A Comparative Study of Histogram Equalization Based Image Enhancement Techniques

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Outline

Introduction

Introducton

Digital image processing is the technology of applying a number of algorithms to process digital image. Basically it includes the following three steps:

- Importing the image via image acquisition tools.
- Analyzing and manipulating the image.
- Output (an altered image or image analysis report).

Introduction

Image enhancement: A collection of techniques that seek to improve the visual appearance of an image or convert images to a form better suited for analysis by human or machine. In general two major approaches, one is **gray level statistics based** and the other is **spatial frequency content** based.

The principle objective of image enhancement is to modify attributes of an image to make it more suitable for a given task and a specific observer.

Introduction

The existing image enhancement techniques can be classified into two catagories:

- Spatial domain enhancement.
- Frequency domain enhancement.

Spatial domain techniques are performed to the image plane itself and are based on direct manipulation of pixels in an image. **Histogram equalization** is a well known spatial domain enhancement technique due to its strong performance and easy algorithm in almost all types of images.

Aims and objectives

- To describe and implement four popular methods of histogram equalization on images with different levels of contrast.
- To compare among these five methods using traditional as well as sophisticated metric.
- To illustrate the application of histogram in the field of image processing.

Basic definitions and notations

Digital image

A digital image is a matrix representation of a two-dimensional image. It can be represented by the following matrix:

$$f(x,y) = \begin{pmatrix} f(0,0) & f(0,1) & ... & f(0,N-1) \\ f(1,0) & f(1,1) & ... & f(1,N-1) \\ . & . & ... & . \\ . & . & ... & . \\ . & . & ... & . \\ f(M-1,0) & f(M-1,1) & ... & f(M-1,N-1) \end{pmatrix} \tag{1}$$

It is advantageous to use a more traditional matrix notation to denote a digital image and its elements:

Basic definitions

$$\mathbf{A} = \begin{pmatrix} a_{0,0} & a_{0,0} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,0} & \cdots & a_{1,N-1} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ a_{M-1,0} & a_{M-1,0} & \cdots & a_{M-1,N-1} \end{pmatrix} \tag{2}$$

where, $a_{i,j} = f(x = i, y = j) = f(i, j)$ and thus equations (1) and (2) are identical.

Basic definitions

Pixel

Each element in the matrix is called an image element, picture element, pixel or pel.Thus a digital image $f(x, y)$ with M rows and N columns contains $M \times N$ number of pixels or image elements. In spatial domain technique for image processing, operation is done on this pixel.

Basic definitions

Neighbors of a pixel

The 4-neighbors of pixel p, $N_4(p)$ are the four pixels located at(shaded square) $(x+1, y)$, $(x-1, y)$, $(x, y+1)$, $(x, y-1)$

(x-1,y-1)	(x-1,y)	(x-1,y+1)
(x, y-1)	(x, y)	(x, y+1)
(x+1,y-1)	(x+1,y)	(x+1,y+1)

Table 1: 8-neighborhood.

The four diagonal neighbors of pixel p, $N_D(p)$ are the four pixels located at
$(x+1, y+1)$, $(x+1, y-1)$, $(x-1, y+1)$, $(x-1, y-1)$ and are denoted by $N_D(p)$. These points, together with the 4-neighbors, are called the 8-neighbors of p, denoted by $N_8(p)$.

Basic definitions

Bit-depth

It explains the number of possible colors from which a particular value can be selected by a pixel. For example: a binary image is an one bit image which can take any of thee two values: 0 or 1 (black or white). An $8 - bit$ gray-scale image can assign one of 256(2^8) colors to a pixel. The number of b bits required to store a digital image of size $M \times N$ with 2^k gray level is,

$$b = M \times N \times k \tag{3}$$

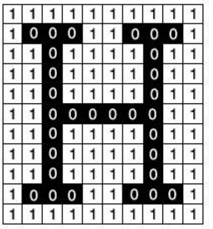

Figure 1: A bi-tonal image, where pixels can take any of the two values namely 0 and1.

