Number Theory Advanced - 2

Dev Karan Singh (devkaran1231)
Expert at codeforces (1817)
5 star at codechef (2040)

Number Theory Advanced - 2

- Primes
  - O(n)
  - O(sqrt(n))
  - Query based? range based
- Sieve of Eratosthenes
- Factors/Prime factorization
  - O(n)
  - O(sqrt(n))
  - Query based?
- Smallest Prime factor sieve (spf sieve)
- Segmented Sieve (what is the need?)

29/5/2023

primes → ?

a number with exactly 2 factors

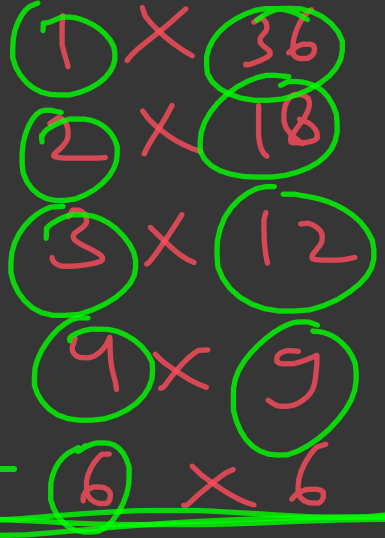
Eg: 17 → 17 × 1

4 → 2 × 2 × 1 ✗

```
int cnt = 0;
for(int i = 1; i <= n; i++){
    if (n % i == 0){
        cnt++;
    }
}
if (cnt == 2) → prime
              → x
```

TC → O(N)

Eg: $n = 36$



① → i

② → n/i

TC → O(√n)

```
if ( n, != 1)
    cnt++;
```

Q given l & r, print all primes between l & r.

