



Recursion & Memoization

Dev Karan Singh (devkaran1231)
Expert at codeforces (1817)
5 star at codechef (2040)

Today's plan

- Print increasing number from 1 to n
- Print decreasing number from n to 1
- Print factorial of a number
- Check if given string is a palindrome or not
- Linear search
- Nth fibonacci number (memoization)
- Print subsequence
- Count subsequence with given sum

→ Next
Monday (23rd)

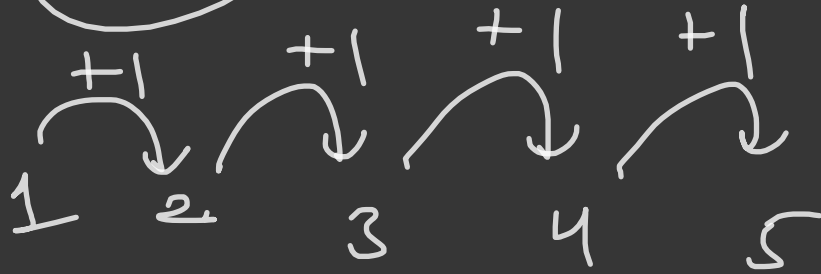
Recursion:

↳ a function calling itself

 $b()$ {
 [] Base case
 [$b()$] body of the function
 }

Q print increasing number from 1 to n

$$n=5$$



$$\begin{array}{r} i=5 \\ \hline n=5 \\ \hline \end{array}$$

```
void print (int i, int n){  
    if (i == n){  
        cout << "endl";  
        return;  
    }  
    cout << i << "  ",  
    print (i+1, n),  
}
```

3



output

1 2 3 4 5

$$i=1 \quad n=5$$

↓ ↗

$$i=2, \quad n=5$$

↓ ↗

$$i=3, \quad n=5$$

↓ ↗

1 2 3

done

i=1

```
print(1, 3) {  
  if (i == n) {  
    count;  
    return;  
  }  
  cout << " ";  
  print(2, 3);  
}
```

i=2

```
print(2, 3) {  
  if (i == n) {  
    count;  
    return;  
  }  
  cout << " ";  
  print(3, 3);  
}
```

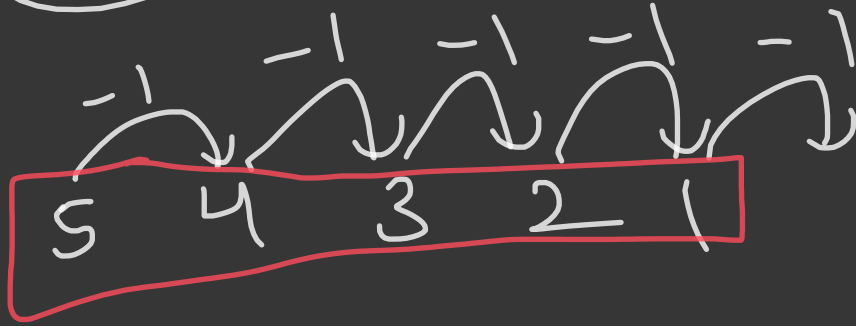
i=3

```
print(3, 3) {  
  if (i == n) {  
    cout << " ";  
    return;  
  }  
  cout << " ";  
  print(4, 3);  
}
```