Basic definitions

Image histogram

Image histogram provides information about brightness and contrast of an image. Discrete function $h(r_k)$ showing the number of occurrences n_k for the kth gray level r_k

$$h(r_k) = n_k \tag{4}$$

A common practice is to normalize a histogram by dividing each of its values by the total number of pixels, denoted by n. So,

$$p(r_k) = \frac{n_k}{n},\ \text{for } k = 0,\ 1,\ ...,\ L-1; \text{ and } \sum_{0}^{L-1} p(r_k) = 1 \tag{5}$$

Image histogram

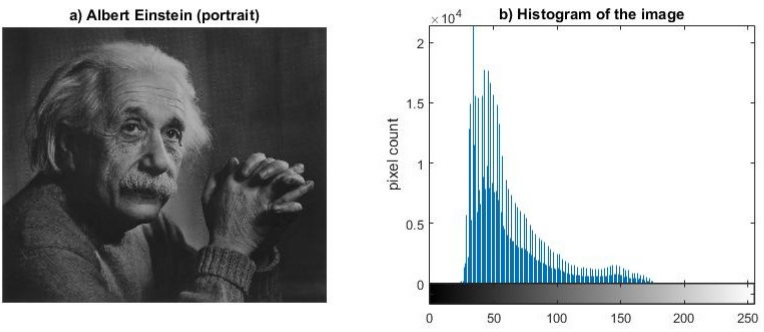

Figure 2: An 8-bit image of Albert Einstein and its histogram

Methodology

Methodology

Four different histogram equalization techniques has been used:

1. Global Histogram Equalization(GHE).
2. Brightness Preserving Bi-Histogram Equalization(BBHE).
3. Equal Area Dualistic Sub Image Histogram Equalization (DSIHE).
4. Recursive Mean Separate Histogram Equalization (RMSHE).

Mathematical formulation of GHE

Let, $\mathbf{X} = \{X(i,j)\}$ is a given image composed in L(for 8-bi image L $= 256$) discrete gray levels denoted as $X_0, X_1, ..., X_{L-1}$

- Calculate the probability density function.

$$p(X_k) = \frac{n_k}{n},\ \text{for} k = 0,\ 1, ...,\ L-1 \tag{6}$$

- Calculate cumulative distribution function by

$$c(x_k) = \sum_{j=0}^{k} p(X_j),\ \text{for}\ k = 0,\ 1, ...,\ L-1 \tag{7}$$

- On the basis of CDF, Define transformation function $f(x)$ as

$$f(X_k) = X_0 + (X_{L-1} - X_0)c(x_k) \tag{8}$$

- Using the transformation function, calculate new intensity values

Global Histogram Equalization(GHE)

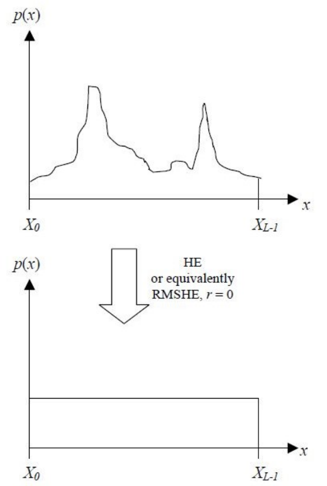

Figure 3: Histogram before and after HE or equivalently, RMSHE, r = 0.

An example of GHE

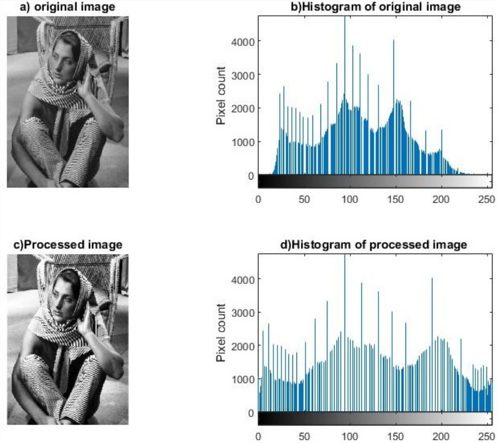

Figure 4: Contrast enhancement based on global histogram equalization.