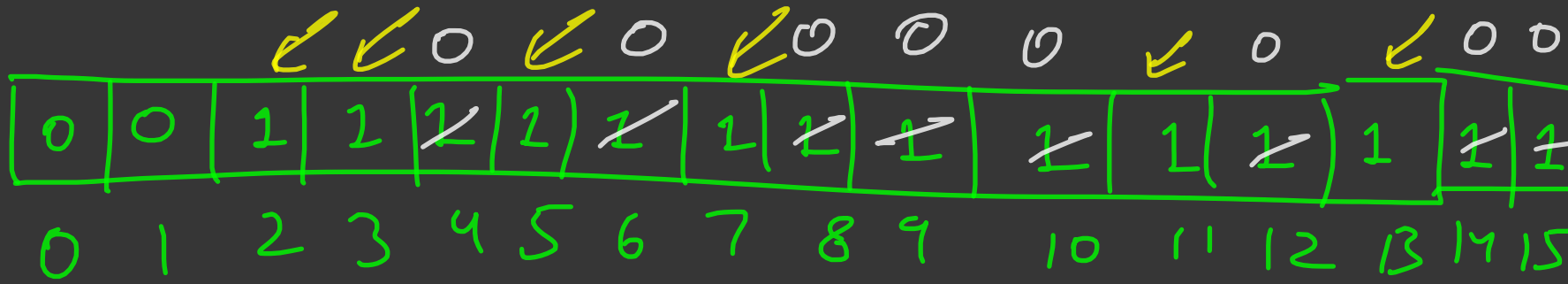
$1 \le l, r \le 10^6$

l to r → i → $o\sqrt{n}$

$10^6 \times 10^3 \rightarrow 10^9 \rightarrow$ TLE

100sec

sieve of Eratosthenes

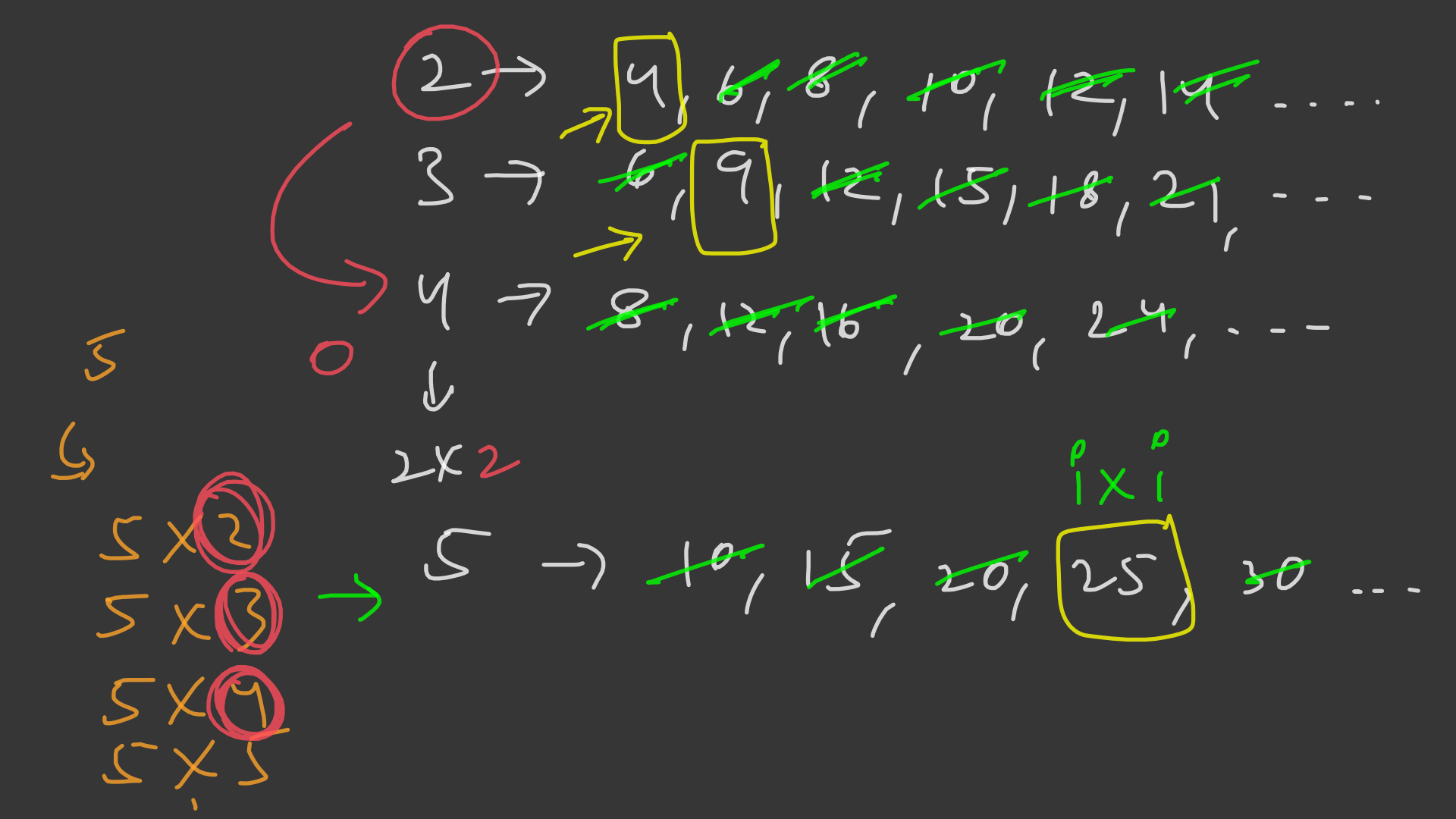


0 → not prime

1 → prime

(2) → 4, 6, 8, 10, 12 ...

[2 to N]


2 → 4, 6, 8, 10, 12, 14 ....
3 → 6, 9, 12, 15, 18, 21, ...
4 → 8, 12, 16, 20, 24, ...
0
5
5 x 2
5 x 3
5 x 4
5 x 5
2x2
5 → 10, 15, 20, 25, 30 ...
p
p
1 X 1

code →

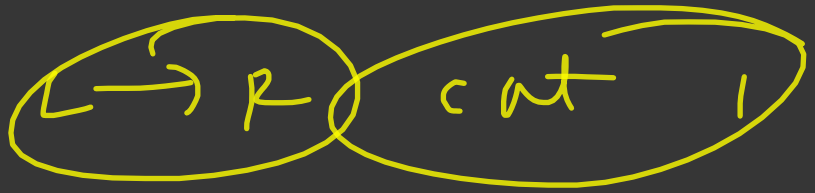
```
int sieve[1000001];
for(int i = 2; i <= 1e6, i++){
    sieve[i] = 1;
}
for(int i = 2; i <= N; i++){
    if(sieve[i] == 0) continue;
    for(int j = 2*i; j <= N; j += i){
        sieve[j] = 0;
    }
}
```

$i \times i \le N$

$j > N$ ⇒ N

$j = i \times i$

$i = \sqrt{N} + 1$



$Tc \rightarrow n \log(\log n)$

→ approx

Prime factorization →

* count of Factors of a number n are in a order $O(n^{1/3})$

36 → [2 x 2 x 3 x 3]

n

```
for (int i=2; i<=n; i++){
    while(n%i==0){
        n=n/i;
        print(i);
    }
}
```

TC → O(n)

$\sqrt{n}$ →

```
for (int i=2, i*i <= N; i++){
    while (n%i == 0){
        n = n/i;
        print(i);
    }
}
```

17 → prime

i = 2x

i = 3x

i = 4x

```
if (n > 1){
    print (n)
}
```

Q q queries given, print prime factorization of each of the q given numbers.

$q \leq 10^5$

$n \leq 10^6$

$q \times \sqrt{n} \rightarrow 10^8$ — TLE

sieve → smallest prime factor

Sieve

spf[i] → smallest prime factor of i

TC → $N \log\log N + q \times \log(n)$

$n/2 \to n/2 \to n/2 \to \dots \to 1$

↳ log(n)

Q/ Assume $n \leq 10^6$ given q queries every query has a number n. Tell me how many numbers b/w 1 to 10^6 have min prime factor as n.

