Instructor Solutions Manual for Physics by Halliday, Resnick, and Krane

Paul Stanley
Beloit College

Volume 1: Chapters 1-24

A Note To The Instructor...

The solutions here are somewhat brief, as they are designed for the instructor, not for the student. Check with the publishers before electronically posting any part of these solutions; website, ftp, or server access *must* be restricted to your students.

I have been somewhat casual about subscripts whenever it is obvious that a problem is one dimensional, or that the choice of the coordinate system is irrelevant to the *numerical* solution. Although this does not change the validity of the answer, it will sometimes obfuscate the approach if viewed by a novice.

There are some *traditional* formula, such as

$$v_x^2 = v_{0x}^2 + 2a_x x,$$

which are not used in the text. The worked solutions use only material from the text, so there may be times when the solution here seems unnecessarily convoluted and drawn out. Yes, I know an easier approach existed. But if it was not in the text, I did not use it here.

I also tried to avoid reinventing the wheel. There are some exercises and problems in the text which build upon previous exercises and problems. Instead of rederiving expressions, I simply refer you to the previous solution.

I adopt a different approach for rounding of significant figures than previous authors; in particular, I usually round intermediate answers. As such, some of my answers will differ from those in the back of the book.

Exercises and Problems which are enclosed in a box also appear in the Student's Solution Manual with considerably more detail and, when appropriate, include discussion on any physical implications of the answer. These student solutions carefully discuss the steps required for solving problems, point out the relevant equation numbers, or even specify where in the text additional information can be found. When two almost equivalent methods of solution exist, often both are presented. You are encouraged to refer students to the Student's Solution Manual for these exercises and problems. However, the material from the Student's Solution Manual must *not* be copied.

Paul Stanley
Beloit College
stanley@clunet.edu

E1-1 (a) Megaphones; (b) Microphones; (c) Decacards (Deck of Cards); (d) Gigalows (Gigolos); (e) Terabulls (Terribles); (f) Decimates; (g) Centipedes; (h) Nanonanettes (?); (i) Picoboos (Peek-a-Boo); (j) Attoboys ('atta boy); (k) Two Hectowithits (To Heck With It); (l) Two Kilomockingbirds (To Kill A Mockingbird, or Tequila Mockingbird).

E1-2 (a) $\$36,000/52$ week $= \$692$/week. (b) $\$10,000,000/(20 \times 12$ month$) = \$41,700$/month. (c) $30 \times 10^9/8 = 3.75 \times 10^9$.

$\boxed{\textbf{E1-3}}$ Multiply out the factors which make up a century.

$$1 \text{ century} = 100 \text{ years} \left(\frac{365 \text{ days}}{1 \text{ year}}\right)\left(\frac{24 \text{ hours}}{1 \text{ day}}\right)\left(\frac{60 \text{ minutes}}{1 \text{ hour}}\right)$$

This gives 5.256×10^7 minutes in a century, so a microcentury is 52.56 minutes.

The percentage difference from Fermi's approximation is (2.56 min)/(50 min) $\times$ 100% or 5.12%.

E1-4 (3000 mi)/(3 hr) = 1000 mi/timezone-hour. There are 24 time-zones, so the circumference is approximately 24×1000 mi $= 24,000$ miles.

E1-5 Actual number of seconds in a year is

$$(365.25 \text{ days})\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 3.1558 \times 10^7 \text{ s}.$$

The percentage error of the approximation is then

$$\frac{3.1416 \times 10^7 \text{ s} - 3.1558 \times 10^7 \text{ s}}{3.1558 \times 10^7 \text{ s}} = -0.45\,\%.$$

E1-6 (a) 10^{-8} seconds per shake means 10^8 shakes per second. There are

$$\left(\frac{365 \text{ days}}{1 \text{ year}}\right)\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 3.1536 \times 10^7 \text{ s/year}.$$

This means there are more shakes in a second.

(b) Humans have existed for a fraction of

$$10^6 \text{ years}/10^{10} \text{ years} = 10^{-4}.$$

That fraction of a day is

$$10^{-4}\,(24 \text{ hr})\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 8.64\,\text{s}.$$

$\boxed{\textbf{E1-7}}$ We'll assume, for convenience only, that the runner with the longer time ran *exactly* one mile. Let the speed of the runner with the shorter time be given by v_1, and call the distance actually ran by this runner d_1. Then $v_1 = d_1/t_1$. Similarly, $v_2 = d_2/t_2$ for the other runner, and $d_2 = 1$ mile.

We want to know when $v_1 > v_2$. Substitute our expressions for speed, and get $d_1/t_1 > d_2/t_2$. Rearrange, and $d_1/d_2 > t_1/t_2$ or $d_1/d_2 > 0.99937$. Then $d_1 > 0.99937$ mile $\times$ (5280 feet/1 mile) or $d_1 > 5276.7$ feet is the condition that the first runner was indeed faster. The first track can be no more than 3.3 feet too short to guarantee that the first runner was faster.

E1-8 We will wait until a day's worth of minutes have been gained. That would be

$$(24\text{ hr})\left(\frac{60\text{ min}}{1\text{ hr}}\right) = 1440\text{ min}.$$

The clock gains one minute per day, so we need to wait 1,440 days, or almost four years. Of course, if it is an older clock with hands that only read 12 hours (instead of 24), then after only 720 days the clock would be correct.

E1-9 First find the "logarithmic average" by

$$\begin{aligned}
\log t_{\text{av}} &= \frac{1}{2}\left(\log(5\times 10^{17}) + \log(6\times 10^{-15})\right), \\
&= \frac{1}{2}\log\left(5\times 10^{17}\times 6\times 10^{-15}\right), \\
&= \frac{1}{2}\log 3000 = \log\left(\sqrt{3000}\right).
\end{aligned}$$

Solve, and $t_{\text{av}} = 54.8$ seconds.

E1-10 After 20 centuries the day would have increased in length by a total of $20\times 0.001\,\text{s} = 0.02\,\text{s}$. The cumulative effect would by the product of the *average* increase and the number of days; that average is half of the maximum, so the cumulative effect is $\frac{1}{2}(2000)(365)(0.02\,\text{s}) = 7300\,\text{s}$. That's about 2 hours.

E1-11 Lunar months are based on the Earth's position, and as the Earth moves around the orbit the Moon has farther to go to complete a phase. In 27.3 days the Moon may have orbited through 360°, but since the Earth moved through $(27.3/365)\times 360^\circ = 27^\circ$ the Moon needs to move 27° farther to catch up. That will take $(27^\circ/360^\circ)\times 27.3$ days $= 2.05$ days, but in that time the Earth would have moved on yet farther, and the moon will need to catch up again. How much farther? $(2.05/365)\times 360^\circ = 2.02^\circ$ which means $(2.02^\circ/360^\circ)\times 27.3$ days $= 0.153$ days. The total so far is 2.2 days longer; we could go farther, but at our accuracy level, it isn't worth it.

E1-12 $(1.9\,\text{m})(3.281\text{ ft}/1.000\,\text{m}) = 6.2$ ft, or just under 6 feet, 3 inches.

E1-13 (a) 100 meters = 328.1 feet (Appendix G), or 328.1/3 = 10.9 yards. This is 28 feet longer than 100 yards, or (28 ft)(0.3048 m/ft) = 8.5 m. (b) A metric mile is $(1500\,\text{m})(6.214\times 10^{-4}\text{ mi/m}) = 0.932$ mi. I'd rather run the metric mile.

E1-14 There are

$$300,000\text{ years}\left(\frac{365.25\text{ days}}{1\text{ year}}\right)\left(\frac{24\text{ hr}}{1\text{ day}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 9.5\times 10^{12}\text{s}$$

that will elapse before the cesium clock is in error by 1 s. This is almost 1 part in 10^{13}. This kind of accuracy with respect to 2572 miles is

$$10^{-13}(2572\text{ mi})\left(\frac{1609\,\text{m}}{1\text{ mi}}\right) = 413\text{ nm}.$$

$\boxed{\textbf{E1-15}}$ The volume of Antarctica is approximated by the area of the base time the height; the area of the base is the area of a semicircle. Then

$$V = Ah = \left(\frac{1}{2}\pi r^2\right) h.$$

The volume is

$$\begin{aligned} V &= \frac{1}{2}(3.14)(2000 \times 1000 \text{ m})^2(3000 \text{ m}) = 1.88 \times 10^{16} \text{ m}^3 \\ &= 1.88 \times 10^{16} \text{ m}^3 \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 1.88 \times 10^{22} \text{ cm}^3. \end{aligned}$$

E1-16 The volume is $(77\times10^4\,\text{m}^2)(26\,\text{m}) = 2.00\times10^7\,\text{m}^3$. This is equivalent to

$$(2.00\times10^7\text{ m}^3)(10^{-3}\text{ km/m})^3 = 0.02\text{ km}^3.$$

E1-17 (a) $C = 2\pi r = 2\pi(6.37 \times 10^3 \text{ km}) = 4.00 \times 10^4$ km. (b) $A = 4\pi r^2 = 4\pi(6.37 \times 10^3 \text{ km})^2 = 5.10 \times 10^8$ km. (c) $V = \frac{4}{3}\pi(6.37 \times 10^3 \text{ km})^3 = 1.08 \times 10^{12} \text{ km}^3$.

E1-18 The conversions: squirrel, 19 km/hr(1000 m/km)/(3600 s/hr) = 5.3 m/s;
rabbit, 30 knots(1.688ft/s/knot)(0.3048 m/ft) = 15 m/s;
snail, 0.030 mi/hr(1609 m/mi)/(3600 s/hr) = 0.013 m/s;
spider, 1.8 ft/s(0.3048 m/ft) = 0.55 m/s;
cheetah, 1.9 km/min(1000 m/km)/(60 s/min) = 32 m/s;
human, 1000 cm/s/(100 cm/m) = 10 m/s;
fox, 1100 m/min/(60 s/min) = 18 m/s;
lion, 1900 km/day(1000 m/km)/(86, 400 s/day) = 22 m/s.
The order is snail, spider, squirrel, human, rabbit, fox, lion, cheetah.

$\boxed{\textbf{E1-19}}$ One light-year is the distance traveled by light in one year, or $(3 \times 10^8 \text{ m/s}) \times (1 \text{ year})$. Then

$$19,200\frac{\text{mi}}{\text{hr}}\left(\frac{\text{light-year}}{(3 \times 10^8 \text{ m/s}) \times (1 \text{ year})}\right)\left(\frac{1609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right)\left(\frac{100 \text{ year}}{1 \text{ century}}\right),$$

which is equal to 0.00286 light-year/century.

E1-20 Start with the British units inverted,

$$\frac{\text{gal}}{30.0 \text{ mi}}\left(\frac{231 \text{ in}^3}{\text{gal}}\right)\left(\frac{1.639 \times 10^{-2} \text{ L}}{\text{in}^3}\right)\left(\frac{\text{mi}}{1.609 \text{ km}}\right) = 7.84 \times 10^{-2} \text{ L/km}.$$

E1-21 (b) A light-year is

$$(3.00 \times 10^5 \text{ km/s})\left(\frac{3600 \text{ s}}{1 \text{ hr}}\right)\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)(365 \text{ days}) = 9.46 \times 10^{12} \text{ km}.$$

A parsec is

$$\frac{1.50 \times 10^8 \text{ km}}{0^\circ\, 0'\, 1''}\left(\frac{360^\circ}{2\pi \text{ rad}}\right) = \frac{1.50 \times 10^8 \text{ km}}{(1/3600)^\circ}\left(\frac{360^\circ}{2\pi \text{ rad}}\right) = 3.09 \times 10^{13} \text{ km}.$$

(a) $(1.5 \times 10^8 \text{ km})/(3.09 \times 10^{13} \text{ km/pc} = 4.85 \times 10^{-6}$ pc. $(1.5 \times 10^8 \text{ km})/(9.46 \times 10^{12} \text{ km/ly}) = 1.59 \times 10^{-5}$ ly.

E1-22 First find the "logarithmic average" by

$$\begin{aligned}
\log d_{\rm av} &= \frac{1}{2}\left(\log(2\times 10^{26}) + \log(1\times 10^{-15})\right),\\
&= \frac{1}{2}\log\left(2\times 10^{26}\times 1\times 10^{-15}\right),\\
&= \frac{1}{2}\log 2\times 10^{11} = \log\left(\sqrt{2\times 10^{11}}\right).
\end{aligned}$$

Solve, and $d_{\rm av} = 450$ km.

$\boxed{\textbf{E1-23}}$ The number of atoms is given by (1 kg)/($1.00783 \times 1.661 \times 10^{-27}$ kg), or 5.974×10^{26} atoms.

E1-24 (a) $(2 \times 1.0 + 16)$u$(1.661 \times 10^{-27}\text{kg}) = 3.0 \times 10^{-26}$kg.
(b) $(1.4 \times 10^{21}\text{kg})/(3.0 \times 10^{-26}\text{kg}) = 4.7 \times 10^{46}$ molecules.

E1-25 The coffee in Paris costs \$18.00 per kilogram, or

$$\$18.00\ \text{kg}^{-1}\left(\frac{0.4536\ \text{kg}}{1\ \text{lb}}\right) = \$8.16\ \text{lb}^{-1}.$$

It is cheaper to buy coffee in New York (at least according to the physics textbook, that is.)

E1-26 The room volume is $(21 \times 13 \times 12)\text{ft}^3(0.3048\ \text{m/ft})^3 = 92.8\,\text{m}^3$. The mass contained in the room is

$$(92.8\,\text{m}^3)(1.21\,\text{kg/m}^3) = 112\,\text{kg}.$$

$\boxed{\textbf{E1-27}}$ One mole of sugar cubes would have a volume of $N_A \times 1.0$ cm^3, where N_A is the Avogadro constant. Since the volume of a cube is equal to the length cubed, $V = l^3$, then $l = \sqrt[3]{N_A}$ cm $= 8.4 \times 10^7$ cm.

E1-28 The number of seconds in a week is $60 \times 60 \times 24 \times 7 = 6.05 \times 10^5$. The "weight" loss per second is then

$$(0.23\,\text{kg})/(6.05 \times 10^5\,\text{s}) = 3.80 \times 10^{-1}\ \text{mg/s}.$$

$\boxed{\textbf{E1-29}}$ The definition of the meter was wavelengths per meter; the question asks for meters per wavelength, so we want to take the reciprocal. The definition is accurate to 9 figures, so the reciprocal should be written as $1/1,650,763.73 = 6.05780211 \times 10^{-7}\,\text{m} = 605.780211$ nm.

E1-30 (a) $37.76 + 0.132 = 37.89$. (b) $16.264 - 16.26325 = 0.001$.

$\boxed{\textbf{E1-31}}$ The easiest approach is to first solve Darcy's Law for K, and then substitute the known SI units for the other quantities. Then

$$K = \frac{VL}{AHt}\ \text{has units of}\ \frac{(\text{m}^3)\,(\text{m})}{(\text{m}^2)\,(\text{m})\,(\text{s})}$$

which can be simplified to m/s.

E1-32 The Planck length, l_P, is found from

$$\begin{aligned}
[l_\mathrm{P}] &= [c^i][G^j][h^k], \\
\mathrm{L} &= (\mathrm{LT}^{-1})^i(\mathrm{L}^3\mathrm{T}^{-2}\mathrm{M}^{-1})^j(\mathrm{ML}^2\mathrm{T}^{-1})^k, \\
&= \mathrm{L}^{i+3j+2k}\mathrm{T}^{-i-2j-k}\mathrm{M}^{-j+k}.
\end{aligned}$$

Equate powers on each side,

$$\begin{aligned}
\mathrm{L:}\ 1 &= i+3j+2k, \\
\mathrm{T:}\ 0 &= -i-2j-k, \\
\mathrm{M:}\ 0 &= -j+k.
\end{aligned}$$

Then $j = k$, and $i = -3k$, and $1 = 2k$; so $k = 1/2$, $j = 1/2$, and $i = -3/2$. Then

$$\begin{aligned}
[l_\mathrm{P}] &= [c^{-3/2}][G^{1/2}][h^{1/2}], \\
&= (3.00\times10^{8}\,\mathrm{m/s})^{-3/2}(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg})^{1/2}(6.63\times10^{-34}\,\mathrm{kg\cdot m^2/s})^{1/2}, \\
&= 4.05\times10^{-35}\,\mathrm{m}.
\end{aligned}$$

E1-33 The Planck mass, m_P, is found from

$$\begin{aligned}
[m_\mathrm{P}] &= [c^i][G^j][h^k], \\
\mathrm{M} &= (\mathrm{LT}^{-1})^i(\mathrm{L}^3\mathrm{T}^{-2}\mathrm{M}^{-1})^j(\mathrm{ML}^2\mathrm{T}^{-1})^k, \\
&= \mathrm{L}^{i+3j+2k}\mathrm{T}^{-i-2j-k}\mathrm{M}^{-j+k}.
\end{aligned}$$

Equate powers on each side,

$$\begin{aligned}
\mathrm{L:}\ 0 &= i+3j+2k, \\
\mathrm{T:}\ 0 &= -i-2j-k, \\
\mathrm{M:}\ 1 &= -j+k.
\end{aligned}$$

Then $k = j+1$, and $i = -3j-1$, and $0 = -1+2k$; so $k = 1/2$, and $j = -1/2$, and $i = 1/2$. Then

$$\begin{aligned}
[m_\mathrm{P}] &= [c^{1/2}][G^{-1/2}][h^{1/2}], \\
&= (3.00\times10^{8}\,\mathrm{m/s})^{1/2}(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg})^{-1/2}(6.63\times10^{-34}\,\mathrm{kg\cdot m^2/s})^{1/2}, \\
&= 5.46\times10^{-8}\,\mathrm{kg}.
\end{aligned}$$

$\boxed{\textbf{P1-1}}$ There are $24\times60 = 1440$ traditional minutes in a day. The conversion plan is then fairly straightforward

$$822.8\text{ dec. min}\left(\frac{1440\text{ trad. min}}{1000\text{ dec. min}}\right) = 1184.8\text{ trad. min}.$$

This is traditional minutes since midnight, the time in traditional hours can be found by dividing by 60 min/hr, the integer part of the quotient is the hours, while the remainder is the minutes. So the time is 19 hours, 45 minutes, which would be 7:45 pm.

P1-2 (a) By similar triangles, the ratio of the distances is the same as the ratio of the diameters—390:1.

(b) Volume is proportional to the radius (diameter) cubed, or $390^3 = 5.93\times10^7$.

(c) $0.52^\circ(2\pi/360^\circ) = 9.1\times10^{-3}$ rad. The diameter is then $(9.1\times10^{-3}\text{ rad})(3.82\times10^5\text{ km}) =$ 3500 km.

P1-3 (a) The circumference of the Earth is approximately 40,000 km; 0.5 seconds of an arc is $0.5/(60 \times 60 \times 360) = 3.9 \times 10^{-7}$ of a circumference, so the north-south error is $\pm(3.9 \times 10^{-7})(4 \times 10^7\,\mathrm{m}) = \pm 15.6\,\mathrm{m}$. This is a range of 31 m.

(b) The east-west range is smaller, because the distance measured along a latitude is smaller than the circumference by a factor of the cosine of the latitude. Then the range is $31 \cos 43.6^\circ = 22\,\mathrm{m}$.

(c) The tanker is in Lake Ontario, some 20 km off the coast of Hamlin?

P1-4 Your position is determined by the time it takes for your longitude to rotate "underneath" the sun (in fact, that's the way longitude was measured originally as in 5 hours west of the Azores...) the rate the sun sweep over at equator is 25,000 miles/86,400 s = 0.29 miles/second. The correction factor because of latitude is the cosine of the latitude, so the sun sweeps overhead near England at approximately 0.19 mi/s. Consequently a 30 mile accuracy requires an error in time of no more than (30 mi)/(0.19 mi/s) = 158 seconds.

Trip takes about 6 months, so clock accuracy needs to be within $(158\,\mathrm{s})/(180\ \mathrm{day}) = 1.2$ seconds/day.

(b) Same, except 0.5 miles accuracy requires 2.6 s accuracy, so clock needs to be within 0.007 s/day!

P1-5 Let B be breaths/minute while sleeping. Each breath takes in (1.43 g/L)(0.3 L) = 0.429 g; and lets out (1.96 g/L)(0.3 L) = 0.288 g. The net loss is 0.141 g. Multiply by the number of breaths, (8 hr)(60 min./hr)B(0.141 g) = B(67.68 g). I'll take a short nap, and count my breaths, then finish the problem.

I'm back now, and I found my breaths to be 8/minute. So I lose 541 g/night, or about 1 pound.

P1-6 The mass of the water is $(1000\,\mathrm{kg/m^3})(5700\,\mathrm{m^3}) = 5.7 \times 10^6\,\mathrm{kg}$. The rate that water leaks drains out is

$$\frac{(5.7 \times 10^6\,\mathrm{kg})}{(12\ \mathrm{hr})(3600\ \mathrm{s/hr})} = 132\,\mathrm{kg/s}.$$

$\boxed{\textbf{P1-7}}$ Let the radius of the grain be given by r_g. Then the surface area of the grain is $A_g = 4\pi r_g^2$, and the volume is given by $V_g = (4/3)\pi r_g^3$.

If N grains of sand have a total surface area equal to that of a cube 1 m on a edge, then $NA_g = 6\ \mathrm{m}^2$. The total volume V_t of this number of grains of sand is NV_g. Eliminate N from these two expressions and get

$$V_t = NV_g = \frac{(6\ \mathrm{m}^2)}{A_g} V_g = \frac{(6\ \mathrm{m}^2) r_g}{3}.$$

Then $V_t = (2\ \mathrm{m}^2)(50 \times 10^{-6}\ \mathrm{m}) = 1 \times 10^{-4}\ \mathrm{m}^3$.

The mass of a volume V_t is given by

$$1 \times 10^{-4}\ \mathrm{m}^3 \left(\frac{2600\ \mathrm{kg}}{1\ \mathrm{m}^3}\right) = 0.26\ \mathrm{kg}.$$

P1-8 For a cylinder $V = \pi r^2 h$, and $A = 2\pi r^2 + 2\pi r h$. We want to minimize A with respect to changes in r, so

$$\begin{aligned} \frac{dA}{dr} &= \frac{d}{dr}\left(2\pi r^2 + 2\pi r \frac{V}{\pi r^2}\right), \\ &= 4\pi r - 2\frac{V}{r^2}. \end{aligned}$$

Set this equal to zero; then $V = 2\pi r^3$. Notice that $h = 2r$ in this expression.

P1-9 (a) The volume per particle is

$$(9.27 \times 10^{-26}\text{kg})/(7870\,\text{kg/m}^3) = 1.178 \times 10^{-28}\text{m}^3.$$

The radius of the corresponding sphere is

$$r = \sqrt[3]{\frac{3(1.178 \times 10^{-28}\text{m}^3)}{4\pi}} = 1.41 \times 10^{-10}\text{m}.$$

Double this, and the spacing is 282 pm.

(b) The volume per particle is

$$(3.82 \times 10^{-26}\text{kg})/(1013\,\text{kg/m}^3) = 3.77 \times 10^{-29}\text{m}^3.$$

The radius of the corresponding sphere is

$$r = \sqrt[3]{\frac{3(3.77 \times 10^{-29}\text{m}^3)}{4\pi}} = 2.08 \times 10^{-10}\text{m}.$$

Double this, and the spacing is 416 pm.

P1-10 (a) The area of the plate is (8.43 cm)(5.12 cm) = 43.2 cm^2. (b) (3.14)(3.7 cm)^2 = 43 cm^2.

$\boxed{\textbf{E2-1}}$ Add the vectors as is shown in Fig. 2-4. If $\vec{\mathbf{a}}$ has length $a = 4$ m and $\vec{\mathbf{b}}$ has length $b = 3$ m then the sum is given by $\vec{\mathbf{s}}$. The cosine law can be used to find the magnitude s of $\vec{\mathbf{s}}$,

$$s^2 = a^2 + b^2 - 2ab\cos\theta,$$

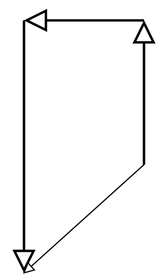
where θ is the angle between sides a and b in the figure.

(a) $(7\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = -1.0$, and $\theta = 180°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing in the same direction.

(b) $(1\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = 1.0$, and $\theta = 0°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing in the opposite direction.

(c) $(5\text{ m})^2 = (4\text{ m})^2 + (3\text{ m})^2 - 2(4\text{ m})(3\text{ m})\cos\theta$, so $\cos\theta = 0$, and $\theta = 90°$. This means that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are pointing at right angles to each other.

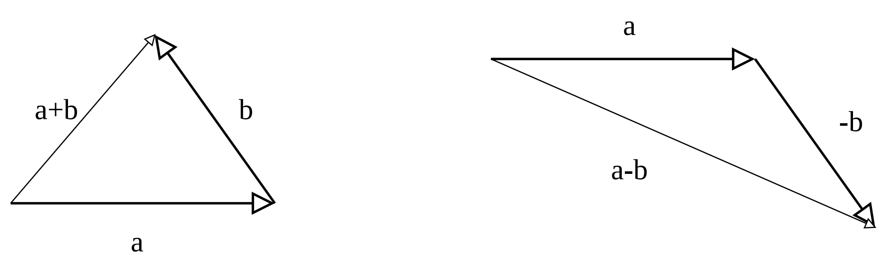
E2-2 (a) Consider the figures below.



(b) Net displacement is 2.4 km west, $(5.2 - 3.1 = 2.1)$ km south. A bird would fly

$$\sqrt{2.4^2 + 2.1^2}\text{ km} = 3.2\text{ km}.$$

E2-3 Consider the figure below.



E2-4 (a) The components are $(7.34)\cos(252°) = -2.27\hat{\mathbf{i}}$ and $(7.34)\sin(252°) = -6.98\hat{\mathbf{j}}$.

(b) The magnitude is $\sqrt{(-25)^2 + (43)^2} = 50$; the direction is $\theta = \tan^{-1}(43/-25) = 120°$. We did need to choose the correct quadrant.

$\boxed{\textbf{E2-5}}$ The components are given by the trigonometry relations

$$O = H\sin\theta = (3.42\text{ km})\sin 35.0° = 1.96\text{ km}$$

and

$$A = H\cos\theta = (3.42\text{ km})\cos 35.0° = 2.80\text{ km}.$$

The stated angle is measured from the east-west axis, counter clockwise from east. So O is measured against the north-south axis, with north being positive; A is measured against east-west with east being positive.

Since her individual steps are displacement vectors which are only north-south or east-west, she must eventually take enough north-south steps to equal 1.96 km, and enough east-west steps to equal 2.80 km. Any individual step can only be along one or the other direction, so the minimum total will be 4.76 km.

E2-6 Let $\vec{\mathbf{r}}_\mathrm{f} = 124\hat{\mathbf{i}}$ km and $\vec{\mathbf{r}}_\mathrm{i} = (72.6\hat{\mathbf{i}} + 31.4\hat{\mathbf{j}})$ km. Then the ship needs to travel

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} = (51.4\hat{\mathbf{i}} + 31.4\hat{\mathbf{j}})\text{ km}.$$

Ship needs to travel $\sqrt{51.4^2 + 31.4^2}$ km $= 60.2$ km in a direction $\theta = \tan^{-1}(31.4/51.4) = 31.4°$ west of north.

$\boxed{\textbf{E2-7}}$ (a) In unit vector notation we need only add the components; $\vec{\mathbf{a}}+\vec{\mathbf{b}} = (5\hat{\mathbf{i}}+3\hat{\mathbf{j}})+(-3\hat{\mathbf{i}}+2\hat{\mathbf{j}}) = (5-3)\hat{\mathbf{i}} + (3+2)\hat{\mathbf{j}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$.

(b) If we define $\vec{\mathbf{c}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$ and write the magnitude of $\vec{\mathbf{c}}$ as c, then $c = \sqrt{c_x^2 + c_y^2} = \sqrt{2^2 + 5^2} = 5.39$. The direction is given by $\tan\theta = c_y/c_x$ which gives an angle of 68.2°, measured counterclockwise from the positive x-axis.

E2-8 (a) $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (4-1)\hat{\mathbf{i}} + (-3+1)\hat{\mathbf{j}} + (1+4)\hat{\mathbf{k}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$.

(b) $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (4 - -1)\hat{\mathbf{i}} + (-3-1)\hat{\mathbf{j}} + (1-4)\hat{\mathbf{k}} = 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$.

(c) Rearrange, and $\vec{\mathbf{c}} = \vec{\mathbf{b}} - \vec{\mathbf{a}}$, or $\vec{\mathbf{b}} - \vec{\mathbf{a}} = (-1-4)\hat{\mathbf{i}} + (1 - -3)\hat{\mathbf{j}} + (4-1)\hat{\mathbf{k}} = -5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.

E2-9 (a) The magnitude of $\vec{\mathbf{a}}$ is $\sqrt{4.0^2 + (-3.0)^2} = 5.0$; the direction is $\theta = \tan^{-1}(-3.0/4.0) = 323°$.

(b) The magnitude of $\vec{\mathbf{b}}$ is $\sqrt{6.0^2 + 8.0^3} = 10.0$; the direction is $\theta = \tan^{-1}(6.0/8.0) = 36.9°$.

(c) The resultant vector is $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (4.0 + 6.0)\hat{\mathbf{i}} + (-3.0 + 8.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ is $\sqrt{(10.0)^2 + (5.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(5.0/10.0) = 26.6°$.

(d) The resultant vector is $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (4.0 - 6.0)\hat{\mathbf{i}} + (-3.0 - 8.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ is $\sqrt{(-2.0)^2 + (-11.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(-11.0/-2.0) = 260°$.

(e) The resultant vector is $\vec{\mathbf{b}} - \vec{\mathbf{a}} = (6.0 - 4.0)\hat{\mathbf{i}} + (8.0 - -3.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{b}} - \vec{\mathbf{a}}$ is $\sqrt{(2.0)^2 + (11.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(11.0/2.0) = 79.7°$.

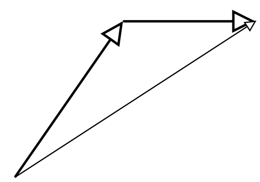
E2-10 (a) Find components of $\vec{\mathbf{a}}$; $a_x = (12.7)\cos(28.2°) = 11.2$, $a_y = (12.7)\sin(28.2°) = 6.00$. Find components of $\vec{\mathbf{b}}$; $b_x = (12.7)\cos(133°) = -8.66$, $b_y = (12.7)\sin(133°) = 9.29$. Then

$$\vec{\mathbf{r}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} = (11.2 - 8.66)\hat{\mathbf{i}} + (6.00 + 9.29)\hat{\mathbf{j}} = 2.54\hat{\mathbf{i}} + 15.29\hat{\mathbf{j}}.$$

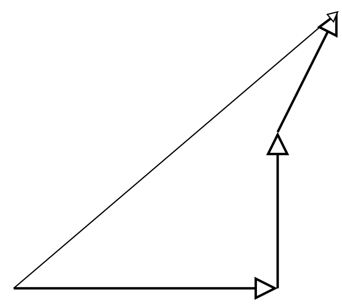
(b) The magnitude of $\vec{\mathbf{r}}$ is $\sqrt{2.54^2 + 15.29^2} = 15.5$.

(c) The angle is $\theta = \tan^{-1}(15.29/2.54) = 80.6°$.

E2-11 Consider the figure below.



E2-12 Consider the figure below.



$\boxed{\textbf{E2-13}}$ Our axes will be chosen so that $\hat{\mathbf{i}}$ points toward 3 O'clock and $\hat{\mathbf{j}}$ points toward 12 O'clock.
(a)
The two relevant positions are $\vec{\mathbf{r}}_\mathrm{i} = (11.3\ \mathrm{cm})\hat{\mathbf{i}}$ and $\vec{\mathbf{r}}_\mathrm{f} = (11.3\ \mathrm{cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned} \Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} \\ &= (11.3\ \mathrm{cm})\hat{\mathbf{j}} - (11.3\ \mathrm{cm})\hat{\mathbf{i}}. \end{aligned}$$

(b)
The two relevant positions are now $\vec{\mathbf{r}}_\mathrm{i} = (11.3\ \mathrm{cm})\hat{\mathbf{j}}$ and $\vec{\mathbf{r}}_\mathrm{f} = (-11.3\ \mathrm{cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned} \Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} \\ &= (11.3\ \mathrm{cm})\hat{\mathbf{j}} - (-11.3\ \mathrm{cm})\hat{\mathbf{j}} \\ &= (22.6\ \mathrm{cm})\hat{\mathbf{j}}. \end{aligned}$$

(c)
The two relevant positions are now $\vec{\mathbf{r}}_\mathrm{i} = (-11.3\ \mathrm{cm})\hat{\mathbf{j}}$ and $\vec{\mathbf{r}}_\mathrm{f} = (-11.3\ \mathrm{cm})\hat{\mathbf{j}}$. Then

$$\begin{aligned} \Delta\vec{\mathbf{r}} &= \vec{\mathbf{r}}_\mathrm{f} - \vec{\mathbf{r}}_\mathrm{i} \\ &= (-11.3\ \mathrm{cm})\hat{\mathbf{j}} - (-11.3\ \mathrm{cm})\hat{\mathbf{j}} \\ &= (0\ \mathrm{cm})\hat{\mathbf{j}}. \end{aligned}$$

E2-14 (a) The components of $\vec{\mathbf{r}}_1$ are

$$r_{1x} = (4.13\,\mathrm{m})\cos(225^\circ) = -2.92\,\mathrm{m}$$

and

$$r_{1y} = (4.13\,\mathrm{m})\sin(225^\circ) = -2.92\,\mathrm{m}.$$

The components of $\vec{\mathbf{r}}_2$ are

$$r_{1x} = (5.26\,\mathrm{m})\cos(0^\circ) = 5.26\,\mathrm{m}$$

and

$$r_{1y} = (5.26\,\mathrm{m})\sin(0^\circ) = 0\,\mathrm{m}.$$

The components of $\vec{\mathbf{r}}_3$ are

$$r_{1x} = (5.94\,\mathrm{m})\cos(64.0^\circ) = 2.60\,\mathrm{m}$$

and

$$r_{1y} = (5.94\,\mathrm{m})\sin(64.0^\circ) = 5.34\,\mathrm{m}.$$

(b) The resulting displacement is

$$\left[(-2.92 + 5.26 + 2.60)\hat{\mathbf{i}} + (-2.92 + 0 + 5.34)\hat{\mathbf{j}}\right]\,\mathrm{m} = (4.94\hat{\mathbf{i}} + 2.42\hat{\mathbf{j}})\,\mathrm{m}.$$

(c) The magnitude of the resulting displacement is $\sqrt{4.94^2 + 2.42^2}\,\mathrm{m} = 5.5\,\mathrm{m}$. The direction of the resulting displacement is $\theta = \tan^{-1}(2.42/4.94) = 26.1^\circ$. (d) To bring the particle back to the starting point we need only reverse the answer to (c); the magnitude will be the same, but the angle will be 206°.

E2-15 The components of the initial position are

$$r_{1x} = (12,000\ \mathrm{ft})\cos(40^\circ) = 9200\ \mathrm{ft}$$

and

$$r_{1y} = (12,000\ \mathrm{ft})\sin(40^\circ) = 7700\ \mathrm{ft}.$$

The components of the final position are

$$r_{2x} = (25,8000\ \mathrm{ft})\cos(163^\circ) = -24,700\ \mathrm{ft}$$

and

$$r_{2y} = (25,800\ \mathrm{ft})\sin(163^\circ) = 7540\ \mathrm{ft}.$$

The displacement is

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \left[(-24,700 - 9,200)\hat{\mathbf{i}} + (7,540 - 9,200)\hat{\mathbf{j}})\right] = (-33900\hat{\mathbf{i}} - 1660\hat{\mathbf{j}})\ \mathrm{ft}.$$

E2-16 (a) The displacement vector is $\vec{\mathbf{r}} = (410\hat{\mathbf{i}} - 820\hat{\mathbf{j}})$ mi, where positive x is east and positive y is north. The magnitude of the displacement is $\sqrt{(410)^2 + (-820)^2}$ mi $= 920$ mi. The direction is $\theta = \tan^{-1}(-820/410) = 300^\circ$.

(b) The average velocity is the displacement divided by the *total* time, 2.25 hours. Then

$$\vec{\mathbf{v}}_{\mathrm{av}} = (180\hat{\mathbf{i}} - 360\hat{\mathbf{j}})\ \mathrm{mi/hr}.$$

(c) The average speed is total distance over total time, or $(410+820)/(2.25)$ mi/hr $= 550$ mi/hr.

$\boxed{\textbf{E2-17}}$ (a) Evaluate $\vec{\mathbf{r}}$ when $t = 2$ s.

$$\begin{aligned}
\vec{\mathbf{r}} &= [(2\text{ m/s}^3)t^3 - (5\text{ m/s})t]\hat{\mathbf{i}} + [(6\text{ m}) - (7\text{ m/s}^4)t^4]\hat{\mathbf{j}} \\
&= [(2\text{ m/s}^3)(2\text{ s})^3 - (5\text{ m/s})(2\text{ s})]\hat{\mathbf{i}} + [(6\text{ m}) - (7\text{ m/s}^4)(2\text{ s})^4]\hat{\mathbf{j}} \\
&= [(16\text{ m}) - (10\text{ m})]\hat{\mathbf{i}} + [(6\text{ m}) - (112\text{ m})]\hat{\mathbf{j}} \\
&= [(6\text{ m})]\hat{\mathbf{i}} + [-(106\text{ m})]\hat{\mathbf{j}}.
\end{aligned}$$

(b) Evaluate:

$$\begin{aligned}
\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} &= [(2\text{ m/s}^3)3t^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(7\text{ m/s}^4)4t^3]\hat{\mathbf{j}} \\
&= [(6\text{ m/s}^3)t^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)t^3]\hat{\mathbf{j}}.
\end{aligned}$$

Into this last expression we now evaluate $\vec{\mathbf{v}}(t = 2\text{ s})$ and get

$$\begin{aligned}
\vec{\mathbf{v}} &= [(6\text{ m/s}^3)(2\text{ s})^2 - (5\text{ m/s})]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)(2\text{ s})^3]\hat{\mathbf{j}} \\
&= [(24\text{ m/s}) - (5\text{ m/s})]\hat{\mathbf{i}} + [-(224\text{ m/s})]\hat{\mathbf{j}} \\
&= [(19\text{ m/s})]\hat{\mathbf{i}} + [-(224\text{ m/s})]\hat{\mathbf{j}},
\end{aligned}$$

for the velocity $\vec{\mathbf{v}}$ when $t = 2$ s.

(c) Evaluate

$$\begin{aligned}
\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} &= [(6\text{ m/s}^3)2t]\hat{\mathbf{i}} + [-(28\text{ m/s}^4)3t^2]\hat{\mathbf{j}} \\
&= [(12\text{ m/s}^3)t]\hat{\mathbf{i}} + [-(84\text{ m/s}^4)t^2]\hat{\mathbf{j}}.
\end{aligned}$$

Into this last expression we now evaluate $\vec{\mathbf{a}}(t = 2\text{ s})$ and get

$$\begin{aligned}
\vec{\mathbf{a}} &= [(12\text{ m/s}^3)(2\text{ s})]\hat{\mathbf{i}} + [-(84\text{ m/s}^4)(2\ 2)^2]\hat{\mathbf{j}} \\
&= [(24\text{ m/s}^2)]\hat{\mathbf{i}} + [-(336\text{ m/s}^2)]\hat{\mathbf{j}}.
\end{aligned}$$

E2-18 (a) Let *ui* point north, $\hat{\mathbf{j}}$ point east, and $\hat{\mathbf{k}}$ point up. The displacement is $(8.7\hat{\mathbf{i}} + 9.7\hat{\mathbf{j}} + 2.9\hat{\mathbf{k}})$ km. The average velocity is found by dividing each term by 3.4 hr; then

$$\vec{\mathbf{v}}_{\text{av}} = (2.6\hat{\mathbf{i}} + 2.9\hat{\mathbf{j}} + 0.85)\text{ km/hr}.$$

The magnitude of the average velocity is $\sqrt{2.6^2 + 2.9^2 + 0.85^2}$ km/hr $= 4.0$ km/hr.

(b) The horizontal velocity has a magnitude of $\sqrt{2.6^2 + 2.9^2}$ km/hr $= 3.9$ km/hr. The angle with the horizontal is given by $\theta = \tan^{-1}(0.85/3.9) = 13^\circ$.