Q print decreasing from n to 1

M1

n=5

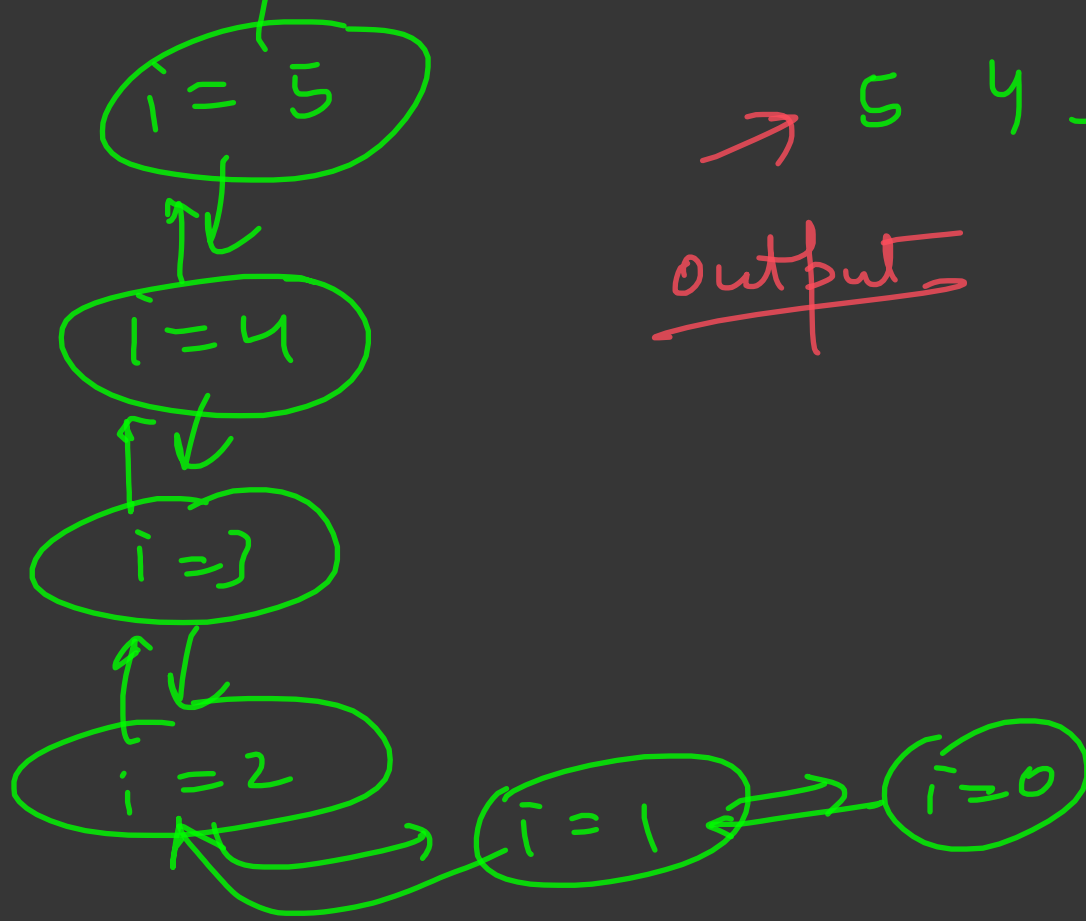


recursion
end



```
void print(int i){  
    if (i==0){  
        return;  
    }  
    cout << i;  
    print(i-1);  
}
```

3



→ 5 4 3 2 1
output

M2

```
void print (int i, int n) {  
    if (i == n) {  
        cout << " ";  
        return;  
    }  
    print (i+1, n);  
    cout << " ";  
}
```

$n=3$

$i=1$

```
print(1, 3) {  
  if (i == n) {  
    cout  
    return,  
  }  
  print(2, 3)  
  cout << i;  
}
```

$i=2$

```
print(2, 3) {  
  if (i == n) {  
    cout  
    return,  
  }  
  print(3, 3)  
  cout << i;  
}
```

$i=3$

```
print(3, 3) {  
  if (i == n) {  
    cout << i,  
    return;  
  }  
  print(4, 3)  
  cout << i,  
}
```