[mp[spf[i]]++]

#

	Globally	Inside function
int/double	10^7	10^6
char		
bool	10^8	10^7

$1 \le L, R \le 1e^7$

↳ spz ✓ meuč ✓

$1 \le L, R \le 1e^8$

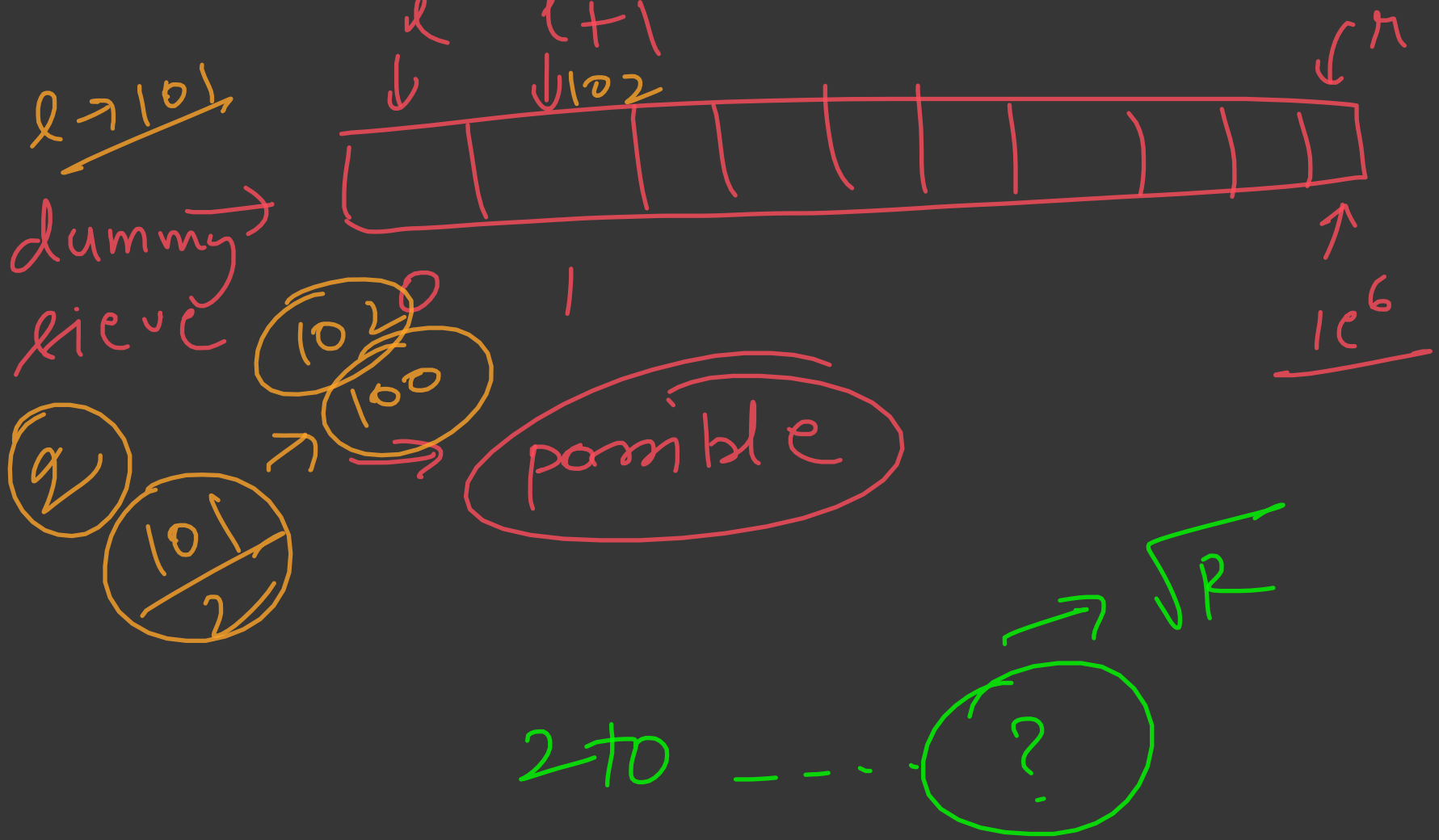
✓

Q. $1 \le L, R \le 10^9$

How many primes are there in the range L to R?

→ $1 \le R - L \le 10^6$

a ≥ 1


l
l+1
r
l → 101
102
dummy
sieve
1
10^6
102
100
101
2
possible
$\sqrt{R}$
2 to ---- ?

→ only primes till $\sqrt{R}$ will be required to mark numbers till R in the sieve

49

#

7

2 to 70

↳ primes

l = 1035

R = 5000