E2-19 (a) The derivative of the velocity is

$$\vec{\mathbf{a}} = [(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t]\hat{\mathbf{i}}$$

so the acceleration at $t = 3\,\text{s}$ is $\vec{\mathbf{a}} = (-18.0\,\text{m/s}^2)\hat{\mathbf{i}}$. (b) The acceleration is zero when $(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t = 0$, or $t = 0.75\,\text{s}$. (c) The velocity is *never* zero; there is no way to "cancel" out the y component. (d) The speed equals $10\,\text{m/s}$ when $10 = \sqrt{v_x^2 + 8^2}$, or $v_x = \pm 6.0\,\text{m/s}$. This happens when $(6.0\,\text{m/s}^2) - (8.0\,\text{m/s}^3)t = \pm 6.0\,\text{m/s}$, or when $t = 0\,\text{s}$.

E2-20 If v is constant then so is $v^2 = v_x^2 + v_y^2$. Take the derivative;

$$2v_x \frac{d}{dt} v_x + 2v_y \frac{d}{dt} v_y = 2(v_x a_x + v_y a_y).$$

But if the value is constant the derivative is zero.

$\boxed{\textbf{E2-21}}$ Let the actual flight time, as measured by the passengers, be T. There is some time difference between the two cities, call it $\Delta T =$ Namulevu time - Los Angeles time. The ΔT will be positive if Namulevu is east of Los Angeles. The time in Los Angeles can then be found from the time in Namulevu by subtracting ΔT.

The actual time of flight from Los Angeles to Namulevu is then the difference between when the plane lands (LA times) and when the plane takes off (LA time):

$$\begin{aligned} T &= (18{:}50 - \Delta T) - (12{:}50) \\ &= 6{:}00 - \Delta T, \end{aligned}$$

where we have written times in 24 hour format to avoid the AM/PM issue. The return flight time can be found from

$$\begin{aligned} T &= (18{:}50) - (1{:}50 - \Delta T) \\ &= 17{:}00 + \Delta T, \end{aligned}$$

where we have again changed to LA time for the purpose of the calculation.

(b) Now we just need to solve the two equations and two unknowns.

$$\begin{aligned} 17{:}00 + \Delta T &= 6{:}00 - \Delta T \\ 2\Delta T &= 6{:}00 - 17{:}00 \\ \Delta T &= -5{:}30. \end{aligned}$$

Since this is a negative number, Namulevu is located *west* of Los Angeles.

(a) $T = 6{:}00 - \Delta T = 11 : 30$, or eleven and a half hours.

(c) The distance traveled by the plane is given by $d = vt =$ (520 mi/hr)(11.5 hr) = 5980 mi. We'll draw a circle around Los Angeles with a radius of 5980 mi, and then we look for where it intersects with longitudes that would belong to a time zone ΔT away from Los Angeles. Since the Earth rotates once every 24 hours and there are 360 longitude degrees, then each hour corresponds to 15 longitude degrees, and then Namulevu must be located approximately $15^\circ \times 5.5 = 83^\circ$ west of Los Angeles, or at about longitude 160 east. The location on the globe is then latitude 5°, in the vicinity of Vanuatu.

When this exercise was originally typeset the times for the outbound and the inbound flights were inadvertently switched. I suppose that we could blame this on the airlines; nonetheless, when the answers were prepared for the back of the book the reversed numbers put Namulevu *east* of Los Angeles. That would put it in either the North Atlantic or Brazil.

E2-22 There is a three hour time zone difference. So the flight is seven hours long, but it takes 3 hr 51 min for the sun to travel same distance. Look for when the sunset distance has caught up with plane:

$$\begin{aligned} d_{\text{sunset}} &= d_{\text{plane}}, \\ v_{\text{sunset}}(t - 1{:}35) &= v_{\text{plane}} t, \\ (t - 1{:}35)/3{:}51 &= t/7{:}00, \end{aligned}$$

so $t = 3{:}31$ into flight.

$\boxed{\textbf{E2-23}}$ The distance is

E2-23 The distance is

$$d = vt = (112\text{ km/hr})(1\,\text{s})/(3600\text{ s/hr}) = 31\,\text{m}.$$

E2-24 The time taken for the ball to reach the plate is

$$t = \frac{d}{v} = \frac{(18.4\,\text{m})}{(160\text{ km/hr})}(3600\text{ s/hr})/(1000\text{ m/km}) = 0.414\,\text{s}.$$

$\boxed{\textbf{E2-25}}$ Speed is distance traveled divided by time taken; this is equivalent to the inverse of the slope of the line in Fig. 2-32. The line appears to pass through the origin and through the point (1600 km, 80×10^6 y), so the speed is $v = 1600\text{ km}/80 \times 10^6\text{ y} = 2 \times 10^{-5}$ km/y. Converting,

$$v = 2 \times 10^{-5}\text{km/y}\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right) = 2\text{ cm/y}$$

E2-26 (a) For Maurice Greene $v_{\text{av}} = (100\,\text{m})/(9.81\,\text{m}) = 10.2\,\text{m/s}$. For Khalid Khannouchi,

$$v_{\text{av}} = \frac{(26.219\text{ mi})}{(2.0950\text{ hr})}\left(\frac{1609\,\text{m}}{1\text{ mi}}\right)\left(\frac{1\text{hr}}{3600\,\text{s}}\right) = 5.594\,\text{m/s}.$$

(b) If Maurice Greene ran the marathon with an average speed equal to his average sprint speed then it would take him

$$t = \frac{(26.219\text{ mi})}{10.2\,\text{m/s}}\left(\frac{1609\,\text{m}}{1\text{ mi}}\right)\left(\frac{1\text{hr}}{3600\,\text{s}}\right) = 1.149\text{ hr},$$

or 1 hour, 9 minutes.

E2-27 The time saved is the difference,

$$\Delta t = \frac{(700\text{ km})}{(88.5\text{ km/hr})} - \frac{(700\text{ km})}{(104.6\text{ km/hr})} = 1.22\text{ hr},$$

which is about 1 hour 13 minutes.

E2-28 The ground elevation will increase by 35 m in a horizontal distance of

$$x = (35.0\,\text{m})/\tan(4.3^\circ) = 465\,\text{m}.$$

The plane will cover that distance in

$$t = \frac{(0.465\text{ km})}{(1300\text{ km/hr})}\left(\frac{3600\,\text{s}}{1\text{hr}}\right) = 1.3\,\text{s}.$$

$\boxed{\textbf{E2-29}}$ Let $v_1 = 40$ km/hr be the speed up the hill, t_1 be the time taken, and d_1 be the distance traveled in that time. We similarly define $v_2 = 60$ km/hr for the down hill trip, as well as t_2 and d_2. Note that $d_2 = d_1$.

$v_1 = d_1/t_1$ and $v_2 = d_2/t_2$. $v_{\rm av} = d/t$, where d total distance and t is the total time. The total distance is $d_1 + d_2 = 2d_1$. The total time t is just the sum of t_1 and t_2, so

$$\begin{aligned}
v_{\rm av} &= \frac{d}{t} \\
&= \frac{2d_1}{t_1 + t_2} \\
&= \frac{2d_1}{d_1/v_1 + d_2/v_2} \\
&= \frac{2}{1/v_1 + 1/v_2},
\end{aligned}$$

Take the reciprocal of both sides to get a simpler looking expression

$$\frac{2}{v_{\rm av}} = \frac{1}{v_1} + \frac{1}{v_2}.$$

Then the average speed is 48 km/hr.

E2-30 (a) Average speed is *total* distance divided by *total* time. Then

$$v_{\rm av} = \frac{(240\ {\rm ft}) + (240\ {\rm ft})}{(240\ {\rm ft})/(4.0\ {\rm ft/s}) + (240\ {\rm ft})/(10\ {\rm ft/s})} = 5.7\ {\rm ft/s}.$$

(b) Same approach, but different information given, so

$$v_{\rm av} = \frac{(60\ {\rm s})(4.0\ {\rm ft/s}) + (60\ {\rm s})(10\ {\rm ft/s})}{(60\ {\rm s}) + (60\ {\rm s})} = 7.0\ {\rm ft/s}.$$

E2-31 The distance traveled is the total area under the curve. The "curve" has four regions: (I) a triangle from 0 to 2 s; (II) a rectangle from 2 to 10 s; (III) a trapezoid from 10 to 12 s; and (IV) a rectangle from 12 to 16 s.

The area underneath the curve is the sum of the areas of the four regions.

$$d = \frac{1}{2}(2\,{\rm s})(8\,{\rm m/s}) + (8.0\,{\rm s})(8\,{\rm m/s}) + \frac{1}{2}(2\,{\rm s})(8\,{\rm m/s} + 4\,{\rm m/s}) + (4.0\,{\rm s})(4\,{\rm m/s}) = 100\,{\rm m}.$$

E2-32 The acceleration is the slope of a velocity-time curve,

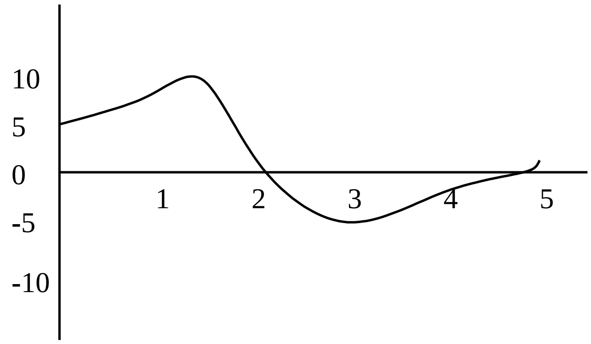
$$a = \frac{(8\,{\rm m/s}) - (4\,{\rm m/s})}{(10\,{\rm s}) - (12\,{\rm s})} = -2\,{\rm m/s^2}.$$

$\boxed{\textbf{E2-33}}$ The initial velocity is $\vec{\mathbf{v}}_{\rm i} = (18\ {\rm m/s})\hat{\mathbf{i}}$, the final velocity is $\vec{\mathbf{v}}_{\rm f} = (-30\ {\rm m/s})\hat{\mathbf{i}}$. The average acceleration is then

$$\vec{\mathbf{a}}_{\rm av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_{\rm f} - \vec{\mathbf{v}}_{\rm i}}{\Delta t} = \frac{(-30\ {\rm m/s})\hat{\mathbf{i}} - (18\ {\rm m/s})\hat{\mathbf{i}}}{2.4\ {\rm s}},$$

which gives $\vec{\mathbf{a}}_{\rm av} = (-20.0\ {\rm m/s^2})\hat{\mathbf{i}}$.

E2-34 Consider the figure below.



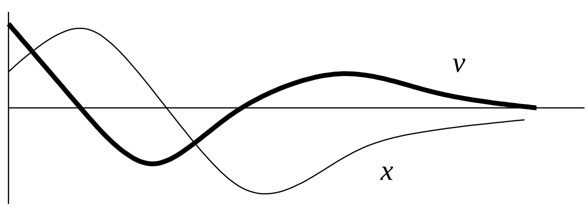
E2-35 (a) Up to A $v_x > 0$ and is constant. From A to B v_x is decreasing, but still positive. From B to C $v_x = 0$. From C to D $v_x < 0$, but $|v_x|$ is decreasing.

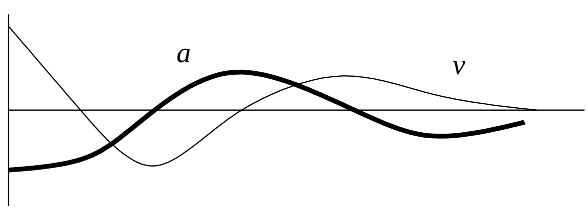
(b) No. Constant acceleration would appear as (part of) a parabola; but it would be challenging to distinguish between a parabola and an almost parabola.

E2-36 (a) Up to A $v_x > 0$ and is decreasing. From A to B $v_x = 0$. From B to C $v_x > 0$ and is increasing. From C to D $v_x > 0$ and is constant.

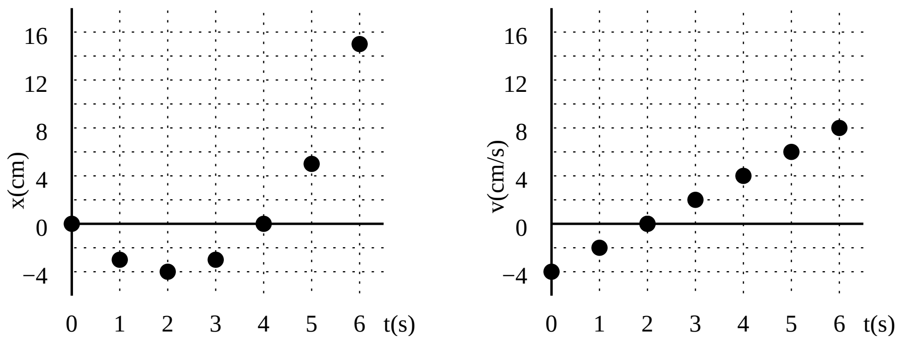
(b) No. Constant acceleration would appear as (part of) a parabola; but it would be challenging to distinguish between a parabola and an almost parabola.

$\boxed{\textbf{E2-37}}$ Consider the figure below.





E2-38 Consider the figure below.



The acceleration is a constant 2 cm/s/s during the entire time interval.

E2-39 (a) A must have units of $\mathrm{m/s^2}$. B must have units of $\mathrm{m/s^3}$.

(b) The maximum positive x position occurs when $v_x = 0$, so

$$v_x = \frac{dx}{dt} = 2At - 3Bt^2$$

implies $v_x = 0$ when either $t = 0$ or $t = 2A/3B = 2(3.0\,\mathrm{m/s^2})/3(1.0\,\mathrm{m/s^3}) = 2.0\,\mathrm{s}$.

(c) Particle starts from rest, then travels in positive direction until $t = 2\,\mathrm{s}$, a distance of

$$x = (3.0\,\mathrm{m/s^2})(2.0\,\mathrm{s})^2 - (1.0\,\mathrm{m/s^3})(2.0\,\mathrm{s})^3 = 4.0\,\mathrm{m}.$$

Then the particle moves back to a final position of

$$x = (3.0\,\mathrm{m/s^2})(4.0\,\mathrm{s})^2 - (1.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^3 = -16.0\,\mathrm{m}.$$

The total path followed was $4.0\,\mathrm{m} + 4.0\,\mathrm{m} + 16.0\,\mathrm{m} = 24.0\,\mathrm{m}$.

(d) The displacement is $-16.0\,\mathrm{m}$ as was found in part (c).

(e) The velocity is $v_x = (6.0\,\mathrm{m/s^2})t - (3.0\,\mathrm{m/s^3})t^2$. When $t = 0$, $v_x = 0.0\,\mathrm{m/s}$. When $t = 1.0\,\mathrm{s}$, $v_x = 3.0\,\mathrm{m/s}$. When $t = 2.0\,\mathrm{s}$, $v_x = 0.0\,\mathrm{m/s}$. When $t = 3.0\,\mathrm{s}$, $v_x = -9.0\,\mathrm{m/s}$. When $t = 4.0\,\mathrm{s}$, $v_x = -24.0\,\mathrm{m/s}$.

(f) The acceleration is the time derivative of the velocity,

$$a_x = \frac{dv_x}{dt} = (6.0\,\mathrm{m/s^2}) - (6.0\,\mathrm{m/s^3})t.$$

When $t = 0\,\mathrm{s}$, $a_x = 6.0\,\mathrm{m/s^2}$. When $t = 1.0\,\mathrm{s}$, $a_x = 0.0\,\mathrm{m/s^2}$. When $t = 2.0\,\mathrm{s}$, $a_x = -6.0\,\mathrm{m/s^2}$. When $t = 3.0\,\mathrm{s}$, $a_x = -12.0\,\mathrm{m/s^2}$. When $t = 4.0\,\mathrm{s}$, $a_x = -18.0\,\mathrm{m/s^2}$.

(g) The distance traveled was found in part (a) to be $-20\,\mathrm{m}$ The average speed during the time interval is then $v_{x,\mathrm{av}} = (-20\,\mathrm{m})/(2.0\,\mathrm{s}) = -10\,\mathrm{m/s}$.

E2-40 $v_{0x} = 0$, $v_x = 360$ km/hr $= 100\,\mathrm{m/s}$. Assuming constant acceleration the average velocity will be

$$v_{x,\mathrm{av}} = \frac{1}{2}(100\,\mathrm{m/s} + 0) = 50\,\mathrm{m/s}.$$

The time to travel the distance of the runway at this average velocity is

$$t = (1800\,\mathrm{m})/(50\,\mathrm{m/s}) = 36\,\mathrm{s}.$$

The acceleration is

$$a_x = 2x/t^2 = 2(1800\,\mathrm{m})/(36.0\,\mathrm{s})^2 = 2.78\,\mathrm{m/s^2}.$$

$\boxed{\textbf{E2-41}}$ (a) Apply Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (3.0 \times 10^7 \text{ m/s}) &= (0) + (9.8 \text{ m/s}^2)t, \\ 3.1 \times 10^6 \text{ s} &= t. \end{aligned}$$

(b) Apply Eq. 2-28 using an initial position of $x_0 = 0$,

$$\begin{aligned} x &= x_0 + v_{0x} + \frac{1}{2}a_x t^2, \\ x &= (0) + (0) + \frac{1}{2}(9.8 \text{ m/s}^2)(3.1 \times 10^6 \text{ s})^2, \\ x &= 4.7 \times 10^{13} \text{ m}. \end{aligned}$$

E2-42 $v_{0x} = 0$ and $v_x = 27.8\,\text{m/s}$. Then

$$t = (v_x - v_{0x})/a = ((27.8\,\text{m/s}) - (0))\,/(50\,\text{m/s}^2) = 0.56\,\text{s}.$$

I want that car.

E2-43 The muon will travel for t seconds before it comes to a rest, where t is given by

$$t = (v_x - v_{0x})/a = \big((0) - (5.20 \times 10^6\text{m/s})\big)\,/(-1.30 \times 10^{14}\text{m/s}^2) = 4 \times 10^{-8}\text{s}.$$

The distance traveled will be

$$x = \frac{1}{2}a_x t^2 + v_{0x}t = \frac{1}{2}(-1.30 \times 10^{14}\text{m/s}^2)(4 \times 10^{-8}\text{s})^2 + (5.20 \times 10^6\text{m/s})(4 \times 10^{-8}\text{s}) = 0.104\,\text{m}.$$

E2-44 The average velocity of the electron was

$$v_{x,\text{av}} = \frac{1}{2}(1.5 \times 10^5 \text{ m/s} + 5.8 \times 10^6 \text{ m/s}) = 3.0 \times 10^6 \text{ m/s}.$$

The time to travel the distance of the runway at this average velocity is

$$t = (0.012\,\text{m})/(3.0 \times 10^6 \text{ m/s}) = 4.0 \times 10^{-9} \text{ s}.$$

The acceleration is

$$a_x = (v_x - v_{0x})/t = ((5.8 \times 10^6 \text{ m/s}) - (1.5 \times 10^5 \text{ m/s}))/(4.0 \times 10^{-9} \text{ s}) = 1.4 \times 10^{15}\text{m/s}^2.$$

$\boxed{\textbf{E2-45}}$ It will be easier to solve the problem if we change the units for the initial velocity,

$$v_{0x} = 1020\frac{\text{km}}{\text{hr}}\left(\frac{1000 \text{ m}}{\text{km}}\right)\left(\frac{\text{hr}}{3600 \text{ s}}\right) = 283\frac{\text{m}}{\text{s}},$$

and then applying Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (0) &= (283 \text{ m/s}) + a_x(1.4 \text{ s}), \\ -202 \text{ m/s}^2 &= a_x. \end{aligned}$$

The problem asks for this in terms of g, so

$$-202 \text{ m/s}^2 \left(\frac{g}{9.8 \text{ m/s}^2}\right) = 21g.$$

E2-46 Change miles to feet and hours to seconds. Then $v_x = 81$ ft/s and $v_{0x} = 125$ ft/s. The time is then

$$t = ((81 \text{ ft/s}) - (125 \text{ ft/s}))/(-17 \text{ ft/s}^2) = 2.6\,\text{s}.$$

E2-47 (a) The time to stop is

$$t = ((0 \text{ m/s}) - (24.6 \text{ m/s}))/(-4.92 \text{ m/s}^2) = 5.00\,\text{s}.$$

(b) The distance traveled is

$$x = \frac{1}{2}a_x t^2 + v_{0x}t = \frac{1}{2}(-4.92 \text{ m/s}^2)(5.00\,\text{s})^2 + (24.6 \text{ m/s})(5.00\,\text{s}) = 62\,\text{m}.$$

E2-48 Answer part (b) first. The average velocity of the arrow while decelerating is

$$v_{y,\text{av}} = \frac{1}{2}((0) + (260 \text{ ft/s})) = 130 \text{ ft/s}.$$

The time for the arrow to travel 9 inches (0.75 feet) is

$$t = (0.75 \text{ ft})/(130 \text{ ft/s}) = 5.8 \times 10^{-3}\text{s}.$$

(a) The acceleration of the arrow is then

$$a_y = (v_y - v_{0y})/t = ((0) - (260 \text{ ft/s}))/(5.8 \times 10^{-3}\text{s}) = -4.5 \times 10^4 \text{ ft/s}^2.$$

$\boxed{\textbf{E2-49}}$ The problem will be somewhat easier if the units are consistent, so we'll write the maximum speed as

$$1000\frac{\text{ft}}{\text{min}}\left(\frac{\text{min}}{60 \text{ s}}\right) = 16.7\,\frac{\text{ft}}{\text{s}}.$$

(a) We can find the time required for the acceleration from Eq. 2-26,

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ (16.7 \text{ ft/s}) &= (0) + (4.00 \text{ ft/s}^2)t, \\ 4.18 \text{ s} &= t. \end{aligned}$$

And from this and Eq 2-28 we can find the distance

$$\begin{aligned} x &= x_0 + v_{0x} + \frac{1}{2}a_x t^2, \\ x &= (0) + (0) + \frac{1}{2}(4.00 \text{ ft/s}^2)(4.18 \text{ s})^2, \\ x &= 34.9 \text{ ft}. \end{aligned}$$

(b) The motion of the elevator is divided into three parts: acceleration from rest, constant speed motion, and deceleration to a stop. The total distance is given at 624 ft and in part (a) we found the distance covered during acceleration was 34.9 ft. By symmetry, the distance traveled during deceleration should also be 34.9 ft. The distance traveled at constant speed is then $(624 - 34.9 - 34.9)$ ft $= 554$ ft. The time required for the constant speed portion of the trip is found from Eq. 2-22, rewritten as

$$\Delta t = \frac{\Delta x}{v} = \frac{554 \text{ ft}}{16.7 \text{ ft/s}} = 33.2 \text{ s}.$$

The total time for the trip is the sum of times for the three parts: accelerating (4.18 s), constant speed (33.2 s), and decelerating (4.18 s). The total is 41.6 seconds.

E2-50 (a) The deceleration is found from

$$a_x = \frac{2}{t^2}(x - v_0 t) = \frac{2}{(4.0\,\mathrm{s})^2}((34\,\mathrm{m}) - (16\,\mathrm{m/s})(4.0\,\mathrm{s})) = -3.75\,\mathrm{m/s}^2.$$

(b) The impact speed is

$$v_x = v_{0x} + a_x t = (16\,\mathrm{m/s}) + (-3.75\,\mathrm{m/s}^2)(4.0\,\mathrm{s}) = 1.0\,\mathrm{m/s}.$$

E2-51 Assuming the drops fall from rest, the time to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1700\,\mathrm{m})}{(-9.8\,\mathrm{m/s}^2)}} = 19\,\mathrm{s}.$$

The velocity of the falling drops would be

$$v_y = a_y t = (-9.8\,\mathrm{m/s}^2)(19\,\mathrm{s}) = 190\,\mathrm{m/s},$$

or about 2/3 the speed of sound.

E2-52 Solve the problem out of order.

(b) The time to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-120\,\mathrm{m})}{(-9.8\,\mathrm{m/s}^2)}} = 4.9\,\mathrm{s}.$$

(a) The speed at which the elevator hits the ground is

$$v_y = a_y t = (-9.8\,\mathrm{m/s}^2)(4.9\,\mathrm{s}) = 48\,\mathrm{m/s}.$$

(d) The time to fall half-way is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-60\,\mathrm{m})}{(-9.8\,\mathrm{m/s}^2)}} = 3.5\,\mathrm{s}.$$

(c) The speed at the half-way point is

$$v_y = a_y t = (-9.8\,\mathrm{m/s}^2)(3.5\,\mathrm{s}) = 34\,\mathrm{m/s}.$$

$\boxed{\textbf{E2-53}}$ The initial velocity of the "dropped" wrench would be zero. I choose vertical to be along the y axis with up as positive, which is the convention of Eq. 2-29 and Eq. 2-30. It turns out that it is much easier to solve part (b) before solving part (a).

(b) We solve Eq. 2-29 for the time of the fall.

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (-24.0\text{ m/s}) &= (0) - (9.8\text{ m/s}^2)t, \\ 2.45\text{ s} &= t. \end{aligned}$$

(a) Now we can use Eq. 2-30 to find the height from which the wrench fell.

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ (0) &= y_0 + (0)(2.45\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(2.45\text{ s})^2, \\ 0 &= y_0 - 29.4\text{ m} \end{aligned}$$

We have set $y = 0$ to correspond to the final position of the wrench: on the ground. This results in an initial position of $y_0 = 29.4$ m; it is positive because the wrench was dropped from a point *above* where it landed.

E2-54 (a) It is easier to solve the problem from the point of view of an object which falls from the highest point. The time to fall from the highest point is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-53.7\,\text{m})}{(-9.81\,\text{m/s}^2)}} = 3.31\,\text{s}.$$

The speed at which the object hits the ground is

$$v_y = a_y t = (-9.81\,\text{m/s}^2)(3.31\,\text{s}) = -32.5\,\text{m/s}.$$

But the motion is symmetric, so the object must have been launched up with a velocity of $v_y = 32.5\,\text{m/s}$.

(b) Double the previous answer; the time of flight is 6.62 s.

E2-55 (a) The time to fall the first 50 meters is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-50\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 3.2\,\text{s}.$$

(b) The *total* time to fall 100 meters is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-100\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 4.5\,\text{s}.$$

The time to fall through the second 50 meters is the difference, 1.3 s.

E2-56 The rock returns to the ground with an equal, but opposite, velocity. The acceleration is then

$$a_y = ((-14.6\,\text{m/s}) - (14.6\,\text{m/s}))/(7.72\,\text{s}) = 3.78\,\text{m/s}^2.$$

That would put them on Mercury.

$\boxed{\textbf{E2-57}}$ (a) Solve Eq. 2-30 for the initial velocity. Let the distances be measured from the ground so that $y_0 = 0$.

$$\begin{aligned}
y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\
(36.8 \text{ m}) &= (0) + v_{0y}(2.25 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(2.25 \text{ s})^2, \\
36.8 \text{ m} &= v_{0y}(2.25 \text{ s}) - 24.8 \text{ m}, \\
27.4 \text{ m/s} &= v_{0y}.
\end{aligned}$$

(b) Solve Eq. 2-29 for the velocity, using the result from part (a).

$$\begin{aligned}
v_y &= v_{0y} - gt, \\
v_y &= (27.4 \text{ m/s}) - (9.8 \text{ m/s}^2)(2.25 \text{ s}), \\
v_y &= 5.4 \text{ m/s}.
\end{aligned}$$

(c) We need to solve Eq. 2-30 to find the height to which the ball rises, but we don't know how long it takes to get there. So we first solve Eq. 2-29, because we do know the velocity at the highest point ($v_y = 0$).

$$\begin{aligned}
v_y &= v_{0y} - gt, \\
(0) &= (27.4 \text{ m/s}) - (9.8 \text{ m/s}^2)t, \\
2.8 \text{ s} &= t.
\end{aligned}$$

And then we find the height to which the object rises,

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ y &= (0) + (27.4 \text{ m/s})(2.8 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(2.8 \text{ s})^2, \\ y &= 38.3\text{m}. \end{aligned}$$

This is the height as measured from the ground; so the ball rises $38.3 - 36.8 = 1.5$ m above the point specified in the problem.

E2-58 The time it takes for the ball to fall 2.2 m is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-2.2\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 0.67\,\text{s}.$$

The ball hits the ground with a velocity of

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(0.67\,\text{s}) = -6.6\,\text{m/s}.$$

The ball then bounces up to a height of 1.9 m. It is easier to solve the falling part of the motion, and then apply symmetry. The time is would take to fall is

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1.9\,\text{m})}{(-9.8\,\text{m/s}^2)}} = 0.62\,\text{s}.$$

The ball hits the ground with a velocity of

$$v_y = a_y t = (-9.8\,\text{m/s}^2)(0.62\,\text{s}) = -6.1\,\text{m/s}.$$

But we are interested in when the ball moves up, so $v_y = 6.1\,\text{m/s}$.

The acceleration while in contact with the ground is

$$a_y = ((6.1\,\text{m/s}) - (-6.6\,\text{m/s}))/(0.096\,\text{s}) = 130\,\text{m/s}^2.$$

E2-59 The position as a function of time for the first object is

$$y_1 = -\frac{1}{2}gt^2,$$

The position as a function of time for the second object is

$$y_2 = -\frac{1}{2}g(t - 1\,\text{s})^2$$

The difference,

$$\Delta y = y_2 - y_1 = frac12g\left((2\,\text{s})t - 1\right),$$

is the set equal to $10\,\text{m}$, so $t = 1.52\,\text{s}$.

E2-60 Answer part (b) first.

(b) Use the quadratic equation to solve

$$(-81.3\,\text{m}) = \frac{1}{2}(-9.81\,\text{m/s}^2)t^2 + (12.4\,\text{m/s})t$$

for time. Get $t = -3.0\,\text{s}$ and $t = 5.53\,\text{s}$. Keep the positive answer.

(a) Now find final velocity from

$$v_y = (-9.8\,\text{m/s}^2)(5.53\,\text{s}) + (12.4\,\text{m/s}) = -41.8\,\text{m/s}.$$

$\boxed{\textbf{E2-61}}$ The total time the pot is visible is 0.54 s; the pot is visible for 0.27 s on the way down. We'll define the initial position as the highest point and make our measurements from there. Then $y_0 = 0$ and $v_{0y} = 0$. Define t_1 to be the time at which the *falling* pot passes the top of the window y_1, then $t_2 = t_1 + 0.27$ s is the time the pot passes the bottom of the window $y_2 = y_1 - 1.1$ m. We have two equations we can write, both based on Eq. 2-30,

$$\begin{aligned} y_1 &= y_0 + v_{0y}t_1 - \frac{1}{2}gt_1^2, \\ y_1 &= (0) + (0)t_1 - \frac{1}{2}gt_1^2, \end{aligned}$$

and

$$\begin{aligned} y_2 &= y_0 + v_{0y}t_2 - \frac{1}{2}gt_2^2, \\ y_1 - 1.1 \text{ m} &= (0) + (0)t_2 - \frac{1}{2}g(t_1 + 0.27 \text{ s})^2, \end{aligned}$$

Isolate y_1 in this last equation and then set the two expressions equal to each other so that we can solve for t_1,

$$\begin{aligned} -\frac{1}{2}gt_1^2 &= 1.1 \text{ m} - \frac{1}{2}g(t_1 + 0.27 \text{ s})^2, \\ -\frac{1}{2}gt_1^2 &= 1.1 \text{ m} - \frac{1}{2}g(t_1^2 + [0.54 \text{ s}]t_1 + 0.073 \text{ s}^2), \\ 0 &= 1.1 \text{ m} - \frac{1}{2}g([0.54 \text{ s}]t_1 + 0.073 \text{ s}^2). \end{aligned}$$

This last line can be directly solved to yield $t_1 = 0.28$ s as the time when the falling pot passes the top of the window. Use this value in the first equation above and we can find $y_1 = -\frac{1}{2}(9.8 \text{ m/s}^2)(0.28 \text{ s})^2 = -0.38$ m. The negative sign is because the top of the window is beneath the highest point, so the pot must have risen to 0.38 m above the top of the window.

P2-1 (a) The net shift is $\sqrt{(22\,\text{m})^2 + (17\,\text{m})^2)} = 28\,\text{m}$.
(b) The vertical displacement is $(17\,\text{m})\sin(52°) = 13\,\text{m}$.

P2-2 Wheel "rolls" through half of a turn, or $\pi r = 1.41\,\text{m}$. The vertical displacement is $2r = 0.90\,\text{m}$. The net displacement is

$$\sqrt{(1.41\,\text{m})^2 + (0.90\,\text{m})^2} = 1.67\,\text{m}0.$$

The angle is

$$\theta = \tan^{-1}(0.90\,\text{m})/(1.41\,\text{m}) = 33°.$$

$\boxed{\textbf{P2-3}}$ We align the coordinate system so that the origin corresponds to the starting position of the fly and that all positions inside the room are given by positive coordinates.
(a) The displacement vector can just be written,

$$\Delta\vec{\mathbf{r}} = (10 \text{ ft})\hat{\mathbf{i}} + (12 \text{ ft})\hat{\mathbf{j}} + (14 \text{ ft})\hat{\mathbf{k}}.$$

(b) The magnitude of the displacement vector is $|\Delta\vec{\mathbf{r}}| = \sqrt{10^2 + 12^2 + 14^2}$ ft= 21 ft.

(c) The straight line distance between two points is the shortest possible distance, so the length of the path taken by the fly must be greater than or equal to 21 ft.

(d) If the fly walks it will need to cross two faces. The shortest path will be the diagonal across these two faces. If the lengths of sides of the room are l_1, l_2, and l_3, then the diagonal length across two faces will be given by

$$\sqrt{(l_1 + l_2)^2 + l_3^2},$$

where we want to choose the l_i from the set of 10 ft, 12 ft, and 14 ft that will minimize the length. The minimum distance is when $l_1 = 10$ ft, $l_2 = 12$ ft, and $l_3 = 14$. Then the minimal distance the fly would *walk* is 26 ft.

P2-4 Choose vector $\vec{\mathbf{a}}$ to lie on the x axis. Then $\vec{\mathbf{a}} = a\hat{\mathbf{i}}$ and $\vec{\mathbf{b}} = b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}}$ where $b_x = b\cos\theta$ and $b_y = b\sin\theta$. The sum then has components

$$r_x = a + b\cos\theta \text{ and } r_y = b\sin\theta.$$

Then

$$\begin{aligned} r^2 &= (a + b\cos\theta)^2 + (b\sin\theta)^2, \\ &= a^2 + 2ab\cos\theta + b^2. \end{aligned}$$

P2-5 (a) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{(35.0 \text{ mi/hr})(t/2) + (55.0 \text{ mi/hr})(t/2)}{(t/2) + (t/2)} = 45.0 \text{ mi/hr}.$$

(b) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{(d/2) + (d/2)}{(d/2)/(35.0 \text{ mi/hr}) + (d/2)/(55.0 \text{ mi/hr})} = 42.8 \text{ mi/hr}.$$

(c) Average speed is total distance divided by total time. Then

$$v_{\text{av}} = \frac{d + d}{(d)/(45.0 \text{ mi/hr}) + (d)/(42.8 \text{ mi/hr})} = 43.9 \text{ mi/hr}$$

P2-6 (a) We'll do just one together. How about $t = 2.0\,\text{s}$?

$$x = (3.0\,\text{m/s})(2.0\,\text{s}) + (-4.0\,\text{m/s}^2)(2.0\,\text{s})^2 + (1.0\,\text{m/s}^3)(2.0\,\text{s})^3 = -2.0\,\text{m}.$$

The rest of the values are, starting from $t = 0$, $x = 0.0$ m, 0.0 m, -2.0 m, 0.0 m, and 12.0 m.

(b) Always final minus initial. The answers are $x_\text{f} - x_\text{i} = -2.0\,\text{m} - 0.0\,\text{m} = -2.0\,\text{m}$ and $x_\text{f} - x_\text{i} = 12.0\,\text{m} - 0.0\,\text{m} = 12.0\,\text{m}$.

(c) Always displacement divided by (change in) time.

$$v_{\text{av}} = \frac{(12.0\,\text{m}) - (-2.0\,\text{m})}{(4.0\,\text{s}) - (2.0\,\text{s})} = 7.0\,\text{m/s},$$

and

$$v_{\text{av}} = \frac{(0.0\,\text{m}) - (0.0\,\text{m})}{(3.0\,\text{s}) - (0.0\,\text{s})} = 0.0\,\text{m/s}.$$

$\boxed{\textbf{P2-7}}$ (a) Assume the bird has no size, the trains have some separation, and the bird is just leaving one of the trains. The bird will be able to fly from one train to the other *before* the two trains collide, regardless of how close together the trains are. After doing so, the bird is now on the other train, the trains are still separated, so once again the bird can fly between the trains before they collide. This process can be repeated every time the bird touches one of the trains, so the bird will make an infinite number of trips between the trains.

(b) The trains collide in the middle; therefore the trains collide after (51 km)/(34 km/hr) = 1.5 hr. The bird was flying with constant speed this entire time, so the distance flown by the bird is (58 km/hr)(1.5 hr) = 87 km.

P2-8 (a) Start with a perfect square:

$$\begin{aligned}
(v_1 - v_2)^2 &> 0, \\
v_1^2 + v_2^2 &> 2v1v2, \\
(v_1^2 + v_2^2)t_1t_2 &> 2v_1v_2t_1t_2, \\
d_1^2 + d_2^2 + (v_1^2 + v_2^2)t_1t_2 &> d_1^2 + d_2^2 + 2v_1v_2t_1t_2, \\
(v_1^2t_1 + v_2^2t_2)(t_1 + t_2) &> (d_1 + d_2)^2, \\
\frac{v_1^2t_1 + v_2^2t_2}{d_1 + d_2} &> \frac{d_1 + d_2}{t_1 + t_2}, \\
\frac{v_1d_1 + v_2d_2}{d_1 + d_2} &> \frac{v_1t_1 + v_2t_2}{t_1 + t_2}
\end{aligned}$$

Actually, it only works if $d_1 + d_2 > 0$!

(b) If $v_1 = v_2$.

P2-9 (a) The average velocity during the time interval is $v_{\text{av}} = \Delta x/\Delta t$, or

$$v_{\text{av}} = \frac{(A + B(3\,\text{s})^3) - (A + B(2\,\text{s})^3)}{(3\,\text{s}) - (2\,\text{s})} = (1.50\ \text{cm/s}^3)(19\,\text{s}^3)/(1\,\text{s}) = 28.5\ \text{cm/s}.$$

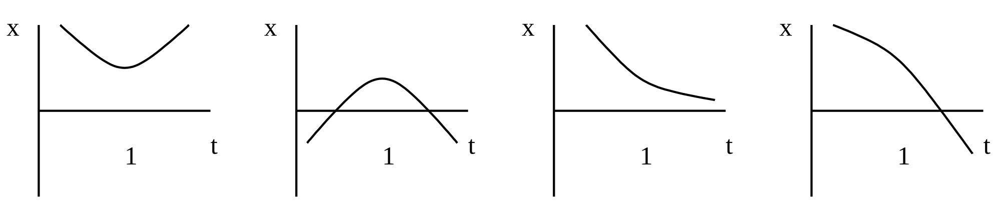
(b) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(2\,\text{s})^2 = 18$ cm/s.
(c) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(3\,\text{s})^2 = 40.5$ cm/s.
(d) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(2.5\,\text{s})^2 = 28.1$ cm/s.
(e) The midway position is $(x_\text{f} + x_\text{i})/2$, or

$$x_{\text{mid}} = A + B[(3\,\text{s})^3 + (2\,\text{s})^3)]/2 = A + (17.5\,\text{s}^3)B.$$

This occurs when $t = \sqrt[3]{(17.5\,\text{s}^3)}$. The instantaneous velocity at this point is

$$v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(\sqrt[3]{(17.5\,\text{s}^3)})^2 = 30.3\ \text{cm/s}.$$

P2-10 Consider the figure below.



