Combinatorics

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Combinatorics

- ncr
- How to deal with overflow
- ncr template
- Tricks
- Questions
  - Unique Paths
  - K - special cells
- Stars and Bars concept
  - Different ways to represent n as a sum of k non zero integers
  - Different ways to represent n as a sum of k non neg integers
  - Marbles

# #

No of ways

n_{C_r} → to select r objects from given n objects

↳ $\frac{n!}{(n-r)!(r)!}$

$$n_{C_r} = n_{C_{n-r}}$$

↳ (1)

5_{C_1} → $\frac{5!}{4!\,1!}$ → (5)

→ formula
→ precomp
→ compute & print.

Q given q queries for every query given n and r. Print $^{n}C_{r}$

$1 \le r \le n \le 10^5$ $Q \le 10^5$

Q cal $^{n}C_{r}$

$$\frac{n!}{(n-r)!(r)!}$$

$T_c \rightarrow O(N)$

QXN → TLE

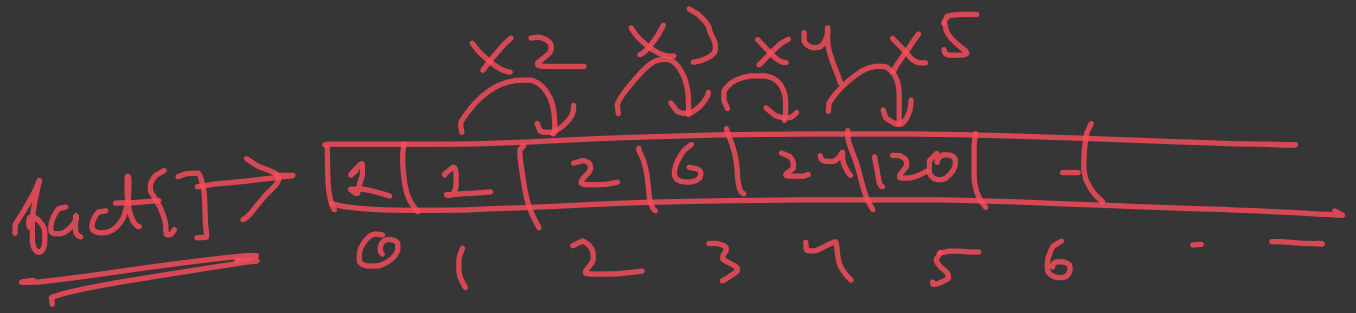
n

c_n

$n \rightarrow 1e^5$

$n \leq n$

$1e^5$



O(1) → $\left[\frac{fact[n]}{fact[r] \times fact[n-r]}\right]$

% mod

$n \rightarrow 20$

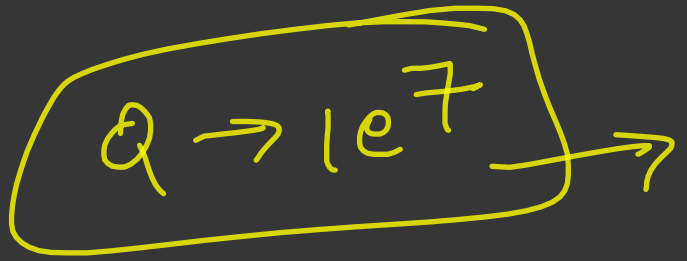
$r \rightarrow 10$

20 C 10 → fact[20]

$TC \rightarrow O(N) + O(Q \times \log mod)$

$1e^5 + 1e5 \times 20$

$\rightarrow O(1e^6)$



$TC \rightarrow 1e^5 + 1e7 \times 20$

$\rightarrow O(1e^8)$

TLE

$Q \rightarrow 1e7$

$Tc \rightarrow 1e5 \times 20 + 1e7$

$O(N) \times \log(mod) + O(Q)$

$$\frac{fact[n]}{fact[r] \times fact[n-r]}$$

$\frac{1}{fact[i]}$

$\downarrow$ mod

$$modinv[i] = (fact[i]^{-1}) \% mod$$

Q

$^{n}C_{r}$

print the answer with mod.

the answer is guaranteed to be $\leq 10^9 + 7$ / 10^{18}

int

long long int

$N \rightarrow 10^5$

n → small

n → close to n

$${}^{n}C_{2} \quad {}^{n}C_{5} \quad \cdots \quad {}^{n}C_{n-2} \quad {}^{n}C_{n-4}$$

n!

$${}^{10}C_{4} \rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \quad (1 \times 2 \times 3 \times 4 \times 5 \times 6)}$$