Brightness Preserving Bi-Histogram Equalization(BBHE)

- Partitions Histogram in two sub-histograms and equalize them independently.
- Proposed to minimize mean intensity change.
- Ultimate goal is to preserve brightness and enhance contrast.
- Image parameters such as mean gray-scale level used for partitioning.

Equal Area Dualistic Sub Image Histogram Equalization (DSIHE).

This method is also known as Dualistic Sub Image Histogram Equalization(DSIHE).

- Image parameters such as median grayscale level used for partitioning.
- The input image is decomposed into two sub-images, being one dark and one bright.
- Then applies Histogram Equalization on two sub-images.

Mathematical Formulation for BBHE and DSIHE

- Input image $X(i,j)$ with gray levels 0 to 255.
- Image $X(i,j)$ is segmented by a section with gray level of X_m
- X_m (mean in case of BBHE and median in case of DSIHE)
- The image is decomposed into two sub images X_L and X_U.
- $$\mathbf{X} = \mathbf{X}_L \cup \mathbf{X}_U$$

  where

  $$\mathbf{X}_L = \{X(i,j) | X(i,j) \leq X_m, \forall X(i,j) \in \mathbf{X}\}$$

  and

  $$\mathbf{X}_U = \{X(i,j) | X(i,j) > X_m, \forall X(i,j) \in \mathbf{X}\}$$

Mathematical Formulation for BBHE and DSIHE

- X_L is composed by gray level of $\{l_0, l_1, ..., l_m\}$, X_U is composed by gray level of $\{l_{m+1}, l_{m+2}, ..., l_{L-1}\}$
- Respective probability density functions of the sub-images are:

$$p_L(X_k) = \frac{n_L^k}{n_L},\ \text{for } k =\ 0,\ 1, ...,\ m$$

and

$$p_U(X_k) = \frac{n_U^k}{n_U},\ \text{for } k =\ m+1,\ m+2, ...,\ L-1$$

Mathematical Formulation for BBHE and DSIHE

- n_L^k and n_U^k are the numbers of X_k
- $n_L = \sum_{k=0}^{m} n_L^k$, $n_U = \sum_{k=m+1}^{L-1} n_U^k$
- The respective cumulative density function for $\mathbf{X}_L$ and $\mathbf{X}_U$ are : $c_L(x) = \sum_{j=0}^{k} p_L(X_j)$ and $c_U(x) = \sum_{j=m+1}^{L-1} p_U(X_j)$
- Transformation function defined for exploiting the cumulative density functions: $f_L(x) = X_0 + (X_m - X_0)c_L(x)$ and $f_U(x) = X_{m+1} + (X_{L-1} - X_{m+1})c_U(x)$

Mathematical Formulation for BBHE and DSIHE

- Based on these transform functions, the decomposed sub-image are equalized independently.
- The composition of resulting equalized sub-images constitutes the output of BBHE or DSIHE as $\mathbf{Y} = \{Y(i,j)\}$ $= \mathrm{f}_L(\mathbf{X}_L) \cup f_U(\mathbf{X}_U)$ where $f_L(\mathbf{X}_L) = \{f_L(X(i,j)|\forall X(i,j) \in \mathbf{X}_L)\}$ and $f_U(\mathbf{X}_U) = \{f_U(X(i,j)|\forall X(i,j) \in \mathbf{X}_U)\}$

Algorithm for BBHE

- Obtain the original image.
- Get histogram of the original image.
- Calculate mean of the histogram.
- Divide the histogram on the basis of the mean in two parts.
- Equalize each part independently using PDF and CDF.
- Combine both sub-images for the processed image

Algorithm for BBHE

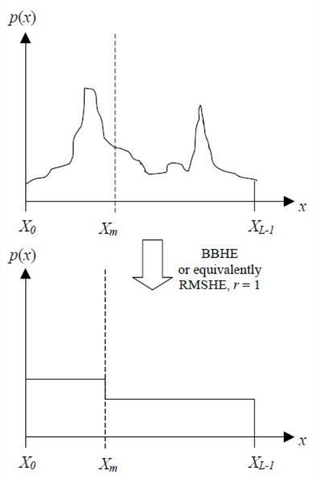

Figure 5: Histogram before and after BBHE or equivalently, RMSHE, r = 1.