(a) (b) (c) (d)

$\boxed{\textbf{P2-11}}$ (a) The average velocity is displacement divided by change in time,

$$v_{\text{av}} = \frac{(2.0 \text{ m/s}^3)(2.0 \text{ s})^3 - (2.0 \text{ m/s}^3)(1.0 \text{ s})^3}{(2.0 \text{ s}) - (1.0 \text{ s})} = \frac{14.0 \text{ m}}{1.0 \text{ s}} = 14.0 \text{ m/s}.$$

The average acceleration is the change in velocity. So we need an expression for the velocity, which is the time derivative of the position,

$$v = \frac{dx}{dt} = \frac{d}{dt}(2.0 \text{ m/s}^3)t^3 = (6.0 \text{ m/s}^3)t^2.$$

From this we find average acceleration

$$a_{\text{av}} = \frac{(6.0 \text{ m/s}^3)(2.0 \text{ s})^2 - (6.0 \text{ m/s}^3)(1.0 \text{ s})^2}{(2.0 \text{ s}) - (1.0 \text{ s})} = \frac{18.0 \text{ m/s}}{1.0 \text{ s}} = 18.0 \text{ m/s}^2.$$

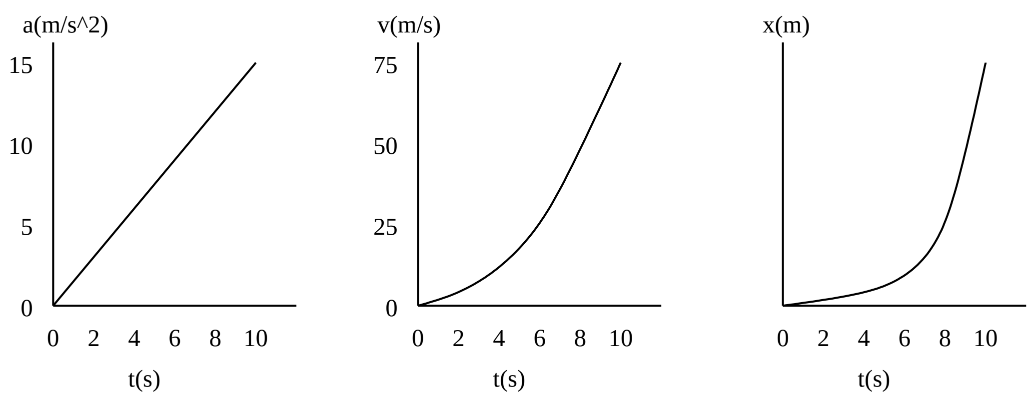
(b) The instantaneous velocities can be found directly from $v = (6.0 \text{ m/s}^2)t^2$, so $v(2.0 \text{ s}) = 24.0$ m/s and $v(1.0 \text{ s}) = 6.0$ m/s. We can get an expression for the instantaneous acceleration by taking the time derivative of the velocity

$$a = \frac{dv}{dt} = \frac{d}{dt}(6.0 \text{ m/s}^3)t^2 = (12.0 \text{ m/s}^3)t.$$

Then the instantaneous accelerations are $a(2.0 \text{ s}) = 24.0 \text{ m/s}^2$ and $a(1.0 \text{ s}) = 12.0 \text{ m/s}^2$

(c) Since the motion is monotonic we expect the average quantities to be somewhere between the instantaneous values at the endpoints of the time interval. Indeed, that is the case.

P2-12 Consider the figure below.



P2-13 Start with $v_{\text{f}} = v_{\text{i}} + at$, but $v_{\text{f}} = 0$, so $v_{\text{i}} = -at$, then

$$x = \frac{1}{2}at^2 + v_{\text{i}}t = \frac{1}{2}at^2 - at^2 = -\frac{1}{2}at^2,$$

so $t = \sqrt{-2x/a} = \sqrt{-2(19.2 \text{ ft})/(-32 \text{ ft/s}^2)} = 1.10\,\text{s}$. Then $v_{\text{i}} = -(-32 \text{ ft/s}^2)(1.10\,\text{s}) = 35.2$ ft/s. Converting,

$$35.2 \text{ ft/s}(1/5280 \text{ mi/ft})(3600 \text{ s/h}) = 24 \text{ mi/h}.$$

P2-14 (b) The average speed during while traveling the 160 m is

$$v_{\mathrm{av}} = (33.0\,\mathrm{m/s} + 54.0\,\mathrm{m/s})/2 = 43.5\,\mathrm{m/s}.$$

The time to travel the 160 m is $t = (160\,\mathrm{m})/(43.5\,\mathrm{m/s}) = 3.68\,\mathrm{s}$.

(a) The acceleration is

$$a = \frac{2x}{t^2} - \frac{2v_i}{t} = \frac{2(160\,\mathrm{m})}{(3.68\,\mathrm{s})^2} - \frac{2(33.0\,\mathrm{m/s})}{(3.68\,\mathrm{s})} = 5.69\,\mathrm{m/s^2}.$$

(c) The time required to get up to a speed of 33 m/s is

$$t = v/a = (33.0\,\mathrm{m/s})/(5.69\,\mathrm{m/s^2}) = 5.80\,\mathrm{s}.$$

(d) The distance moved from start is

$$d = \frac{1}{2}at^2 = \frac{1}{2}(5.69\,\mathrm{m/s^2})(5.80\,\mathrm{s})^2 = 95.7\,\mathrm{m}.$$

P2-15 (a) The distance traveled during the reaction time happens at constant speed; $t_{\mathrm{reac}} = d/v = (15\,\mathrm{m})/(20\,\mathrm{m/s}) = 0.75\,\mathrm{s}$.

(b) The braking distance is proportional to the speed squared (look at the numbers!) and in this case is $d_{\mathrm{brake}} = v^2/(20\,\mathrm{m/s^2})$. Then $d_{\mathrm{brake}} = (25\,\mathrm{m/s})^2/(20\,\mathrm{m/s^2}) = 31.25\,\mathrm{m}$. The reaction time distance is $d_{\mathrm{reac}} = (25\,\mathrm{m/s})(0.75\,\mathrm{s}) = 18.75\,\mathrm{m}$. The stopping distance is then 50 m.

P2-16 (a) For the car $x_{\mathrm{c}} = a_{\mathrm{c}}t^2/2$. For the truck $x_{\mathrm{t}} = v_{\mathrm{t}}t$. Set both x_i to the same value, and then substitute the time from the truck expression:

$$x = a_{\mathrm{c}}t^2/2 = a_{\mathrm{c}}(x/v_{\mathrm{t}})^2/2,$$

or

$$x = 2{v_{\mathrm{t}}}^2/a_{\mathrm{c}} = 2(9.5\,\mathrm{m/s})^2/(2.2\,\mathrm{m/s}) = 82\,\mathrm{m}.$$

(b) The speed of the car will be given by $v_{\mathrm{c}} = a_{\mathrm{c}}t$, or

$$v_{\mathrm{c}} = a_{\mathrm{c}}t = a_{\mathrm{c}}x/v_{\mathrm{t}} = (2.2\,\mathrm{m/s})(82\,\mathrm{m})/(9.5\,\mathrm{m/s}) = 19\,\mathrm{m/s}.$$

P2-17 The runner covered a distance d_1 in a time interval t_1 during the acceleration phase and a distance d_2 in a time interval t_2 during the constant speed phase. Since the runner started from rest we know that the constant speed is given by $v = at_1$, where a is the runner's acceleration.

The distance covered during the acceleration phase is given by

$$d_1 = \frac{1}{2}at_1^2.$$

The distance covered during the constant speed phase can also be found from

$$d_2 = vt_2 = at_1t_2.$$

We want to use these two expressions, along with $d_1 + d_2 = 100$ m and $t_2 = (12.2\text{ s}) - t_1$, to get

$$\begin{aligned}
100\text{ m} &= d_1 + d_2 = \frac{1}{2}at_1^2 + at_1(12.2\text{ s} - t_1),\\
&= -\frac{1}{2}at_1^2 + a(12.2\text{ s})t_1,\\
&= -(1.40\text{ m/s}^2)t_1^2 + (34.2\text{ m/s})t_1.
\end{aligned}$$

This last expression is quadratic in t_1, and is solved to give $t_1 = 3.40\,\text{s}$ or $t_1 = 21.0\,\text{s}$. Since the race only lasted 12.2 s we can ignore the second answer.

(b) The distance traveled during the acceleration phase is then

$$d_1 = \frac{1}{2}at_1^2 = (1.40 \text{ m/s}^2)(3.40\,\text{s})^2 = 16.2\,\text{m}.$$

P2-18 (a) The ball will return to the ground with the same speed it was launched. Then the total time of flight is given by

$$t = (v_\text{f} - v_\text{i})/g = (-25\,\text{m/s} - 25\,\text{m/s})/(9.8\,\text{m/s}^2) = 5.1\,\text{s}.$$

(b) For small quantities we can think in terms of derivatives, so

$$\delta t = (\delta v_\text{f} - \delta v_\text{i})/g,$$

or $\tau = 2\epsilon/g$.

P2-19 Use $y = -gt^2/2$, but only keep the absolute value. Then $y_{50} = (9.8\,\text{m/s}^2)(0.05\,\text{s})^2/2 =$ 1.2 cm; $y_{100} = (9.8\,\text{m/s}^2)(0.10\,\text{s})^2/2 = 4.9$ cm; $y_{150} = (9.8\,\text{m/s}^2)(0.15\,\text{s})^2/2 = 11$ cm; $y_{200} = (9.8\,\text{m/s}^2)(0.20\,\text{s})^2/2 = 20$ cm; $y_{250} = (9.8\,\text{m/s}^2)(0.25\,\text{s})^2/2 = 31$ cm.

P2-20 The truck will move 12 m in $(12\,\text{m})/(55 \text{ km/h}) = 0.785\,\text{s}$. The apple will fall $y = -gt^2/2 = -(9.81\,\text{m/s}^2)(0.785\,\text{s})^2/2 = -3.02\,\text{m}$.

$\boxed{\textbf{P2-21}}$ The rocket travels a distance $d_1 = \frac{1}{2}at_1^2 = \frac{1}{2}(20 \text{ m/s}^2)(60 \text{ s})^2 = 36,000$ m during the acceleration phase; the rocket velocity at the end of the acceleration phase is $v = at = (20 \text{ m/s}^2)(60 \text{ s}) =$ 1200 m/s. The second half of the trajectory can be found from Eqs. 2-29 and 2-30, with $y_0 = 36,000$ m and $v_{0y} = 1200$ m/s.

(a) The highest point of the trajectory occurs when $v_y = 0$, so

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (0) &= (1200 \text{ m/s}) - (9.8 \text{ m/s}^2)t, \\ 122 \text{ s} &= t. \end{aligned}$$

This time is used to find the height to which the rocket rises,

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ &= (36000 \text{ m}) + (1200 \text{ m/s})(122\text{s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(122 \text{ s})^2 = 110000 \text{ m}. \end{aligned}$$

(b) The easiest way to find the total time of flight is to solve Eq. 2-30 for the time when the rocket has returned to the ground. Then

$$\begin{aligned} y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \\ (0) &= (36000 \text{ m}) + (1200 \text{ m/s})t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2. \end{aligned}$$

This quadratic expression has two solutions for t; one is negative so we don't need to worry about it, the other is $t = 270\,\text{s}$. This is the free-fall part of the problem, to find the total time we need to add on the 60 seconds of accelerated motion. The total time is then 330 seconds.

P2-22 (a) The time required for the player to "fall" from the highest point a distance of $y = 15$ cm is $\sqrt{2y/g}$; the total time spent in the top 15 cm is twice this, or $2\sqrt{2y/g} = 2\sqrt{2(0.15\,\mathrm{m})/(9.81\,\mathrm{m/s})} = 0.350\,\mathrm{s}$.

(b) The time required for the player to "fall" from the highest point a distance of 76 cm is $\sqrt{2(0.76\,\mathrm{m})/(9.81\,\mathrm{m/s})} = 0.394\,\mathrm{s}$, the time required for the player to fall from the highest point a distance of $(76 - 15 = 61)$ cm is $\sqrt{2(0.61\,\mathrm{m})/g} = 0.353\,\mathrm{s}$. The time required to fall the bottom 15 cm is the difference, or 0.041 s. The time spent in the bottom 15 cm is twice this, or 0.081 s.

P2-23 (a) The average speed between A and B is $v_{\mathrm{av}} = (v + v/2)/2 = 3v/4$. We can also write $v_{\mathrm{av}} = (3.0\,\mathrm{m})/\Delta t = 3v/4$. Finally, $v/2 = v - g\Delta t$. Rearranging, $v/2 = g\Delta t$. Combining all of the above,

$$\frac{v}{2} = g\left(\frac{4(3.0\,\mathrm{m})}{3v}\right) \quad \text{or } v^2 = (8.0\,\mathrm{m})g.$$

Then $v = \sqrt{(8.0\,\mathrm{m})(9.8\,\mathrm{m/s^2})} = 8.85\,\mathrm{m/s}$.

(b) The time to the highest point above B is $v/2 = gt$, so the distance above B is

$$y = -\frac{g}{2}t^2 + \frac{v}{2}t = -\frac{g}{2}\left(\frac{v}{2g}\right)^2 + \frac{v}{2}\left(\frac{v}{2g}\right) = \frac{v^2}{8g}.$$

Then $y = (8.85\,\mathrm{m/s})^2/(8(9.8\,\mathrm{m/s^2})) = 1.00\,\mathrm{m}$.

P2-24 (a) The time in free fall is $t = \sqrt{-2y/g} = \sqrt{-2(-145\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 5.44\,\mathrm{s}$.

(b) The speed at the bottom is $v = -gt = -(9.81\,\mathrm{m/s^2})(5.44\,\mathrm{s}) = -53.4\,\mathrm{m/s}$.

(c) The time for deceleration is given by $v = -25gt$, or $t = -(-53.4\,\mathrm{m/s})/(25 \times 9.81\ /\mathrm{s^2}) = 0.218\,\mathrm{s}$. The distance through which deceleration occurred is

$$y = \frac{25g}{2}t^2 + vt = (123\,\mathrm{m/s^2})(0.218\,\mathrm{s})^2 + (-53.4\,\mathrm{m/s})(0.218\,\mathrm{s}) = -5.80\,\mathrm{m}.$$

$\boxed{\textbf{P2-25}}$ Find the time she fell from Eq. 2-30,

$$(0\text{ ft}) = (144\text{ ft}) + (0)t - \frac{1}{2}(32\text{ ft/s}^2)t^2,$$

which is a simple quadratic with solutions $t = \pm 3.0$ s. Only the positive solution is of interest. Use this time in Eq. 2-29 to find her speed when she hit the ventilator box,

$$v_y = (0) - (32\text{ ft/s}^2)(3.0\text{ s}) = -96\text{ ft/s}.$$

This becomes the initial velocity for the deceleration motion, so her average speed during deceleration is given by Eq. 2-27,

$$v_{\mathrm{av},y} = \frac{1}{2}(v_y + v_{0y}) = \frac{1}{2}((0) + (-96\text{ ft/s})) = -48\text{ ft/s}.$$

This average speed, used with the distance of 18 in (1.5 ft), can be used to find the time of deceleration

$$v_{\mathrm{av},y} = \Delta y/\Delta t,$$

and putting numbers into the expression gives $\Delta t = 0.031$ s. We actually used $\Delta y = -1.5$ ft, where the negative sign indicated that she was still moving downward. Finally, we use this in Eq. 2-26 to find the acceleration,

$$(0) = (-96\text{ ft/s}) + a(0.031\text{ s}),$$

which gives $a = +3100$ ft/s^2. In terms of g this is $a = 97g$, which can be found by multiplying through by $1 = g/(32\text{ ft/s}^2)$.

P2-26 Let the speed of the disk when it comes into contact with the ground be v_1; then the average speed during the deceleration process is $v_1/2$; so the time taken for the deceleration process is $t_1 = 2d/v_1$, where $d = -2\,\text{mm}$. But d is also give by $d = at_1^2/2 + v_1 t_1$, so

$$d = \frac{100g}{2}\left(\frac{2d}{v_1}\right)^2 + v_1\left(\frac{2d}{v_1}\right) = 200g\frac{d^2}{v_1^2} + 2d,$$

or $v_1^2 = -200gd$. The negative signs *are* necessary!.

The disk was dropped from a height $h = -y$ and it first came into contact with the ground when it had a speed of v_1. Then the average speed is $v_1/2$, and we can repeat most of the above (except $a = -g$ instead of $100g$), and then the time to fall is $t_2 = 2y/v_1$,

$$y = \frac{g}{2}\left(\frac{2y}{v_1}\right)^2 + v_1\left(\frac{2y}{v_1}\right) = 2g\frac{y^2}{v_1^2} + 2y,$$

or $v_1^2 = -2gy$. The negative signs *are* necessary!.

Equating, $y = 100d = 100(-2\,\text{mm}) = -0.2\,\text{m}$, so $h = 0.2\,\text{m}$. Note that although $100g$'s sounds like plenty, you still shouldn't be dropping your hard disk drive!

P2-27 Measure from the feet! Jim is 2.8 cm tall in the photo, so 1 cm on the photo is 60.7 cm in real-life. Then Jim has fallen a distance $y_1 = -3.04\,\text{m}$ while Clare has fallen a distance $y_2 = -5.77\,\text{m}$. Clare jumped first, and the time she has been falling is t_2; Jim jumped seconds, the time he has been falling is $t_1 = t_2 - \Delta t$. Then $y_2 = -gt_2^2/2$ and $y_1 = -gt_1^2/2$, or $t_2 = \sqrt{-2y_2/g} = \sqrt{-2(-5.77\,\text{m})/(9.81\,\text{m/s}^2)} = 1.08\,\text{s}$ and $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(3.04\,\text{m})/(9.81\,\text{m/s}^2)} = 0.79\,\text{s}$. So Jim waited 0.29 s.

P2-28 (a) Assuming she starts from rest and has a speed of v_1 when she opens her chute, then her average speed while falling freely is $v_1/2$, and the time taken to fall $y_1 = -52.0\,\text{m}$ is $t_1 = 2y_1/v_1$. Her speed v_1 is given by $v_1 = -gt_1$, or $v_1^2 = -2gy_1$. Then $v_1 = -\sqrt{-2(9.81\,\text{m/s}^2)(-52.0\,\text{m})} = -31.9\,\text{m/s}$. We must use the *negative* answer, because she fall down! The time in the air is then $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(52.0\,\text{m})/(9.81\,\text{m/s}^2)} = 3.26\,\text{s}$.

Her final speed is $v_2 = -2.90\,\text{m/s}$, so the time for the deceleration is $t_2 = (v_2 - v_1)/a$, where $a = 2.10\,\text{m/s}^2$. Then $t_2 = (-2.90\,\text{m/s} - -31.9\,\text{m/s})/(2.10\,\text{m/s}^2) = 13.8\,\text{s}$.

Finally, the total time of flight is $t = t_1 + t_2 = 3.26\,\text{s} + 13.8\,\text{s} = 17.1\,\text{s}$.

(b) The distance fallen during the deceleration phase is

$$y_2 = -\frac{g}{2}t_2^2 + v_1 t_2 = -\frac{(2.10\,\text{m/s}^2)}{2}(13.8\,\text{s})^2 + (-31.9\,\text{m/s})(13.8\,\text{s}) = -240\,\text{m}.$$

The total distance fallen is $y = y_1 + y_2 = -52.0\,\text{m} - 240\,\text{m} = -292\,\text{m}$. It is negative because she was falling down.

P2-29 Let the speed of the bearing be v_1 at the top of the windows and v_2 at the bottom. These speeds are related by $v_2 = v_1 - gt_{12}$, where $t_{12} = 0.125\,\text{s}$ is the time between when the bearing is at the top of the window and at the bottom of the window. The average speed is $v_{\text{av}} = (v_1 + v_2)/2 = v_1 - gt_{12}/2$. The distance traveled in the time t_{12} is $y_{12} = -1.20\,\text{m}$, so

$$y_{12} = v_{\text{av}} t_{12} = v_1 t_{12} - gt_{12}^2/2,$$

and expression that can be solved for v_1 to yield

$$v_1 = \frac{y_{12} + gt_{12}^2/2}{t_{12}} = \frac{(-1.20\,\text{m}) + (9.81\,\text{m/s}^2)(0.125\,\text{s})^2/2}{(0.125\,\text{s})} = -8.99\,\text{m/s}.$$

Now that we know v_1 we can find the height of the building above the top of the window. The time the object has fallen to get to the top of the window is $t_1 = -v_1/g = -(-8.99\,\mathrm{m/s})/(9.81\,\mathrm{m/s^2}) = 0.916\,\mathrm{m}$.

The total time for falling is then $(0.916\,\mathrm{s}) + (0.125\,\mathrm{s}) + (1.0\,\mathrm{s}) = 2.04\,\mathrm{s}$. Note that we remembered to divide the last time by two! The total distance from the top of the building to the bottom is then

$$y = -gt^2/2 = -(9.81\,\mathrm{m/s^2})(2.04\,\mathrm{s})^2/2 = 20.4\,\mathrm{m}.$$

P2-30 Each ball falls from a highest point through a distance of 2.0 m in

$$t = \sqrt{-2(2.0\,\mathrm{m})/(9.8\,\mathrm{m/s^2})} = 0.639\,\mathrm{s}.$$

The time each ball spends in the air is twice this, or 1.28 s. The frequency of tosses per ball is the reciprocal, $f = 1/T = 0.781\,\mathrm{s}^{-1}$. There are five ball, so we multiply this by 5, but there are two hands, so we then divide that by 2. The tosses per hand per second then requires a factor 5/2, and the tosses per hand per minute is 60 times this, or 117.

$\boxed{\textbf{P2-31}}$ Assume each hand can toss n objects per second. Let τ be the amount of time that any one object is in the air. Then $2n\tau$ is the number of objects that are in the air at any time, where the "2" comes from the fact that (most?) jugglers have two hands. We'll estimate n, but τ can be found from Eq. 2-30 for an object which falls a distance h from rest:

$$0 = h + (0)t - \frac{1}{2}gt^2,$$

solving, $t = \sqrt{2h/g}$. But τ is twice this, because the object had to go up before it could come down. So the number of objects that can be juggled is

$$4n\sqrt{2h/g}$$

We estimate $n = 2$ tosses/second. So the maximum number of objects one could juggle to a height h would be

$$3.6\sqrt{h/\mathrm{meters}}.$$

P2-32 (a) We need to look up the height of the leaning tower to solve this! If the height is $h = 56\,\mathrm{m}$, then the time taken to fall a distance $h = -y_1$ is $t_1 = \sqrt{-2y_1/g} = \sqrt{-2(-56\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 3.4\,\mathrm{s}$. The second object, however, has only fallen a a time $t_2 = t_1 - \Delta t = 3.3\,\mathrm{s}$, so the distance the second object falls is $y_2 = -gt_2^2/2 = -(9.81\,\mathrm{m/s^2})(3.3\,\mathrm{s})^2/2 = 53.4$. The difference is $y_1 - y_2 = 2.9\,\mathrm{m}$.

(b) If the vertical separation is $\Delta y = 0.01\,\mathrm{m}$, then we can approach this problem in terms of differentials,

$$\delta y = at\,\delta t,$$

so $\delta t = (0.01\,\mathrm{m})/[((9.81\,\mathrm{m/s^2})(3.4\,\mathrm{s})] = 3\times 10^{-4}\,\mathrm{s}$.

P2-33 Use symmetry, and focus on the path from the highest point downward. Then $\Delta t_U = 2t_U$, where t_U is the time from the highest point to the upper level. A similar expression exists for the lower level, but replace U with L. The distance from the highest point to the upper level is $y_U = -gt_U^2/2 = -g(\Delta t_U/2)^2/2$. The distance from the highest point to the lower level is $y_L = -gt_L^2/2 = -g(\Delta t_L/2)^2/2$. Now $H = y_U - y_L = -g\Delta t_U^2/8 - -g\Delta t_L^2/8$, which can be written as

$$g = \frac{8H}{\Delta t_L^2 - \Delta t_U^2}.$$

E3-1 The Earth orbits the sun with a speed of 29.8 km/s. The distance to Pluto is 5900×10^6 km. The time it would take the Earth to reach the orbit of Pluto is

$$t = (5900\times10^6 \text{ km})/(29.8 \text{ km/s}) = 2.0\times10^8 \text{ s},$$

or 6.3 years!

E3-2 (a) $a = F/m = (3.8\,\text{N})/(5.5\,\text{kg}) = 0.69\,\text{m/s}^2$.
(b) $t = v_\text{f}/a = (5.2\,\text{m/s})/(0.69\,\text{m/s}^2) = 7.5\,\text{s}$.
(c) $x = at^2/2 = (0.69\,\text{m/s}^2)(7.5\,\text{s})^2/2 = 20\,\text{m}$.

$\boxed{\textbf{E3-3}}$ Assuming constant acceleration we can find the average speed during the interval from Eq. 2-27

$$v_{\text{av},x} = \frac{1}{2}(v_x + v_{0x}) = \frac{1}{2}\left((5.8\times10^6 \text{ m/s}) + (0)\right) = 2.9\times10^6 \text{ m/s}.$$

From this we can find the time spent accelerating from Eq. 2-22, since $\Delta x = v_{\text{av},x}\Delta t$. Putting in the numbers $\Delta t = 5.17\times10^{-9}$s. The acceleration is then

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(5.8\times10^6 \text{ m/s}) - (0)}{(5.17\times10^{-9}\text{s})} = 1.1\times10^{15}\text{m/s}^2.$$

The net force on the electron is from Eq. 3-5,

$$\sum F_x = ma_x = (9.11\times10^{-31}\text{kg})(1.1\times10^{15}\text{m/s}^2) = 1.0\times10^{-15}\,\text{N}.$$

E3-4 The average speed while decelerating is $v_\text{av} = 0.7\times10^7$m/s. The time of deceleration is $t = x/v_\text{av} = (1.0\times10^{-14}\text{m})/(0.7\times10^7\text{m/s}) = 1.4\times10^{-21}\,\text{s}$. The deceleration is $a = \Delta v/t = (-1.4\times10^7\text{m/s})/(1.4\times10^{-21}\,\text{s}) = -1.0\times10^{28}\text{m/s}^2$. The force is $F = ma = (1.67\times10^{-27}\text{kg})(1.0\times10^{28}\text{m/s}^2) = 17\,\text{N}$.

$\boxed{\textbf{E3-5}}$ The *net* force on the sled is 92 N−90 N= 2 N; subtract because the forces are in opposite directions. Then

$$a_x = \frac{\sum F_x}{m} = \frac{(2\,\text{N})}{(25\,\text{kg})} = 8.0\times10^{-2}\text{m/s}^2.$$

E3-6 53 km/hr is 14.7 m/s. The average speed while decelerating is $v_\text{av} = 7.4\,\text{m/s}$. The time of deceleration is $t = x/v_\text{av} = (0.65\,\text{m})/(7.4\,\text{m/s}) = 8.8\times10^{-2}\,\text{s}$. The deceleration is $a = \Delta v/t = (-14.7\,\text{m/s})/(8.8\times10^{-2}\,\text{s}) = -17\times10^2\text{m/s}^2$. The force is $F = ma = (39\text{kg})(1.7\times10^2\text{m/s}^2) = 6600\,\text{N}$.

E3-7 Vertical acceleration is $a = F/m = (4.5\times10^{-15}\text{N})/(9.11\times10^{-31}\text{kg}) = 4.9\times10^{15}\text{m/s}^2$. The electron moves horizontally 33 mm in a time $t = x/v_x = (0.033\,\text{m})/(1.2\times10^7\text{m/s}) = 2.8\times10^{-9}\,\text{s}$. The vertical distance deflected is $y = at^2/2 = (4.9\times10^{15}\text{m/s}^2)(2.8\times10^{-9}\,\text{s})^2/2 = 1.9\times10^{-2}\text{m}$.

E3-8 (a) $a = F/m = (29\,\text{N})/(930\,\text{kg}) = 3.1\times10^{-2}\,\text{m/s}^2$.
(b) $x = at^2/2 = (3.1\times10^{-2}\,\text{m/s}^2)(86400\,\text{s})^2/2 = 1.2\times10^8\,\text{m}$.
(c) $v = at = (3.1\times10^{-2}\,\text{m/s}^2)(86400\,\text{s}) = 2700\,\text{m/s}$.

$\boxed{\textbf{E3-9}}$ Write the expression for the motion of the first object as $\sum F_x = m_1 a_{1x}$ and that of the second object as $\sum F_x = m_2 a_{2x}$. In both cases there is only one force, F, on the object, so $\sum F_x = F$. We will solve these for the mass as $m_1 = F/a_1$ and $m_2 = F/a_2$. Since $a_1 > a_2$ we can conclude that $m_2 > m_1$

(a) The acceleration of and object with mass $m_2 - m_1$ under the influence of a single force of magnitude F would be

$$a = \frac{F}{m_2 - m_1} = \frac{F}{F/a_2 - F/a_1} = \frac{1}{1/(3.30\,\mathrm{m/s^2}) - 1/(12.0\,\mathrm{m/s^2})},$$

which has a numerical value of $a = 4.55$ m/s^2.

(b) Similarly, the acceleration of an object of mass $m_2 + m_1$ under the influence of a force of magnitude F would be

$$a = \frac{1}{1/a_2 + 1/a_1} = \frac{1}{1/(3.30\,\mathrm{m/s^2}) + 1/(12.0\,\mathrm{m/s^2})},$$

which is the same as part (a) except for the sign change. Then $a = 2.59$ m/s^2.

E3-10 (a) The required acceleration is $a = v/t = 0.1c/t$. The required force is $F = ma = 0.1mc/t$. Then

$$F = 0.1(1200\times10^3\,\mathrm{kg})(3.00\times10^8\mathrm{m/s})/(2.59\times10^5\mathrm{s}) = 1.4\times10^8\mathrm{N},$$

and

$$F = 0.1(1200\times10^3\,\mathrm{kg})(3.00\times10^8\mathrm{m/s})/(5.18\times10^6\mathrm{s}) = 6.9\times10^6\mathrm{N},$$

(b) The distance traveled during the acceleration phase is $x_1 = at_1^2/2$, the time required to travel the remaining distance is $t_2 = x_2/v$ where $x_2 = d - x_1$. d is 5 light-months, or $d = (3.00\times 10^8\mathrm{m/s})(1.30\times10^7\mathrm{s}) = 3.90\times10^{15}\mathrm{m}$. Then

$$t = t_1 + t_2 = t_1 + \frac{d - x_1}{v} = t_1 + \frac{2d - at_1^2}{2v} = t_1 + \frac{2d - vt_1}{2v}.$$

If t_1 is 3 days, then

$$t = (2.59\times10^5\mathrm{s}) + \frac{2(3.90\times10^{15}\mathrm{m}) - (3.00\times10^7\mathrm{m/s})(2.59\times10^5\mathrm{s})}{2(3.00\times10^7\mathrm{m/s})} = 1.30\times10^8\mathrm{s} = 4.12\text{ yr},$$

if t_1 is 2 months, then

$$t = (5.18\times10^6\mathrm{s}) + \frac{2(3.90\times10^{15}\mathrm{m}) - (3.00\times10^7\mathrm{m/s})(5.18\times10^6\mathrm{s})}{2(3.00\times10^7\mathrm{m/s})} = 1.33\times10^8\mathrm{s} = 4.20\text{ yr},$$

$\boxed{\textbf{E3-11}}$ (a) The net force on the second block is given by

$$\sum F_x = m_2 a_{2x} = (3.8\,\mathrm{kg})(2.6\,\mathrm{m/s^2}) = 9.9\,\mathrm{N}.$$

There is only one (relevant) force on the block, the force of block 1 on block 2.

(b) There is only one (relevant) force on block 1, the force of block 2 on block 1. By Newton's third law this force has a magnitude of 9.9 N. Then Newton's second law gives $\sum F_x = -9.9$ N$= m_1 a_{1x} = (4.6$ kg$)a_{1x}$. So $a_{1x} = -2.2$ m/s^2 at the instant that $a_{2x} = 2.6$ m/s^2.

E3-12 (a) $W = (5.00$ lb$)(4.448\,\mathrm{N/lb}) = 22.2\,\mathrm{N}$; $m = W/g = (22.2\,\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 2.26\,\mathrm{kg}$.

(b) $W = (240$ lb$)(4.448\,\mathrm{N/lb}) = 1070\,\mathrm{N}$; $m = W/g = (1070\,\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 109\,\mathrm{kg}$.

(c) $W = (3600$ lb$)(4.448\,\mathrm{N/lb}) = 16000\,\mathrm{N}$; $m = W/g = (16000\,\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 1630\,\mathrm{kg}$.

E3-13 (a) $W = (1420.00 \text{ lb})(4.448\,\text{N/lb}) = 6320\,\text{N}$; $m = W/g = (6320\,\text{N})/(9.81\,\text{m/s}^2) = 644\,\text{kg}$.
(b) $m = 412\,\text{kg}$; $W = mg = (412\,\text{kg})(9.81\,\text{m/s}^2) = 4040\,\text{N}$.

E3-14 (a) $W = mg = (75.0\,\text{kg})(9.81\,\text{m/s}^2) = 736\,\text{N}$.
(b) $W = mg = (75.0\,\text{kg})(3.72\,\text{m/s}^2) = 279\,\text{N}$.
(c) $W = mg = (75.0\,\text{kg})(0\,\text{m/s}^2) = 0\,\text{N}$.
(d) The mass is 75.0 kg at all locations.

$\boxed{\textbf{E3-15}}$ If $g = 9.81$ m/s^2, then $m = W/g = (26.0 \text{ N})/(9.81 \text{ m/s}^2) = 2.65$ kg.
(a) Apply $W = mg$ again, but now $g = 4.60$ m/s^2, so at this point $W = (2.65 \text{ kg})(4.60 \text{ m/s}^2) =$ 12.2 N.
(b) If there is no gravitational force, there is no weight, because $g = 0$. There is still mass, however, and that mass is still 2.65 kg.

E3-16 Upward force balances weight, so $F = W = mg = (12000\,\text{kg})(9.81\,\text{m/s}^2) = 1.2\times10^5\text{N}$.

E3-17 Mass is $m = W/g$; net force is $F = ma$, or $F = Wa/g$. Then

$$F = (3900 \text{ lb})(13 \text{ ft/s}^2)/(32 \text{ ft/s}^2) = 1600 \text{ lb}.$$

E3-18 $a = \Delta v/\Delta t = (450\,\text{m/s})/(1.82\,\text{s}) = 247\,\text{m/s}^2$. Net force is $F = ma = (523\,\text{kg})(247\,\text{m/s}^2) = 1.29\times10^5\text{N}$.

$\boxed{\textbf{E3-19}}$ $\sum F_x = 2(1.4\times10^5 \text{ N}) = ma_x$. Then $m = 1.22\times10^5$ kg and

$$W = mg = (1.22\times10^5\text{kg})(9.81\,\text{m/s}^2) = 1.20\times10^6\,\text{N}.$$

E3-20 Do part (b) first; there must be a 10 lb force to support the mass. Now do part (a), but cover up the left hand side of both pictures. If you can't tell which picture is which, then they must both be 10 lb!

E3-21 (b) Average speed during deceleration is 40 km/h, or 11 m/s. The time taken to stop the car is then $t = x/v_{\text{av}} = (61\,\text{m})/(11\,\text{m/s}) = 5.6\,\text{s}$.
(a) The deceleration is $a = \Delta v/\Delta t = (22\,\text{m/s})/(5.6\,\text{s}) = 3.9\,\text{m/s}^2$. The braking force is $F = ma = Wa/g = (13,000\,\text{N})(3.9\,\text{m/s}^2)/(9.81\,\text{m/s}^2) = 5200\,\text{N}$.
(d) The deceleration is same; the time to stop the car is then $\Delta t = \Delta v/a = (11\,\text{m/s})/(3.9\,\text{m/s}^2) = 2.8\,s$.
(c) The distance traveled during stopping is $x = v_{\text{av}}t = (5.6\,\text{m/s})(2.8\,\text{s}) = 16\,\text{m}$.

E3-22 Assume acceleration of free fall is $9.81\,\text{m/s}^2$ at the altitude of the meteor. The net force is $F_{\text{net}} = ma = (0.25\,\text{kg})(9.2\,m/\text{s}^2) = 2.30\,\text{N}$. The weight is $W = mg = (0.25\,kg)(9.81\,\text{m/s}^2) = 2.45\,\text{N}$. The retarding force is $F_{\text{net}} - W = (2.3\,\text{N}) - (2.45\,\text{N}) = -0.15\,\text{N}$.

$\boxed{\textbf{E3-23}}$ (a) Find the time during the "jump down" phase from Eq. 2-30.

$$(0 \text{ m}) = (0.48 \text{ m}) + (0)t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2,$$

which is a simple quadratic with solutions $t = \pm0.31$ s. Use this time in Eq. 2-29 to find his speed when he hit ground,

$$v_y = (0) - (9.8 \text{ m/s}^2)(0.31 \text{ s}) = -3.1 \text{ m/s}.$$

This becomes the initial velocity for the deceleration motion, so his average speed during deceleration is given by Eq. 2-27,

$$v_{\text{av},y} = \frac{1}{2}(v_y + v_{0y}) = \frac{1}{2}((0) + (-3.1 \text{ m/s})) = -1.6 \text{ m/s}.$$

This average speed, used with the distance of -2.2 cm (-0.022 m), can be used to find the time of deceleration

$$v_{\text{av},y} = \Delta y/\Delta t,$$

and putting numbers into the expression gives $\Delta t = 0.014$ s. Finally, we use this in Eq. 2-26 to find the acceleration,

$$(0) = (-3.1 \text{ m/s}) + a(0.014 \text{ s}),$$

which gives $a = 220 \text{ m/s}^2$.

(b) The average *net* force on the man is

$$\sum F_y = ma_y = (83\,\text{kg})(220\,\text{m/s}^2) = 1.8\times 10^4\text{N}.$$

E3-24 The average speed of the salmon while decelerating is 4.6 ft/s. The time required to stop the salmon is then $t = x/v_{\text{av}} = (0.38 \text{ ft})/(4.6 \text{ ft/s}) = 8.3\times 10^{-2}$s. The deceleration of the salmon is $a = \Delta v/\Delta t = (9.2 \text{ ft/s})/(8.2\text{-}2\text{s}) = 110 \text{ ft/s}^2$. The force on the salmon is then $F = Wa/g = (19 \text{ lb})(110 \text{ ft/s}^2)/(32 \text{ ft/s}^2) = 65$ lb.

E3-25 From appendix G we find 1 lb = 4.448 N; so the weight is (100 lb)(4.448 N/1 lb) = 445 N; similarly the cord will break if it pulls upward on the object with a force greater than 387 N. The mass of the object is $m = W/g = (445 \text{ N})/(9.8 \text{ m/s}^2) = 45$ kg.

There are two vertical forces on the 45 kg object, an upward force from the cord F_{OC} (which has a maximum value of 387 N) and a downward force from gravity F_{OG}. Then $\sum F_y = F_{OC} - F_{OG} = (387 \text{ N}) - (445 \text{ N}) = -58$ N. Since the net force is negative, the object must be accelerating downward according to

$$a_y = \sum F_y/m = (-58\,\text{N})/(45\,\text{kg}) = -1.3\,\text{m/s}^2.$$

E3-26 (a) Constant speed means no acceleration, hence no net force; this means the weight is balanced by the force from the scale, so the scale reads 65 N.

(b) Net force on mass is $F_{\text{net}} = ma = Wa/g = (65\,\text{N})(-2.4\,\text{m/s}^2)/(9.81\,\text{m/s}^2) = -16\,\text{N}$.. Since the weight is 65 N, the scale must be exerting a force of $(-16\,\text{N}) - (-65\,\text{N}) = 49\,\text{N}$.

E3-27 The magnitude of the net force is $W - R = (1600\,\text{kg})(9.81\,\text{m/s}^2) - (3700\,\text{N}) = 12000\,\text{N}$. The acceleration is then $a = F/m = (12000\,\text{N})/(1600\,\text{kg}) = 7.5\,\text{m/s}^2$. The time to fall is

$$t = \sqrt{2y/a} = \sqrt{2(-72\,\text{m})/(-7.5\,\text{m/s}^2)} = 4.4\,\text{s}.$$

The final speed is $v = at = (-7.5\,\text{m/s}^2)(4.4\,\text{s}) = 33\,\text{m/s}$. Get better brakes, eh?

E3-28 The average speed during the acceleration is 140 ft/s. The time for the plane to travel 300 ft is

$$t = x/v_{\text{av}} = (300 \text{ ft})/(140 \text{ ft/s}) = 2.14\,\text{s}.$$

The acceleration is then

$$a = \Delta v/\Delta t = (280 \text{ ft/s})/(2.14\,\text{s}) = 130 \text{ ft/s}^2.$$

The net force on the plane is $F = ma = Wa/g = (52000 \text{ lb})(130 \text{ ft/s}^2)/(32 \text{ ft/s}^2) = 2.1\times 10^5$lb.