$n!$ $(n-n)!$

$$= \frac{10 \times 9 \times 8 \times 7}{4!}$$

```
// ans = 1;
for(int i = 1; i <= min(n, n-n); i++){
    ans = (ans * (n-i+1)) / i;
}
```

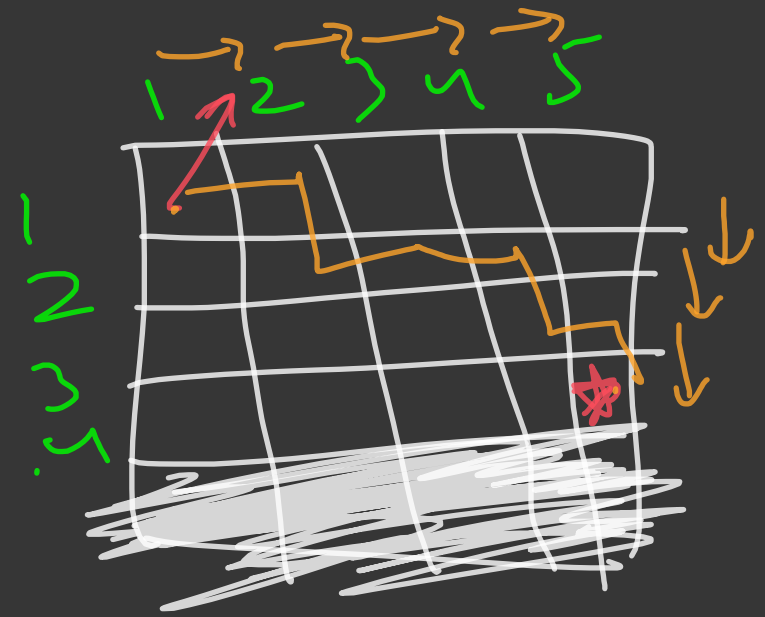
$n \to 10$
$r \to 4$

$$ans \to 1 \times \frac{10}{1} \times \frac{9}{2} \times \frac{8}{3} \times \frac{7}{4}$$

$$^{n}C_{r} \to \min(r, n-r)$$

$^{n}C_{n} \to$ Not pre

precomp.



M = 5

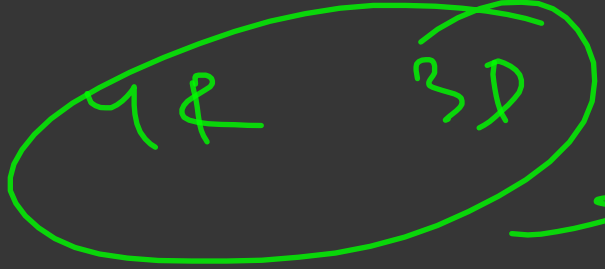
N x M

rows cols.

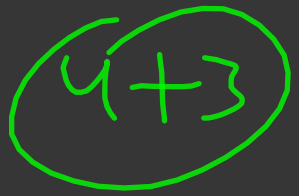
(N-1) steps down

(M-1) steps Right

R D R R D R R



→ path.