output: 3 2 1

Q return factorial of a number (n)
{ if (i == 0) {

$$0! = 1$$

$$1! = [0!] \times 1$$

$$2! = [1!] \times 2$$

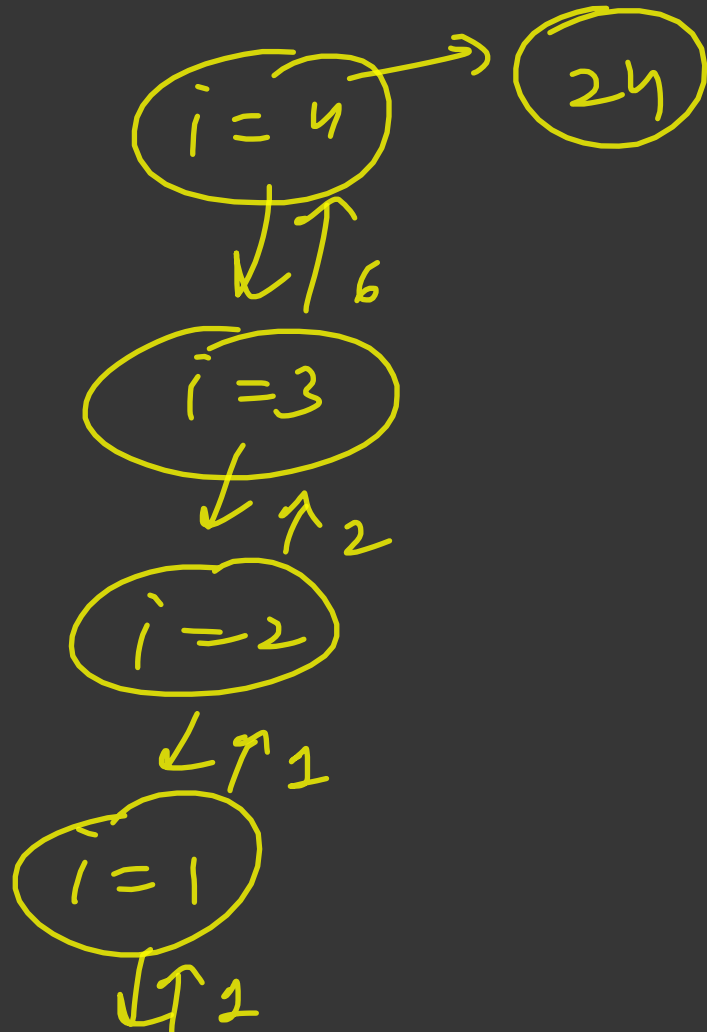
$$3! = [2!] \times 3$$

return 1,

}

return fact(i-1) * i;

}



Q

$n=7$

$i=0$

j

i

✓ ✓ ✓ ✓ ✓ ✓ ✓
0 1 2 3 4 5 6
a b c d c b a

$i \neq j \ (j < i) \{$

return true;

$\}$

$s[i] \neq s[j] \{$

```
        return false,  
    }  
    return  
        rec(i+1, j-1);
```

Que Linear search

$x = 11$

arr \rightarrow $[3, 7, 9, 11, 5, 4]$

$i = 0$ n arr

$find(i = 0, n, arr, x) \{$

$if (i == n) \{$

$return false;$

$\}$

$if (arr[i] == x) \{$

$return true;$

3

return^{find}(i+1, n, an, x);