The force exerted by the catapult is then $2.1\times 10^5\text{lb} - 2.4\times 10^4 \text{ lb} = 1.86\times 10^5$ lb.

E3-29 (a) The acceleration of a hovering rocket is 0, so the net force is zero; hence the thrust must equal the weight. Then $T = W = mg = (51000\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 5.0\times10^5\mathrm{N}$.

(b) If the rocket accelerates upward then the net force is $F = ma = (51000\,\mathrm{kg})(18\,\mathrm{m/s^2}) = 9.2\times10^5\,\mathrm{N}$. Now $F_{\mathrm{net}} = T - W$, so $T = 9.2\times10^5\,\mathrm{N} + 5.0\times10^5\mathrm{N} = 1.42\times10^6\mathrm{N}$.

E3-30 (a) Net force on parachute + person system is $F_{\mathrm{net}} = ma = (77\,\mathrm{kg} + 5.2\,\mathrm{kg})(-2.5\,\mathrm{s^2}) = -210\,\mathrm{N}$. The weight of the system is $W = mg = (77\,\mathrm{kg}+5.2\,\mathrm{kg})(9.81\,\mathrm{s^2}) = 810\,\mathrm{N}$. If P is the upward force of the air on the system (parachute) then $P = F_{\mathrm{net}} + W = (-210\,\mathrm{N}) + (810\,\mathrm{N}) = 600\,\mathrm{N}$.

(b) The net force on the *parachute* is $F_{\mathrm{net}} = ma = (5.2\,\mathrm{kg})(-2.5\,\mathrm{s^2}) = -13\,\mathrm{N}$. The weight of the parachute is $W = mg = (5.2\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 51\,\mathrm{N}$. If D is the downward force of the person on the parachute then $D = -F_{\mathrm{net}} - W + P = -(-13\,\mathrm{N}) - (51\,\mathrm{N}) + 600\,\mathrm{N} = 560\,\mathrm{N}$.

$\boxed{\textbf{E3-31}}$ (a) The *total* mass of the helicopter+car system is 19,500 kg; and the only other force acting on the system is the force of gravity, which is

$$W = mg = (19,500\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 1.91\times10^5\,\mathrm{N}.$$

The force of gravity is directed down, so the net force on the system is $\sum F_y = F_{BA} - (1.91\times10^5$ N). The net force can also be found from Newton's second law: $\sum F_y = ma_y = (19,500$ kg)(1.4 m/s^2) $= 2.7\times10^4$ N. Equate the two expressions for the net force, $F_{BA} - (1.91\times10^5$ N) $= 2.7\times10^4$ N, and solve; $F_{BA} = 2.2\times10^5$ N.

(b) Repeat the above steps except: (1) the system will consist only of the car, and (2) the upward force on the car comes from the supporting cable only F_{CC}. The weight of the car is $W = mg = (4500$ kg)(9.8 m/s^2) $= 4.4\times10^4$ N. The net force is $\sum F_y = F_{CC} - (4.4\times10^4$ N), it can also be written as $\sum F_y = ma_y = (4500$ kg)(1.4 m/s^2) $= 6300$ N. Equating, $F_{CC} = 50,000$ N.

P3-1 (a) The acceleration is $a = F/m = (2.7\times10^{-5}\mathrm{N})/(280\,\mathrm{kg}) = 9.64\times10^{-8}\mathrm{m/s^2}$. The displacement (from the original trajectory) is

$$y = at^2/2 = (9.64\times10^{-8}\mathrm{m/s^2})(2.4\,\mathrm{s})^2/2 = 2.8\times10^{-7}\mathrm{m}.$$

(b) The acceleration is $a = F/m = (2.7\times10^{-5}\mathrm{N})/(2.1\,\mathrm{kg}) = 1.3\times10^{-5}\mathrm{m/s^2}$. The displacement (from the original trajectory) is

$$y = at^2/2 = (1.3\times10^{-5}\mathrm{m/s^2})(2.4\,\mathrm{s})^2/2 = 3.7\times10^{-5}\mathrm{m}.$$

P3-2 (a) The acceleration of the sled is $a = F/m = (5.2\,\mathrm{N})/(8.4\,\mathrm{kg}) = 0.62\,\mathrm{m/s^2}$.

(b) The acceleration of the girl is $a = F/m = (5.2\,\mathrm{N})/(40\,kg) = 0.13\,\mathrm{m/s^2}$.

(c) The distance traveled by girl is $x_1 = a_1t^2/2$; the distance traveled by the sled is $x_2 = a_2t^2/2$. The two meet when $x_1 + x_2 = 15\,\mathrm{m}$. This happens when $(a_1 + a_2)t^2 = 30\,\mathrm{m}$. They then meet when $t = \sqrt{(30\,\mathrm{m})/(0.13\,\mathrm{m/s^2} + 0.62\,\mathrm{m/s^2})} = 6.3\,\mathrm{s}$. The girl moves $x_1 = (0.13\,\mathrm{m/s^2})(6.3\,\mathrm{s})^2/2 = 2.6\,\mathrm{m}$.

$\boxed{\textbf{P3-3}}$ (a) Start with block one. It starts from rest, accelerating through a distance of 16 m in a time of 4.2 s. Applying Eq. 2-28,

$$\begin{aligned} x &= x_0 + v_{0x}t + \frac{1}{2}a_xt^2, \\ -16\ \mathrm{m} &= (0) + (0)(4.2\ \mathrm{s}) + \frac{1}{2}a_x(4.2\ \mathrm{s})^2, \end{aligned}$$

find the acceleration to be $a_x = -1.8$ m/s^2.

Now for the second block. The acceleration of the second block is identical to the first for much the same reason that all objects fall with approximately the same acceleration.

(b) The initial and final velocities are related by a sign, then $v_x = -v_{0x}$ and Eq. 2-26 becomes

$$\begin{aligned} v_x &= v_{0x} + a_x t, \\ -v_{0x} &= v_{0x} + a_x t, \\ -2v_{0x} &= (-1.8 \text{ m/s}^2)(4.2 \text{ s}). \end{aligned}$$

which gives an initial velocity of $v_{0x} = 3.8$ m/s.

(c) Half of the time is spent coming down from the highest point, so the time to "fall" is 2.1 s. The distance traveled is found from Eq. 2-28,

$$x = (0) + (0)(2.1\,\text{s}) + \frac{1}{2}(-1.8\,\text{m/s}^2)(2.1\,\text{s})^2 = -4.0\,\text{m}.$$

P3-4 (a) The weight of the engine is $W = mg = (1400\,\text{kg})(9.81\,\text{m/s}^2) = 1.37\times10^4\,\text{N}$. If each bolt supports 1/3 of this, then the force on a bolt is 4600 N.

(b) If engine accelerates up at $2.60\,\text{m/s}^2$, then net force on the engine is

$$F_{\text{net}} = ma = (1400\,\text{kg})(2.60\,\text{m/s}^2) = 3.64\times10^3\,\text{N}.$$

The upward force from the bolts must then be

$$B = F_{\text{net}} + W = (3.64\times10^3\,\text{N}) + (1.37\times10^4\,\text{N}) = 1.73\times10^4\text{N}.$$

The force per bolt is one third this, or 5800 N.

P3-5 (a) If craft descends with constant speed then net force is zero, so thrust balances weight. The weight is then 3260 N.

(b) If the thrust is 2200 N the net force is $2200\,\text{N} - 3260\,\text{N} = -1060\,\text{N}$. The mass is then $m = F/a = (-1060\,\text{N})/(-0.390\,\text{m/s}^2) = 2720\,\text{kg}$.

(c) The acceleration due to gravity is $g = W/m = (3260\,\text{N})/(2720\,\text{kg}) = 1.20\,\text{m/s}^2$.

P3-6 The weight is originally Mg. The net force is originally $-Ma$. The upward force is then originally $B = Mg - Ma$. The goal is for a net force of $(M-m)a$ and a weight $(M-m)g$. Then

$$(M-m)a = B - (M-m)g = Mg - Ma - Mg + mg = mg - Ma,$$

or $m = 2Ma/(a+g)$.

$\boxed{\textbf{P3-7}}$ (a) Consider all three carts as one system. Then

$$\begin{aligned} \sum F_x &= m_{\text{total}} a_x, \\ 6.5 \text{ N} &= (3.1\,\text{kg} + 2.4\,\text{kg} + 1.2\,\text{kg})a_x, \\ 0.97 \text{ m/s}^2 &= a_x. \end{aligned}$$

(b) Now choose your system so that it only contains the third car. Then

$$\sum F_x = F_{23} = m_3 a_x = (1.2\,\text{kg})(0.97\,\text{m/s}^2).$$

The unknown can be solved to give $F_{23} = 1.2\,\text{N}$ directed to the right.

(c) Consider a system involving the second and third carts. Then $\sum F_x = F_{12}$, so Newton's law applied to the system gives

$$F_{12} = (m_2 + m_3)a_x = (2.4\,\text{kg} + 1.2\,\text{kg})(0.97\,\text{m/s}^2) = 3.5\,\text{N}.$$

P3-8 (a) $F = ma = (45.2\,\text{kg} + 22.8\,\text{kg} + 34.3\,\text{kg})(1.32\,\text{m/s}^2) = 135\,\text{N}$.

(b) Consider only m_3. Then $F = ma = (34.3\,\text{kg})(1.32\,\text{m/s}^2) = 45.3\,\text{N}$.

(c) Consider m_2 and m_3. Then $F = ma = (22.8\,\text{kg} + 34.3\,\text{kg})(1.32\,\text{m/s}^2) = 75.4\,\text{N}$.

P3-9 (c) The net force on each link is the same, $F_{\text{net}} = ma = (0.100\,\text{kg})(2.50\,\text{m/s}^2) = 0.250\,\text{N}$.

(a) The weight of each link is $W = mg = (0.100\,\text{kg})(9.81\,\text{m/s}^2) = 0.981\,\text{N}$. On each link (except the top or bottom link) there is a weight, an upward force from the link above, and a downward force from the link below. Then $F_{\text{net}} = U - D - W$. Then $U = F_{\text{net}} + W + D = (0.250\,\text{N}) + (0.981\,\text{N}) + D = 1.231\,\text{N} + D$. For the bottom link $D = 0$. For the bottom link, $U = 1.23\,\text{N}$. For the link above, $U = 1.23\,\text{N} + 1.23\,\text{N} = 2.46\,\text{N}$. For the link above, $U = 1.23\,\text{N} + 2.46\,\text{N} = 3.69\,\text{N}$. For the link above, $U = 1.23\,\text{N} + 3.69\,\text{N} = 4.92\,\text{N}$.

(b) For the top link, the upward force is $U = 1.23\,\text{N} + 4.92\,\text{N} = 6.15\,\text{N}$.

P3-10 (a) The acceleration of the two blocks is $a = F/(m_1 + m_2)$ The net force on block 2 is from the force of contact, and is

$$P = m_2 a = F m_2/(m_1 + m_2) = (3.2\,\text{N})(1.2\,\text{kg})/(2.3\,\text{kg} + 1.2\,\text{kg}) = 1.1\,\text{N}.$$

(b) The acceleration of the two blocks is $a = F/(m_1 + m_2)$ The net force on block 1 is from the force of contact, and is

$$P = m_1 a = F m_1/(m_1 + m_2) = (3.2\,\text{N})(2.3\,\text{kg})/(2.3\,\text{kg} + 1.2\,\text{kg}) = 2.1\,\text{N}.$$

Not a third law pair, eh?

$\boxed{\textbf{P3-11}}$ (a) Treat the system as including both the block and the rope, so that the mass of the system is $M + m$. There is one (relevant) force which acts on the system, so $\sum F_x = P$. Then Newton's second law would be written as $P = (M + m)a_x$. Solve this for a_x and get $a_x = P/(M + m)$.

(b) Now consider only the block. The horizontal force doesn't act on the block; instead, there is the force of the rope on the block. We'll assume that force has a magnitude R, and this is the *only* (relevant) force on the block, so $\sum F_x = R$ for the net force on the block.. In this case Newton's second law would be written $R = Ma_x$. Yes, a_x is the same in part (a) and (b); the acceleration of the block is the same as the acceleration of the block + rope. Substituting in the results from part (a) we find

$$R = \frac{M}{M + m} P.$$

E4-1 (a) The time to pass between the plates is $t = x/v_x = (2.3 \text{ cm})/(9.6{\times}10^8 \text{ cm/s}) = 2.4{\times}10^{-9}\text{s}$.

(b) The vertical displacement of the beam is then $y = a_y t^2/2 == (9.4 \times 10^{16} \text{ cm/s}^2)(2.4 \times 10^{-9}\text{s})^2/2 = 0.27$ cm.

(c) The velocity components are $v_x = 9.6 \times 10^8$ cm/s and $v_y = a_y t = (9.4 \times 10^{16} \text{ cm/s}^2)(2.4 \times 10^{-9}\text{s}) = 2.3{\times}10^8\text{cm/s}$.

E4-2 $\vec{\mathbf{a}} = \Delta\vec{\mathbf{v}}/\Delta t = -(6.30\hat{\mathbf{i}} - 8.42\hat{\mathbf{j}})(\text{m/s})/(3\,\text{s}) = (-2.10\hat{\mathbf{i}} + 2.81\hat{\mathbf{j}})(\text{m/s}^2)$.

$\boxed{\textbf{E4-3}}$ (a) The velocity is given by

$$\begin{aligned} \frac{d\vec{\mathbf{r}}}{dt} &= \frac{d}{dt}\left(A\hat{\mathbf{i}}\right) + \frac{d}{dt}\left(Bt^2\hat{\mathbf{j}}\right) + \frac{d}{dt}\left(Ct\hat{\mathbf{k}}\right), \\ \vec{\mathbf{v}} &= (0) + 2Bt\hat{\mathbf{j}} + C\hat{\mathbf{k}}. \end{aligned}$$

(b) The acceleration is given by

$$\begin{aligned} \frac{d\vec{\mathbf{v}}}{dt} &= \frac{d}{dt}\left(2Bt\hat{\mathbf{j}}\right) + \frac{d}{dt}\left(C\hat{\mathbf{k}}\right), \\ \vec{\mathbf{a}} &= (0) + 2B\hat{\mathbf{j}} + (0). \end{aligned}$$

(c) Nothing exciting happens in the x direction, so we will focus on the yz plane. The trajectory in this plane is a parabola.

E4-4 (a) Maximum x is when $v_x = 0$. Since $v_x = a_x t + v_{x,0}$, $v_x = 0$ when $t = -v_{x,0}/a_x = -(3.6\,\text{m/s})/(-1.2\,\text{m/s}^2) = 3.0\,\text{s}$.

(b) Since $v_x = 0$ we have $|\vec{\mathbf{v}}| = |v_y|$. But $v_y = a_y t + v_{y,0} = -(1.4\,\text{m/s})(3.0\,\text{s}) + (0) = -4.2\,\text{m/s}$. Then $|\vec{\mathbf{v}}| = 4.2\,\text{m/s}$.

(c) $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2 + \vec{\mathbf{v}}_0 t$, so

$$\vec{\mathbf{r}} = [-(0.6\,\text{m/s}^2)\hat{\mathbf{i}} - (0.7\,\text{m/s}^2)\hat{\mathbf{j}}](3.0\,\text{s})^2 + [(3.6\,\text{m/s})\hat{\mathbf{i}}](3.0\,\text{s}) = (5.4\,\text{m})\hat{\mathbf{i}} - (6.3\,\text{m})\hat{\mathbf{j}}.$$

E4-5 $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (3.7\,\text{N})\hat{\mathbf{j}} + (4.3\,\text{N})\hat{\mathbf{i}}$. Then $\vec{\mathbf{a}} = \vec{\mathbf{F}}/m = (0.71\,\text{m/s}^2)\hat{\mathbf{j}} + (0.83\,\text{m/s}^2)\hat{\mathbf{i}}$.

E4-6 (a) The acceleration is $\vec{\mathbf{a}} = \vec{\mathbf{F}}/m = (2.2\,\text{m/s}^2)\hat{\mathbf{j}}$. The velocity after 15 seconds is $\vec{\mathbf{v}} = \vec{\mathbf{a}}t + \vec{\mathbf{v}}_{\mathbf{0}}$, or

$$\vec{\mathbf{v}} = [(2.2\,\text{m/s}^2)\hat{\mathbf{j}}](15\,\text{s}) + [(42\,\text{m/s})\hat{\mathbf{i}}] = (42\,\text{m/s})\hat{\mathbf{i}} + (33\,\text{m/s})\hat{\mathbf{j}}.$$

(b) $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2 + \vec{\mathbf{v}}_0 t$, so

$$\vec{\mathbf{r}} = [(1.1\,\text{m/s}^2)\hat{\mathbf{j}}](15\,\text{s})^2 + [(42\,\text{m/s})\hat{\mathbf{i}}](15\,\text{s}) = (630\,\text{m})\hat{\mathbf{i}} + (250\,\text{m})\hat{\mathbf{j}}.$$

$\boxed{\textbf{E4-7}}$ The block has a weight $W = mg = (5.1 \text{ kg})(9.8 \text{ m/s}^2) = 50$ N.

(a) Initially $P = 12$ N, so $P_y = (12\,\text{N})\sin(25°) = 5.1$ N and $P_x = (12\,\text{N})\cos(25°) = 11$ N. Since the upward component is less than the weight, the block doesn't leave the floor, and a normal force will be present which will make $\sum F_y = 0$. There is only one contribution to the horizontal force, so $\sum F_x = P_x$. Newton's second law then gives $a_x = P_x/m = (11 \text{ N})/(5.1 \text{ kg}) = 2.2 \text{ m/s}^2$.

(b) As P is increased, so is P_y; eventually P_y will be large enough to overcome the weight of the block. This happens just after $P_y = W = 50$ N, which occurs when $P = P_y/\sin\theta = 120$ N.

(c) Repeat part (a), except now $P = 120$ N. Then $P_x = 110$ N, and the acceleration of the block is $a_x = P_x/m = 22 \text{ m/s}^2$.

E4-8 (a) The block has weight $W = mg = (96.0\,\text{kg})(9.81\,\text{m/s}^2) = 942\,\text{N}$. $P_x = (450\,\text{N})\cos(38°) = 355\,\text{N}$; $P_y = (450\,\text{N})\sin(38°) = 277\,\text{N}$. Since $P_y < W$ the crate stays on the floor and there is a normal force $N = W - P_y$. The net force in the x direction is $F_x = P_x - (125\,\text{N}) = 230\,\text{N}$. The acceleration is $a_x = F_x/m = (230\,\text{N})/(96.0\,\text{kg}) = 2.40\,\text{m/s}^2$.

(b) The block has mass $m = W/g = (96.0\,\text{N})/(9.81\,\text{m/s}^2) = 9.79\,\text{kg}$. $P_x = (450\,\text{N})\cos(38°) = 355\,\text{N}$; $P_y = (450\,\text{N})\sin(38°) = 277\,\text{N}$. Since $P_y > W$ the crate lifts off of the floor! The net force in the x direction is $F_x = P_x - (125\,\text{N}) = 230\,\text{N}$. The x acceleration is $a_x = F_x/m = (230\,\text{N})/(9.79\,\text{kg}) = 23.5\,\text{m/s}^2$. The net force in the y direction is $F_y = P_y - W = 181\,\text{N}$. The y acceleration is $a_y = F_y/m = (181\,\text{N})/(9.79\,\text{kg}) = 18.5\,\text{m/s}^2$. Wow.

E4-9 Let y be perpendicular and x be parallel to the incline. Then $P = 4600\,\text{N}$;

$$P_x = (4600\,\text{N})\cos(27°) = 4100\,\text{N};$$

$$P_y = (4600\,\text{N})\sin(27°) = 2090\,\text{N}.$$

The weight of the car is $W = mg = (1200\,\text{kg})(9.81\,\text{m/s}^2) = 11800\,\text{N}$;

$$W_x = (11800\,\text{N})\sin(18°) = 3650\,\text{N};$$

$$W_y = (11800\,\text{N})\cos(18°) = 11200\,\text{N}.$$

Since $W_y > P_y$ the car stays on the incline. The net force in the x direction is $F_x = P_x - W_x = 450\,\text{N}$. The acceleration in the x direction is $a_x = F_x/m = (450\,\text{N})/(1200\,\text{kg}) = 0.375\,\text{m/s}^2$. The distance traveled in 7.5 s is $x = a_x t^2/2 = (0.375\,\text{m/s}^2)(7.5\,\text{s})^2/2 = 10.5\,\text{m}$.

E4-10 Constant speed means zero acceleration, so net force is zero. Let y be perpendicular and x be parallel to the incline. The weight is $W = mg = (110\,\text{kg})(9.81\,\text{m/s}^2) = 1080\,\text{N}$; $W_x = W\sin(34°)$; $W_y = W\cos(34°)$. The push F has components $F_x = F\cos(34°)$ and $F_y = -F\sin(34°)$. The y components will balance after a normal force is introduced; the x components will balance if $F_x = W_x$, or $F = W\tan(34°) = (1080\,\text{N})\tan(34°) = 730\,\text{N}$.

$\boxed{\textbf{E4-11}}$ If the x axis is parallel to the river and the y axis is perpendicular, then $\vec{\mathbf{a}} = 0.12\hat{\mathbf{i}}\,\text{m/s}^2$. The net force on the barge is

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (9500\,\text{kg})(0.12\hat{\mathbf{i}}\,\text{m/s}^2) = 1100\hat{\mathbf{i}}\,\text{N}.$$

The force exerted on the barge by the horse has components in both the x and y direction. If $P = 7900$ N is the magnitude of the pull and $\theta = 18°$ is the direction, then $\vec{\mathbf{P}} = P\cos\theta\hat{\mathbf{i}} + P\sin\theta\hat{\mathbf{j}} = (7500\hat{\mathbf{i}} + 2400\hat{\mathbf{j}})$ N.

Let the force exerted on the barge by the water be $\vec{\mathbf{F_w}} = F_{w,x}\hat{\mathbf{i}} + F_{w,y}\hat{\mathbf{j}}$. Then $\sum F_x = (7500$ N$) + F_{w,x}$ and $\sum F_y = (2400$ N$) + F_{w,y}$. But we already found $\sum \vec{\mathbf{F}}$, so

$$\begin{aligned} F_x = 1100\text{ N} &= 7500\text{ N} + F_{w,x}, \\ F_y = 0 &= 2400\text{ N} + F_{w,y}. \end{aligned}$$

Solving, $F_{w,x} = -6400$ N and $F_{w,y} = -2400$ N. The magnitude is found by $F_w = \sqrt{F_{w,x}^2 + F_{w,y}^2} = 6800$ N.

E4-12 (a) Let y be perpendicular and x be parallel to the direction of motion of the plane. Then $W_x = mg\sin\theta$; $W_y = mg\cos\theta$; $m = W/g$. The plane is accelerating in the x direction, so $a_x = 2.62\,\mathrm{m/s^2}$; the net force is in the x direction, where $F_x = ma_x$. But $F_x = T - W_x$, so

$$T = F_x + W_x = W\frac{a_x}{g} + W\sin\theta = (7.93\times10^4\mathrm{N})\left[\frac{(2.62\,\mathrm{m/s^2})}{(9.81\,\mathrm{m/s^2})} + \sin(27^\circ)\right] = 5.72\times10^4\mathrm{N}.$$

(b) There is no motion in the y direction, so

$$L = W_y = (7.93\times10^4\mathrm{N})\cos(27^\circ) = 7.07\times10^4\mathrm{N}.$$

$\boxed{\textbf{E4-13}}$ (a) The ball rolled off horizontally so $v_{0y} = 0$. Then

$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2, \\ (-4.23\text{ ft}) &= (0)t - \frac{1}{2}(32.2\text{ ft/s}^2)t^2, \end{aligned}$$

which can be solved to yield $t = 0.514$ s.

(b) The initial velocity in the x direction can be found from $x = v_{0x}t$; rearranging, $v_{0x} = x/t = (5.11\text{ ft})/(0.514\text{ s}) = 9.94$ ft/s. Since there is no y component to the velocity, then the initial speed is $v_0 = 9.94$ ft/s.

E4-14 The electron travels for a time $t = x/v_x$. The electron "falls" vertically through a distance $y = -gt^2/2$ in that time. Then

$$y = -\frac{g}{2}\left(\frac{x}{v_x}\right)^2 = -\frac{(9.81\,\mathrm{m/s^2})}{2}\left(\frac{(1.0\,\mathrm{m})}{(3.0\times10^7\mathrm{m/s})}\right)^2 = -5.5\times10^{-15}\,\mathrm{m}.$$

E4-15 (a) The dart "falls" vertically through a distance $y = -gt^2/2 = -(9.81\,\mathrm{m/s^2})(0.19\,\mathrm{s})^2/2 = -0.18\,\mathrm{m}$.

(b) The dart travels horizontally $x = v_xt = (10\,\mathrm{m/s})(0.19\,\mathrm{s}) = 1.9\,\mathrm{m}$.

E4-16 The initial velocity components are

$$v_{x,0} = (15\,\mathrm{m/s})\cos(20^\circ) = 14\,\mathrm{m/s}$$

and

$$v_{y,0} = -(15\,\mathrm{m/s})\sin(20^\circ) = -5.1\,\mathrm{m/s}.$$

(a) The horizontal displacement is $x = v_xt = (14\,\mathrm{m/s})(2.3\,\mathrm{s}) = 32\,\mathrm{m}$.

(b) The vertical displacement is

$$y = -gt^2/2 + v_{y,0}t = -(9.81\,\mathrm{m/s^2})(2.3\,\mathrm{s})^2/2 + (-5.1\,\mathrm{m/s})(2.3\,\mathrm{s}) = -38\,\mathrm{m}.$$

$\boxed{\textbf{E4-17}}$ Find the time in terms of the the initial y component of the velocity:

$$\begin{aligned} v_y &= v_{0y} - gt, \\ (0) &= v_{0y} - gt, \\ t &= v_{0y}/g. \end{aligned}$$

Use this time to find the highest point:

$$
\begin{aligned}
y &= v_{0y}t - \frac{1}{2}gt^2, \\
y_{\max} &= v_{0y}\left(\frac{v_{0y}}{g}\right) - \frac{1}{2}g\left(\frac{v_{0y}}{g}\right)^2, \\
&= \frac{v_{0y}^2}{2g}.
\end{aligned}
$$

Finally, we know the initial y component of the velocity from Eq. 2-6, so $y_{\max} = (v_0 \sin\phi_0)^2/2g$.

E4-18 The horizontal displacement is $x = v_x t$. The vertical displacement is $y = -gt^2/2$. Combining, $y = -g(x/v_x)^2/2$. The edge of the nth step is located at $y = -nw$, $x = nw$, where $w = 2/3$ ft. If $|y| > nw$ when $x = nw$ then the ball hasn't hit the step. Solving,

$$
\begin{aligned}
g(nw/v_x)^2/2 &< nw, \\
gnw/v_x^2 &< 2, \\
n &< 2v_x^2/(gw) = 2(5.0\text{ ft/s})^2/[(32\text{ ft/s}^2)(2/3\text{ ft})] = 2.34.
\end{aligned}
$$

Then the ball lands on the third step.

E4-19 (a) Start from the observation point 9.1 m above the ground. The ball will reach the highest point when $v_y = 0$, this will happen at a time t after the observation such that $t = v_{y,0}/g = (6.1\,\text{m/s})/(9.81\,\text{m/s}^2) = 0.62\,\text{s}$. The vertical displacement (from the ground) will be

$$
y = -gt^2/2 + v_{y,0}t + y_0 = -(9.81\,\text{m/s}^2)(0.62\,\text{s})^2/2 + (6.1\,\text{m/s})(0.62\,\text{s}) + (9.1\,\text{m}) = 11\,\text{m}.
$$

(b) The time for the ball to return to the ground from the highest point is $t = \sqrt{2y_{\max}/g} = \sqrt{2(11\,\text{m})/(9.81\,\text{m/s}^2)} = 1.5\,\text{s}$. The total time of flight is twice this, or 3.0 s. The horizontal distance traveled is $x = v_x t = (7.6\,\text{m/s})(3.0\,\text{s}) = 23\,\text{m}$.

(c) The velocity of the ball just prior to hitting the ground is

$$
\vec{\mathbf{v}} = \vec{\mathbf{a}}t + \vec{\mathbf{v}}_0 = (-9.81\,\text{m/s}^2)\hat{\mathbf{j}}(1.5\,\text{s}) + (7.6\,\text{m/s})\hat{\mathbf{i}} = 7.6\,\text{m/s}\hat{\mathbf{i}} - 15\,\text{m/s}uj.
$$

The magnitude is $\sqrt{7.6^2 + 15^2}(\text{m/s}) = 17\,\text{m/s}$. The direction is

$$
\theta = \arctan(-15/7.6) = -63^\circ.
$$

E4-20 Focus on the time it takes the ball to get to the plate, assuming it traveled in a straight line. The ball has a "horizontal" velocity of 135 ft/s. Then $t = x/v_x = (60.5\text{ ft})/(135\text{ ft/s}) = 0.448\,\text{s}$. The ball will "fall" a vertical distance of $y = -gt^2/2 = -(32\text{ ft/s}^2)(0.448\,\text{s})^2/2 = -3.2$ ft. That's in the strike zone.

E4-21 Since $R \propto 1/g$ one can write $R_2/R_1 = g_1/g_2$, or

$$
\Delta R = R_2 - R_1 = R_1\left(1 - \frac{g_1}{g_2}\right) = (8.09\,\text{m})\left[1 - \frac{(9.7999\,\text{m/s}^2)}{(9.8128\,\text{m/s}^2)}\right] = 1.06\text{ cm}.
$$

E4-22 If initial position is $\vec{\mathbf{r}}_0 = 0$, then final position is $\vec{\mathbf{r}} = (13\text{ ft})\hat{\mathbf{i}} + (3\text{ ft})\hat{\mathbf{j}}$. The initial velocity is $\vec{\mathbf{v}}_0 = v\cos\theta\hat{\mathbf{i}} + v\sin\theta\hat{\mathbf{j}}$. The horizontal equation is $(13\text{ ft}) = v\cos\theta t$; the vertical equation is $(3\text{ ft}) = -(g/2)t^2 + v\sin\theta t$. Rearrange the vertical equation and then divide by the horizontal equation to get

$$\frac{3\text{ ft} + (g/2)t^2}{(13\text{ ft})} = \tan\theta,$$

or

$$t^2 = [(13\text{ ft})\tan(55°) - (3\text{ ft})][2/(32\,\text{m/s}^2)] = 0.973\,\text{s}^2,$$

or $t = 0.986\,\text{s}$. Then $v = (13\text{ ft})/(\cos(55°)(0.986\,\text{s})) = 23\text{ ft/s}$.

E4-23 $v_x = x/t = (150\text{ ft})/(4.50\,\text{s}) = 33.3\text{ ft/s}$. The time to the highest point is half the hang time, or 2.25 s. The vertical speed when the ball hits the ground is $v_y = -gt = -(32\text{ ft/s}^2)(2.25\,\text{s}) = 72.0\text{ ft/s}$. Then the initial vertical velocity is 72.0 ft/s. The magnitude of the initial velocity is $\sqrt{72^2 + 33^2}(\text{ft/s}) = 79\text{ ft/s}$. The direction is

$$\theta = \arctan(72/33) = 65°.$$

E4-24 (a) The magnitude of the initial velocity of the projectile is $v = 264\text{ ft/s}$. The projectile was in the air for a time t where

$$t = \frac{x}{v_x} = \frac{x}{v\cos\theta} = \frac{(2300\text{ ft})}{(264\text{ ft/s})\cos(-27°)} = 9.8\,\text{s}.$$

(b) The height of the plane was $-y$ where

$$-y = gt^2/2 - v_{y,0}t = (32\text{ ft/s})(9.8\,\text{s})^2/2 - (264\text{ ft/s})\sin(-27°)(9.8\,\text{s}) = 2700\text{ ft}.$$

$\boxed{\textbf{E4-25}}$ Define the point the ball leaves the racquet as $\vec{\mathbf{r}} = 0$.

(a) The initial conditions are given as $v_{0x} = 23.6\text{ m/s}$ and $v_{0y} = 0$. The time it takes for the ball to reach the horizontal location of the net is found from

$$\begin{aligned} x &= v_{0x}t, \\ (12\text{ m}) &= (23.6\text{ m/s})t, \\ 0.51\text{ s} &= t, \end{aligned}$$

Find how far the ball has moved horizontally in this time:

$$y = v_{0y}t - \frac{1}{2}gt^2 = (0)(0.51\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(0.51\text{ s})^2 = -1.3\text{ m}.$$

Did the ball clear the net? The ball started 2.37 m above the ground and "fell" through a distance of 1.3 m by the time it arrived at the net. So it is still 1.1 m above the ground and 0.2 m above the net.

(b) The initial conditions are now given by $v_{0x} = (23.6\text{ m/s})(\cos[-5.0°]) = 23.5\text{ m/s}$ and $v_{0y} = (23.6\text{ m/s})(\sin[-5.0°]) = -2.1\text{ m/s}$. Now find the time to reach the net just as done in part (a):

$$t = x/v_{0x} = (12.0\text{ m})/(23.5\text{ m/s}) = 0.51\text{ s}.$$

Find the vertical position of the ball when it arrives at the net:

$$y = v_{0y}t - \frac{1}{2}gt^2 = (-2.1\text{ m/s})(0.51\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(0.51\text{ s})^2 = -2.3\text{ m}.$$

Did the ball clear the net? Not this time; it started 2.37 m above the ground and then passed the net 2.3 m lower, or only 0.07 m above the ground.

E4-26 The initial speed of the ball is given by $v = \sqrt{gR} = \sqrt{(32\ \text{ft/s}^2)(350\ \text{ft})} = 106$ ft/s. The time of flight from the batter to the wall is

$$t = x/v_x = (320\ \text{ft})/[(106\ \text{ft/s})\cos(45^\circ)] = 4.3\,\text{s}.$$

The height of the ball at that time is given by $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, or

$$y = (4\ \text{ft}) + (106\ \text{ft/s})\sin(45^\circ)(4.3\,\text{s}) - (16\ \text{ft/s}^2)(4.3\,\text{s})^2 = 31\ \text{ft},$$

clearing the fence by 7 feet.

E4-27 The ball lands $x = (358\ \text{ft}) + (39\ \text{ft})\cos(28^\circ) = 392$ ft from the initial position. The ball lands $y = (39\ \text{ft})\sin(28^\circ) - (4.60\ \text{ft}) = 14$ ft above the initial position. The horizontal equation is $(392\ \text{ft}) = v\cos\theta t$; the vertical equation is $(14\ \text{ft}) = -(g/2)t^2 + v\sin\theta t$. Rearrange the vertical equation and then divide by the horizontal equation to get

$$\frac{14\ \text{ft} + (g/2)t^2}{(392\ \text{ft})} = \tan\theta,$$

or

$$t^2 = [(392\ \text{ft})\tan(52^\circ) - (14\ \text{ft})][2/(32\,\text{m/s}^2)] = 30.5\,\text{s}^2,$$

or $t = 5.52$ s. Then $v = (392\ \text{ft})/(\cos(52^\circ)(5.52\,\text{s})) = 115$ ft/s.

E4-28 Since ball is traveling at 45° when it returns to the same level from which it was thrown for maximum range, then we can assume it actually traveled ≈ 1.6 m. farther than it would have had it been launched from the ground level. This won't make a big difference, but that means that $R = 60.0\,\text{m} - 1.6\,\text{m} = 58.4\,\text{m}$. If v_0 is initial speed of ball thrown directly up, the ball rises to the highest point in a time $t = v_0/g$, and that point is $y_{\text{max}} = gt^2/2 = v_0^2/(2g)$ above the launch point. But $v_0^2 = gR$, so $y_{\text{max}} = R/2 = (58.4\,\text{m})/2 = 29.2\,\text{m}$. To this we add the 1.60 m point of release to get 30.8 m.

$\boxed{\textbf{E4-29}}$ The net force on the pebble is zero, so $\sum F_y = 0$. There are only two forces on the pebble, the force of gravity W and the force of the water on the pebble F_{PW}. These point in opposite directions, so $0 = F_{PW} - W$. But $W = mg = (0.150\ \text{kg})(9.81\ \text{m/s}^2) = 1.47$ N. Since $F_{PW} = W$ in this problem, the force of the water on the pebble must also be 1.47 N.

E4-30 Terminal speed is when drag force equal weight, or $mg = bv_{\text{T}}{}^2$. Then $v_{\text{T}} = \sqrt{mg/b}$.

$\boxed{\textbf{E4-31}}$ Eq. 4-22 is

$$v_y(t) = v_{\text{T}}\left(1 - e^{-bt/m}\right),$$

where we have used Eq. 4-24 to substitute for the terminal speed. We want to solve this equation for time when $v_y(t) = v_{\text{T}}/2$, so

$$\begin{aligned}
\tfrac{1}{2}v_{\text{T}} &= v_{\text{T}}\left(1 - e^{-bt/m}\right),\\
\tfrac{1}{2} &= \left(1 - e^{-bt/m}\right),\\
e^{-bt/m} &= \tfrac{1}{2}\\
bt/m &= -\ln(1/2)\\
t &= \tfrac{m}{b}\ln 2
\end{aligned}$$

E4-32 The terminal speed is 7 m/s for a raindrop with $r = 0.15$ cm. The mass of this drop is $m = 4\pi\rho r^3/3$, so

$$b = \frac{mg}{v_{\mathrm{T}}} = \frac{4\pi(1.0\times10^{-3}\mathrm{kg/cm^3})(0.15\ \mathrm{cm})^3(9.81\,\mathrm{m/s^2})}{3(7\,\mathrm{m/s})} = 2.0\times10^{-5}\mathrm{kg/s}.$$

E4-33 (a) The speed of the train is $v = 9.58\,\mathrm{m/s}$. The drag force on one car is $f = 243(9.58)\,\mathrm{N} = 2330\,\mathrm{N}$. The total drag force is $23(2330\,\mathrm{N}) = 5.36\times10^4\mathrm{N}$. The net force exerted on the cars (treated as a single entity) is $F = ma = 23(48.6\times10^3\mathrm{kg})(0.182\,\mathrm{m/s^2}) = 2.03\times10^5\mathrm{N}$. The pull of the locomotive is then $P = 2.03\times10^5\mathrm{N} + 5.36\times10^4\mathrm{N} = 2.57\times10^5\mathrm{N}$.

(b) If the locomotive is pulling the cars at constant speed up an incline then it must exert a force on the cars equal to the sum of the drag force and the parallel component of the weight. The drag force is the same in each case, so the parallel component of the weight is $W_{||} = W\sin\theta = 2.03\times10^5\mathrm{N} = ma$, where a is the acceleration from part (a). Then

$$\theta = \arcsin(a/g) = \arcsin[(0.182\,\mathrm{m/s^2})/(9.81\,\mathrm{m/s^2})] = 1.06^\circ.$$

E4-34 (a) $a = v^2/r = (2.18\times10^6\mathrm{m/s})^2/(5.29\times10^{-11}\mathrm{m}) = 8.98\times10^{22}\mathrm{m/s^2}$.

(b) $F = ma = (9.11\times10^{-31}\mathrm{kg})(8.98\times10^{22}\mathrm{m/s^2}) = 8.18\times10^{-8}\mathrm{N}$, toward the center.

E4-35 (a) $v = \sqrt{ra_{\mathrm{c}}} = \sqrt{(5.2\ \mathrm{m})(6.8)(9.8\ \mathrm{m/s^2})} = 19$ m/s.

(b) Use the fact that one revolution corresponds to a length of $2\pi r$:

$$19\frac{\mathrm{m}}{\mathrm{s}}\left(\frac{1\ \mathrm{rev}}{2\pi(5.2\ \mathrm{m})}\right)\left(\frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right) = 35\frac{\mathrm{rev}}{\mathrm{min}}.$$

E4-36 (a) $v = 2\pi r/T = 2\pi(15\,\mathrm{m})/(12\,\mathrm{s}) = 7.85\,\mathrm{m/s}$. Then $a = v^2/r = (7.85\,\mathrm{m/s})^2/(15\,\mathrm{m}) = 4.11\,\mathrm{m/s^2}$, directed toward center, which is down.

(b) Same arithmetic as in (a); direction is still toward center, which is now up.