An example of BBHE

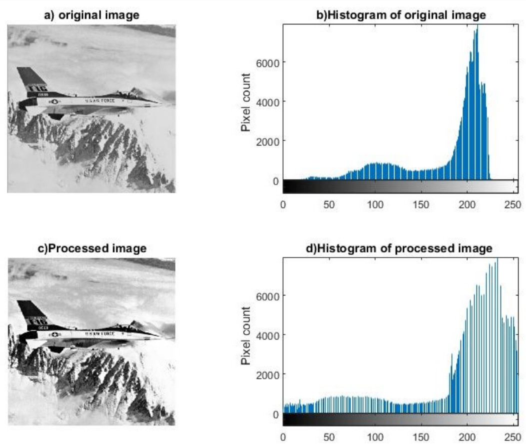

Figure 6: Contrast enhancement based on brightness preserving bi-histogram equalization.

Algorithm for DSIHE

- Obtain the original image.
- Get histogram of the original image.
- Calculate median of the histogram.
- Divide the histogram on the basis of the median in two parts.
- Equalize each part independently using PDF and CDF.
- Combine both sub-images for the processed image

An example of DSIHE

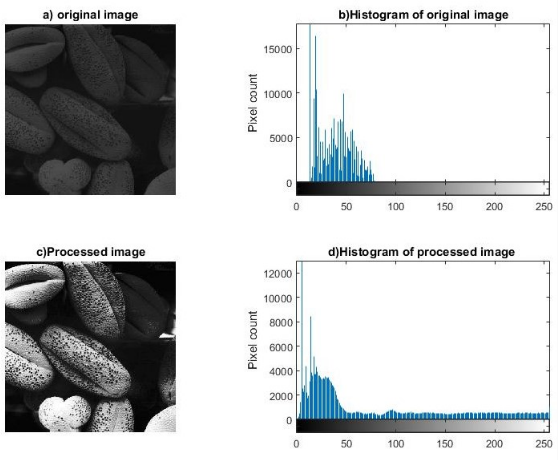

Figure 7: Contrast enhancement based on dualistic sub-image histogram equalization.

Recursive Mean Separate Histogram Equalization (RMSHE)

- Generalization of HE and BBHE in term of brightness preservation
- Recursively separating the input histogram based on the mean

Recursive Mean Separate Histogram Equalization (RMSHE)

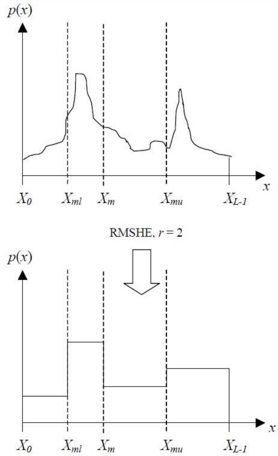

Figure 8: Recursive mean separated histogram equalization with recursion level r=2

An example of RMSHE

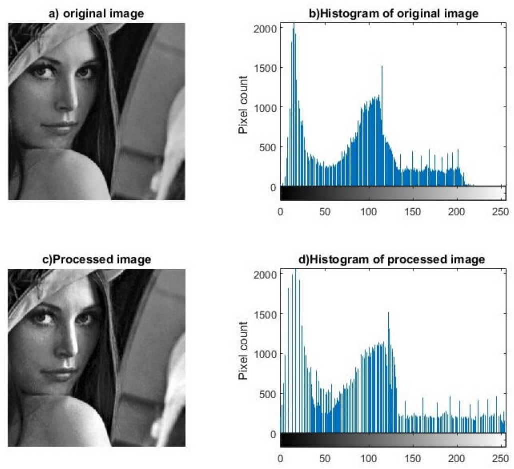

Figure 9: Contrast enhancement based on recursive mean separate histogram equalization.

Results and discussion

Quality assessment

The following measurement are used to make comparison among the histogram equalization techniques

- **Mean squared error (MSE)** is the average of squared intensity differences distorted and reference image pixels. Lower value of MSE means that the image is of good quality.
- **Peak signal to noise ratio(PSNR)** is the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. It varies between 25 to 40 dB. Higher value of PSNR is good
- **Structural similarity index(SSIM)** varies between 0 to 1. The value 1 means, the image is of best quality.