↳

an → 3, 7, 9, 11, 5, 4

x = 11

↑ true

↑

$i = 0$

↓ ↑ true

$i = 1$

↓ ↑ true

$i = 2$

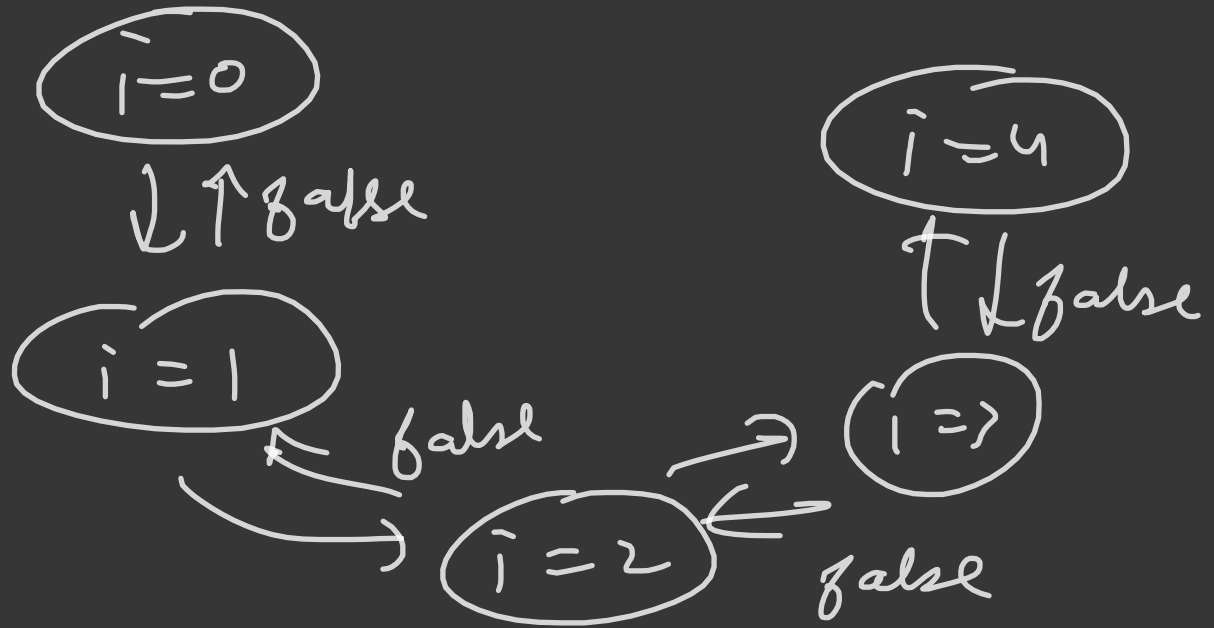
↔
true

$i = 3$

arr → [1, 2, 3, 4] $x = 5$
 0 1 2 3
 ↑ false

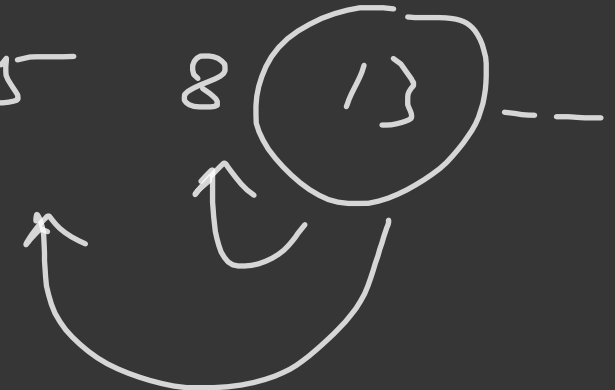
TC $\rightarrow O(N)$

space $\rightarrow O(N)$



Q return n^{th} fibonacci number

0 1 1 2 3 5 8 13 --



```
int fibo(int n){  
    if (n ≤ 1){  
        return n;  
    }  
}
```

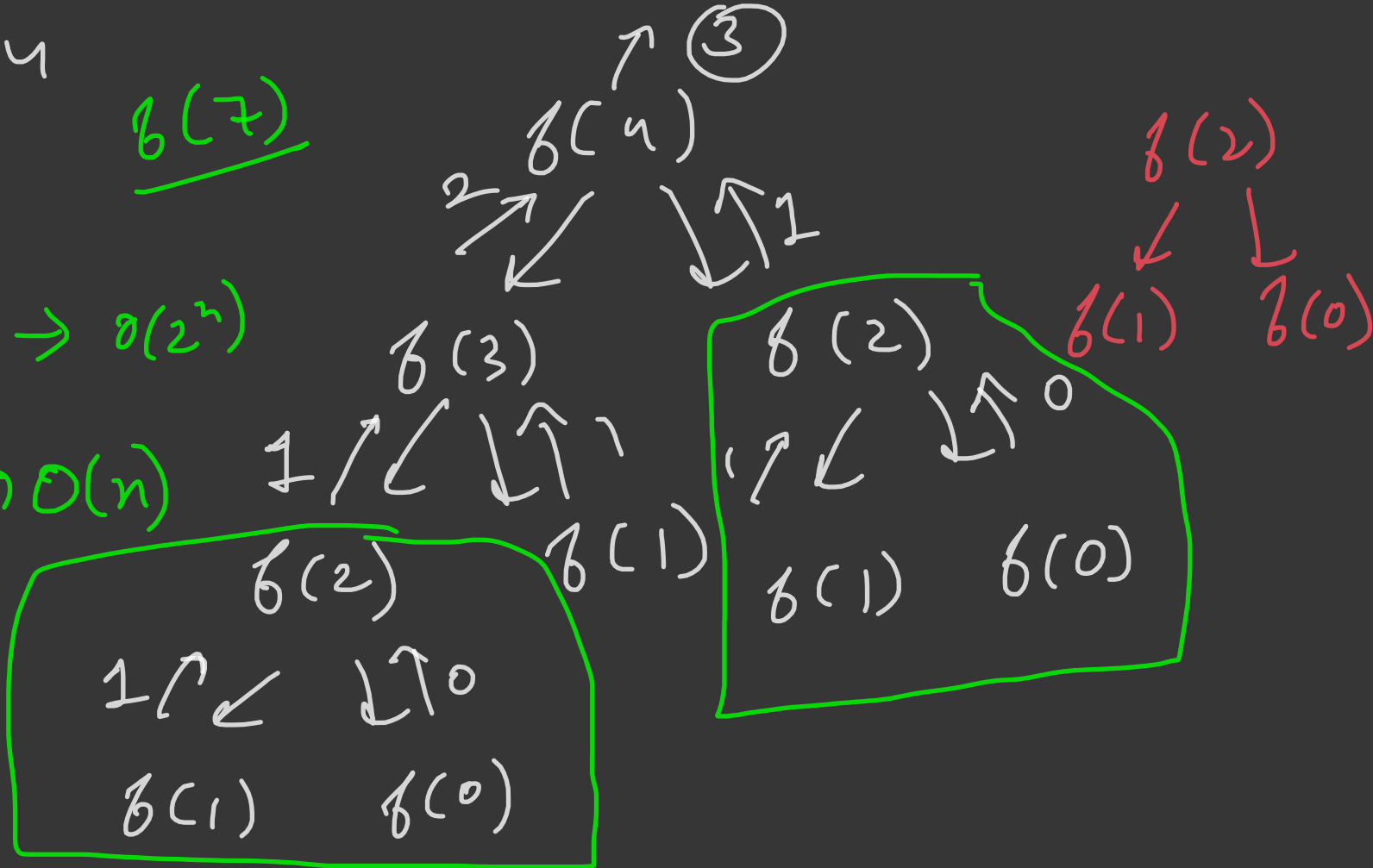
→ return (fibo($n-1$) + fibo($n-2$));