(c) The magnitude of the net force in both (a) and (b) is $F = ma = (75\,\mathrm{kg})(4.11\,\mathrm{m/s^2}) = 310\,\mathrm{N}$. The weight of the person is the same in both parts: $W = mg = (75\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 740\,\mathrm{N}$. At the top the net force is down, the weight is down, so the Ferris wheel is pushing up with a force of $P = W - F = (740\,\mathrm{N}) - (310\,\mathrm{N}) = 430\,\mathrm{N}$. At the bottom the net force is up, the weight is down, so the Ferris wheel is pushing up with a force of $P = W + F = (740\,\mathrm{N}) + (310\,\mathrm{N}) = 1050\,\mathrm{N}$.

E4-37 (a) $v = 2\pi r/T = 2\pi(20\times10^3\mathrm{m})/(1.0\,\mathrm{s}) = 1.26\times10^5\mathrm{m/s}$.

(b) $a = v^2/r = (1.26\times10^5\mathrm{m/s})^2/(20\times10^3\mathrm{m}) = 7.9\times10^5\mathrm{m/s^2}$.

E4-38 (a) $v = 2\pi r/T = 2\pi(6.37\times10^6\mathrm{m})/(86400\,\mathrm{s}) = 463\,\mathrm{m/s}$. $a = v^2/r = (463\,\mathrm{m/s})^2/(6.37\times10^6\mathrm{m}) = 0.034\,\mathrm{m/s^2}$.

(b) The net force on the object is $F = ma = (25.0\,\mathrm{kg})(0.034\,\mathrm{m/s^2}) = 0.85\,\mathrm{N}$. There are two forces on the object: a force up from the scale (S), and the weight down, $W = mg = (25.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 245\,\mathrm{N}$. Then $S = F + W = 245\,\mathrm{N} + 0.85\,\mathrm{N} = 246\,\mathrm{N}$.

E4-39 Let $\Delta x = 15$ m be the length; $t_w = 90$ s, the time to walk the stalled Escalator; $t_s = 60$ s, the time to ride the moving Escalator; and t_m, the time to walk up the moving Escalator.

The walking speed of the person relative to a fixed Escalator is $v_{we} = \Delta x/t_w$; the speed of the Escalator relative to the ground is $v_{eg} = \Delta x/t_s$; and the speed of the walking person relative to the ground on a moving Escalator is $v_{wg} = \Delta x/t_m$. But these three speeds are related by $v_{wg} = v_{we}+v_{eg}$. Combine all the above:

$$\begin{aligned}
v_{wg} &= v_{we} + v_{eg}, \\
\frac{\Delta x}{t_m} &= \frac{\Delta x}{t_w} + \frac{\Delta x}{t_s}, \\
\frac{1}{t_m} &= \frac{1}{t_w} + \frac{1}{t_s}.
\end{aligned}$$

Putting in the numbers, $t_m = 36$ s.

E4-40 Let v_w be the walking speed, v_s be the sidewalk speed, and $v_\mathrm{m} = v_\mathrm{w} + v_\mathrm{s}$ be Mary's speed while walking on the moving sidewalk. All three cover the same distance x, so $v_i = x/t_i$, where i is one of w, s, or m. Put this into the Mary equation, and

$$1/t_\mathrm{m} = 1/t_\mathrm{w} + 1/t_\mathrm{s} = 1/(150\,\mathrm{s}) + 1/(70\,\mathrm{s}) = 1/48\,\mathrm{s}.$$

E4-41 If it takes longer to fly westward then the speed of the plane (relative to the ground) westward must be less than the speed of the plane eastward. We conclude that the jet-stream must be blowing east. The speed of the plane relative to the ground is $v_\mathrm{e} = v_\mathrm{p} + v_\mathrm{j}$ when going east and $v_\mathrm{w} = v_\mathrm{p} - v_\mathrm{j}$ when going west. In either case the distance is the same, so $x = v_i t_i$, where i is e or w. Since $t_\mathrm{w} - t_\mathrm{e}$ is given, we can write

$$t_\mathrm{w} - t_\mathrm{e} = \frac{x}{v_\mathrm{p} - v_\mathrm{j}} - \frac{x}{v_\mathrm{p} + v_\mathrm{j}} = x\frac{2v_\mathrm{j}}{{v_\mathrm{p}}^2 - {v_\mathrm{j}}^2}.$$

Solve the quadratic if you want, but since $v_\mathrm{j} \ll v_\mathrm{p}$ we can neglect it in the denominator and

$$v_\mathrm{j} = {v_\mathrm{p}}^2(0.83\text{ h})/(2x) = (600\text{ mi/h})^2(0.417\text{ h})/(2700\text{ mi}) = 56\text{ mi/hr}.$$

E4-42 The horizontal component of the rain drop's velocity is 28 m/s. Since $v_x = v\sin\theta$, $v = (28\,\mathrm{m/s})/\sin(64^\circ) = 31\,\mathrm{m/s}$.

$\boxed{\textbf{E4-43}}$ (a) The position of the bolt relative to the elevator is y_{be}, the position of the bolt relative to the shaft is y_{bs}, and the position of the elevator relative to the shaft is y_{es}. Zero all three positions at $t = 0$; at this time $v_{0,bs} = v_{0,es} = 8.0$ ft/s.

The three equations describing the positions are

$$\begin{aligned}
y_{bs} &= v_{0,bs}t - \frac{1}{2}gt^2, \\
y_{es} &= v_{0,es}t + \frac{1}{2}at^2, \\
y_{be} + r_{es} &= r_{bs},
\end{aligned}$$

where $a = 4.0$ m/s^2 is the upward acceleration of the elevator. Rearrange the last equation and solve for y_{be}; get $y_{be} = -\frac{1}{2}(g+a)t^2$, where advantage was taken of the fact that the initial velocities are the same.

Then

$$t = \sqrt{-2y_{be}/(g+a)} = \sqrt{-2(-9.0\text{ ft})/(32\text{ ft/s}^2 + 4\text{ ft/s}^2)} = 0.71\,\mathrm{s}$$

(b) Use the expression for y_{bs} to find how the bolt moved relative to the shaft:

$$y_{bs} = v_{0,bs}t - \frac{1}{2}gt^2 = (8.0\text{ ft})(0.71\text{ s}) - \frac{1}{2}(32\text{ ft/s}^2)(0.71\text{ s})^2 = -2.4\text{ ft}.$$

E4-44 The speed of the plane relative to the ground is $v_{\rm pg} = (810 \text{ km})/(1.9 \text{ h}) = 426$ km/h. The velocity components of the plane relative to the air are $v_N = (480 \text{ km/h})\cos(21^\circ) = 448$ km/h and $v_E = (480 \text{ km/h})\sin(21^\circ) = 172$ km/h. The wind must be blowing with a component of 172 km/h to the west and a component of $448 - 426 = 22$ km/h to the south.

E4-45 (a) Let $\hat{\mathbf{i}}$ point east and $\hat{\mathbf{j}}$ point north. The velocity of the torpedo is $\vec{\mathbf{v}} = (50 \text{ km/h})\hat{\mathbf{i}}\sin\theta + (50 \text{ km/h})\hat{\mathbf{j}}\cos\theta$. The initial coordinates of the battleship are then $\vec{\mathbf{r}}_0 = (4.0 \text{ km})\hat{\mathbf{i}}\sin(20^\circ) + (4.0 \text{ km})\hat{\mathbf{j}}\cos(20^\circ) = (1.37 \text{ km})\hat{\mathbf{i}} + (3.76 \text{ km})\hat{\mathbf{j}}$. The final position of the battleship is $\vec{\mathbf{r}} = (1.37 \text{ km} + 24 \text{ km/h}t)\hat{\mathbf{i}} + (3.76 \text{ km})\hat{\mathbf{j}}$, where t is the time of impact. The final position of the torpedo is the same, so

$$[(50 \text{ km/h})\hat{\mathbf{i}}\sin\theta + (50 \text{ km/h})\hat{\mathbf{j}}\cos\theta]t = (1.37 \text{ km} + 24 \text{ km/h}t)\hat{\mathbf{i}} + (3.76 \text{ km})\hat{\mathbf{j}},$$

or

$$[(50 \text{ km/h})\sin\theta]t - 24 \text{ km/h}t = 1.37 \text{ km}$$

and

$$[(50 \text{ km/h})\cos\theta]t = 3.76 \text{ km}.$$

Dividing the top equation by the bottom and rearranging,

$$50\sin\theta - 24 = 18.2\cos\theta.$$

Use any trick you want to solve this. I used Maple and found $\theta = 46.8^\circ$.

(b) The time to impact is then $t = 3.76 \text{ km}/[(50 \text{ km/h})\cos(46.8^\circ)] = 0.110$ h, or 6.6 minutes.

P4-1 Let $\vec{\mathbf{r}}_A$ be the position of particle of particle A, and $\vec{\mathbf{r}}_B$ be the position of particle B. The equations for the motion of the two particles are then

$$\begin{aligned}
\vec{\mathbf{r}}_A &= \vec{\mathbf{r}}_{0,A} + \vec{\mathbf{v}}t, \\
&= d\hat{\mathbf{j}} + vt\hat{\mathbf{i}}; \\
\vec{\mathbf{r}}_B &= \frac{1}{2}\vec{\mathbf{a}}t^2, \\
&= \frac{1}{2}a(\sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}})t^2.
\end{aligned}$$

A collision will occur if there is a time when $\vec{\mathbf{r}}_A = \vec{\mathbf{r}}_B$. Then

$$d\hat{\mathbf{j}} + vt\hat{\mathbf{i}} = \frac{1}{2}a(\sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}})t^2,$$

but this is really *two* equations: $d = \frac{1}{2}at^2\cos\theta$ and $vt = \frac{1}{2}at^2\sin\theta$.

Solve the second one for t and get $t = 2v/(a\sin\theta)$. Substitute that into the first equation, and then rearrange,

$$\begin{aligned}
d &= \frac{1}{2}at^2\cos\theta, \\
d &= \frac{1}{2}a\left(\frac{2v}{a\sin\theta}\right)^2\cos\theta, \\
\sin^2\theta &= \frac{2v}{ad}\cos\theta, \\
1 - \cos^2\theta &= \frac{2v^2}{ad}\cos\theta, \\
0 &= \cos^2\theta + \frac{2v^2}{ad}\cos\theta - 1.
\end{aligned}$$

This last expression is quadratic in $\cos\theta$. It simplifies the solution if we define $b = 2v/(ad) = 2(3.0\ \text{m/s})^2/([0.4\ \text{m/s}^2][30\ \text{m}]) = 1.5$, then

$$\cos\theta = \frac{-b \pm \sqrt{b^2+4}}{2} = -0.75 \pm 1.25.$$

Then $\cos\theta = 0.5$ and $\theta = 60°$.

P4-2 (a) The acceleration of the ball is $\vec{\mathbf{a}} = (1.20\,\text{m/s}^2)\hat{\mathbf{i}} - (9.81\,\text{m/s}^2)\hat{\mathbf{j}}$. Since $\vec{\mathbf{a}}$ is constant the trajectory is given by $\vec{\mathbf{r}} = \vec{\mathbf{a}}t^2/2$, since $\vec{\mathbf{v}}_0 = 0$ and we choose $\vec{\mathbf{r}}_0 = 0$. This is a straight line trajectory, with a direction given by $\vec{\mathbf{a}}$. Then

$$\theta = \arctan(9.81/1.20) = 83.0°.$$

and $R = (39.0\,\text{m})/\tan(83.0°) = 4.79\,\text{m}$. It will be useful to find $H = (39.0\,\text{m})/\sin(83.0°) = 39.3\,\text{m}$.

(b) The magnitude of the acceleration of the ball is $a = \sqrt{9.81^2 + 1.20^2}(\text{m/s}^2) = 9.88\,\text{m/s}^2$. The time for the ball to travel down the hypotenuse of the figure is then $t = \sqrt{2(39.3\,\text{m})/(9.88\,\text{m/s}^2)} = 2.82\,\text{s}$.

(c) The magnitude of the speed of the ball at the bottom will then be

$$v = at = (9.88\,\text{m/s}^2)(2.82\,\text{s}) = 27.9\,\text{m/s}.$$

P4-3 (a) The rocket thrust is $\vec{\mathbf{T}} = (61.2\ \text{kN})\cos(58.0°)\hat{\mathbf{i}} + (61.2\ \text{kN})\sin(58.0°)\hat{\mathbf{i}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 51.9\ \text{kN}\hat{\mathbf{j}}$. The net force on the rocket is the $\vec{\mathbf{F}} = \vec{\mathbf{T}} + \vec{\mathbf{W}}$, or

$$\vec{\mathbf{F}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 51.9\ \text{kN}\hat{\mathbf{j}} - (3030\,\text{kg})(9.81\,\text{m/s}^2)\hat{\mathbf{j}} = 32.4\ \text{kN}\hat{\mathbf{i}} + 22.2\ \text{kN}\hat{\mathbf{j}}.$$

The acceleration (until rocket cut-off) is this net force divided by the mass, or

$$\vec{\mathbf{a}} = 10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}}.$$

The position at rocket cut-off is given by

$$\begin{aligned}\vec{\mathbf{r}} &= \vec{\mathbf{a}}t^2/2 = (10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}})(48.0\,\text{s})^2/2,\\ &= 1.23\times10^4\text{m}\hat{\mathbf{i}} + 8.44\times10^3\text{m}\hat{\mathbf{j}}.\end{aligned}$$

The altitude at rocket cut-off is then 8.44 km.

(b) The velocity at rocket cut-off is

$$\vec{\mathbf{v}} = \vec{\mathbf{a}}t = (10.7\text{m/s}^2\hat{\mathbf{i}} + 7.33\text{m/s}^2\hat{\mathbf{j}})(48.0\,\text{s}) = 514\,\text{m/s}\hat{\mathbf{i}} + 352\,\text{m/s}\hat{\mathbf{j}},$$

this becomes the initial velocity for the "free fall" part of the journey. The rocket will hit the ground after t seconds, where t is the solution to

$$0 = -(9.81\,\text{m/s}^2)t^2/2 + (352\,\text{m/s})t + 8.44\times10^3\,\text{m}.$$

The solution is $t = 90.7\,\text{s}$. The rocket lands a horizontal distance of $x = v_x t = (514\,\text{m/s})(90.7\,\text{s}) = 4.66\times10^4$ m beyond the rocket cut-off; the total horizontal distance covered by the rocket is 46.6 km+ 12.3 km = 58.9 km.

P4-4 (a) The horizontal speed of the ball is $v_x = 135$ ft/s. It takes

$$t = x/v_x = (30.0 \text{ ft})/(135 \text{ ft/s}) = 0.222\,\text{s}$$

to travel the 30 feet horizontally, whether the first 30 feet, the last 30 feet, or 30 feet somewhere in the middle.

(b) The ball "falls" $y = -gt^2/2 = -(32 \text{ ft/s}^2)(0.222\,\text{s})^2/2 = -0.789$ ft while traveling the first 30 feet.

(c) The ball "falls" a total of $y = -gt^2/2 = -(32 \text{ ft/s}^2)(0.444\,\text{s})^2/2 = -3.15$ ft while traveling the first 60 feet, so during the last 30 feet it must have fallen $(-3.15 \text{ ft}) - (-0.789 \text{ ft}) = -2.36$ ft.

(d) The distance fallen because of acceleration is not linear in time; the distance moved horizontally is linear in time.

P4-5 (a) The initial velocity of the ball has components

$$v_{x,0} = (25.3\,\text{m/s})\cos(42.0^\circ) = 18.8\,\text{m/s}$$

and

$$v_{y,0} = (25.3\,\text{m/s})\sin(42.0^\circ) = 16.9\,\text{m/s}.$$

The ball is in the air for $t = x/v_x = (21.8\,\text{m})/(18.8\,\text{m/s}) = 1.16\,\text{s}$ before it hits the wall.

(b) $y = -gt^2/2 + v_{y,0}t = -(4.91\,\text{m/s}^2)(1.16\,\text{s})^2 + (16.9\,\text{m/s})(1.16\,\text{s}) = 13.0\,\text{m}$.

(c) $v_x = v_{x,0} = 18.8\,\text{m/s}$. $v_y = -gt + v_{y,0} = -(9.81\,\text{m/s}^2)(1.16\,\text{s}) + (16.9\,\text{m/s}) = 5.52\,\text{m/s}$.

(d) Since $v_y > 0$ the ball is still heading up.

P4-6 (a) The initial vertical velocity is $v_{y,0} = v_0 \sin\phi_0$. The time to the highest point is $t = v_{y,0}/g$. The highest point is $H = gt^2/2$. Combining,

$$H = g(v_0 \sin\phi_0/g)^2/2 = v_0^2 \sin^2\phi_0/(2g).$$

The range is $R = (v_0^2/g)\sin 2\phi_0 = 2(v_0^2/g)\sin\phi_0\cos\phi_0$. Since $\tan\theta = H/(R/2)$, we have

$$\tan\theta = \frac{2H}{R} = \frac{v_0^2 \sin^2\phi_0/g}{2(v_0^2/g)\sin\phi_0\cos\phi_0} = \frac{1}{2}\tan\phi_0.$$

(b) $\theta = \arctan(0.5\tan 45^\circ) = 26.6^\circ$.

$\boxed{\textbf{P4-7}}$ The components of the initial velocity are given by $v_{0x} = v_0\cos\theta = 56$ ft/s and $v_{0y} = v_0\sin\theta = 106$ ft/s where we used $v_0 = 120$ ft/s and $\theta = 62^\circ$.

(a) To find h we need only find out the vertical position of the stone when $t = 5.5$ s.

$$y = v_{0y}t - \frac{1}{2}gt^2 = (106 \text{ ft/s})(5.5 \text{ s}) - \frac{1}{2}(32 \text{ ft/s}^2)(5.5 \text{ s})^2 = 99 \text{ ft}.$$

(b) Look at this as a vector problem:

$$\begin{aligned}
\vec{\mathbf{v}} &= \vec{\mathbf{v}}_0 + \vec{\mathbf{a}}t, \\
&= \left(v_{0x}\hat{\mathbf{i}} + v_{0y}\hat{\mathbf{j}}\right) - g\hat{\mathbf{j}}t, \\
&= v_{0x}\hat{\mathbf{i}} + (v_{0y} - gt)\,\hat{\mathbf{j}}, \\
&= (56 \text{ ft/s})\hat{\mathbf{i}} + \left((106 \text{ ft/s} - (32 \text{ ft/s}^2)(5.5 \text{ s})\right)\hat{\mathbf{j}}, \\
&= (56 \text{ ft/s})\hat{\mathbf{i}} + (-70.0 \text{ ft/s})\hat{\mathbf{j}}.
\end{aligned}$$

The magnitude of this vector gives the speed when $t = 5.5$ s; $v = \sqrt{56^2 + (-70)^2}$ ft/s$=$ 90 ft/s.

(c) Highest point occurs when $v_y = 0$. Solving Eq. 4-9(b) for time; $v_y = 0 = v_{0y} - gt = (106 \text{ ft/s}) - (32 \text{ ft/s}^2)t$; $t = 3.31$ s. Use this time in Eq. 4-10(b),

$$y = v_{0y}t - \frac{1}{2}gt^2 = (106 \text{ ft/s})(3.31 \text{ s}) - \frac{1}{2}(32 \text{ ft/s}^2)(3.31 \text{ s})^2 = 176 \text{ ft}.$$

P4-8 (a) Since $R = (v_0^2/g)\sin 2\phi_0$, it is sufficient to prove that $\sin 2\phi_0$ is the same for both $\sin 2(45^\circ + \alpha)$ and $\sin 2(45^\circ - \alpha)$.

$$\sin 2(45^\circ \pm \alpha) = \sin(90^\circ \pm 2\alpha) = \cos(\pm 2\alpha) = \cos(2\alpha).$$

Since the $\pm$ dropped out, the two quantities are equal.

(b) $\phi_0 = (1/2)\arcsin(Rg/v_0^2) = (1/2)\arcsin((20.0\,\text{m})(9.81\,\text{m/s}^2)/(30.0\,\text{m/s})^2) = 6.3^\circ$. The other choice is $90^\circ - 6.3^\circ = 83.7^\circ$.

$\boxed{\textbf{P4-9}}$ To score the ball must pass the horizontal distance of 50 m with an altitude of no less than 3.44 m. The initial velocity components are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$ where $v_0 = 25$ m/s, and θ is the unknown.

The time to the goal post is $t = x/v_{0x} = x/(v_0 \cos\theta)$.

The vertical motion is given by

$$\begin{aligned} y &= v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{v_0 \cos\theta}\right)^2, \\ &= x\frac{\sin\theta}{\cos\theta} - \frac{gx^2}{2v_0^2}\frac{1}{\cos^2\theta}. \end{aligned}$$

In this last expression y needs to be greater than 3.44 m. In this last expression use

$$\frac{1}{\cos^2\theta} - 1 + 1 = \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta} + 1 = \frac{1 - \cos^2\theta}{\cos^2\theta} + 1 = \frac{\sin^2\theta}{\cos^2\theta} + 1 = \tan^2\theta + 1.$$

This gives for our y expression

$$y = x\tan\theta - \frac{gx^2}{2v_0}\left(\tan^2\theta + 1\right),$$

which can be combined with numbers and constraints to give

$$\begin{aligned} (3.44 \text{ m}) &\le (50 \text{ m})\tan\theta - \frac{(9.8 \text{ m/s}^2)(50 \text{ m})^2}{2(25 \text{ m/s})^2}\left(\tan^2\theta + 1\right), \\ 3.44 &\le 50\tan\theta - 20\left(\tan^2\theta + 1\right), \\ 0 &\le -20\tan^2\theta + 50\tan\theta - 23 \end{aligned}$$

Solve, and $\tan\theta = 1.25 \pm 0.65$, so the allowed kicking angles are between $\theta = 31^\circ$ and $\theta = 62^\circ$.

P4-10 (a) The height of the projectile at the highest point is $H = L\sin\theta$. The amount of time before the projectile hits the ground is $t = \sqrt{2H/g} = \sqrt{2L\sin\theta/g}$. The horizontal distance covered by the projectile in this time is $x = v_x t = v\sqrt{2L\sin\theta/g}$. The horizontal distance to the projectile when it is at the highest point is $x' = L\cos\theta$. The projectile lands at

$$D = x - x' = v\sqrt{2L\sin\theta/g} - L\cos\theta.$$

(b) The projectile will pass overhead if $D > 0$.

P4-11 $v^2 = v_x^2 + v_y^2$. For a projectile v_x is constant, so we need only evaluate $d^2(v_y^2)/dt^2$. The first derivative is $2v_y\,dv_y/dt = -2v_y g$. The derivative of this (the second derivative) is $-2g\,dv_y/dt = 2g^2$.

P4-12 $|\vec{\mathbf{r}}|$ is a maximum when r^2 is a maximum. $r^2 = x^2 + y^2$, or

$$\begin{aligned} r^2 &= (v_{x,0}t)^2 + (-gt^2/2 + v_{y,0}t)^2, \\ &= (v_0 t \cos\phi_0)^2 + \left(v_0 t \sin\phi_0 - gt^2/2\right)^2, \\ &= v_0^2 t^2 - v_0 g t^3 \sin\phi_0 + g^2 t^4/4. \end{aligned}$$

We want to look for the condition which will allow dr^2/dt to vanish. Since

$$dr^2/dt = 2v_0^2 t - 3v_0 g t^2 \sin\phi_0 + g^2 t^3$$

we can focus on the quadratic discriminant, $b^2 - 4ac$, which is

$$9v_0^2 g^2 \sin^2\phi_0 - 8v_0^2 g^2,$$

a quantity which will only be greater than zero if $9\sin^2\phi_0 > 8$. The critical angle is then

$$\phi_c = \arcsin(\sqrt{8/9}) = 70.5^\circ.$$

$\boxed{\textbf{P4-13}}$ There is a downward force on the balloon of 10.8 kN from gravity and an upward force of 10.3 kN from the buoyant force of the air. The resultant of these two forces is 500 N down, but since the balloon is descending at constant speed so the net force on the balloon must be zero. This is possible because there is a drag force on the balloon of $D = bv^2$, this force is directed upward. The magnitude must be 500 N, so the constant b is

$$b = \frac{(500\,\mathrm{N})}{(1.88\,\mathrm{m/s})^2} = 141\,\mathrm{kg/m}.$$

If the crew drops 26.5 kg of ballast they are "lightening" the balloon by

$$(26.5\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 260\,\mathrm{N}.$$

This reduced the weight, but not the buoyant force, so the drag force at constant speed will now be $500\,\mathrm{N} - 260\,\mathrm{N} = 240\,\mathrm{N}$.

The new constant downward speed will be

$$v = \sqrt{D/b} = \sqrt{(240\,\mathrm{N})/(141\,\mathrm{kg/m})} = 1.30\,\mathrm{m/s}.$$

P4-14 The constant b is

$$b = (500\,\mathrm{N})/(1.88\,\mathrm{m/s}) = 266\,\mathrm{N\cdot s/m}.$$

The drag force after "lightening" the load will still be 240 N. The new downward speed will be

$$v = D/b = (240\,\mathrm{N})/(266\,\mathrm{N\cdot s/m}) = 0.902\,\mathrm{m/s}.$$

P4-15 (a) Initially $v_0 = 0$, so $D = 0$, the only force is the weight, so $a = -g$.

(b) After some time the acceleration is zero, then $W = D$, or $b{v_\mathrm{T}}^2 = mg$, or $v_\mathrm{T} = \sqrt{mg/b}$.

(c) When $v = v_\mathrm{T}/2$ the drag force is $D = b{v_\mathrm{T}}^2/4 = mg/4$, so the net force is $F = D - W = -3mg/4$. The acceleration is the $a = -3g/4$.

P4-16 (a) The net force on the barge is $F = -D = -bv$, this results in a differential equation $m\,dv/dt = -bv$, which can be written as

$$\begin{aligned} dv/v &= -(b/m)dt, \\ \int dv/v &= -(b/m)\int dt, \\ \ln(v_\mathrm{f}/v_\mathrm{i}) = -bt/m. \end{aligned}$$

Then $t = (m/b)\ln(v_\mathrm{i}/v_\mathrm{f})$.

(b) $t = [(970\,\mathrm{kg})/(68\,\mathrm{N}\cdot\mathrm{s/m})]\ln(32/8.3) = 19\,\mathrm{s}$.

$\boxed{\textbf{P4-17}}$ (a) The acceleration is the time derivative of the velocity,

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{mg}{b}\left(1 - e^{-bt/m}\right)\right) = \frac{mg}{b}\frac{b}{m}e^{-bt/m},$$

which can be simplified as $a_y = ge^{-bt/m}$. For large t this expression approaches 0; for small t the exponent can be expanded to give

$$a_y \approx g\left(1 - \frac{bt}{m}\right) = g - v_\mathrm{T}t,$$

where in the last line we made use of Eq. 4-24.

(b) The position is the integral of the velocity,

$$\begin{aligned} \int_0^t v_y\,dt &= \int_0^t \left(\frac{mg}{b}\left(1 - e^{-bt/m}\right)\right)\,dt, \\ \int_0^t \frac{dy}{dt}\,dt &= \frac{mg}{b}\left(t - (-m/b)e^{-bt/m}\right)\Big|_0^t, \\ \int_0^y dy &= v_\mathrm{T}\left(t + \frac{v_\mathrm{T}}{g}\left(e^{-v_\mathrm{T}t/g} - 1\right)\right), \\ y &= v_\mathrm{T}\left(t + \frac{v_\mathrm{T}}{g}\left(e^{-v_\mathrm{T}t/g} - 1\right)\right). \end{aligned}$$

P4-18 (a) We have $v_y = v_\mathrm{T}(1 - e^{-bt/m})$ from Eq. 4-22; this can be substituted into the last line of the solution for P4-17 to give

$$y_{95} = v_\mathrm{T}\left(t - \frac{v_y}{g}\right).$$

We can also rearrange Eq. 4-22 to get $t = -(m/b)\ln(1 - v_y/v_\mathrm{T})$, so

$$y_{95} = {v_\mathrm{T}}^2/g\left(-\ln(1 - v_y/v_\mathrm{T}) - \frac{v_y}{v_\mathrm{T}}\right).$$

But $v_y/v_\mathrm{T} = 0.95$, so

$$y_{95} = {v_\mathrm{T}}^2/g\,(-\ln(0.05) - 0.95) = {v_\mathrm{T}}^2/g\,(\ln 20 - 19/20)\,.$$

(b) $y_{95} = (42\,\mathrm{m/s})^2/(9.81\,\mathrm{m/s^2})(2.05) = 370\,\mathrm{m}$.

P4-19 (a) Convert units first. $v = 86.1\,\mathrm{m/s}$, $a = 0.05(9.81\,\mathrm{m/s^2}) = 0.491\,\mathrm{m/s^2}$. The minimum radius is $r = v^2/a = (86.1\,\mathrm{m/s})/(0.491\,\mathrm{m/s^2}) = 15$ km.

(b) $v = \sqrt{ar} = \sqrt{(0.491\,\mathrm{m/s^2})(940\,\mathrm{m})} = 21.5\,\mathrm{m/s}$. That's 77 km/hr.

P4-20 (a) The position is given by $\vec{\mathbf{r}} = R \sin \omega t\, \hat{\mathbf{i}} + R(1 - \cos \omega t)\hat{\mathbf{j}}$, where $\omega = 2\pi/(20\,\mathrm{s}) = 0.314\,\mathrm{s}^{-1}$ and $R = 3.0\,\mathrm{m}$. When $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{r}} = (3.0\,\mathrm{m})\hat{\mathbf{i}} + (3.0\,\mathrm{m})\hat{\mathbf{j}}$; when $t = 7.5\,\mathrm{s}$ $\vec{\mathbf{r}} = (2.1\,\mathrm{m})\hat{\mathbf{i}} + (5.1\,\mathrm{m})\hat{\mathbf{j}}$; when $t = 10\,\mathrm{s}$ $\vec{\mathbf{r}} = (6.0\,\mathrm{m})\hat{\mathbf{j}}$. These vectors have magnitude 4.3 m, 5.5 m and 6.0 m, respectively. The vectors have direction 45°, 68° and 90° respectively.

(b) $\Delta\vec{\mathbf{r}} = (-3.0\,\mathrm{m})\hat{\mathbf{i}} + (3.0\,\mathrm{m})\hat{\mathbf{j}}$, which has magnitude 4.3 m and direction 135°.

(c) $v_{\mathrm{av}} = \Delta r/\Delta t = (4.3\,\mathrm{m})/(5.0\,\mathrm{s}) = 0.86\,\mathrm{m/s}$. The direction is the same as $\Delta\vec{\mathbf{r}}$.

(d) The velocity is given by $\vec{\mathbf{v}} = R\omega \cos \omega t\, \hat{\mathbf{i}} + R\omega \sin \omega t\, \hat{\mathbf{j}}$. At $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{v}} = (0.94\,\mathrm{m/s})\hat{\mathbf{j}}$; at $t = 10\,\mathrm{s}$ $\vec{\mathbf{v}} = (-0.94\,\mathrm{m/s})\hat{\mathbf{i}}$.

(e) The acceleration is given by $\vec{\mathbf{a}} = -R\omega^2 \sin \omega t\, \hat{\mathbf{i}} + R\omega^2 \cos \omega t\, \hat{\mathbf{j}}$. At $t = 5.0\,\mathrm{s}$ $\vec{\mathbf{a}} = (-0.30\,\mathrm{m/s^2})\hat{\mathbf{i}}$; at $t = 10\,\mathrm{s}$ $\vec{\mathbf{a}} = (-0.30\,\mathrm{m/s^2})\hat{\mathbf{j}}$.

P4-21 Start from where the stone lands; in order to get there the stone fell through a vertical distance of 1.9 m while moving 11 m horizontally. Then

$$y = -\frac{1}{2}gt^2 \text{ which can be written as } t = \sqrt{\frac{-2y}{g}}.$$

Putting in the numbers, $t = 0.62\,\mathrm{s}$ is the time of flight from the moment the string breaks. From this time find the horizontal velocity,

$$v_x = \frac{x}{t} = \frac{(11\,\mathrm{m})}{(0.62\,\mathrm{s})} = 18\,\mathrm{m/s}.$$

Then the centripetal acceleration is

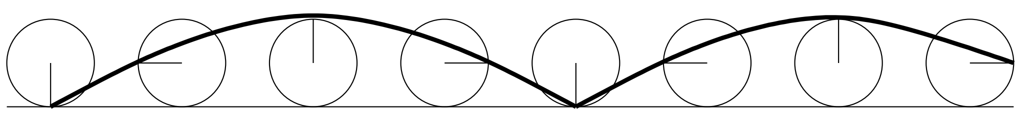
$$a_c = \frac{v^2}{r} = \frac{(18\,\mathrm{m/s})^2}{(1.4\,\mathrm{m})} = 230\,\mathrm{m/s^2}.$$

P4-22 (a) The path traced out by her feet has circumference $c_1 = 2\pi r \cos 50°$, where r is the radius of the earth; the path traced out by her head has circumference $c_2 = 2\pi(r + h)\cos 50°$, where h is her height. The difference is $\Delta c = 2\pi h \cos 50° = 2\pi(1.6\,\mathrm{m})\cos 50° = 6.46\,\mathrm{m}$.

(b) $a = v^2/r = (2\pi r/T)^2/r = 4\pi^2 r/T^2$. Then $\Delta a = 4\pi^2 \Delta r/T^2$. Note that $\Delta r = h\cos\theta$! Then

$$\Delta a = 4\pi^2(1.6\,\mathrm{m})\cos 50°/(86400\,\mathrm{s})^2 = 5.44\times10^{-9}\mathrm{m/s^2}.$$

P4-23 (a) A cycloid looks something like this:



(b) The position of the particle is given by

$$\vec{\mathbf{r}} = (R\sin\omega t + \omega R t)\hat{\mathbf{i}} + (R\cos\omega t + R)\hat{\mathbf{j}}.$$

The maximum value of y occurs whenever $\cos\omega t = 1$. The minimum value of y occurs whenever $\cos\omega t = -1$. At either of those times $\sin\omega t = 0$.

The velocity is the derivative of the displacement vector,

$$\vec{\mathbf{v}} = (R\omega\cos\omega t + \omega R)\hat{\mathbf{i}} + (-R\omega\sin\omega t)\hat{\mathbf{j}}.$$

When y is a maximum the velocity simplifies to

$$\vec{\mathbf{v}} = (2\omega R)\hat{\mathbf{i}} + (0)\hat{\mathbf{j}}.$$

When y is a minimum the velocity simplifies to

$$\vec{\mathbf{v}} = (0)\hat{\mathbf{i}} + (0)\hat{\mathbf{j}}.$$

The acceleration is the derivative of the velocity vector,

$$\vec{\mathbf{a}} = (-R\omega^2 \sin \omega t)\hat{\mathbf{i}} + (-R\omega^2 \cos \omega t)\hat{\mathbf{j}}.$$

When y is a maximum the acceleration simplifies to

$$\vec{\mathbf{a}} = (0)\hat{\mathbf{i}} + (-R\omega^2)\hat{\mathbf{j}}.$$

When y is a minimum the acceleration simplifies to

$$\vec{\mathbf{a}} = (0)\hat{\mathbf{i}} + (R\omega^2)\hat{\mathbf{j}}.$$

P4-24 (a) The speed of the car is $v_c = 15.3\,\mathrm{m/s}$. The snow appears to fall with an angle $\theta = \arctan(15.3/7.8) = 63°$.

(b) The apparent speed is $\sqrt{(15.3)^2 + (7.8)^2}(\mathrm{m/s}) = 17.2\,\mathrm{m/s}$.

P4-25 (a) The decimal angles are 89.994250° and 89.994278°. The earth moves in the orbit around the sun with a speed of $v = 2.98\times10^4\mathrm{m/s}$ (Appendix C). The speed of light is then between $c = (2.98\times10^4\mathrm{m/s})\tan(89.994250°) = 2.97\times10^8\mathrm{m/s}$ and $c = (2.98\times10^4\mathrm{m/s})\tan(89.994278°) = 2.98\times10^8\mathrm{m/s}$. This method is *highly* sensitive to rounding. Calculating the orbital speed from the radius and period of the Earth's orbit will likely result in different answers!

P4-26 (a) Total distance is $2l$, so $t_0 = 2l/v$.

(b) Assume wind blows east. Time to travel out is $t_1 = l/(v+u)$, time to travel back is $t_2 = l/(v-u)$. Total time is sum, or

$$t_E = \frac{l}{v+u} + \frac{l}{v-u} = \frac{2lv}{v^2-u^2} = \frac{t_0}{1-u^2/v^2}.$$

If wind blows west the times reverse, but the result is otherwise the same.

(c) Assume wind blows north. The airplane will still have a speed of v relative to the wind, but it will need to fly with a heading away from east. The speed of the plane relative to the ground will be $\sqrt{v^2-u^2}$. This will be the speed even when it flies west, so

$$t_N = \frac{2l}{\sqrt{v^2-u^2}} = \frac{t_0}{\sqrt{1-u^2/v^2}}.$$

(d) If $u > v$ the wind sweeps the plane along in one general direction only; it can never fly back. Sort of like a black hole event horizon.

$\boxed{\textbf{P4-27}}$ The velocity of the police car with respect to the ground is $\vec{\mathbf{v}}_{pg} = -76\mathrm{km/h}\hat{\mathbf{i}}$. The velocity of the motorist with respect the ground is $\vec{\mathbf{v}}_{mg} = -62\ \mathrm{km/h}\hat{\mathbf{j}}$.

The velocity of the motorist with respect to the police car is given by solving

$$\vec{\mathbf{v}}_{mg} = \vec{\mathbf{v}}_{mp} + \vec{\mathbf{v}}_{pg},$$

so $\vec{\mathbf{v}}_{mp} = 76\mathrm{km/h}\hat{\mathbf{i}} - 62\ \mathrm{km/h}\hat{\mathbf{j}}$. This velocity has magnitude

$$v_{mp} = \sqrt{(76\mathrm{km/h})^2 + (-62\ \mathrm{km/h})^2} = 98\ \mathrm{km/h}.$$

The direction is

$$\theta = \arctan(-62 \text{ km/h})/(76\text{km/h}) = -39°,$$

but that is relative to $\hat{\mathbf{i}}$. We want to know the direction relative to the line of sight. The line of sight is

$$\alpha = \arctan(57\,\text{m})/(41\,\text{m}) = -54°$$

relative to $\hat{\mathbf{i}}$, so the answer must be 15°.

P4-28 (a) The velocity of the plane with respect to the air is $\vec{\mathbf{v}}_{pa}$; the velocity of the air with respect to the ground is $\vec{\mathbf{v}}_{ag}$, the velocity of the plane with respect to the ground is $\vec{\mathbf{v}}_{pg}$. Then $\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pa} + \vec{\mathbf{v}}_{ag}$. This can be represented by a triangle; since the sides are given we can find the angle between $\vec{\mathbf{v}}_{ag}$ and $\vec{\mathbf{v}}_{pg}$ (points north) using the cosine law

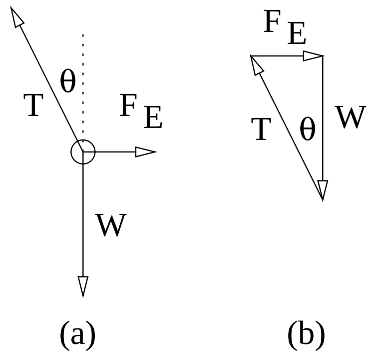
$$\theta = \arccos\left(\frac{(135)^2 - (135)^2 - (70)^2}{-2(135)(70)}\right) = 75°.$$

(b) The direction of $\vec{\mathbf{v}}_{pa}$ can also be found using the cosine law,

$$\theta = \arccos\left(\frac{(70)^2 - (135)^2 - (135)^2}{-2(135)(135)}\right) = 30°.$$

$\boxed{\textbf{E5-1}}$ There are three forces which act on the charged sphere— an electric force, F_E, the force of gravity, W, and the tension in the string, T. All arranged as shown in the figure on the right below.

(a) Write the vectors so that they geometrically show that the sum is zero, as in the figure on the left below. Now $W = mg = (2.8 \times 10^{-4}\text{ kg})(9.8\text{ m/s}^2) = 2.7 \times 10^{-3}$ N. The magnitude of the electric force can be found from the tangent relationship, so $F_E = W \tan\theta = (2.7 \times 10^{-3}\text{ N})\tan(33°) = 1.8 \times 10^{-3}$ N.