→ 7

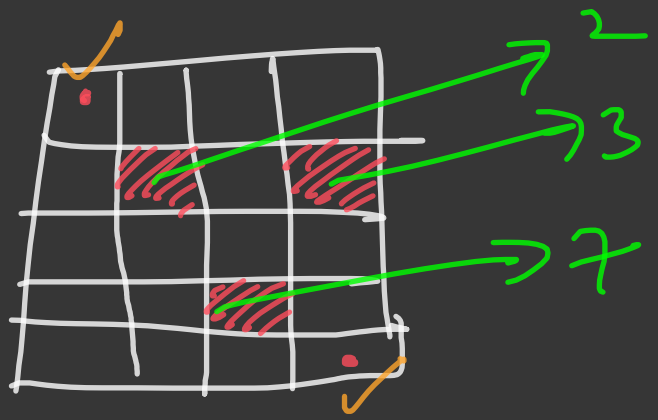
R D D D R R R

→ Ans

↓

$(N-1)$ D & $(M-1)$ R

$N-1+M-1 \rightarrow N+M-2$

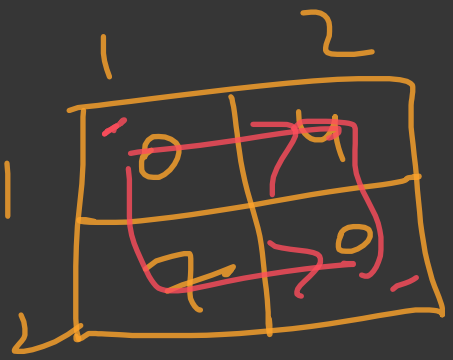


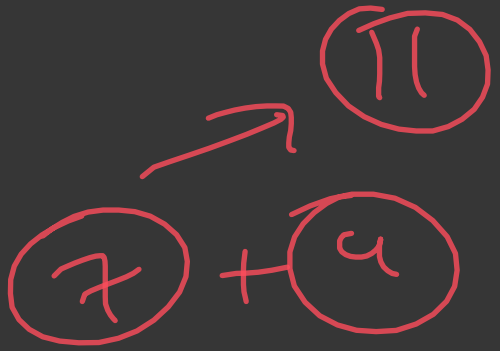
Eg:

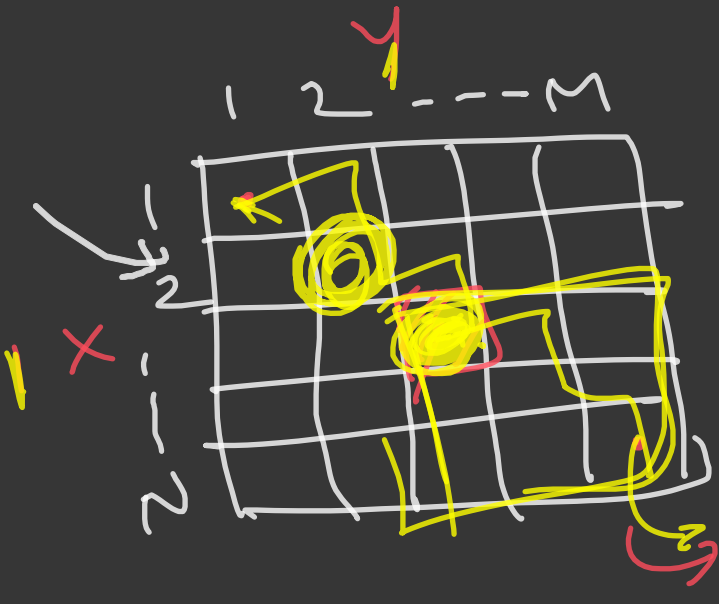
2 2 2

1 2 4

2 1 7







(N-X+1, M-Y+1)

$$P_i \times \binom{x+y-2}{x-1} \times \binom{N+M-x-y}{N-x}$$

Q

n = 5

K = 3

↳

(2, 2, 1) → 5

1 _ 1 _ 1 _ 1 _ 1

1, 1, 3 → 5

1, 3, 1 → 5

$^{n-1}C_{K-1}$ → Ans.