Global histogram equalization

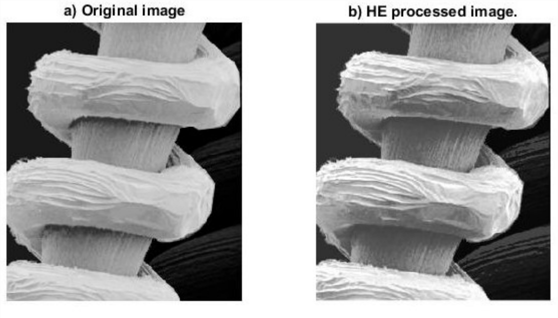

Figure 10: Contrast enhancement based on histogram equalization.

Brightness preserving bi-histogram equalization

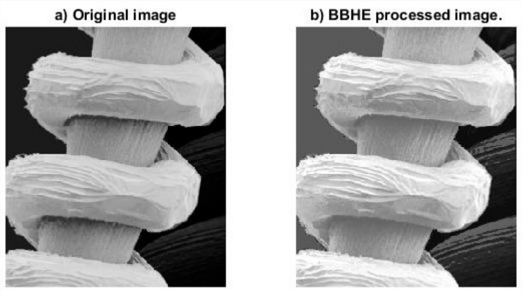

Figure 11: Contrast enhancement based on brightness preserving bi-histogram equalization.

Dualistic sub-image histogram equalization

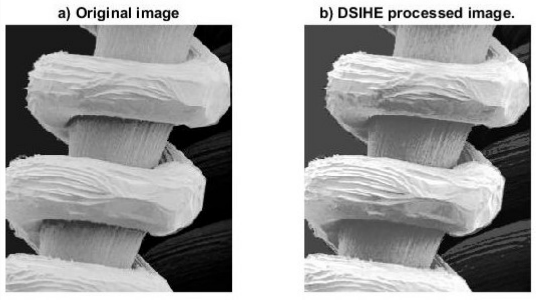

Figure 12: Contrast enhancement based on dualistic sub-image histogram equalization.

Recursive mean separate histogram equalization

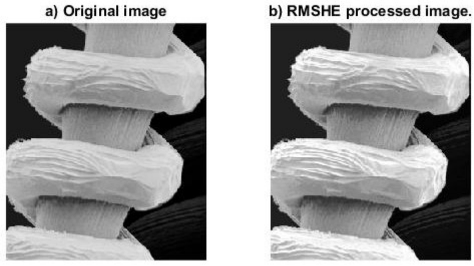

Figure 13: Contrast enhancement based on recursive mean separate histogram equalization.

Experimental results

Simulation results for 'Tungsten-filament' and 'Barbara' are presented in table 2 and 3. and

Methods	Mean	SD	SSIM	MSE	PSNR
Tungsten filament	128.11	75.31	–	–	–
GHE	127.71	73.5	0.79991	478.83	21.32
BBHE	150.5	69.05	0.80593	843.65	18.86
DSHE	140.43	72.94	0.79856	533.68	20.85
RMSHE	133.99	79.97	0.90909	139.46	26.68

Table 2: Comparison of various histogram equalization methods using objective image quality measures

Methods	Mean	SD	SSIM	MSE	PSNR
Barbara	111.5	48.15	–	–	–
GHE	127.48	73.88	0.875	969.18	18.26
BBHE	118.44	73.77	0.868	782.22	19.19
DSIHE	117.94	73.77	0.867	777.89	19.22
RMSHE	115.93	61.01	0.937	243.36	24.26

Table 3: Comparison of various histogram equalization methods using objective image quality measures

Conclusion

Conclusion

- The experimental results shows that RMSHE processed 'Tungsten-filament' image has lowest MSE , highest PSNR and highest SSIM among these four techniques.
- Similar result shows for 'Barbara' image.

So, recursive mean separate histogram equalization(RMSHE) performs well according to this performance measure as well as visual assessment.

Appendix

Reference i

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