(b) The tension can be found from the cosine relation, so

$$T = W/\cos\theta = (2.7 \times 10^{-3}\,\text{N})/\cos(33°) = 3.2 \times 10^{-3}\,\text{N}.$$

E5-2 (a) The net force on the elevator is $F = ma = Wa/g = (6200\text{ lb})(3.8\text{ ft/s}^2)/(32\text{ ft/s}^2) =$ 740 lb. Positive means up. There are two force on the elevator: a weight W down and a tension from the cable T up. Then $F = T - W$ or $T = F + W = (740\text{ lb}) + (6200\text{ lb}) = 6940$ lb.

(b) If the elevator acceleration is down then $F = -740$ lb; consequently $T = F+W = (-740\text{ lb})+(6200\text{ lb}) = 5460$ lb.

E5-3 (a) The tension T is up, the weight W is down, and the net force F is in the direction of the acceleration (up). Then $F = T - W$. But $F = ma$ and $W = mg$, so

$$m = T/(a+g) = (89\,\text{N})/[(2.4\,\text{m/s}^2) + (9.8\,\text{m/s}^2)] = 7.3\,\text{kg}.$$

(b) $T = 89\,$N. The direction of velocity is unimportant. In both (a) and (b) the acceleration is up.

E5-4 The average speed of the elevator during deceleration is $v_{\text{av}} = 6.0\,\text{m/s}$. The time to stop the elevator is then $t = (42.0\,\text{m})/(6.0\,\text{m/s}) = 7.0\,\text{s}$. The deceleration is then $a = (12.0\,\text{m/s})/(7.0\,\text{s}) = 1.7\,\text{m/s}^2$. Since the elevator is moving downward but slowing down, then the acceleration is up, which will be positive.

The net force on the elevator is $F = ma$; this is equal to the tension T minus the weight W. Then

$$T = F + W = ma + mg = (1600\,\text{kg})[(1.7\,\text{m/s}^2) + (9.8\,\text{m/s}^2)] = 1.8\times10^4\text{N}.$$

$\boxed{\textbf{E5-5}}$ (a) The magnitude of the man's acceleration is given by

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = \frac{(110\text{ kg}) - (74\text{ kg})}{(110\text{ kg}) + (74\text{ kg})} g = 0.2g,$$

and is directed down. The time which elapses while he falls is found by solving $y = v_{0y}t + \frac{1}{2}a_y t^2$, or, with numbers, $(-12\text{ m}) = (0)t + \frac{1}{2}(-0.2g)t^2$ which has the solutions $t = \pm 3.5$ s. The velocity with which he hits the ground is then $v = v_{0y} + a_y t = (0) + (-0.2g)(3.5\text{ s}) = -6.9$ m/s.

(b) Reducing the speed can be accomplished by reducing the acceleration. We can't change Eq. 5-4 without also changing one of the assumptions that went into it. Since the man is hoping to reduce the speed with which he hits the ground, it makes sense that he might want to climb up the rope.

E5-6 (a) Although it might be the monkey which does the work, the upward force to lift him still comes from the tension in the rope. The minimum tension to lift the log is $T = W_1 = m_1 g$. The net force on the monkey is $T - W_\mathrm{m} = m_\mathrm{m} a$. The acceleration of the monkey is then

$$a = (m_\mathrm{l} - m_\mathrm{m})g/m_\mathrm{m} = [(15\,\mathrm{kg}) - (11\,\mathrm{kg})](9.8\,\mathrm{m/s^2})/(11\,\mathrm{kg}) = 3.6\,\mathrm{m/s^2}.$$

(b) Atwood's machine!

$$a = (m_\mathrm{l} - m_\mathrm{m})g/(m_\mathrm{l} + m_\mathrm{m}) = [(15\,\mathrm{kg}) - (11\,\mathrm{kg})](9.8\,\mathrm{m/s^2})/[(15\,\mathrm{kg}) + (11\,\mathrm{kg})] = 1.5\,\mathrm{m/s}.$$

(c) Atwood's machine!

$$T = 2m_\mathrm{l} m_\mathrm{m} g/(m_\mathrm{l} + m_\mathrm{m}) = 2(15\,\mathrm{kg})(11\,\mathrm{kg})(9.8\,\mathrm{m/s^2})/[(15\,\mathrm{kg}) + (11\,\mathrm{kg})] = 120\,\mathrm{N}.$$

E5-7 The weight of each car has two components: a component parallel to the cables $W_{||} = W \sin\theta$ and a component normal to the cables $W_\perp$.. The normal component is "balanced" by the supporting cable. The parallel component acts with the pull cable.

In order to accelerate a car up the incline there must be a net force up with magnitude $F = ma$. Then $F = T_\mathrm{above} - T_\mathrm{below} - W_{||}$, or

$$\Delta T = ma + mg\sin\theta = (2800\,\mathrm{kg})[(0.81\,\mathrm{m/s^2}) + (9.8\,\mathrm{m/s^2})\sin(35^\circ)] = 1.8\times 10^4\mathrm{N}.$$

E5-8 The tension in the cable is T, the weight of the man + platform system is $W = mg$, and the net force on the man + platform system is $F = ma = Wa/g = T - W$. Then

$$T = Wa/g + W = W(a/g + 1) = (180\text{ lb} + 43\text{ lb})[(1.2\text{ ft/s}^2)/(32\text{ ft/s}^2) + 1] = 231\text{ lb}.$$

$\boxed{\textbf{E5-9}}$ See Sample Problem 5-8. We need only apply the (unlabeled!) equation

$$\mu_s = \tan\theta$$

to find the egg angle. In this case $\theta = \tan^{-1}(0.04) = 2.3^\circ$.

E5-10 (a) The maximum force of friction is $F = \mu_\mathrm{s} N$. If the rear wheels support half of the weight of the automobile then $N = W/2 = mg/2$. The maximum acceleration is then

$$a = F/m = \mu_\mathrm{s} N/m = \mu_\mathrm{s} g/2.$$

(b) $a = (0.56)(9.8\,\mathrm{m/s^2})/2 = 2.7\,\mathrm{m/s^2}$.

E5-11 The maximum force of friction is $F = \mu_\mathrm{s} N$. Since there is no motion in the y direction the magnitude of the normal force must equal the weight, $N = W = mg$. The maximum acceleration is then

$$a = F/m = \mu_\mathrm{s} N/m = \mu_\mathrm{s} g = (0.95)(9.8\,\mathrm{m/s^2}) = 9.3\,\mathrm{m/s^2}.$$

E5-12 There is no motion in the vertical direction, so $N = W = mg$. Then $\mu_\mathrm{k} = F/N = (470\,\mathrm{N})/[(9.8\,\mathrm{m/s^2})(79\,\mathrm{kg})] = 0.61$.

$\boxed{\textbf{E5-13}}$ A 75 kg mass has a weight of $W = (75\,\text{kg})(9.8\,\text{m/s}^2) = 735\,\text{N}$, so the force of friction on each end of the bar must be 368 N. Then

$$F \geq \frac{f_s}{\mu_s} = \frac{(368\,\text{N})}{(0.41)} = 900\,\text{N}.$$

E5-14 (a) There is no motion in the vertical direction, so $N = W = mg$.

To get the box moving you must overcome static friction and push with a force of $P \geq \mu_\text{s} N = (0.41)(240\,\text{N}) = 98\,\text{N}$.

(b) To keep the box moving at constant speed you must push with a force equal to the kinetic friction, $P = \mu_\text{k} N = (0.32)(240\,\text{N}) = 77\,\text{N}$.

(c) If you push with a force of 98 N on a box that experiences a (kinetic) friction of 77 N, then the net force on the box is 21 N. The box will accelerate at

$$a = F/m = Fg/W = (21\,\text{N})(9.8\,\text{m/s}^2)/(240\,\text{N}) = 0.86\,\text{m/s}^2.$$

E5-15 (a) The maximum braking force is $F = \mu_\text{s} N$. There is no motion in the vertical direction, so $N = W = mg$. Then $F = \mu_\text{s} mg = (0.62)(1500\,\text{kg})(9.8\,\text{m/s}^2) = 9100\,\text{N}$.

(b) Although we still use $F = \mu_\text{s} N$, $N \neq W$ on an incline! The weight has two components; one which is parallel to the surface and the other which is perpendicular. Since there is no motion perpendicular to the surface we must have $N = W_\perp = W \cos\theta$. Then

$$F = \mu_\text{s} mg \cos\theta = (0.62)(1500\,\text{kg})(9.8\,\text{m/s}^2)\cos(8.6°) = 9000\,\text{N}.$$

E5-16 $\mu_s = \tan\theta$ is the condition for an object to sit without slipping on an incline. Then $\theta = \arctan(0.55) = 29°$. The angle should be reduced by 13°.

$\boxed{\textbf{E5-17}}$ (a) The force of static friction is less than $\mu_s N$, where N is the normal force. Since the crate isn't moving up or down, $\sum F_y = 0 = N - W$. So in this case $N = W = mg = (136$ kg$)(9.81$ m/s$^2) = 1330$ N. The force of static friction is less than or equal to $(0.37)(1330\,\text{N}) = 492\,\text{N}$; moving the crate will require a force greater than or equal to 492 N.

(b) The second worker could lift upward with a force L, reducing the normal force, and hence reducing the force of friction. If the first worker can move the block with a 412 N force, then $412 \geq \mu_s N$. Solving for N, the normal force needs to be less than 1110 N. The crate doesn't move off the table, so then $N + L = W$, or $L = W - N = (1330\text{ N}) - (1110\text{ N}) = 220$ N.

(c) Or the second worker can help by adding a push so that the total force of both workers is equal to 492 N. If the first worker pushes with a force of 412 N, the second would need to push with a force of 80 N.

E5-18 The coefficient of static friction is $\mu_\text{s} = \tan(28.0°) = 0.532$. The acceleration is $a = 2(2.53\,\text{m})/(3.92\,\text{s})^2 = .329\,\text{m/s}^2$. We will need to insert a negative sign since this is downward.

The weight has two components: a component parallel to the plane, $W_{||} = mg\sin\theta$; and a component perpendicular to the plane, $W_\perp = mg\cos\theta$. There is no motion perpendicular to the plane, so $N = W_\perp$. The kinetic friction is then $f = \mu_\text{k} N = \mu_\text{k} mg\cos\theta$. The net force parallel to the plane is $F = ma = f - W_{||} = \mu_\text{k} mg\cos\theta - mg\sin\theta$. Solving this for μ_k,

$$\begin{aligned}
\mu_\text{k} &= (a + g\sin\theta)/(g\cos\theta), \\
&= [(-0.329\,\text{m/s}^2) + (9.81\,\text{m/s}^2)\sin(28.0°)]/[((9.81\,\text{m/s}^2)\cos(28.0°)] = 0.494.
\end{aligned}$$

E5-19 The acceleration is $a = -2d/t^2$, where $d = 203\,\text{m}$ is the distance down the slope and t is the time to make the run.

The weight has two components: a component parallel to the incline, $W_{||} = mg\sin\theta$; and a component perpendicular to the incline, $W_\perp = mg\cos\theta$. There is no motion perpendicular tot he plane, so $N = W_\perp$. The kinetic friction is then $f = \mu_\text{k} N = \mu_\text{k} mg\cos\theta$. The net force parallel to the plane is $F = ma = f - W_{||} = \mu_\text{k} mg\cos\theta - mg\sin\theta$. Solving this for μ_k,

$$\begin{aligned}\mu_\text{k} &= (a + g\sin\theta)/(g\cos\theta),\\ &= (g\sin\theta - 2d/t^2)/(g\cos\theta).\end{aligned}$$

If $t = 61\,\text{s}$, then

$$\mu_\text{k} = \frac{(9.81\,\text{m/s}^2)\sin(3.0^\circ) - 2(203\,\text{m})/(61\,\text{s})^2}{(9.81\,\text{m/s}^2)\cos(3.0^\circ)} = 0.041;$$

if $t = 42\,\text{s}$, then

$$\mu_\text{k} = \frac{(9.81\,\text{m/s}^2)\sin(3.0^\circ) - 2(203\,\text{m})/(42\,\text{s})^2}{(9.81\,\text{m/s}^2)\cos(3.0^\circ)} = 0.029.$$

E5-20 (a) If the block slides down with constant velocity then $a = 0$ and $\mu_\text{k} = \tan\theta$. Not only that, but the force of kinetic friction must be equal to the parallel component of the weight, $f = W_{||}$. If the block is projected up the ramp then the net force is now $2W_{||} = 2mg\sin\theta$. The deceleration is $a = 2g\sin\theta$; the block will travel a time $t = v_0/a$ before stopping, and travel a distance

$$d = -at^2/2 + v_0 t = -a(v_0/a)^2/2 + v_0(v_0/a) = v_0^2/(2a) = v_0^2/(4g\sin\theta)$$

before stopping.

(b) Since $\mu_\text{k} < \mu_\text{s}$, the incline is *not* steep enough to get the block moving again once it stops.

$\boxed{\textbf{E5-21}}$ Let a_1 be acceleration down frictionless incline of length l, and t_1 the time taken. The a_2 is acceleration down "rough" incline, and $t_2 = 2t_1$ is the time taken. Then

$$l = \frac{1}{2}a_1 t_1^2 \text{ and } l = \frac{1}{2}a_2(2t_1)^2.$$

Equate and find $a_1/a_2 = 4$.

There are two force which act on the ice when it sits on the frictionless incline. The normal force acts perpendicular to the surface, so it doesn't contribute any components parallel to the surface. The force of gravity has a component parallel to the surface, given by

$$W_{||} = mg\sin\theta,$$

and a component perpendicular to the surface given by

$$W_\perp = mg\cos\theta.$$

The acceleration down the frictionless ramp is then

$$a_1 = \frac{W_{||}}{m} = g\sin\theta.$$

When friction is present the force of kinetic friction is $f_k = \mu_k N$; since the ice doesn't move perpendicular to the surface we also have $N = W_\perp$; and finally the acceleration down the ramp is

$$a_2 = \frac{W_{||} - f_k}{m} = g(\sin\theta - \mu\cos\theta).$$

Previously we found the ratio of a_1/a_2, so we now have

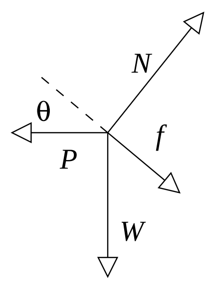
$$\begin{aligned}
\sin\theta &= 4\sin\theta - 4\mu\cos\theta, \\
\sin 33^\circ &= 4\sin 33^\circ - 4\mu\cos 33^\circ, \\
\mu &= 0.49.
\end{aligned}$$

E5-22 (a) The static friction between A and the table must be equal to the weight of block B to keep A from sliding. This means $m_B g = \mu_s(m_A + m_C)g$, or $m_c = m_B/\mu_s - m_A = (2.6\,\mathrm{kg})/(0.18) - (4.4\,\mathrm{kg}) = 10\,\mathrm{kg}$.

(b) There is no up/down motion for block A, so $f = \mu_k N = \mu_k m_A g$. The net force on the system containing blocks A and B is $F = W_B - f = m_B g - \mu_k m_A g$; the acceleration of this system is then

$$a = g\frac{m_B - \mu_k m_A}{m_A + m_B} = (9.\,\mathrm{m/s^2})\frac{(2.6\,\mathrm{kg}) - (0.15)(4.4\,\mathrm{kg})}{(2.6\,\mathrm{kg}) + (4.4\,\mathrm{kg})} = 2.7\,\mathrm{m/s^2}.$$

$\boxed{\textbf{E5-23}}$ There are four forces on the block— the force of gravity, $W = mg$; the normal force, N; the horizontal push, P, and the force of friction, f. Choose the coordinate system so that components are either parallel (x-axis) to the plane or perpendicular (y-axis) to it. $\theta = 39^\circ$. Refer to the figure below.



The magnitudes of the x components of the forces are $W_x = W\sin\theta$, $P_x = P\cos\theta$ and f; the magnitudes of the y components of the forces are $W_y = W\cos\theta$, $P_y = P\sin\theta$.

(a) We consider the first the case of the block moving up the ramp; then f is directed down. Newton's second law for each set of components then reads as

$$\begin{aligned}
\sum F_x &= P_x - f - W_x = P\cos\theta - f - W\sin\theta = ma_x, \\
\sum F_y &= N - P_y - W_y = N - P\sin\theta - W\cos\theta = ma_y
\end{aligned}$$

Then the second equation is easy to solve for N

$$N = P\sin\theta + W\cos\theta = (46\ \mathrm{N})\sin(39^\circ) + (4.8\ \mathrm{kg})(9.8\ \mathrm{m/s^2})\cos(39^\circ) = 66\ \mathrm{N}.$$

The force of friction is found from $f = \mu_k N = (0.33)(66\ \mathrm{N}) = 22\ \mathrm{N}$. This is directed down the incline while the block is moving up. We can now find the acceleration in the x direction.

$$\begin{aligned}
ma_x &= P\cos\theta - f - W\sin\theta, \\
&= (46\,\mathrm{N})\cos(39^\circ) - (22\,\mathrm{N}) - (4.8\,\mathrm{kg})(9.8\,\mathrm{m/s^2})\sin(39^\circ) = -16\,\mathrm{N}.
\end{aligned}$$

So the block is slowing down, with an acceleration of magnitude $3.3\ \mathrm{m/s^2}$.

(b) The block has an initial speed of $v_{0x} = 4.3$ m/s; it will rise until it stops; so we can use $v_y = 0 = v_{0y} + a_y t$ to find the time to the highest point. Then $t = (v_y - v_{0y})/a_y = -(-4.3\ \mathrm{m/s})/(3.3\ \mathrm{m/s^2} = 1.3$ s. Now that we know the time we can use the other kinematic relation to find the distance

$$y = v_{0y}t + \frac{1}{2}a_y t^2 = (4.3\ \mathrm{m/s})(1.3\ \mathrm{s}) + \frac{1}{2}(-3.3\ \mathrm{m/s^2})(1.3\ \mathrm{s})^2 = 2.8\ \mathrm{m}$$

(c) When the block gets to the top it *might* slide back down. But in order to do so the frictional force, which is now directed up the ramp, must be sufficiently small so that $f + P_x \leq W_x$. Solving for f we find $f \leq W_x - P_x$ or, using our numbers from above, $f \leq -6$ N. Is this possible? No, so the block will not slide back down the ramp, *even if the ramp were frictionless,* while the horizontal force is applied.

E5-24 (a) The horizontal force needs to overcome the maximum static friction, so $P \geq \mu_s N = \mu_s mg = (0.52)(12\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 61\,\mathrm{N}$.

(b) If the force acts upward from the horizontal then there are two components: a horizontal component $P_x = P\cos\theta$ and a vertical component $P_y = P\sin\theta$. The normal force is now given by $W = P_y + N$; consequently the maximum force of static friction is now $\mu_s N = \mu_s(mg - P\sin\theta)$. The block will move only if $P\cos\theta \geq \mu_s(mg - P\sin\theta)$, or

$$P \geq \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta} = \frac{(0.52)(12\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{\cos(62^\circ) + (0.52)\sin(62^\circ)} = 66\,\mathrm{N}.$$

(c) If the force acts downward from the horizontal then $\theta = -62^\circ$, so

$$P \geq \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta} = \frac{(0.52)(12\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{\cos(-62^\circ) + (0.52)\sin(-62^\circ)} = 5900\,\mathrm{N}.$$

E5-25 (a) If the tension acts upward from the horizontal then there are two components: a horizontal component $T_x = T\cos\theta$ and a vertical component $T_y = T\sin\theta$. The normal force is now given by $W = T_y + N$; consequently the maximum force of static friction is now $\mu_s N = \mu_s(W - T\sin\theta)$. The crate will move only if $T\cos\theta \geq \mu_s(W - T\sin\theta)$, or

$$P \geq \frac{\mu_s W}{\cos\theta + \mu_s\sin\theta} = \frac{(0.52)(150\text{ lb})}{\cos(17^\circ) + (0.52)\sin(17^\circ)} = 70\text{ lb}.$$

(b) Once the crate starts to move then the net force on the crate is $F = T_x - f$. The acceleration is then

$$\begin{aligned} a &= \frac{g}{W}[T\cos\theta - \mu_k(W - T\sin\theta)], \\ &= \frac{(32\text{ ft/s}^2)}{(150\text{ lb})}\{(70\text{ lb})\cos(17^\circ) - (0.35)[(150\text{ lb}) - (70\text{ lb})\sin(17^\circ)]\}, \\ &= 4.6\text{ ft/s}^2. \end{aligned}$$

E5-26 If the tension acts upward from the horizontal then there are two components: a horizontal component $T_x = T\cos\theta$ and a vertical component $T_y = T\sin\theta$. The normal force is now given by $W = T_y + N$; consequently the maximum force of static friction is now $\mu_s N = \mu_s(W - T\sin\theta)$. The crate will move only if $T\cos\theta \geq \mu_s(W - T\sin\theta)$, or

$$W \leq T\cos\theta/\mu_s + T\sin\theta.$$

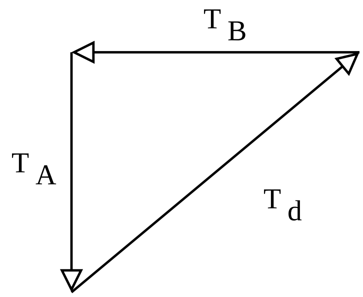
We want the maximum, so we find $dW/d\theta$,

$$dW/d\theta = -(T/\mu_s)\sin\theta + T\cos\theta,$$

which equals zero when $\mu_s = \tan\theta$. For this problem $\theta = \arctan(0.35) = 19^\circ$, so

$$W \leq (1220\,\mathrm{N})\cos(19^\circ)/(0.35) + (1220\,\mathrm{N})\sin(19^\circ) = 3690\,\mathrm{N}.$$

E5-27 The three force on the know above A must add to zero. Construct a vector diagram: $\vec{\mathbf{T}}_A + \vec{\mathbf{T}}_B + \vec{\mathbf{T}}_d = 0$, where $\vec{\mathbf{T}}_d$ refers to the diagonal rope. T_A and T_B must be related by $T_A = T_B \tan\theta$, where $\theta = 41°$.



There is no up/down motion of block B, so $N = W_B$ and $f = \mu_s W_B$. Since block B is at rest $f = T_B$. Since block A is at rest $W_A = T_A$. Then

$$W_A = W_B(\mu_s \tan\theta) = (712\,\mathrm{N})(0.25)\tan(41°) = 155\,\mathrm{N}.$$

E5-28 (a) Block 2 doesn't move up/down, so $N = W_2 = m_2 g$ and the force of friction on block 2 is $f = \mu_k m_2 g$. Block 1 is on a frictionless incline; only the component of the weight parallel to the surface contributes to the motion, and $W_{||} = m_1 g \sin\theta$. There are two relevant forces on the two mass system. The effective net force is the of magnitude $W_{||} - f$, so the acceleration is

$$a = g\frac{m_1 \sin\theta - \mu_k m_2}{m_1 + m_2} = (9.81\,\mathrm{m/s^2})\frac{(4.20\,kg)\sin(27°) - (0.47)(2.30\,\mathrm{kg})}{(4.20\,\mathrm{kg}) + (2.30\,\mathrm{kg})} = 1.25\,\mathrm{m/s^2}.$$

The blocks accelerate down the ramp.

(b) The net force on block 2 is $F = m_2 a = T - f$. The tension in the cable is then

$$T = m_2 a + \mu_k m_2 g = (2.30\,\mathrm{kg})[(1.25\,\mathrm{m/s^2}) + (0.47)(9.81\,\mathrm{m/s^2})] = 13.5\,\mathrm{N}.$$

$\boxed{\textbf{E5-29}}$ This problem is similar to Sample Problem 5-7, except now there is friction which can act on block B. The relevant equations are now for block B

$$N - m_B g \cos\theta = 0$$

and

$$T - m_B g \sin\theta \pm f = m_B a,$$

where the sign in front of f depends on the direction in which block B is moving. If the block is moving up the ramp then friction is directed down the ramp, and we would use the negative sign. If the block is moving down the ramp then friction will be directed up the ramp, and then we will use the positive sign. Finally, if the block is stationary then friction we be in such a direction as to make $a = 0$.

For block A the relevant equation is

$$m_A g - T = m_A a.$$

Combine the first two equations with $f = \mu N$ to get

$$T - m_B g \sin\theta \pm \mu m_B g \cos\theta = m_B a.$$

Take some care when interpreting friction for the static case, since the static value of μ yields the maximum possible static friction force, which is not necessarily the actual static frictional force.

Combine this last equation with the block A equation,

$$m_A g - m_A a - m_B g \sin\theta \pm \mu m_B g \cos\theta = m_B a,$$

and then rearrange to get

$$a = g\frac{m_A - m_B \sin\theta \pm \mu m_B \cos\theta}{m_A + m_B}.$$

For convenience we will use metric units; then the masses are $m_A = 13.2\,\text{kg}$ and $m_B = 42.6\,\text{kg}$. In addition, $\sin 42^\circ = 0.669$ and $\cos 42^\circ = 0.743$.

(a) If the blocks are originally at rest then

$$m_A - m_B \sin\theta = (13.2\,\text{kg}) - (42.6\,\text{kg})(0.669) = -15.3\,\text{kg}$$

where the negative sign indicates that block B would slide downhill if there were no friction.

If the blocks are originally at rest we need to consider static friction, so the last term can be as large as

$$\mu m_B \cos\theta = (.56)(42.6\,\text{kg})(0.743) = 17.7\,\text{kg}.$$

Since this quantity is larger than the first static friction would be large enough to stop the blocks from accelerating if they are at rest.

(b) If block B is moving up the ramp we use the negative sign, and the acceleration is

$$a = (9.81\,\text{m/s}^2)\frac{(13.2\,\text{kg}) - (42.6\,\text{kg})(0.669) - (.25)(42.6\,\text{kg})(0.743)}{(13.2\,\text{kg}) + (42.6\,\text{kg})} = -4.08\,\text{m/s}^2.$$

where the negative sign means down the ramp. The block, originally moving up the ramp, will slow down and stop. Once it stops the static friction takes over and the results of part (a) become relevant.

(c) If block B is moving down the ramp we use the positive sign, and the acceleration is

$$a = (9.81\,\text{m/s}^2)\frac{(13.2\,\text{kg}) - (42.6\,\text{kg})(0.669) + (.25)(42.6\,\text{kg})(0.743)}{(13.2\,\text{kg}) + (42.6\,\text{kg})} = -1.30\,\text{m/s}^2.$$

where the negative sign means down the ramp. This means that if the block is moving down the ramp it will continue to move down the ramp, faster and faster.

E5-30 The weight can be resolved into a component parallel to the incline, $W_{||} = W\sin\theta$ and a component that is perpendicular, $W_\perp = W\cos\theta$. There are two normal forces on the crate, one from each side of the trough. By symmetry we expect them to have equal magnitudes; since they both act perpendicular to their respective surfaces we expect them to be perpendicular to each other. They must add to equal the perpendicular component of the weight. Since they are at right angles and equal in magnitude, this yields $N^2 + N^2 = {W_\perp}^2$, or $N = W_\perp/\sqrt{2}$.

Each surface contributes a frictional force $f = \mu_\text{k} N = \mu_\text{k} W_\perp/\sqrt{2}$; the total frictional force is then twice this, or $\sqrt{2}\mu_\text{k} W_\perp$. The net force on the crate is $F = W\sin\theta - \sqrt{2}\mu_\text{k} W\cos\theta$ down the ramp. The acceleration is then

$$a = g(\sin\theta - \sqrt{2}\mu_\text{k}\cos\theta).$$

E5-31 The normal force between the top slab and the bottom slab is $N = W_\text{t} = m_\text{t}g$. The force of friction between the top and the bottom slab is $f \le \mu N = \mu m_\text{t} g$. We don't yet know if the slabs slip relative to each other, so we don't yet know what kind of friction to consider.

The acceleration of the top slab is

$$a_\text{t} = (110\,\text{N})/(9.7\,\text{kg}) - \mu(9.8\,\text{m/s}^2) = 11.3\,\text{m/s}^2 - \mu(9.8\,\text{m/s}^2).$$

The acceleration of the bottom slab is

$$a_\text{b} = \mu(9.8\,\text{m/s}^2)(9.7,\text{kg})/(42\,\text{kg}) = \mu(2.3\,\text{m/s}^2).$$

Can these two be equal? Only if $\mu \ge 0.93$. Since the static coefficient is less than this, the block slide. Then $a_\text{t} = 7.6\,\text{m/s}^2$ and $a_\text{b} = 0.87\,\text{m/s}^2$.

E5-32 (a) Convert the speed to ft/s: $v = 88$ ft/s. The acceleration is

$$a = v^2/r = (88 \text{ ft/s})^2/(25 \text{ ft}) = 310 \text{ ft/s}^2.$$

(b) $a = 310 \text{ ft/s}^2 g/(32 \text{ ft/s}^2) = 9.7g$.

$\boxed{\textbf{E5-33}}$ (a) The force required to keep the car in the turn is $F = mv^2/r = Wv^2/(rg)$, or

$$F = (10700\,\text{N})(13.4\,\text{m/s})^2/[(61.0\,\text{m})(9.81\,\text{m/s}^2)] = 3210\,\text{N}.$$

(b) The coefficient of friction required is $\mu_\text{s} = F/W = (3210\,\text{N})/(10700\,\text{N}) = 0.300$.

E5-34 (a) The proper banking angle is given by

$$\theta = \arctan\frac{v^2}{Rg} = \arctan\frac{(16.7\,\text{m/s})^2}{(150\,\text{m})(9.8\,\text{m/s}^2)} = 11^\circ.$$

(b) If the road is not banked then the force required to keep the car in the turn is $F = mv^2/r = Wv^2/(Rg)$ and the required coefficient of friction is

$$\mu_\text{s} = F/W = \frac{v^2}{Rg} = \frac{(16.7\,\text{m/s})^2}{(150\,\text{m})(9.8\,\text{m/s}^2)} = 0.19.$$

E5-35 (a) This conical pendulum makes an angle $\theta = \arcsin(0.25/1.4) = 10^\circ$ with the vertical. The pebble has a speed of

$$v = \sqrt{Rg\tan\theta} = \sqrt{(0.25\,\text{m})(9.8\,\text{m/s}^2)\tan(10^\circ)} = 0.66\,\text{m/s}.$$

(b) $a = v^2/r = (0.66\,\text{m/s})^2/(0.25\,\text{m}) = 1.7\,\text{m/s}^2$.
(c) $T = mg/cos\theta = (0.053\,\text{kg})(9.8\,\text{m/s}^2)/\cos(10^\circ) = 0.53\,\text{N}$.

E5-36 Ignoring air friction (there must be a forward component to the friction!), we have a normal force upward which is equal to the weight: $N = mg = (85\,\text{kg})(9.8\,\text{m/s}^2) = 833\,\text{N}$. There is a sideways component to the friction which is equal tot eh centripetal force, $F = mv^2/r = (85\,\text{kg})(8.7\,\text{m/s})^2/(25\,\text{m}) = 257\,\text{N}$. The magnitude of the net force of the road on the person is

$$F = \sqrt{(833\,\text{N})^2 + (257\,\text{N})^2} = 870\,\text{N},$$

and the direction is $\theta = \arctan(257/833) = 17^\circ$ off of vertical.

E5-37 (a) The speed is $v = 2\pi r f = 2\pi(5.3\times10^{-11}\,\mathrm{m})(6.6\times10^{15}/\mathrm{s}) = 2.2\times10^{6}\mathrm{m/s}$.

(b) The acceleration is $a = v^2/r = (2.2\times10^{6}\mathrm{m/s})^2/(5.3\times10^{-11}\mathrm{m}) = 9.1\times10^{22}\,\mathrm{m/s^2}$.

(c) The net force is $F = ma = (9.1\times10^{-31}\,\mathrm{kg})(9.1\times10^{22}\,\mathrm{m/s^2}) = 8.3\times10^{-8}\mathrm{N}$.

E5-38 The basket has speed $v = 2\pi r/t$. The basket experiences a frictional force $F = mv^2/r = m(2\pi r/t)^2/r = 4\pi^2 mr/t^2$. The coefficient of static friction is $\mu_\mathrm{s} = F/N = F/W$. Combining,

$$\mu_\mathrm{s} = \frac{4\pi^2 r}{gt^2} = \frac{4\pi^2(4.6\,\mathrm{m})}{(9.8\,\mathrm{m/s^2})(24\,\mathrm{s})^2} = 0.032.$$

$\boxed{\textbf{E5-39}}$ There are two forces on the hanging cylinder: the force of the cord pulling up T and the force of gravity $W = Mg$. The cylinder is at rest, so these two forces must balance, or $T = W$. There are three forces on the disk, but only the force of the cord on the disk T is relevant here, since there is no friction or vertical motion.

The disk undergoes circular motion, so $T = mv^2/r$. We want to solve this for v and then express the answer in terms of m, M, r, and G.

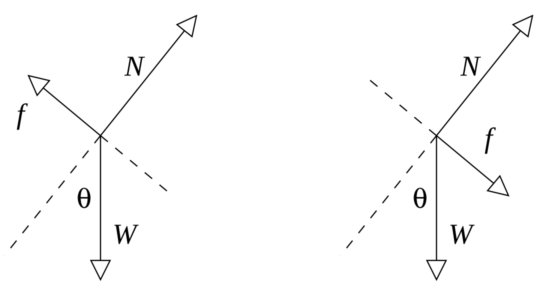
$$v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{Mgr}{m}}.$$

E5-40 (a) The frictional force stopping the car is $F = \mu_\mathrm{s} N = \mu_\mathrm{s} mg$. The deceleration of the car is then $a = \mu_\mathrm{s} g$. If the car is moving at $v = 13.3\,\mathrm{m/s}$ then the average speed while decelerating is half this, or $v_\mathrm{av} = 6.7\,\mathrm{m/s}$. The time required to stop is $t = x/v_\mathrm{av} = (21\,\mathrm{m})/(6.7\,\mathrm{m/s}) = 3.1\,\mathrm{s}$. The deceleration is $a = (13.3\,\mathrm{m/s})/(3.1\,\mathrm{s}) = 4.3\,\mathrm{m/s^2}$. The coefficient of friction is $\mu_\mathrm{s} = a/g = (4.3\,\mathrm{m/s^2})/(9.8\,\mathrm{m/s^2}) = 0.44$.

(b) The acceleration is the same as in part (a), so $r = v^2/a = (13.3\,\mathrm{m/s})^2/(4.3\,\mathrm{m/s^2}) = 41\,\mathrm{m}$.

$\boxed{\textbf{E5-41}}$ There are three forces to consider: the normal force of the road on the car N; the force of gravity on the car W; and the frictional force on the car f. The acceleration of the car in circular motion is toward the center of the circle; this means the *net* force on the car is horizontal, toward the center. We will arrange our coordinate system so that r is horizontal and z is vertical. Then the components of the normal force are $N_r = N\sin\theta$ and $N_z = N\cos\theta$; the components of the frictional force are $f_r = f\cos\theta$ and $f_z = f\sin\theta$.

The direction of the friction depends on the speed of the car. The figure below shows the two force diagrams.



The turn is designed for 95 km/hr, at this speed a car should require *no* friction to stay on the road. Using Eq. 5-17 we find that the banking angle is given by

$$\tan\theta_b = \frac{v^2}{rg} = \frac{(26\ \mathrm{m/s})^2}{(210\ \mathrm{m})(9.8\ \mathrm{m/s^2})} = 0.33,$$

for a bank angle of $\theta_b = 18°$.

(a) On the rainy day traffic is moving at 14 m/s. This is slower than the rated speed, so any frictional force must be directed up the incline. Newton's second law is then

$$\begin{aligned}\sum F_r &= N_r - f_r = N\sin\theta - f\cos\theta = \frac{mv^2}{r},\\ \sum F_z &= N_z + f_z - W = N\cos\theta + f\sin\theta - mg = 0.\end{aligned}$$

We can substitute $f = \mu_s N$ to find the minimum value of μ_s which will keep the cars from slipping. There will then be two equations and two unknowns, μ_s and N. Solving for N,

$$N\left(\sin\theta - \mu_s\cos\theta\right) = \frac{mv^2}{r} \text{ and } N\left(\cos\theta + \mu_s\sin\theta\right) = mg.$$

Combining,

$$\left(\sin\theta - \mu_s\cos\theta\right)mg = \left(\cos\theta + \mu_s\sin\theta\right)\frac{mv^2}{r}$$

Rearrange,

$$\mu_s = \frac{gr\sin\theta - v^2\cos\theta}{gr\cos\theta + v^2\sin\theta}.$$

We know all the numbers. Put them in and we'll find $\mu_s = 0.22$

(b) Now the frictional force will point the other way, so Newton's second law is now

$$\begin{aligned}\sum F_r &= N_r + f_r = N\sin\theta + f\cos\theta = \frac{mv^2}{r},\\ \sum F_z &= N_z - f_z - W = N\cos\theta - f\sin\theta - mg = 0.\end{aligned}$$

The bottom equation can be rearranged to show that

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}.$$

This can be combined with the top equation to give

$$mg\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{mv^2}{r}.$$

We can solve this final expression for v using all our previous numbers and get $v = 35$ m/s. That's about 130 km/hr.

E5-42 (a) The net force on the person at the top of the Ferris wheel is $mv^2/r = W - N_\mathrm{t}$, pointing down. The net force on the bottom is still mv^2/r, but this quantity now equals $N_\mathrm{b} - W$ and is point up. Then $N_\mathrm{b} = 2W - N_\mathrm{t} = 2(150 \text{ lb}) - (125 \text{ lb}) = 175$ lb.

(b) Doubling the speed would quadruple the net force, so the new scale reading at the top would be (150 lb) – 4[(150 lb) – (125 lb)] = 50 lb. Wee!

E5-43 The net force on the object when it is *not* sliding is $F = mv^2/r$; the speed of the object is $v = 2\pi rf$ (f is rotational frequency here), so $F = 4\pi^2 mrf^2$. The coefficient of friction is then at least $\mu_\mathrm{s} = F/W = 4\pi^2 rf^2/g$. If the object stays put when the table rotates at $33\frac{1}{3}$ rev/min then

$$\mu_\mathrm{s} \geq 4\pi^2(0.13\,\mathrm{m})(33.3/60\,/\mathrm{s})^2/(9.8\,\mathrm{m/s^2}) = 0.16.$$

If the object slips when the table rotates at 45.0 rev/min then

$$\mu_\mathrm{s} \leq 4\pi^2(0.13\,\mathrm{m})(45.0/60\,/\mathrm{s})^2/(9.8\,\mathrm{m/s^2}) = 0.30.$$

E5-44 This is effectively a banked highway problem if the pilot is flying correctly.

$$r = \frac{v^2}{g\tan\theta} = \frac{(134\,\text{m/s})^2}{(9.8\,\text{m/s}^2)\tan(38.2°)} = 2330\,\text{m}.$$

E5-45 (a) Assume that frigate bird flies as well as a pilot. Then this is a banked highway problem. The speed of the bird is given by $v^2 = gr\tan\theta$. But there is also $vt = 2\pi r$, so $2\pi v^2 = gvt\tan\theta$, or

$$v = \frac{gt\tan\theta}{2\pi} = \frac{(9.8\,\text{m/s}^2)(13\,\text{s})\tan(25°)}{2\pi} = 9.5\,\text{m/s}.$$

(b) $r = vt/(2\pi) = (9.5\,\text{m/s})(13\,\text{s})/(2\pi) = 20\,\text{m}$.

E5-46 (a) The radius of the turn is $r = \sqrt{(33\,\text{m})^2 - (18\,\text{m})^2} = 28\,\text{m}$. The speed of the plane is $v = 2\pi rf = 2\pi(28\,\text{m})(4.4/60\,/\text{s}) = 13\,\text{m/s}$. The acceleration is $a = v^2/r = (13\,\text{m/s})^2/(28\,\text{m}) = 6.0\,m/\text{s}^2$.

(b) The tension has two components: $T_x = T\cos\theta$ and $T_y = T\sin\theta$. In this case $\theta = \arcsin(18/33) = 33°$. All of the centripetal force is provided for by T_x, so

$$T = (0.75\,\text{kg})(6.0\,\text{m/s}^2)/\cos(33°) = 5.4\,\text{N}.$$

(c) The lift is balanced by the weight and T_y. The lift is then

$$T_y + W = (5.4\,\text{N})\sin(33°) + (0.75\,\text{kg})(9.8\,\text{m/s}^2) = 10\,\text{N}.$$

E5-47 (a) The acceleration is $a = v^2/r = 4\pi^2 r/t^2 = 4\pi^2(6.37\times10^6\text{m})/(8.64\times10^4\text{s})^2 = 3.37\times10^{-2}\text{m/s}^2$. The centripetal force on the standard kilogram is $F = ma = (1.00\,\text{kg})(3.37\times10^{-2}\text{m/s}^2) = 0.0337\,\text{N}$.