Q: different ways to represent n as sum of k

non-negative integers

Eg: $n = 5$, $k = 3$

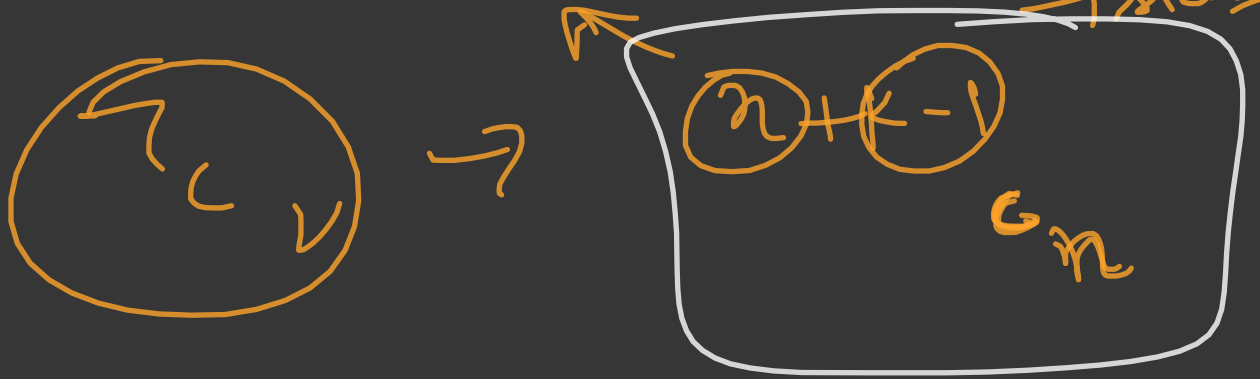
→ unity

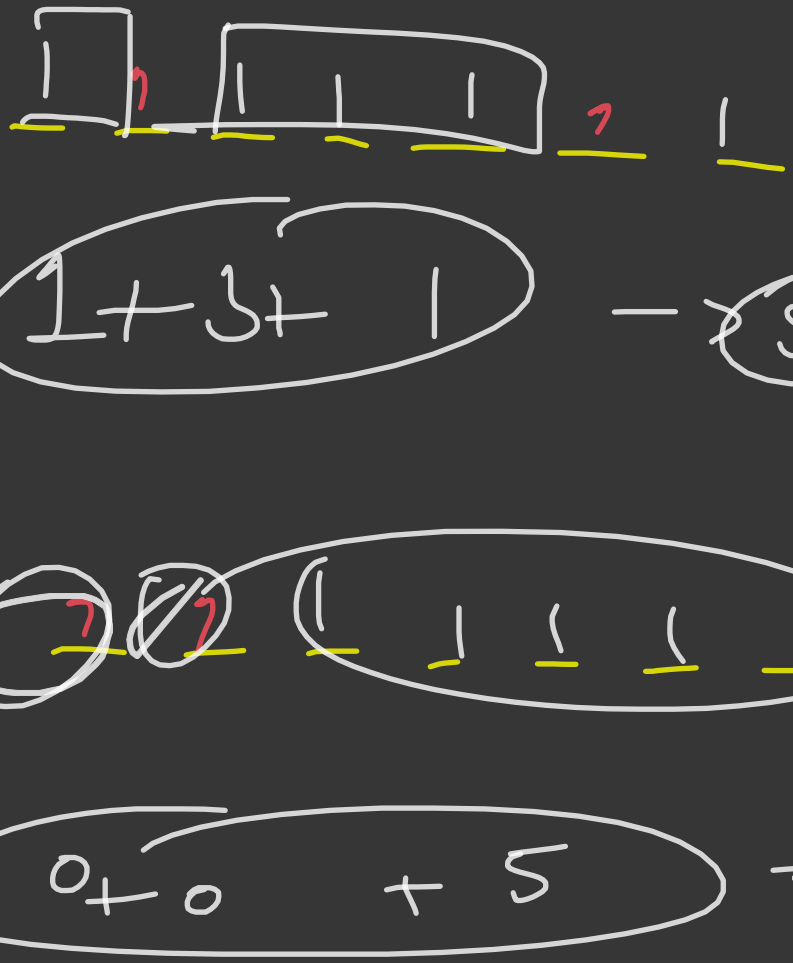
Bars

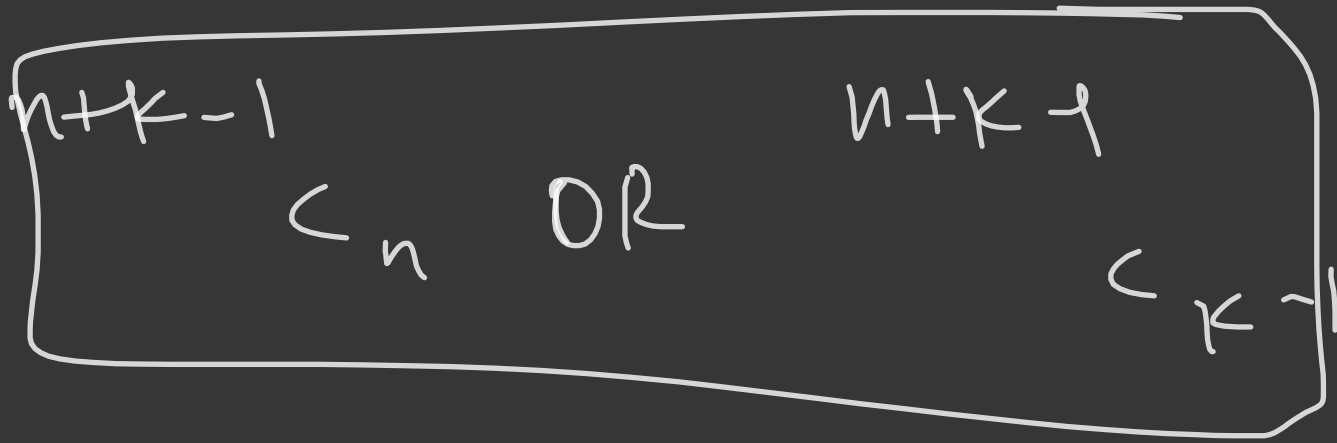
(a, b, c)

7 places chose 2 or 5

Bars from them




1 1 1 1 1 1
1 + 3 + 1
5
1 1 1 1 1
0 + 0 + 5
5


n+k-1
n+k-1
C n
OR
C k-1