$$n = 4$$

$$\underline{f(7)}$$

$$\textcircled{TC} \rightarrow O(2^n)$$

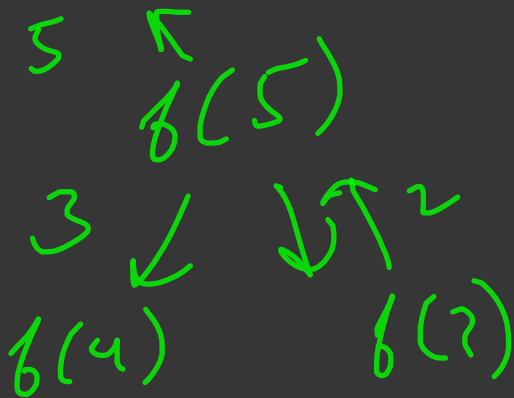
$$\textcircled{SC} \rightarrow O(n)$$



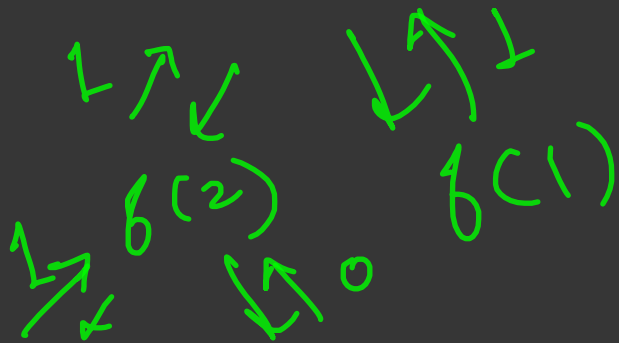
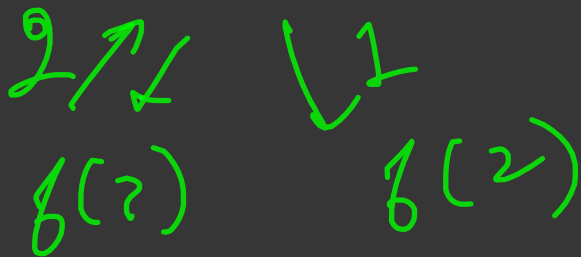
0	1	1	2	3	5
0	1	2	3	4	5

→

dp



$$2^n \rightarrow n$$



$$TC \rightarrow O(n)$$

$$SC \rightarrow O(n)$$

$f(1)$ $f(0)$

```
int fibo (int n) {  
    if (n <= 1) {  
        return n;
```



```
    }  
    ① if (dp[n] != -1) {  
        ② return dp[n];
```


3 }

$dp[n] = \text{fib}(n-1) + \text{fib}(n-2)$

return $dp[n]$,
}

an \rightarrow [2, 7, 4, 5, 1]

depends
on question

Que

subarray



contiguous
part of
the
array

v/s

subsequence v/s



Non contiguous
but
in order

Relative
positions
must be
same

subset



Anything
any order