(b) The tension in the balance would be $T = W - F = (9.80\,\text{N}) - (0.0337\,\text{N}) = 9.77\,\text{N}$.

E5-48 (a) $v = 4(0.179\,\text{m/s}^4)(7.18\,\text{s})^3 - 2(2.08\,\text{m/s}^2)(7.18\,\text{s}) = 235\,\text{m/s}$.

(b) $a = 12(0.179\,\text{m/s}^4)(7.18\,\text{s})^2 - 2(2.08\,\text{m/s}^2) = 107\,\text{m/s}^2$.

(c) $F = ma = (2.17\,\text{kg})(107\,\text{m/s}^2) = 232\,\text{N}$.

$\boxed{\textbf{E5-49}}$ The force only has an x component, so we can use Eq. 5-19 to find the velocity.

$$v_x = v_{0x} + \frac{1}{m}\int_0^t F_x\,dt = v_0 + \frac{F_0}{m}\int_0^t (1 - t/T)\;dt$$

Integrating,

$$v_x = v_0 + a_0\left(t - \frac{1}{2T}t^2\right)$$

Now put this expression into Eq. 5-20 to find the position as a function of time

$$x = x_0 + \int_0^t v_x\,dt = \int_0^t \left(v_{0x} + a_0\left(t - \frac{1}{2T}t^2\right)\right)\;dt$$

Integrating,

$$x = v_0T + a_0\left(\frac{1}{2}T^2 - \frac{1}{6T}T^3\right) = v_0T + a_0\frac{T^2}{3}.$$

Now we can put $t = T$ into the expression for v.

$$v_x = v_0 + a_0\left(T - \frac{1}{2T}T^2\right) = v_0 + a_0T/2.$$

P5-1 (a) There are two forces which accelerate block 1: the tension, T, and the parallel component of the weight, $W_{||,1} = m_1 g \sin\theta_1$. Assuming the block accelerates to the right,

$$m_1 a = m_1 g \sin\theta_1 - T.$$

There are two forces which accelerate block 2: the tension, T, and the parallel component of the weight, $W_{||,2} = m_2 g \sin\theta_2$. Assuming the block 1 accelerates to the right, block 2 must also accelerate to the right, and

$$m_2 a = T - m_2 g \sin\theta_2.$$

Add these two equations,

$$(m_1 + m_2)a = m_1 g \sin\theta_1 - m_2 g \sin\theta_2,$$

and then rearrange:

$$a = \frac{m_1 g \sin\theta_1 - m_2 g \sin\theta_2}{m_1 + m_2}.$$

Or, take the two net force equations, divide each side by the mass, and set them equal to each other:

$$g \sin\theta_1 - T/m_1 = T/m_2 - g \sin\theta_2.$$

Rearrange,

$$T\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = g \sin\theta_1 + g \sin\theta_2,$$

and then rearrange again:

$$T = \frac{m_1 m_2 g}{m_1 + m_2}(\sin\theta_1 + \sin\theta_2).$$

(b) The negative sign we get in the answer means that block 1 accelerates up the ramp.

$$a = \frac{(3.70\,\text{kg})\sin(28^\circ) - (4.86\,\text{kg})\sin(42^\circ)}{(3.70\,\text{kg}) + (4.86\,\text{kg})}(9.81\,\text{m/s}^2) = -1.74\,\text{m/s}^2.$$

$$T = \frac{(3.70\,\text{kg})(4.86\,\text{kg})(9.81\,\text{m/s}^2)}{(3.70\,\text{kg}) + (4.86\,\text{kg})}[\sin(28^\circ) + \sin(42^\circ)] = 23.5\,\text{N}.$$

(c) No acceleration happens when $m_2 = (3.70\,\text{kg})\sin(28^\circ)/\sin(42^\circ) = 2.60\,\text{kg}$. If m_2 is more massive than this m_1 will accelerate up the plane; if m_2 is less massive than this m_1 will accelerate down the plane.

P5-2 (a) Since the pulley is massless, $F = 2T$. The largest value of T that will allow block 2 to remain on the floor is $T \leq W_2 = m_2 g$. So $F \leq 2(1.9\,\text{kg})(9.8\,\text{m/s}^2) = 37\,\text{N}$.

(b) $T = F/2 = (110\,\text{N})/2 = 55\,\text{N}$.

(c) The net force on block 1 is $T - W_1 = (55\,\text{N}) - (1.2\,\text{kg})(9.8\,\text{m/s}^2) = 43\,\text{N}$. This will result in an acceleration of $a = (43\,\text{N})/(1.2\,\text{kg}) = 36\,\text{m/s}^2$.

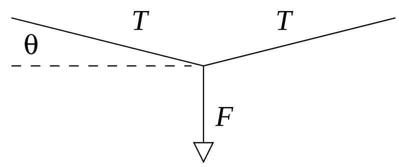
$\boxed{\textbf{P5-3}}$ As the string is pulled the two masses will move together so that the configuration will look like the figure below. The point where the force is applied to the string is massless, so $\sum F = 0$ at that point. We can take advantage of this fact and the figure below to find the tension in the cords, $F/2 = T\cos\theta$. The factor of 1/2 occurs because only 1/2 of F is contained in the right triangle that has T as the hypotenuse. From the figure we can find the x component of the force on one mass to be $T_x = T\sin\theta$. Combining,

$$T_x = \frac{F}{2}\frac{\sin\theta}{\cos\theta} = \frac{F}{2}\tan\theta.$$

But the tangent is equal to

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{x}{\sqrt{L^2 - x^2}}$$

And now we have the answer in the book.



What happens when $x = L$? Well, a_x is infinite according to this expression. Since that could only happen if the tension in the string were infinite, then there must be some other physics that we had previously ignored.

P5-4 (a) The force of static friction can be as large as $f \leq \mu_s N = (0.60)(12 \text{ lb}) = 7.2$ lb. That is more than enough to hold the block up.

(b) The force of static friction is actually only large enough to hold up the block: $f = 5.0$ lb. The magnitude of the force of the wall on the block is then $F_{\text{bw}} = \sqrt{(5.0)^2 + (12.0)^2}\text{lb} = 13$ lb.

P5-5 (a) The weight has two components: normal to the incline, $W_\perp = mg\cos\theta$ and parallel to the incline, $W_{||} = mg\sin\theta$. There is no motion perpendicular to the incline, so $N = W_\perp = mg\cos\theta$. The force of friction on the block is then $f = \mu N = \mu mg\cos\theta$, where we use whichever μ is appropriate. The net force on the block is $F - f - W_{||} = F \pm \mu mg\cos\theta - mg\sin\theta$.

To hold the block in place we use μ_s and friction will point *up* the ramp so the $\pm$ is $+$, and

$$F = (7.96\,\text{kg})(9.81\,\text{m/s}^2)[\sin(22.0°) - (0.25)\cos(22.0°)] = 11.2\,\text{N}.$$

(b) To find the minimum force to begin sliding the block up the ramp we still have static friction, but now friction points *down*, so

$$F = (7.96\,\text{kg})(9.81\,\text{m/s}^2)[\sin(22.0°) + (0.25)\cos(22.0°)] = 47.4\,\text{N}.$$

(c) To keep the block sliding up at constant speed we have kinetic friction, so

$$F = (7.96\,\text{kg})(9.81\,\text{m/s}^2)[\sin(22.0°) + (0.15)\cos(22.0°)] = 40.1\,\text{N}.$$

P5-6 The sand will slide if the cone makes an angle greater than θ where $\mu_s = \tan\theta$. But $\tan\theta = h/R$ or $h = R\tan\theta$. The volume of the cone is then

$$Ah/3 = \pi R^2 h/3 = \pi R^3 \tan\theta/3 = \pi\mu_s R^3/3.$$

$\boxed{\textbf{P5-7}}$ There are four forces on the broom: the force of gravity $W = mg$; the normal force of the floor N; the force of friction f; and the applied force from the person P (the book calls it F). Then

$$\begin{aligned} \sum F_x &= P_x - f = P\sin\theta - f = ma_x, \\ \sum F_y &= N - P_y - W = N - P\cos\theta - mg = ma_y = 0 \end{aligned}$$

Solve the second equation for N,

$$N = P\cos\theta + mg.$$

(a) If the mop slides at constant speed $f = \mu_k N$. Then

$$P \sin\theta - f = P\sin\theta - \mu_k\left(P\cos\theta + mg\right) = 0.$$

We can solve this for P (which was called F in the book);

$$P = \frac{\mu mg}{\sin\theta - \mu_k \cos\theta}.$$

This is the force required to push the broom at constant speed.

(b) Note that P becomes negative (or infinite) if $\sin\theta \le \mu_k \cos\theta$. This occurs when $\tan\theta_c \le \mu_k$. If this happens the mop stops moving, to get it started again you must overcome the static friction, but this is impossible if $\tan\theta_0 \le \mu_s$

P5-8 (a) The condition to slide is $\mu_s \le \tan\theta$. In this case, $(0.63) > \tan(24°) = 0.445$.

(b) The normal force on the slab is $N = W_\perp = mg\cos\theta$. There are *three* forces parallel to the surface: friction, $f = \mu_s N = \mu_s mg\cos\theta$; the parallel component of the weight, $W_{||} = mg\sin\theta$, and the force F. The block will slide if these don't balance, or

$$F > \mu_s mg\cos\theta - mg\sin\theta = (1.8\times10^7\text{kg})(9.8\,\text{m/s}^2)[(0.63)\cos(24°) - \sin(24°)] = 3.0\times10^7\text{N}.$$

$\boxed{\textbf{P5-9}}$ To hold up the smaller block the frictional force between the larger block and smaller block must be as large as the weight of the smaller block. This can be written as $f = mg$. The normal force of the larger block on the smaller block is N, and the frictional force is given by $f \le \mu_s N$. So the smaller block won't fall if $mg \le \mu_s N$.

There is only one horizontal force on the large block, which is the normal force of the small block on the large block. Newton's third law says this force has a magnitude N, so the acceleration of the large block is $N = Ma$.

There is only one horizontal force on the two block system, the force F. So the acceleration of this system is given by $F = (M+m)a$. The two accelerations are equal, otherwise the blocks won't stick together. Equating, then, gives $N/M = F/(M+m)$.

We can combine this last expression with $mg \le \mu_s N$ and get

$$mg \le \mu_s F\frac{M}{M+m}$$

or

$$F \ge \frac{g(M+m)m}{\mu_s M} = \frac{(9.81\,\text{m/s}^2)(88\,\text{kg} + 16\,\text{kg})(16\,\text{kg})}{(0.38)(88\,\text{kg})} = 490\,\text{N}$$

P5-10 The normal force on the ith block is $N_i = m_i g\cos\theta$; the force of friction on the ith block is then $f_i = \mu_i m_i g\cos\theta$. The parallel component of the weight on the ith block is $W_{||,i} = m_i g\sin\theta$.

(a) The net force on the system is

$$F = \sum_i m_i g(\sin\theta - \mu_i\cos\theta).$$

Then

$$\begin{aligned} a &= (9.81\,\text{m/s}^2)\frac{(1.65\,\text{kg})(\sin 29.5° - 0.226\cos 29.5°) + (3.22\,\text{kg})(\sin 29.5° - 0.127\cos 29.5°)}{(1.65\,\text{kg}) + (3.22\,\text{kg})}, \\ &= 3.46\,\text{m/s}^2. \end{aligned}$$

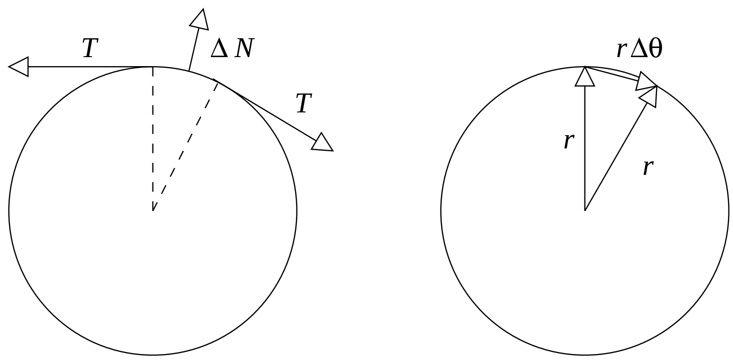
(b) The net force on the lower mass is $m_2 a = W_{||,2} - f_2 - T$, so the tension is

$$T = (9.81\,\text{m/s}^2)(3.22\,\text{kg})(\sin 29.5° - 0.127\cos 29.5°) - (3.22\,\text{kg})(3.46\,\text{m/s}^2) = 0.922\,\text{N}.$$

(c) The acceleration will stay the same, since the system is still the same. Reversing the order of the masses can only result in a reversing of the tension: it is still 0.992 N, but is now negative, meaning compression.

P5-11 The rope wraps around the dowel and there is a contribution to the frictional force Δf from each small segment of the rope where it touches the dowel. There is also a normal force ΔN at each point where the contact occurs. We can find ΔN much the same way that we solve the circular motion problem.

In the figure on the left below we see that we can form a triangle with long side T and short side ΔN. In the figure on the right below we see a triangle with long side r and short side $r\Delta\theta$. These triangles are similar, so $r\Delta\theta/r = \Delta N/T$.



Now $\Delta f = \mu\Delta N$ and $T(\theta) + \Delta f \approx T(\theta + \Delta\theta)$. Combining, and taking the limit as $\Delta\theta \to 0$, $dT = df$

$$\int \frac{1}{\mu}\frac{dT}{T} = \int d\theta$$

integrating both sides of this expression,

$$\begin{aligned}
\int \frac{1}{\mu}\frac{dT}{T} &= \int d\theta, \\
\frac{1}{\mu}\ln T|_{T_1}^{T_2} &= \pi, \\
T_2 &= T_1 e^{\pi\mu}.
\end{aligned}$$

In this case T_1 is the weight and T_2 is the downward force.

P5-12 Answer this out of order!

(b) The maximum static friction between the blocks is 12.0 N; the maximum acceleration of the top block is then $a = F/m = (12.0\,\mathrm{N})/(4.40\,\mathrm{kg}) = 2.73\,\mathrm{m/s^2}$.

(a) The net force on a system of two blocks that will accelerate them at $2.73\,\mathrm{m/s^2}$ is $F = (4.40\,\mathrm{kg} + 5.50\,\mathrm{kg})(2.73\,\mathrm{m/s^2}) = 27.0\,\mathrm{N}$.

(c) The coefficient of friction is $\mu_\mathrm{s} = F/N = F/mg = (12.0\,\mathrm{N})/[(4.40\,\mathrm{kg})(9.81\,\mathrm{m/s^2})] = 0.278$.

P5-13 The speed is $v = 23.6\,\mathrm{m/s}$.

(a) The average speed while stopping is half the initial speed, or $v_\mathrm{av} = 11.8\,\mathrm{m/s}$. The time to stop is $t = (62\,\mathrm{m})/(11.8\,\mathrm{m/s}) = 5.25\,\mathrm{s}$. The rate of deceleration is $a = (23.6\,\mathrm{m/s})/(5.25\,\mathrm{s}) = 4.50\,\mathrm{m/s^2}$. The stopping force is $F = ma$; this is related to the frictional force by $F = \mu_\mathrm{s} mg$, so $\mu_\mathrm{s} = a/g = (4.50\,\mathrm{m/s^2})/(9.81\,\mathrm{m/s^2}) = 0.46$.

(b) Turning,

$$a = v^2/r = (23.6\,\text{m/s})^2/(62\,\text{m}) = 8.98\,\text{m/s}^2.$$

Then $\mu_\text{s} = a/g = (8.98\,\text{m/s}^2)/(9.81\,\text{m/s}^2) = 0.92$.

P5-14 (a) The net force on car as it travels at the top of a circular hill is $F_\text{net} = mv^2/r = W - N$; in this case we are told $N = W/2$, so $F_\text{net} = W/2 = (16000\,\text{N})/2 = 8000\,\text{N}$. When the car travels through the bottom valley the net force at the bottom is $F_\text{net} = mv^2/r = N - W$. Since the magnitude of v, r, and hence F_net is the same in both cases,

$$N = W/2 + W = 3W/2 = 3(16000\,\text{N})/2 = 24000\,\text{N}$$

at the bottom of the valley.

(b) You leave the hill when $N = 0$, or

$$v = \sqrt{rg} = \sqrt{(250\,\text{m})(9.81\,\text{m/s}^2)} = 50\,\text{m/s}.$$

(c) At this speed $F_\text{net} = W$, so at the bottom of the valley $N = 2W = 32000\,\text{N}$.

P5-15 (a) $v = 2\pi r/t = 2\pi(0.052\,\text{m})(3/3.3\,\text{s}) = 0.30\,\text{m/s}$.

(b) $a = v^2/r = (0.30\,\text{m/s})^2/(0.052\,\text{m}) = 1.7\,\text{m/s}^2$, toward center.

(c) $F = ma = (0.0017\,\text{kg})(1.7\,\text{m/s}^2) = 2.9\times10^{-3}\text{N}$.

(d) If the coin can be as far away as r before slipping, then

$$\mu_\text{s} = F/mg = (2\pi r/t)^2/(rg) = 4\pi^2 r/(t^2 g) = 4\pi^2(0.12\,\text{m})/[(3/3.3\,\text{s})^2(9.8\,\text{m/s}^2)] = 0.59.$$

P5-16 (a) Whether you assume constant speed or constant energy, the tension in the string must be the greatest at the bottom of the circle, so that's where the string will break.

(b) The net force on the stone at the bottom is $T - W = mv^2/r$. Then

$$v = \sqrt{rg[T/W-1]} = \sqrt{(2.9\text{ ft})(32\text{ ft/s}^2)[(9.2\text{ lb})/(0.82\text{ lb}) - 1]} = 31\text{ ft/s}.$$

$\boxed{\textbf{P5-17}}$ (a) In order to keep the ball moving in a circle there must be a net centripetal force F_c directed horizontally toward the rod. There are only *three* forces which act on the ball: the force of gravity, $W = mg = (1.34\,\text{kg})(9.81\,\text{m/s}^2) = 13.1\,\text{N}$; the tension in the top string $T_1 = 35.0\,\text{N}$, and the tension in the bottom string, T_2.

The components of the force from the tension in the top string are

$$T_{1,x} = (35.0\,\text{N})\cos 30^\circ = 30.3\,\text{N} \text{ and } T_{1,y} = (35.0\,\text{N})\sin 30^\circ = 17.5\,\text{N}.$$

The vertical components *do* balance, so

$$T_{1,y} + T_{2,y} = W,$$

or $T_{2,y} = (13.1\,\text{N}) - (17.5\,\text{N}) = -4.4\,\text{N}$. From this we can find the tension in the bottom string,

$$T_2 = T_{2,y}/\sin(-30^\circ) = 8.8\,\text{N}.$$

(b) The net force on the object will be the sum of the two horizontal components,

$$F_c = (30.3\,\text{N}) + (8.8\,\text{N})\cos 30^\circ = 37.9\,\text{N}.$$

(c) The speed will be found from

$$\begin{aligned} v &= \sqrt{a_c r} = \sqrt{F_c r/m}, \\ &= \sqrt{(37.9\,\text{m})(1.70\,\text{m})\sin 60^\circ/(1.34\,\text{kg})} = 6.45\,\text{m/s}. \end{aligned}$$

P5-18 The net force on the cube is $F = mv^2/r$. The speed is $2\pi r\omega$. (Note that we are using ω in a non-standard way!) Then $F = 4\pi^2 mr\omega^2$. There are three forces to consider: the normal force of the funnel on the cube N; the force of gravity on the cube W; and the frictional force on the cube f. The acceleration of the cube in circular motion is toward the center of the circle; this means the *net* force on the cube is horizontal, toward the center. We will arrange our coordinate system so that r is horizontal and z is vertical. Then the components of the normal force are $N_r = N\sin\theta$ and $N_z = N\cos\theta$; the components of the frictional force are $f_r = f\cos\theta$ and $f_z = f\sin\theta$.

The direction of the friction depends on the speed of the cube; it will point up if ω is small and down if ω is large.

(a) If ω is small, Newton's second law is

$$\begin{aligned}\sum F_r &= N_r - f_r = N\sin\theta - f\cos\theta = 4\pi^2 mr\omega^2,\\ \sum F_z &= N_z + f_z - W = N\cos\theta + f\sin\theta - mg = 0.\end{aligned}$$

We can substitute $f = \mu_s N$. Solving for N,

$$N\left(\cos\theta + \mu_s\sin\theta\right) = mg.$$

Combining,

$$4\pi^2 r\omega^2 = g\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}.$$

Rearrange,

$$\omega = \frac{1}{2\pi}\sqrt{\frac{g}{r}\,\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}}.$$

This is the minimum value.

(b) Now the frictional force will point the other way, so Newton's second law is now

$$\begin{aligned}\sum F_r &= N_r + f_r = N\sin\theta + f\cos\theta = 4\pi^2 mr\omega^2,\\ \sum F_z &= N_z - f_z - W = N\cos\theta - f\sin\theta - mg = 0.\end{aligned}$$

We've swapped + and - signs, so

$$\omega = \frac{1}{2\pi}\sqrt{\frac{g}{r}\,\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}}$$

is the maximum value.

P5-19 (a) The radial distance from the axis of rotation at a latitude L is $R\cos L$. The speed in the circle is then $v = 2\pi R\cos L/T$. The net force on a hanging object is $F = mv^2/(R\cos L) = 4\pi^2 mR\cos L/T^2$. This net force is *not* directed toward the center of the earth, but is instead directed toward the axis of rotation. It makes an angle L with the Earth's vertical. The tension in the cable must then have two components: one which is vertical (compared to the Earth) and the other which is horizontal. If the cable makes an angle θ with the vertical, then $T_{||} = T\sin\theta$ and $T_\perp = T\cos\theta$. Then $T_{||} = F_{||}$ and $W - T_\perp = F_\perp$. Written with a little more detail,

$$T\sin\theta = 4\pi^2 mR\cos L\,\sin L/T^2 \approx T\theta,$$

and

$$T\cos\theta = 4\pi^2 mR\cos^2 L/T^2 + mg \approx T.$$

But $4\pi^2 R\cos^2 L/T^2 \ll g$, so it can be ignored in the last equation compared to g, and $T \approx mg$. Then from the first equation,

$$\theta = 2\pi^2 R\sin 2L/(gT^2).$$

(b) This is a maximum when $\sin 2L$ is a maximum, which happens when $L = 45°$. Then

$$\theta = 2\pi^2(6.37\times 10^6\mathrm{m})/[(9.8\,\mathrm{m/s^2})(86400\,\mathrm{s})^2] = 1.7\times 10^{-3}\ \mathrm{rad}.$$

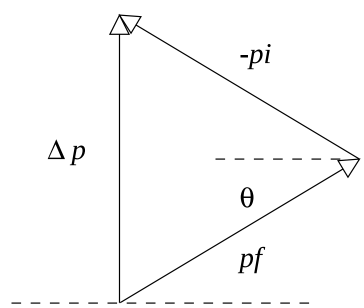
(c) The deflection at both the equator and the poles would be zero.

P5-20 $a = (F_0/m)e^{-t/T}$. Then $v = \int_0^t a\,dt = (F_0T/m)e^{-t/T}$, and $x = \int_0^t v\,dt = (F_0T^2/m)e^{-t/T}$.
(a) When $t = T$ $v = (F_0T/m)e^{-1} = 0.368(F_0T/m)$.
(b) When $t = T$ $x = (F_0T^2/m)e^{-1} = 0.368(F_0T^2/m)$.

E6-1 (a) $v_1 = (m_2/m_1)v_2 = (2650\,\text{kg}/816\,\text{kg})(16.0\text{ km/h}) = 52.0\text{ km/h}$.
(b) $v_1 = (m_2/m_1)v_2 = (9080\,\text{kg}/816\,\text{kg})(16.0\text{ km/h}) = 178\text{ km/h}$.

E6-2 $\vec{\mathbf{p}}_\text{i} = (2000\,\text{kg})(40\text{ km/h})\hat{\mathbf{j}} = 8.00\times10^4\text{kg}\cdot\text{km/h}\hat{\mathbf{j}}$. $\vec{\mathbf{p}}_\text{f} = (2000\,\text{kg})(50\text{ km/h})\hat{\mathbf{i}} = 1.00\times10^5\text{kg}\cdot\text{km/h}\hat{\mathbf{i}}$. $\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_\text{f} - \vec{\mathbf{p}}_\text{i} = 1.00\times10^5\text{kg}\cdot\text{km/h}\hat{\mathbf{i}} - 8.00\times10^4\text{kg}\cdot\text{km/h}\hat{\mathbf{j}}$. $\Delta p = \sqrt{(\Delta p_x)^2 + (\Delta p_y)^2} = 1.28\times10^5\text{kg}\cdot\text{km/h}$. The direction is 38.7° south of east.

$\boxed{\textbf{E6-3}}$ The figure below shows the initial and final momentum vectors arranged to geometrically show $\vec{\mathbf{p}}_\text{f} - \vec{\mathbf{p}}_\text{i} = \Delta\vec{\mathbf{p}}$. We can use the cosine law to find the length of $\Delta\vec{\mathbf{p}}$.



The angle $\alpha = 42° + 42°$, $p_\text{i} = mv = (4.88\,\text{kg})(31.4\,\text{m/s}) = 153\,\text{kg}\cdot\text{m/s}$. Then the magnitude of $\Delta\vec{\mathbf{p}}$ is

$$\Delta p = \sqrt{(153\,\text{kg}\cdot\text{m/s})^2 + (153\,\text{kg}\cdot\text{m/s})^2 - 2(153\,\text{kg}\cdot\text{m/s})^2\cos(84°)} = 205\,\text{kg}\cdot\text{m/s},$$

directed up from the plate. By symmetry it must be perpendicular.

E6-4 The change in momentum is $\Delta p = -mv = -(2300\,\text{kg})(15\,\text{m/s}) = -3.5\times10^4\text{kg}\cdot\text{m/s}$. The average force is $F = \Delta p/\Delta t = (-3.5\times10^4\text{kg}\cdot\text{m/s})/(0.54\,\text{s}) = -6.5\times10^4\text{N}$.

E6-5 (a) The change in momentum is $\Delta p = (-mv) - mv$; the average force is $F = \Delta p/\Delta t = -2mv/\Delta t$.
(b) $F = -2(0.14\,\text{kg})(7.8\,\text{m/s})/(3.9\times10^{-3}\text{s}) = 560\,\text{N}$.

E6-6 (a) $J = \Delta p = (0.046\,\text{kg})(52.2\,\text{m/s}) - 0 = 2.4\,\text{N}\cdot\text{s}$.
(b) The impulse imparted to the club is opposite that imparted to the ball.
(c) $F = \Delta p/\Delta t = (2.4\,\text{N}\cdot\text{s})/(1.20\times10^{-3}\text{s}) = 2000\,\text{N}$.

$\boxed{\textbf{E6-7}}$ Choose the coordinate system so that the ball is only moving along the x axis, with away from the batter as positive. Then $p_{\text{f}x} = mv_{\text{f}x} = (0.150\,\text{kg})(61.5\,\text{m/s}) = 9.23$ kg·m/s and $p_{\text{i}x} = mv_{\text{i}x} = (0.150\,\text{kg})(-41.6\,\text{m/s}) = -6.24$ kg·m/s. The impulse is given by $J_x = p_{\text{f}x} - p_{\text{i}x} = 15.47$ kg·m/s. We can find the average force by application of Eq. 6-7:

$$F_{\text{av},x} = \frac{J_x}{\Delta t} = \frac{(15.47\text{ kg}\cdot\text{m/s})}{(4.7\times10^{-3}\text{ s})} = 3290\,\text{N}.$$

E6-8 The magnitude of the impulse is $J = F\delta t = (-984\,\text{N})(0.0270\,\text{s}) = -26.6\,\text{N}\cdot\text{s}$. Then $p_\text{f} = p_\text{i} + \Delta p$, so

$$v_\text{f} = \frac{(0.420\,\text{kg})(13.8\,\text{m/s}) + (-26.6\,\text{N}\cdot\text{s})}{(0.420\,\text{kg})} = -49.5\,\text{m/s}.$$

The ball moves backward!

E6-9 The change in momentum of the ball is $\Delta p = (mv) - (-mv) = 2mv = 2(0.058\,\text{kg})(32\,\text{m/s}) = 3.7\,\text{kg}\cdot\text{m/s}$. The impulse is the area under a force - time graph; for the trapezoid in the figure this area is $J = F_\text{max}(2\,\text{ms} + 6\,\text{ms})/2 = (4\,\text{ms})F_\text{max}$. Then $F_\text{max} = (3.7\,\text{kg}\cdot\text{m/s})/(4\,\text{ms}) = 930\,\text{N}$.

E6-10 The final speed of each object is given by $v_i = J/m_i$, where i refers to which object (as opposed to "initial"). The object are going in different directions, so the relative speed will be the sum. Then

$$v_\text{rel} = v_1 + v_2 = (300\,\text{N}\cdot\text{s})[1/(1200\,kg) + 1/(1800\,\text{kg})] = 0.42\,\text{m/s}.$$

$\boxed{\textbf{E6-11}}$ Use Simpson's rule. Then the area is given by

$$\begin{aligned} J_x &= \frac{1}{3}h\left(f_0 + 4f_1 + 2f_2 + 4f_3 + ... + 4f_13 + f_14\right), \\ &= \frac{1}{3}(0.2\,\text{ms})\left(200 + 4\cdot 800 + 2\cdot 1200 ...\,\text{N}\right) \end{aligned}$$

which gives $J_x = 4.28$ kg·m/s.

Since the impulse is the change in momentum, and the ball started from rest, $p_{\text{f}x} = J_x + p_{\text{i}x} = 4.28$ kg·m/s. The final velocity is then found from $v_x = p_x/m = 8.6$ m/s.

E6-12 (a) The average speed during the the time the hand is in contact with the board is half of the initial speed, or $v_\text{av} = 4.8\,\text{m/s}$. The time of contact is then $t = y/v_\text{av} = (0.028\,\text{m})/(4.8\,\text{m/s}) = 5.8\,\text{ms}$.

(b) The impulse given to the board is the same as the magnitude in the change in momentum of the hand, or $J = (0.54\,\text{kg})(9.5\,\text{m/s}) = 5.1\,\text{N}\cdot\text{s}$. Then $F_\text{av} = (5.1\,\text{N}\cdot\text{s})/(5.8\,\text{ms}) = 880\,\text{N}$.

E6-13 $\Delta p = J = F\Delta t = (3000\,\text{N})(65.0\,\text{s}) = 1.95\times 10^5\text{N}\cdot\text{s}$. The direction of the thrust relative to the velocity doesn't matter in this exercise.

E6-14 (a) $p = mv = (2.14\times 10^{-3}\text{kg})(483\,\text{m/s}) = 1.03\,\text{kg}\cdot\text{m/s}$.

(b) The impulse imparted to the wall in one second is ten times the above momentum, or $J = 10.3\,\text{kg}\cdot\text{m/s}$. The average force is then $F_\text{av} = (10.3\,\text{kg}\cdot\text{m/s})/(1.0\,\text{s}) = 10.3\,\text{N}$.

(c) The average force for each individual particle is $F_\text{av} = (1.03\,\text{kg}\cdot\text{m/s})/(1.25\times 10^{-3}\,\text{s}) = 830\,\text{N}$.

$\boxed{\textbf{E6-15}}$ A transverse direction means at right angles, so the thrusters have imparted a momentum sufficient to direct the spacecraft 100+3400 = 3500 km to the side of the original path. The spacecraft is half-way through the six-month journey, so it has three months to move the 3500 km to the side. This corresponds to a transverse speed of $v = (3500\times 10^3\text{m})/(90\times 86400\,\text{s}) = 0.45\,\text{m/s}$. The required time for the rocket to fire is $\Delta t = (5400\,\text{kg})(0.45\,\text{m/s})/(1200\,\text{N}) = 2.0\,\text{s}$.

E6-16 Total initial momentum is zero, so

$$v_\text{m} = -\frac{m_\text{s}}{m_\text{m}}v_\text{s} = -\frac{m_\text{s}g}{m_\text{m}g}v_\text{s} = -\frac{(0.158\text{ lb})}{(195\text{ lb})}(12.7\text{ ft/s}) = -1.0\times 10^{-2}\text{ ft/s}.$$

$\boxed{\textbf{E6-17}}$ Conservation of momentum:

$$\begin{aligned}
p_{\mathrm{f,m}} + p_{\mathrm{f,c}} &= p_{\mathrm{i,m}} + p_{\mathrm{i,c}},\\
m_{\mathrm{m}} v_{\mathrm{f,m}} + m_{\mathrm{c}} v_{\mathrm{f,c}} &= m_{\mathrm{m}} v_{\mathrm{i,m}} + m_{\mathrm{c}} v_{\mathrm{i,c}},\\
v_{\mathrm{f,c}} - v_{\mathrm{i,c}} &= \frac{m_{\mathrm{m}} v_{\mathrm{i,m}} - m_{\mathrm{m}} v_{\mathrm{f,m}}}{m_{\mathrm{c}}},\\
\Delta v_{\mathrm{c}} &= \frac{(75.2\ \mathrm{kg})(2.33\ \mathrm{m/s}) - (75.2\ \mathrm{kg})(0)}{(38.6\ \mathrm{kg})},\\
&= 4.54\,\mathrm{m/s}.
\end{aligned}$$

The answer is positive; the cart speed *increases*.

E6-18 Conservation of momentum:

$$\begin{aligned}
p_{\mathrm{f,m}} + p_{\mathrm{f,c}} &= p_{\mathrm{i,m}} + p_{\mathrm{i,c}},\\
m_{\mathrm{m}}(v_{\mathrm{f,c}} - v_{\mathrm{rel}}) + m_{\mathrm{c}} v_{\mathrm{f,c}} &= (m_{\mathrm{m}} + m_{\mathrm{c}}) v_{\mathrm{i,c}},\\
(m_{\mathrm{m}} + m_{\mathrm{c}}) v_{\mathrm{f,c}} - m_{\mathrm{m}} v_{\mathrm{rel}} &= (m_{\mathrm{m}} + m_{\mathrm{c}}) v_{\mathrm{i,c}},\\
\Delta v_{\mathrm{c}} &= m_{\mathrm{m}} v_{\mathrm{rel}}/(m_{\mathrm{m}} + m_{\mathrm{c}}),\\
&= w v_{\mathrm{rel}}/(w + W).
\end{aligned}$$

E6-19 Conservation of momentum. Let m refer to motor and c refer to command module:

$$\begin{aligned}
p_{\mathrm{f,m}} + p_{\mathrm{f,c}} &= p_{\mathrm{i,m}} + p_{\mathrm{i,c}},\\
m_{\mathrm{m}}(v_{\mathrm{f,c}} - v_{\mathrm{rel}}) + m_{\mathrm{c}} v_{\mathrm{f,c}} &= (m_{\mathrm{m}} + m_{\mathrm{c}}) v_{\mathrm{i,c}},\\
(m_{\mathrm{m}} + m_{\mathrm{c}}) v_{\mathrm{f,c}} - m_{\mathrm{m}} v_{\mathrm{rel}} &= (m_{\mathrm{m}} + m_{\mathrm{c}}) v_{\mathrm{i,c}},\\
v_{\mathrm{f,c}} &= \frac{m_{\mathrm{m}} v_{\mathrm{rel}} + (m_{\mathrm{m}} + m_{\mathrm{c}}) v_{\mathrm{i,c}}}{(m_{\mathrm{m}} + m_{\mathrm{c}})},\\
&= \frac{4m_{\mathrm{c}}(125\ \mathrm{km/h}) + (4m_{\mathrm{c}} + m_{\mathrm{c}})(3860\ \mathrm{km/h})}{(4m_{\mathrm{c}} + m_{\mathrm{c}})} = 3960\ \mathrm{km/h}.
\end{aligned}$$

E6-20 Conservation of momentum. The block on the left is 1, the other is 2.

$$\begin{aligned}
m_1 v_{1,\mathrm{f}} + m_2 v_{2,\mathrm{f}} &= m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}},\\
v_{1,\mathrm{f}} &= v_{1,\mathrm{i}} + \frac{m_2}{m_1}(v_{2,\mathrm{i}} - v_{2,\mathrm{f}}),\\
&= (5.5\,\mathrm{m/s}) + \frac{(2.4\,\mathrm{kg})}{(1.6\,\mathrm{kg})}[(2.5\,\mathrm{m/s}) - (4.9\,\mathrm{m/s})],\\
&= 1.9\,\mathrm{m/s}.
\end{aligned}$$

E6-21 Conservation of momentum. The block on the left is 1, the other is 2.

$$\begin{aligned}
m_1 v_{1,\mathrm{f}} + m_2 v_{2,\mathrm{f}} &= m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}},\\
v_{1,\mathrm{f}} &= v_{1,\mathrm{i}} + \frac{m_2}{m_1}(v_{2,\mathrm{i}} - v_{2,\mathrm{f}}),\\
&= (5.5\,\mathrm{m/s}) + \frac{(2.4\,\mathrm{kg})}{(1.6\,\mathrm{kg})}[(-2.5\,\mathrm{m/s}) - (4.9\,\mathrm{m/s})],\\
&= -5.6\,\mathrm{m/s}.
\end{aligned}$$

E6-22 Assume a completely inelastic collision. Call the Earth 1 and the meteorite 2. Then

$$\begin{aligned} m_1 v_{1,\mathrm{f}} + m_2 v_{2,\mathrm{f}} &= m_1 v_{1,\mathrm{i}} + m_2 v_{2,\mathrm{i}}, \\ v_{1,\mathrm{f}} &= \frac{m_2 v_{2,\mathrm{i}}}{m_1 + m_2}, \\ &= \frac{(5\times 10^{10}\mathrm{kg})(7200\,\mathrm{m/s})}{(5.98\times 10^{24}\mathrm{kg}) + (5\times 10^{10}\mathrm{kg})} = 7\times 10^{-11}\mathrm{m/s}. \end{aligned}$$

That's 2 mm/y!

$\boxed{\textbf{E6-23}}$ Conservation of momentum is used to solve the problem:

$$\begin{aligned} P_\mathrm{f} &= P_\mathrm{i}, \\ p_{\mathrm{f,bl}} + p_{\mathrm{f,bu}} &= p_{\mathrm{i,bl}} + p_{\mathrm{i,bu}}, \\ m_\mathrm{bl} v_{\mathrm{f,bl}} + m_\mathrm{bu} v_{\mathrm{f,bu}} &= m_\mathrm{bl} v_{\mathrm{i,bl}} + m_\mathrm{bu} v_{\mathrm{i,bu}}, \\ (715\text{ g}) v_{\mathrm{f,bl}} + (5.18\text{ g})(428\text{ m/s}) &= (715\text{ g})(0) + (5.18\text{ g})(672\text{ m/s}), \end{aligned}$$

which has solution $v_{\mathrm{f,bl}} = 1.77\,\mathrm{m/s}$.

