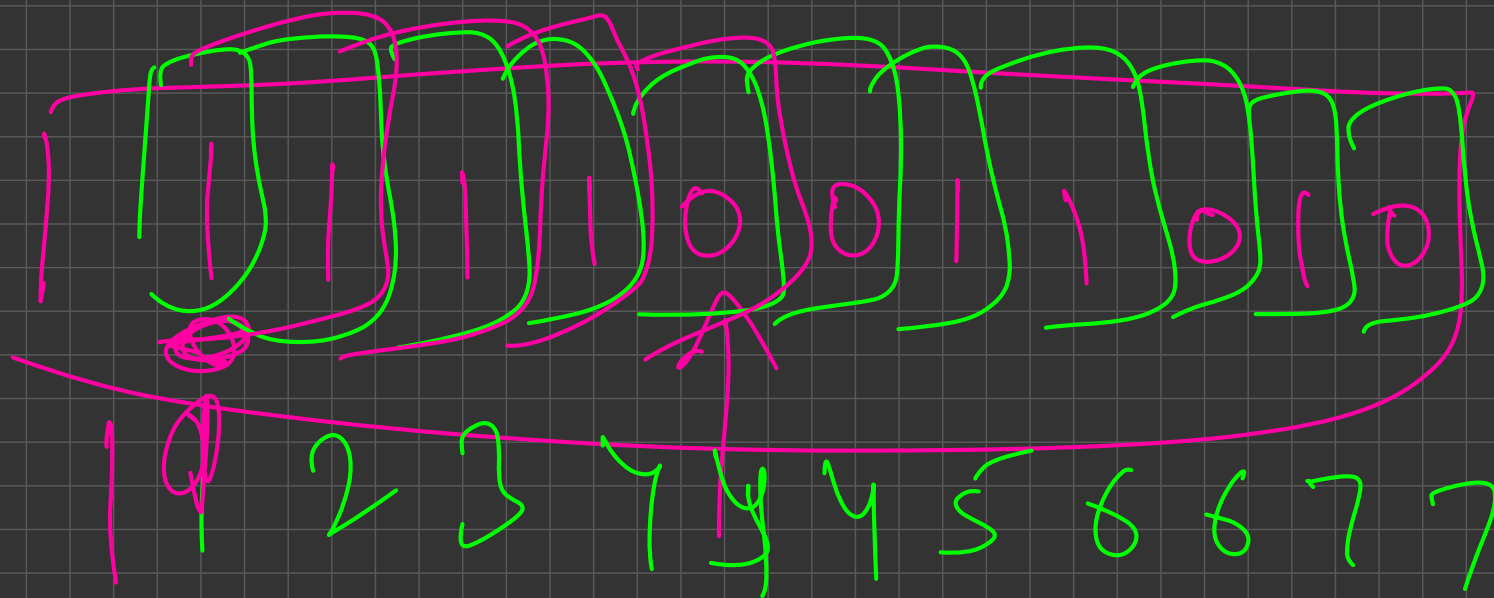
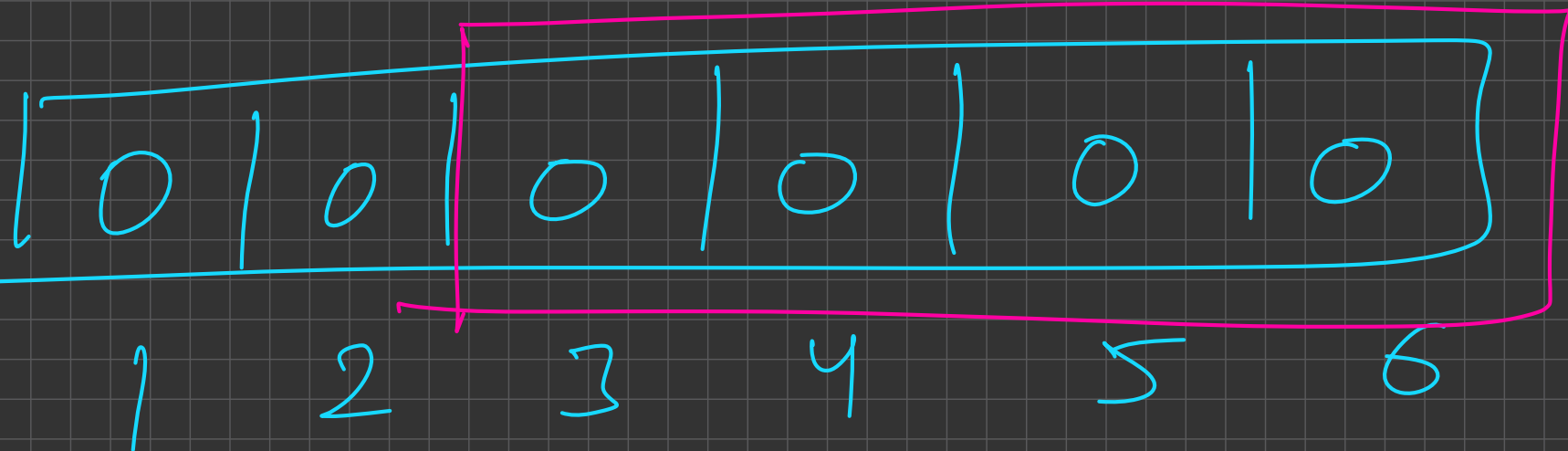
①

②





MST
=



~~sum~~ $\text{sum} < \text{length}$