

# Assignment 1

## Part 1: Theoretical

a)  $x_{\text{even}}[n] = x[n] + x[-n]/2$

$$x_{\text{odd}}[n] = x[n] - x[-n]/2$$

$$x[-n] = \{1, 3, 7, 8, \underline{6}, 1\}$$

$$x_{\text{even}}[n] = \{4.5, \underline{6}, 4.5, 1, 3, 7\}$$

$$x_{\text{odd}}[n] = \{-3.5, \underline{0}, 3.5, 0, 0, 0\}$$

b)  $2 \cdot 1/2\pi = 0.31$  not rational non-periodic

$$3/4 \pi \cdot 1/2 \pi = 3/8 \text{ rational periodic}$$

Periodic + non-periodic = not periodic so the whole signal is non periodic

### c) Time invariance

$$y_1[n] = n \cdot x_1[n]^2 \cdot \cos[\omega_0 n]$$

$$y_1[n-n_0] = (n-n_0) \cdot x_1[n-n_0]^2 \cdot \cos[\omega_0(n-n_0)]$$

$$x_2 = x_1[n-n_0]$$

$$y_2[n] = n \cdot x_2[n]^2 \cdot \cos[\omega_0 n] = n \cdot x_1[n-n_0]^2 \cdot \cos[\omega_0 n]$$

$y_2[n]$  not equal  $y_1[n-n_0]$  so Time variant

### linearity

$$y_1[n] = n \cdot x_1[n]^2 \cdot \cos[\omega_0 n]$$

$$y_2[n] = n \cdot x_2[n]^2 \cdot \cos[\omega_0 n]$$

$$\alpha y_1[n] + \beta y_2[n] = \alpha n \cdot x_1[n]^2 \cdot \cos[\omega_0 n] + \beta n \cdot x_2[n]^2 \cdot \cos[\omega_0 n]$$

$$x_3[n] = \alpha x_1[n] + \beta x_2[n]$$

$$Y_3[n] = n \cdot x_3[n]^2 \cdot \cos[\omega_0 n] = n \cdot (\alpha x_1[n] + \beta x_2[n])^2 \cdot \cos[\omega_0 n]$$

$Y_3[n]$  not equal  $\alpha y_1[n] + \beta y_2[n]$  not linear

## Causality

Casual n depends only on present and past values of the input doesn't depend on future

## Memory

memoryless if the output at time n depends only on  $x[n]$  and not on past or future values memoryless

## BIBO Stability

**Not stable** -> a bounded input  $|x[n]| < B_i < \infty, \forall n$

produces an unbounded output

$$|x[n]|^2 \leq B_i^2, |\cos(\omega_0 n)| \leq 1$$

n **grows unbounded** as  $n \rightarrow \infty$  leading to:  $|y[n]| = |n \cdot x[n]^2 \cdot \cos(\omega_0 n)| \leq n$ .

$$B_i^2 \cdot 1$$

## Question 2

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$$\text{Length} = 4 + 5 - 1 = 8$$

From -1 to 6

$$y[-1] = x[0] \cdot h[-1-0] = 1 \cdot 2 = 2$$

$$y[0] = x[0]h[0] + x[1]h[-1] = 1*2 + (-2)*2 = 2 - 4 = -2$$

$$y[1] = x[0]h[1] + x[1]h[0] + x[2]h[-1] = 1*4 + (-2)*2 + 0*2 = 4 - 4 + 0 = 0$$

$$y[2] = x[0]h[2] + x[1]h[1] + x[2]h[0] = 1*-5 + -2*4 + 0*2 + 3*2 = -5 - 8 + 0 + 6 = -7$$

$$y[3] = x[1]h[2] + x[2]h[1] + x[3]h[0] + x[4]h[-1] = -2*-$$

$$5 + 0*4 + 3*2 + 5*2 = 10 + 0 + 6 + 10 = 26$$

$$y[4] = x[2]h[2] + x[3]h[1] + x[4]h[0] = 0*-5 + 3*4 + 5*2 = 0 + 12 + 10 = 22$$

$$y[5] = x[3]h[2] + x[4]h[1] = (3)(-5) + (5)(4) = -15 + 20 = 5$$

$$y[6] = x[4]h[2] = (5)(-5) = -25$$

$$y[n] = \{2, -2, 0, -7, 26, 22, 5, -25\}$$

### Question 3

$$\text{Length} = 5 + 4 - 1 = 8 \text{ from } -3 \text{ to } 6$$

$$y[-3] = x1[3]x2[0] + x1[4]x2[1] = 5*2 + 3*4 = 10 + 12 = 22$$

$$y[-2] = x1[2]x2[0] + x1[3]x2[1] + x1[4]x2[2] = 1*2 + 5*4 + 3*1 = 2 + 20 + 3 = 25$$

$$y[-1] = x1[1]x2[0] + x1[2]x2[1] + x1[3]x2[2] + x1[4]x2[3] =$$

$$2*2 + 1*4 + 5*1 + 3*8 = 4 + 4 + 5 + 24 = 37$$

$$y[0] = x1[0]x2[0] + x1[1]x2[1] + x1[2]x2[2] + x1[3]x2[3] = 2*2 + 2*4 + 1*$$

$$1 + 5*8 = 4 + 8 + 1 + 40 = 53$$

$$y[1] = x1[0]x2[1] + x1[1]x2[2] + x1[2]x2[3] = 2*4 + 2*1 + 1*8 = 8 + 2 +$$

$$= 18$$

$$y[2]=x_1[0]x_2[2]+x_1[1]x_2[3]=2*1+2*8=2+16=18$$

$$y[3]=x_1[0]x_2[3]=2*8=16$$

$$y[n]=\{22,25,37,\underline{53},18,18,16\}$$

#### Question 4

a)  $x(z)=-5+z^{-1}+0+0+z^{-4}+5z^{-5}$

ROC:  $z \neq 0$

b)  $x(z)=0+z^2+5z^1+5z^0+z^{-1}+0$

ROC:  $z \neq 0$  and  $\infty$

c)  $x(z)=(1/1-1/8z^{-1})+(1/1-1/9z^{-1})$

$x(z)=(z/z-8)+(z/z-9)$  converge if only  $|z|>8$  and  $|z|>9$  so ROC  $|z|>9$