E6-24 The y component of the initial momentum is zero; therefore the magnitudes of the y components of the two particles must be equal after the collision. Then

$$\begin{aligned} m_\alpha v_\alpha \sin\theta_\alpha &= m_\mathrm{O} v_\mathrm{O} \sin\theta_\mathrm{O}, \\ v_\alpha &= \frac{m_\mathrm{O} v_\mathrm{O} \sin\theta_\mathrm{O}}{m_\alpha v_\alpha \sin\theta_\alpha}, \\ &= \frac{(16\text{ u})(1.20\times 10^5\mathrm{m/s})\sin(51^\circ)}{(4.00\text{ u})\sin(64^\circ)} = 4.15\times 10^5\mathrm{m/s}. \end{aligned}$$

E6-25 The total momentum is

$$\begin{aligned} \vec{\mathbf{p}} &= (2.0\,\mathrm{kg})[(15\,\mathrm{m/s})\hat{\mathbf{i}} + (30\,\mathrm{m/s})\hat{\mathbf{j}}] + (3.0\,\mathrm{kg})[(-10\,\mathrm{m/s})\hat{\mathbf{i}} + (5\,\mathrm{m/s})\hat{\mathbf{j}}], \\ &= 75\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}. \end{aligned}$$

The final velocity of B is

$$\begin{aligned} \vec{\mathbf{v}}_{B\mathrm{f}} &= \frac{1}{m_B}(\vec{\mathbf{p}} - m_A \vec{\mathbf{v}}_{A\mathrm{f}}), \\ &= \frac{1}{(3.0\,\mathrm{kg})}\{(75\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}) - (2.0\,\mathrm{kg})[(-6.0\,\mathrm{m/s})\hat{\mathbf{i}} + (30\,\mathrm{m/s})\hat{\mathbf{j}}]\}, \\ &= (4.0\,\mathrm{m/s})\hat{\mathbf{i}} + (5.0\,\mathrm{m/s})\hat{\mathbf{j}}. \end{aligned}$$

E6-26 Assume electron travels in $+x$ direction while neutrino travels in $+y$ direction. Conservation of momentum requires that

$$\vec{\mathbf{p}} = -(1.2\times 10^{-22}\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{i}} - (6.4\times 10^{-23}\mathrm{kg}\cdot\mathrm{m/s})\hat{\mathbf{j}}$$

be the momentum of the nucleus after the decay. This has a magnitude of $p = 1.4\times 10^{-22}\mathrm{kg}\cdot\mathrm{m/s}$ and be directed 152° from the electron.

E6-27 What we know:

$$\begin{aligned}
\vec{\mathbf{p}}_{1,\mathrm{i}} &= (1.50\times10^5\mathrm{kg})(6.20\,\mathrm{m/s})\hat{\mathbf{i}} = 9.30\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}},\\
\vec{\mathbf{p}}_{2,\mathrm{i}} &= (2.78\times10^5\mathrm{kg})(4.30\,\mathrm{m/s})\hat{\mathbf{j}} = 1.20\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}},\\
\vec{\mathbf{p}}_{2,\mathrm{f}} &= (2.78\times10^5\mathrm{kg})(5.10\,\mathrm{m/s})[\sin(18^\circ)\hat{\mathbf{i}} + \cos(18^\circ)\hat{\mathbf{j}}],\\
&= 4.38\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}} + 1.35\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}.
\end{aligned}$$

Conservation of momentum then requires

$$\begin{aligned}
\vec{\mathbf{p}}_{1,\mathrm{f}} &= (9.30\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}) - (4.38\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}})\\
&\quad +(1.20\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}) - (1.35\times10^6\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}),\\
&= 4.92\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}} - 1.50\times10^5\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}.
\end{aligned}$$

This corresponds to a velocity of

$$\vec{\mathbf{v}}_{1,\mathrm{f}} = 3.28\,\mathrm{m/s}\,\hat{\mathbf{i}} - 1.00\,\mathrm{m/s}\,\hat{\mathbf{j}},$$

which has a magnitude of 3.43 m/s directed 17° to the right.

E6-28 $v_\mathrm{f} = -2.1\,\mathrm{m/s}$.

$\boxed{\textbf{E6-29}}$ We want to solve Eq. 6-24 for m_2 given that $v_{1,\mathrm{f}} = 0$ and $v_{1,\mathrm{i}} = -v_{2,\mathrm{i}}$. Making these substitutions

$$\begin{aligned}
(0) &= \frac{m_1 - m_2}{m_1 + m_2}v_{1,\mathrm{i}} + \frac{2m_2}{m_1 + m_2}(-v_{1,\mathrm{i}}),\\
0 &= (m_1 - m_2)v_{1,\mathrm{i}} - (2m_2)v_{1,\mathrm{i}},\\
3m_2 &= m_1
\end{aligned}$$

so $m_2 = 100$ g.

E6-30 (a) Rearrange Eq. 6-27:

$$m_2 = m_1\frac{v_{1\mathrm{i}} - v_{1\mathrm{f}}}{v_{1\mathrm{i}} + v_{1\mathrm{f}}} = (0.342\,\mathrm{kg})\frac{(1.24\,\mathrm{m/s}) - (0.636\,\mathrm{m/s})}{(1.24\,\mathrm{m/s}) + (0.636\,\mathrm{m/s})} = 0.110\,\mathrm{kg}.$$

(b) $v_{2\mathrm{f}} = 2(0.342\,\mathrm{kg})(1.24\,\mathrm{m/s})/(0.342\,\mathrm{kg} + 0.110\,\mathrm{kg}) = 1.88\,\mathrm{m/s}$.

E6-31 Rearrange Eq. 6-27:

$$m_2 = m_1\frac{v_{1\mathrm{i}} - v_{1\mathrm{f}}}{v_{1\mathrm{i}} + v_{1\mathrm{f}}} = (2.0\,\mathrm{kg})\frac{v_{1\mathrm{i}} - v_{1\mathrm{i}}/4}{v_{1\mathrm{i}} + v_{1\mathrm{i}}/4} = 1.2\,\mathrm{kg}.$$

E6-32 I'll multiply all momentum equations by g, then I can use weight directly without converting to mass.

(a) $v_\mathrm{f} = [(31.8\ \mathrm{T})(5.20\ \mathrm{ft/s}) + (24.2\ \mathrm{T})(2.90\ \mathrm{ft/s})]/(31.8\ \mathrm{T} + 24.2\ \mathrm{T}) = 4.21\ \mathrm{ft/s}$.

(b) Evaluate:

$$v_{1\mathrm{f}} = \frac{31.8\ \mathrm{T} - 24.2\ \mathrm{T}}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(5.20\ \mathrm{ft/s}) + \frac{2(24.2\ \mathrm{T})}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(2.90\ \mathrm{ft/s}) = 3.21\ \mathrm{ft/s}.$$

$$v_{2\mathrm{f}} = -\frac{31.8\ \mathrm{T} - 24.2\ \mathrm{T}}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(2.90\ \mathrm{ft/s}) + \frac{2(31.8\ \mathrm{T})}{31.8\ \mathrm{T} + 24.2\ \mathrm{T}}(5.20\ \mathrm{ft/s}) = 5.51\ \mathrm{ft/s}.$$

$\boxed{\textbf{E6-33}}$ Let the initial momentum of the first object be $\vec{\mathbf{p}}_{1,\mathrm{i}} = m\vec{\mathbf{v}}_{1,\mathrm{i}}$, that of the second object be $\vec{\mathbf{p}}_{2,\mathrm{i}} = m\vec{\mathbf{v}}_{2,\mathrm{i}}$, and that of the combined final object be $\vec{\mathbf{p}}_{\mathrm{f}} = 2m\vec{\mathbf{v}}_{\mathrm{f}}$. Then

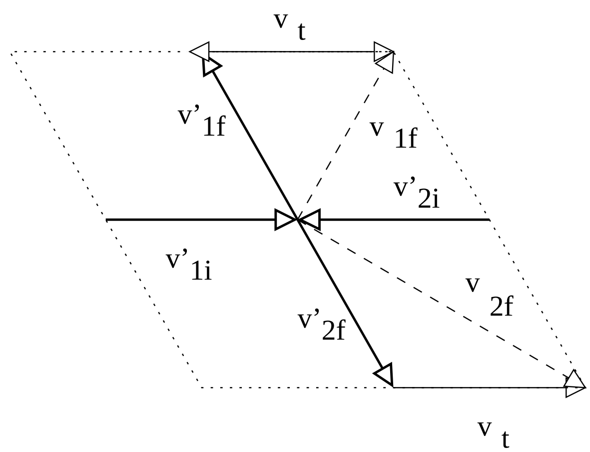
$$\vec{\mathbf{p}}_{1,\mathrm{i}} + \vec{\mathbf{p}}_{2,\mathrm{i}} = \vec{\mathbf{p}}_{\mathrm{f}},$$

implies that we can find a triangle with sides of length $p_{1,\mathrm{i}}$, $p_{2,\mathrm{i}}$, and p_{f}. These lengths are

$$\begin{aligned} p_{1,\mathrm{i}} &= mv_{\mathrm{i}}, \\ p_{2,\mathrm{i}} &= mv_{\mathrm{i}}, \\ p_{\mathrm{f}} &= 2mv_{\mathrm{f}} = 2mv_{\mathrm{i}}/2 = mv_{\mathrm{i}}, \end{aligned}$$

so this is an equilateral triangle. This means the angle between the initial velocities is 120°.

E6-34 We need to change to the center of mass system. Since both particles have the same mass, the conservation of momentum problem is effectively the same as a (vector) conservation of velocity problem. Since one of the particles is originally at rest, the center of mass moves with speed $v_{\mathrm{cm}} = v_{1\mathrm{i}}/2$. In the figure below the center of mass velocities are primed; the transformation velocity is v_t.



Note that since $v_t = v'_{1\mathrm{i}} = v'_{2\mathrm{i}} = v'_{1\mathrm{f}} = v'_{2\mathrm{f}}$ the entire problem can be inscribed in a rhombus. The diagonals of the rhombus are the directions of $v_{1\mathrm{f}}$ and $v_{2\mathrm{f}}$; note that the diagonals of a rhombus are *necessarily* at right angles!

(a) The target proton moves off at 90° to the direction the incident proton moves after the collision, or 26° away from the incident protons original direction.

(b) The y components of the final momenta must be equal, so $v_{2\mathrm{f}} \sin(26°) = v_{1\mathrm{f}} \sin(64°)$, or $v_{2\mathrm{f}} = v_{1\mathrm{f}} \tan(64°)$. The x components must add to the original momentum, so $(514\,\mathrm{m/s}) = v_{2\mathrm{f}} \cos(26°) + v_{1\mathrm{f}} \cos(64°)$, or

$$v_{1\mathrm{f}} = (514\,\mathrm{m/s})/\{\tan(64°)\cos(26°) + \cos(64°)\} = 225\,\mathrm{m/s},$$

and

$$v_{2\mathrm{f}} = (225\,\mathrm{m/s})\tan(64°) = 461\,\mathrm{m/s}.$$

E6-35 $v_{\mathrm{cm}} = \{(3.16\,\mathrm{kg})(15.6\,\mathrm{m/s}) + (2.84\,\mathrm{kg})(-12.2\,\mathrm{m/s})\}/\{(3.16\,\mathrm{kg}) + (2.84\,\mathrm{kg})\} = 2.44\,\mathrm{m/s}$, positive means to the left.

$\boxed{\textbf{P6-1}}$ The force is the change in momentum over change in time; the momentum is the mass time velocity, so

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \Delta v \frac{m}{\Delta t} = 2u\mu,$$

since μ is the mass per unit time.

P6-2 (a) The initial momentum is $\vec{\mathbf{p}}_\mathrm{i} = (1420\,\mathrm{kg})(5.28\,\mathrm{m/s})\hat{\mathbf{j}} = 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}$. After making the right hand turn the final momentum is $\vec{\mathbf{p}}_\mathrm{f} = 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}$. The impulse is $\vec{\mathbf{J}} = 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}} - 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}$, which has magnitude $J = 10600\,\mathrm{kg}\cdot\mathrm{m/s}$.

(b) During the collision the impulse is $\vec{\mathbf{J}} = 0 - 7500\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{i}}$. The magnitude is $J = 7500\,\mathrm{kg}\cdot\mathrm{m/s}$.

(c) The average force is $F = J/t = (10600\,\mathrm{kg}\cdot\mathrm{m/s})/(4.60\,\mathrm{s}) = 2300\,\mathrm{N}$.

(d) The average force is $F = J/t = (7500\,\mathrm{kg}\cdot\mathrm{m/s})/(0.350\,\mathrm{s}) = 21400\,\mathrm{N}$.

P6-3 (a) Only the component of the momentum which is perpendicular to the wall changes. Then

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}} = -2(0.325\,\mathrm{kg})(6.22\,\mathrm{m/s})\sin(33^\circ)\hat{\mathbf{j}} = -2.20\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}}.$$

(b) $\vec{\mathbf{F}} = -\vec{\mathbf{J}}/t = -(-2.20\,\mathrm{kg}\cdot\mathrm{m/s}\,\hat{\mathbf{j}})/(0.0104\,\mathrm{s}) = 212\,\mathrm{N}$.

P6-4 The change in momentum of one bullet is $\Delta p = 2mv = 2(0.0030\,\mathrm{kg})(500\,\mathrm{m/s}) = 3.0\,\mathrm{kg}\cdot\mathrm{m/s}$. The average force is the total impulse in one minute divided by one minute, or

$$F_\mathrm{av} = 100(3.0\,\mathrm{kg}\cdot\mathrm{m/s})/(60\,\mathrm{s}) = 5.0\,\mathrm{N}.$$

P6-5 (a) The volume of a hailstone is $V = 4\pi r^3/3 = 4\pi(0.5\ \mathrm{cm})^3/3 = 0.524\ \mathrm{cm}^3$. The mass of a hailstone is $m = \rho V = (9.2\times10^{-4}\mathrm{kg/cm}^3)(0.524\ \mathrm{cm}^3) = 4.8\times10^{-4}\mathrm{kg}$.

(b) The change in momentum of one hailstone when it hits the ground is

$$\Delta p = (4.8\times10^{-4}\mathrm{kg})(25\,\mathrm{m/s}) = 1.2\times10^{-2}\mathrm{kg}\cdot\mathrm{m/s}.$$

The hailstones fall at 25 m/s, which means that in one second the hailstones in a column 25 m high hit the ground. Over an area of $10\,\mathrm{m}\times20\,\mathrm{m}$ then there would be $(25\,\mathrm{m})(10\,\mathrm{m})(20\,\mathrm{m}) = 500\,\mathrm{m}^3$ worth of hailstones, or 6.00×10^5 hailstones per second striking the surface. Then

$$F_\mathrm{av} = 6.00\times10^5(1.2\times10^{-2}\,\mathrm{kg}\cdot\mathrm{m/s})/(1\,\mathrm{s}) = 7200\,\mathrm{N}.$$

P6-6 Assume the links are *not* connected once the top link is released. Consider the link that starts h above the table; it falls a distance h in a time $t = \sqrt{2h/g}$ and hits the table with a speed $v = gt = \sqrt{2hg}$. When the link hits the table h of the chain is already on the table, and $L - h$ is yet to come. The linear mass density of the chain is M/L, so when this link strikes the table the mass is hitting the table at a rate $dm/dt = (M/L)v = (M/L)\sqrt{2hg}$. The average force required to stop the falling link is then $v\,dm/dt = (M/L)2hg = 2(M/L)hg$. But the weight of the chain that is already on the table is $(M/L)hg$, so the net force on the table is the sum of these two terms, or $F = 3W$.

$\boxed{\textbf{P6-7}}$ The weight of the marbles in the box after a time t is $mgRt$ because Rt is the number of marbles in the box.

The marbles fall a distance h from rest; the time required to fall this distance is $t = \sqrt{2h/g}$, the speed of the marbles when they strike the box is $v = gt = \sqrt{2gh}$. The momentum each marble imparts on the box is then $m\sqrt{2gh}$. If the marbles strike at a rate R then the force required to stop them is $Rm\sqrt{2gh}$.

The reading on the scale is then

$$W = mR(\sqrt{2gh} + gt).$$

This will give a numerical result of

$$(4.60\times10^{-3}\mathrm{kg})(115\,\mathrm{s}^{-1})\left(\sqrt{2(9.81\,\mathrm{m/s^2})(9.62\,\mathrm{m})} + (9.81\,\mathrm{m/s^2})(6.50\,\mathrm{s})\right) = 41.0\,\mathrm{N}.$$

P6-8 (a) $v = (108\,\mathrm{kg})(9.74\,\mathrm{m/s})/(108\,\mathrm{kg} + 1930\,\mathrm{kg}) = 0.516\,\mathrm{m/s}$.

(b) Label the person as object 1 and the car as object 2. Then $m_1v_1 + m_2v_2 = (108\,\mathrm{kg})(9.74\,\mathrm{m/s})$ and $v_1 = v_2 + 0.520\,\mathrm{m/s}$. Combining,

$$v_2 = [1050\,\mathrm{kg}\cdot\mathrm{m/s} - (0.520\,\mathrm{m/s})(108\,\mathrm{kg})]/(108\,\mathrm{kg} + 1930\,\mathrm{kg}) = 0.488\,\mathrm{m/s}.$$

P6-9 (a) It takes a time $t_1 = \sqrt{2h/g}$ to fall $h = 6.5$ ft. An object will be moving at a speed $v_1 = gt_1 = \sqrt{2hg}$ after falling this distance. If there is an inelastic collision with the pile then the two will move together with a speed of $v_2 = Mv_1/(M+m)$ after the collision.

If the pile then stops within $d = 1.5$ inches, then the time of stopping is given by $t_2 = d/(v_2/2) = 2d/v_2$.

For inelastic collisions this corresponds to an average force of

$$F_{\mathrm{av}} = \frac{(M+m)v_2}{t_2} = \frac{(M+m)v_2^2}{2d} = \frac{M^2v_1^2}{2(M+m)d} = \frac{(gM)^2}{g(M+m)}\frac{h}{d}.$$

Note that we multiply through by g to get weights. The numerical result is $F_{\mathrm{av}} = 130$ t.

(b) For an elastic collision $v_2 = 2Mv_1/(M+m)$; the time of stopping is still expressed by $t_2 = 2d/v_2$, but we now know F_{av} instead of d. Then

$$F_{\mathrm{av}} = \frac{mv_2}{t_2} = \frac{mv_2^2}{2d} = \frac{4Mmv_1^2}{(M+m)d} = \frac{2(gM)(gm)}{g(M+m)}\frac{h}{d}.$$

or

$$d = \frac{2(gM)(gm)}{g(M+m)}\frac{h}{F_{\mathrm{av}}},$$

which has a numerical result of $d = 0.51$ inches.

But wait! The weight, which just had an elastic collision, "bounced" off of the pile, and then hit it again. This drives the pile deeper into the earth. The weight hits the pile a second time with a speed of $v_3 = (M-m)/(M+m)v_1$; the pile will (in this second elastic collision) then have a speed of $v_4 = 2M(M+m)v_3 = [(M-m)/(M+m)]v_2$. In other words, we have an infinite series of distances traveled by the pile, and if $\alpha = [(M-m)/(M+m)] = 0.71$, the depth driven by the pile is

$$d_{\mathrm{f}} = d(1 + \alpha^2 + \alpha^4 + \alpha^6 \cdots) = \frac{d}{1-\alpha^2},$$

or $d = 1.03$.

P6-10 The cat jumps off of sled A; conservation of momentum requires that $Mv_{A,1} + m(v_{A,1} + v_c) = 0$, or

$$v_{A,1} = -mv_c/(m+M) = -(3.63\,\mathrm{kg})(3.05\,\mathrm{m/s})/(22.7\,\mathrm{kg} + 3.63\,\mathrm{kg}) = -0.420\,\mathrm{m/s}.$$

The cat lands on sled B; conservation of momentum requires $v_{B,1} = m(v_{A,1} + v_c)/(m+M)$. The cat jumps off of sled B; conservation of momentum is now

$$Mv_{B,2} + m(v_{B,2} - v_c) = m(v_{A,1} + v_c),$$

or

$$v_{B,2} = 2mv_c/(m+M) = (3.63\,\text{kg})[(-0.420\,\text{m/s}) + 2(3.05\,\text{m/s})]/(22.7\,\text{kg} + 3.63\,\text{kg}) = 0.783\,\text{m/s}.$$

The cat then lands on cart A; conservation of momentum requires that $(M+m)v_{A,2} = -Mv_{B,2}$, or

$$v_{A,2} = -(22.7\,\text{kg})(0.783\,\text{m/s})/(22.7\,\text{kg} + 3.63\,\text{kg}) = -0.675\,\text{m/s}.$$

$\boxed{\textbf{P6-11}}$ We align the coordinate system so that west is $+x$ and south is $+y$. The each car contributes the following to the initial momentum

$$\begin{aligned} A &: (2720\text{ lb}/g)(38.5\text{ mi/h})\hat{\mathbf{i}} = 1.05\times10^5\text{ lb}\cdot\text{mi/h}/g\,\hat{\mathbf{i}}, \\ B &: (3640\text{ lb}/g)(58.0\text{ mi/h})\hat{\mathbf{j}} = 2.11\times10^5\text{ lb}\cdot\text{mi/h}/g\,\hat{\mathbf{j}}. \end{aligned}$$

These become the components of the final momentum. The direction is then

$$\theta = \arctan\frac{2.11\times10^5\text{ lb}\cdot\text{mi/h}/g}{1.05\times10^5\text{ lb}\cdot\text{mi/h}/g} = 63.5^\circ,$$

south of west. The magnitude is the square root of the sum of the squares,

$$2.36\times10^5\text{ lb}\cdot\text{mi/h}/g,$$

and we divide this by the mass (6360 lb/g) to get the final speed after the collision: 37.1 mi/h.

P6-12 (a) Ball A must carry off a momentum of $\vec{\mathbf{p}} = m_B v\,\hat{\mathbf{i}} - m_B v/2\,\hat{\mathbf{j}}$, which would be in a direction $\theta = \arctan(-0.5/1) = 27^\circ$ from the original direction of B, or 117° from the final direction.

(b) No.

P6-13 (a) We assume all balls have a mass m. The collision imparts a "sideways" momentum to the cue ball of $m(3.50\,\text{m/s})\sin(65^\circ) = m(3.17\,\text{m/s})$. The other ball must have an equal, but opposite "sideways" momentum, so $-m(3.17\,\text{m/s}) = m(6.75\,\text{m/s})\sin\theta$, or $\theta = -28.0^\circ$.

(b) The final "forward" momentum is

$$m(3.50\,\text{m/s})\cos(65^\circ) + m(6.75\,\text{m/s})\cos(-28^\circ) = m(7.44\,\text{m/s}),$$

so the initial speed of the cue ball would have been 7.44 m/s.

P6-14 Assuming $M \gg m$, Eq. 6-25 becomes

$$v_{2\text{f}} = 2v_{1\text{i}} - v_{1\text{i}} = 2(13\text{ km/s}) - (-12\text{ km/s}) = 38\text{ km/s}.$$

$\boxed{\textbf{P6-15}}$ (a) We get

$$v_{2,\text{f}} = \frac{2(220\text{ g})}{(220\text{ g}) + (46.0\text{ g})}(45.0\,\text{m/s}) = 74.4\,\text{m/s}.$$

(b) Doubling the mass of the clubhead we get

$$v_{2,\text{f}} = \frac{2(440\text{ g})}{(440\text{ g}) + (46.0\text{ g})}(45.0\,\text{m/s}) = 81.5\,\text{m/s}.$$

(c) Tripling the mass of the clubhead we get

$$v_{2,\text{f}} = \frac{2(660\text{ g})}{(660\text{ g}) + (46.0\text{ g})}(45.0\,\text{m/s}) = 84.1\,\text{m/s}.$$

Although the heavier club helps some, the maximum speed to get out of the ball will be less than twice the speed of the club.

P6-16 There will always be at least two collisions. The balls are a, b, and c from left to right. After the first collision between a and b one has

$$v_{b,1} = v_0 \text{ and } v_{a,1} = 0.$$

After the first collision between b and c one has

$$v_{c,1} = 2mv_0/(m + M) \text{ and } v_{b,2} = (m - M)v_0/(m + M).$$

(a) If $m \geq M$ then ball b continue to move to the right (or stops) and there are no more collisions.
(b) If $m < M$ then ball b bounces back and strikes ball a which was at rest. Then

$$v_{a,2} = (m - M)v_0/(m + M) \text{ and } v_{b,3} = 0.$$

P6-17 All three balls are identical in mass and radii? Then balls 2 and 3 will move off at 30° to the initial direction of the first ball. By symmetry we expect balls 2 and 3 to have the same speed.

The problem now is to define an elastic three body collision. It is no longer the case that the balls bounce off with the same speed in the center of mass. One can't even treat the problem as two separate collisions, one right after the other. No amount of momentum conservation laws will help solve the problem; we need some additional physics, but at this point in the text we don't have it.

P6-18 The original speed is v_0 in the lab frame. Let α be the angle of deflection in the cm frame and $\vec{\mathbf{v}}'_1$ be the initial velocity in the cm frame. Then the velocity after the collision in the cm frame is $v'_1 \cos\alpha\,\hat{\mathbf{i}} + v'_1 \sin\alpha\,\hat{\mathbf{j}}$ and the velocity in the lab frame is $(v'_1\cos\alpha + v)\hat{\mathbf{i}} + v'_1\sin\alpha\,\hat{\mathbf{j}}$, where v is the speed of the cm frame. The deflection angle in the lab frame is

$$\theta = \arctan[(v'_1 \sin\alpha)/(v'_1\cos\alpha + v)],$$

but $v = m_1 v_0/(m_1 + m_2)$ and $v'_1 = v_0 - v$ so $v'_1 = m_2 v_0/(m_1 + m_2)$ and

$$\theta = \arctan[(m_2 \sin\alpha)/(m_2\cos\alpha + m_1)].$$

(c) θ is a maximum when $(\cos\alpha + m_1/m_2)/\sin\alpha$ is a minimum, which happens when $\cos\alpha = -m_1/m_2$ if $m_1 \leq m_2$. Then $[(m_2\sin\alpha)/(m_2\cos\alpha + m_1)]$ can have any value between $-\infty$ and ∞, so θ can be between 0 and π.

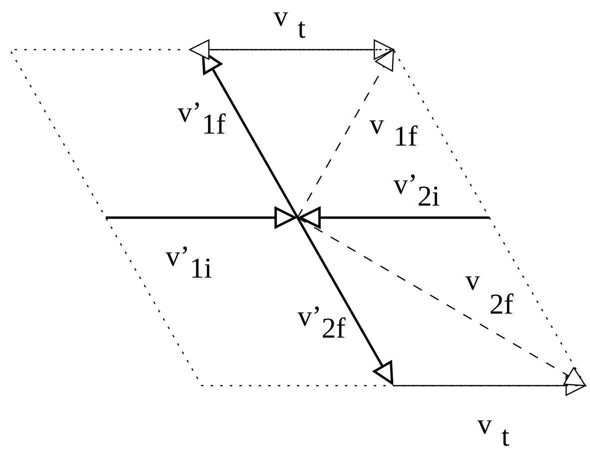
(a) If $m_1 > m_2$ then $(\cos\alpha + m_1/m_2)/\sin\alpha$ is a minimum when $\cos\alpha = -m_2/m_1$, then

$$[(m_2\sin\alpha)/(m_2\cos\alpha + m_1)] = m_2/\sqrt{m_1^2 - m_2^2}.$$

If $\tan\theta = m_2/\sqrt{m_1^2 - m_2^2}$ then m_1 is like a hypotenuse and m_2 the opposite side. Then

$$\cos\theta = \sqrt{m_1^2 - m_2^2}/m_1 = \sqrt{1 - (m_2/m_1)^2}.$$

(b) We need to change to the center of mass system. Since both particles have the same mass, the conservation of momentum problem is effectively the same as a (vector) conservation of velocity problem. Since one of the particles is originally at rest, the center of mass moves with speed $v_{\text{cm}} = v_{1\text{i}}/2$. In the figure below the center of mass velocities are primed; the transformation velocity is v_t.



Note that since $v_t = v'_{1\mathrm{i}} = v'_{2\mathrm{i}} = v'_{1\mathrm{f}} = v'_{2\mathrm{f}}$ the entire problem can be inscribed in a rhombus. The diagonals of the rhombus are the directions of $v_{1\mathrm{f}}$ and $v_{2\mathrm{f}}$; note that the diagonals of a rhombus are *necessarily* at right angles!

$\boxed{\textbf{P6-19}}$ (a) The speed of the bullet after leaving the first block but before entering the second can be determined by momentum conservation.

$$\begin{aligned}
P_\mathrm{f} &= P_\mathrm{i},\\
p_{\mathrm{f,bl}} + p_{\mathrm{f,bu}} &= p_{\mathrm{i,bl}} + p_{\mathrm{i,bu}},\\
m_\mathrm{bl} v_{\mathrm{f,bl}} + m_\mathrm{bu} v_{\mathrm{f,bu}} &= m_\mathrm{bl} v_{\mathrm{i,bl}} + m_\mathrm{bu} v_{\mathrm{i,bu}},\\
(1.78\mathrm{kg})(1.48\,\mathrm{m/s}) + (3.54\times10^{-3}\mathrm{kg})(1.48\,\mathrm{m/s}) &= (1.78\mathrm{kg})(0) + (3.54\times10^{-3}\mathrm{kg})v_{\mathrm{i,bu}},
\end{aligned}$$

which has solution $v_{\mathrm{i,bl}} = 746\,\mathrm{m/s}$.

(b) We do the same steps again, except applied to the first block,

$$\begin{aligned}
P_\mathrm{f} &= P_\mathrm{i},\\
p_{\mathrm{f,bl}} + p_{\mathrm{f,bu}} &= p_{\mathrm{i,bl}} + p_{\mathrm{i,bu}},\\
m_\mathrm{bl} v_{\mathrm{f,bl}} + m_\mathrm{bu} v_{\mathrm{f,bu}} &= m_\mathrm{bl} v_{\mathrm{i,bl}} + m_\mathrm{bu} v_{\mathrm{i,bu}},\\
(1.22\mathrm{kg})(0.63\,\mathrm{m/s}) + (3.54\times10^{-3}\mathrm{kg})(746\,\mathrm{m/s}) &= (1.22\mathrm{kg})(0) + (3.54\times10^{-3}\mathrm{kg})v_{\mathrm{i,bu}},
\end{aligned}$$

which has solution $v_{\mathrm{i,bl}} = 963\,\mathrm{m/s}$.

P6-20 The acceleration of the block down the ramp is $a_1 = g\sin(22^\circ)$. The ramp has a length of $d = h/\sin(22^\circ)$, so it takes a time $t_1 = \sqrt{2d/a_1} = \sqrt{2h/g}/\sin(22^\circ)$ to reach the bottom. The speed when it reaches the bottom is $v_1 = a_1 t_1 = \sqrt{2gh}$. Notice that it is independent of the angle!

The collision is inelastic, so the two stick together and move with an initial speed of $v_2 = m_1 v_1/(m_1 + m_2)$. They slide a distance x before stopping; the average speed while decelerating is $v_\mathrm{av} = v_2/2$, so the stopping time is $t_2 = 2x/v_2$ and the deceleration is $a_2 = v_2/t_2 = v_2^2/(2x)$. If the retarding force is $f = (m_1 + m_2)a_2$, then $f = \mu_\mathrm{k}(m_1 + m_2)g$. Glue it all together and

$$\mu_\mathrm{k} = \frac{m_1^2}{(m_1+m_2)^2}\frac{h}{x} = \frac{(2.0\,\mathrm{kg})^2}{(2.0\,\mathrm{kg}+3.5\,\mathrm{kg})^2}\frac{(0.65\,\mathrm{m})}{(0.57\,\mathrm{m})} = 0.15.$$

P6-21 (a) For an object with initial speed v and deceleration $-a$ which travels a distance x before stopping, the time t to stop is $t = v/a$, the average speed while stopping is $v/2$, and $d = at^2/2$. Combining, $v = \sqrt{2ax}$. The deceleration in this case is given by $a = \mu_\text{k} g$.

Then just after the collision

$$v_A = \sqrt{2(0.130)(9.81\,\text{m/s}^2)(8.20\,\text{m})} = 4.57\,\text{m/s},$$

while

$$v_B = \sqrt{2(0.130)(9.81\,\text{m/s}^2)(6.10\,\text{m})} = 3.94\,\text{m/s},$$

(b) $v_0 = [(1100\,\text{kg})(4.57\,\text{m/s}) + (1400\text{kg})(3.94\,\text{m/s})]/(1400\,\text{kg}) = 7.53\,\text{m/s}.$

E7-1 $x_{\rm cm} = (7.36\times10^{22}\text{kg})(3.82\times10^{8}\text{m})/(7.36\times10^{22}\text{kg} + 5.98\times10^{34}\text{kg}) = 4.64\times10^{6}\text{m}$. This is less than the radius of the Earth.

E7-2 If the particles are l apart then

$$x_1 = m_1 l(m_1 + m_2)$$

is the distance from particle 1 to the center of mass and

$$x_2 = m_2 l(m_1 + m_2)$$

is the distance from particle 2 to the center of mass. Divide the top equation by the bottom and

$$x_1/x_2 = m_1/m_2.$$

$\boxed{\textbf{E7-3}}$ The center of mass velocity is given by Eq. 7-1,

$$\begin{aligned}\vec{\mathbf{v}}_{\rm cm} &= \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2},\\ &= \frac{(2210\text{ kg})(105\text{ km/h}) + (2080\text{ kg})(43.5\text{ km/h})}{(2210\text{ kg}) + (2080\text{ kg})} = 75.2\text{ km/h}.\end{aligned}$$

E7-4 They will meet at the center of mass, so

$$x_{\rm cm} = (65\,\text{kg})(9.7\,\text{m})/(65\,\text{kg} + 42\,\text{kg}) = 5.9\,\text{m}.$$

E7-5 (a) No external forces, center of mass doesn't move.
(b) The collide at the center of mass,

$$x_{\rm cm} = (4.29\,\text{kg})(1.64\,\text{m})/(4.29\,\text{kg} + 1.43\,\text{kg}) = 1.23\,\text{m}.$$

E7-6 The range of the center of mass is

$$R = v_0^2 \sin 2\theta\,/g = (466\,\text{m/s})^2 \sin(2\times 57.4^\circ)/(9.81\,\text{m/s}^2) = 2.01\times10^4\,\text{m}.$$

Half lands directly underneath the highest point, or 1.00×10^4 m. The other piece must land at x, such that

$$2.01\times10^4\,\text{m} = (1.00\times10^4\,\text{m} + x)/2;$$

then $x = 3.02\times10^4$ m.

$\boxed{\textbf{E7-7}}$ The center of mass of the boat + dog doesn't move because there are no external forces on the system. Define the coordinate system so that distances are measured from the shore, so toward the shore is in the negative x direction. The *change* in position of the center of mass is given by

$$\Delta x_{\rm cm} = \frac{m_d\Delta x_d + m_b\Delta x_b}{m_d + m_b} = 0,$$

Both Δx_d and Δx_b are measured with respect to the shore; we are given $\Delta x_{db} = -8.50$ ft, the displacement of the dog with respect to the boat. But

$$\Delta x_d = \Delta x_{db} + \Delta x_b.$$

Since we want to find out about the dog, we'll substitute for the boat's displacement,

$$0 = \frac{m_d \Delta x_d + m_b \left(\Delta x_d - \Delta x_{db}\right)}{m_d + m_b}.$$

Rearrange and solve for Δx_d. Use $W = mg$ and multiply the top and bottom of the expression by g. Then

$$\Delta x_d = \frac{m_b \Delta x_{db}}{m_d + m_b} \frac{g}{g} = \frac{(46.4 \text{ lb})(-8.50 \text{ ft})}{(10.8 \text{ lb}) + (46.4 \text{ lb})} = -6.90 \text{ ft}.$$

The dog is now $21.4 - 6.9 = 14.5$ feet from shore.

E7-8 Richard has too much time on his hands.

The center of mass of the system is x_{cm} away from the center of the boat. Switching seats is effectively the same thing as rotating the canoe through 180°, so the center of mass of the system has moved through a distance of $2x_{\text{cm}} = 0.412\,\text{m}$. Then $x_{\text{cm}} = 0.206\,\text{m}$. Then

$$x_{\text{cm}} = (Ml - ml)/(M + m + m_c) = 0.206\,\text{m},$$

where $l = 1.47\,\text{m}$, $M = 78.4\,\text{kg}$, $m_c = 31.6\,\text{kg}$, and m is Judy's mass. Rearrange,

$$m = \frac{Ml - (M + m_c)x_{\text{cm}}}{l + x_{\text{cm}}} = \frac{(78.4\,\text{kg})(1.47\,\text{m}) - (78.4\,\text{kg} + 31.6\,\text{kg})(0.206\,\text{m})}{(1.47\,\text{m}) + (0.206\,\text{m})} = 55.2\,\text{kg}.$$

E7-9 It takes the man $t = (18.2\,\text{m})/(2.08\,\text{m/s}) = 8.75\,\text{s}$ to walk to the front of the boat. During this time the center of mass of the system has moved forward $x = (4.16\,\text{m/s})(8.75\,\text{s}) = 36.4\,\text{m}$. But in walking forward to the front of the boat the man shifted the center of mass by a distance of $(84.4\,\text{kg})(18.2\,\text{m})/(84.4\,\text{kg} + 425\,\text{kg}) = 3.02\,\text{m}$, so the boat only traveled $36.4\,\text{m} - 3.02\,\text{m} = 33.4\,\text{m}$.

E7-10 Do each coordinate separately.

$$x_{\text{cm}} = \frac{(3\,\text{kg})(0) + (8\,\text{kg})(1\,\text{m}) + (4\,\text{kg})(2\,\text{m})}{(3\,\text{kg}) + (8\,\text{kg}) + (4\,\text{kg})} = 1.07\,\text{m}$$

and

$$y_{\text{cm}} = \frac{(3\,\text{kg})(0) + (8\,\text{kg})(2\,\text{m}) + (4\,\text{kg})(1\,\text{m})}{(3\,\text{kg}) + (8\,\text{kg}) + (4\,\text{kg})} = 1.33\,\text{m}$$

$\boxed{\textbf{E7-11}}$ The center of mass of the three hydrogen atoms will be at the center of the pyramid base. The problem is then reduced to finding the center of mass of the nitrogen atom and the three hydrogen atom triangle. This molecular center of mass must lie on the dotted line in Fig. 7-27.

The location of the plane of the hydrogen atoms can be found from Pythagoras theorem

$$y_{\text{h}} = \sqrt{(10.14\times10^{-11}\text{m})^2 - (9.40\times10^{-11}\text{m})^2} = 3.8\times10^{-11}\text{m}.$$

This distance can be used to find the center of mass of the molecule. From Eq. 7-2,

$$y_{\text{cm}} = \frac{m_{\text{n}} y_{\text{n}} + m_{\text{h}} y_{\text{h}}}{m_{\text{n}} + m_{\text{h}}} = \frac{(13.9 m_{\text{h}})(0) + (3 m_{\text{h}})(3.8\times10^{-11}\text{m})}{(13.9 m_{\text{h}}) + (3 m_{\text{h}})} = 6.75\times10^{-12}\text{m}.$$

E7-12 The velocity components of the center of mass at $t = 1.42\,\text{s}$ are $v_{\text{cm},x} = 7.3\,\text{m/s}$ and $v_{\text{cm},y} = (10.0\,\text{m/s}) - (9.81\,\text{m/s})(1.42\,\text{s}) = -3.93\,\text{m/s}$. Then the velocity components of the "other" piece are

$$v_{2,x} = [(9.6\,\text{kg})(7.3\,\text{m/s}) - (6.5\,\text{kg})(11.4\,\text{m/s})]/(3.1\,\text{kg}) = -1.30\,\text{m/s}.$$

and

$$v_{2,y} = [(9.6\,\text{kg})(-3.9\,\text{m/s}) - (6.5\,\text{kg})(-4.6\,\text{m/s})]/(3.1\,\text{kg}) = -2.4\,\text{m/s}.$$

$\boxed{\textbf{E7-13}}$ The center of mass should lie on the perpendicular bisector of the rod of mass $3M$. We can view the system as having two parts: the heavy rod of mass $3M$ and the two light rods each of mass M. The two light rods have a center of mass at the center of the square.

Both of these center of masses are located along the vertical line of symmetry for the object. The center of mass of the heavy bar is at $y_{\rm h,cm} = 0$, while the *combined* center of mass of the two light bars is at $y_{\rm l,cm} = L/2$, where down is positive. The center of mass of the system is then at

$$y_{\rm cm} = \frac{2My_{\rm l,cm} + 3My_{\rm h,cm}}{2M + 3M} = \frac{2(L/2)}{5} = L/5.$$

E7-14 The two slabs have the same volume and have mass $m_i = \rho_i V$. The center of mass is located at

$$x_{\rm cm} = \frac{m_1 l - m_2 l}{m_1 + m_2} = l\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} = (5.5\ \text{cm})\frac{(7.85\ \text{g/cm}^3) - (2.70\ \text{g/cm}^3)}{(7.85\ \text{g/cm}^3) + (2.70\ \text{g/cm}^3)} = 2.68\ \text{cm}$$

from the boundary inside the iron; it is centered in the y and z directions.

E7-15 Treat the four sides of the box as one thing of mass $4m$ with a mass located $l/2$ above the base. Then the center of mass is

$$z_{\rm cm} = (l/2)(4m)/(4m + m) = 2l/5 = 2(0.4\,\text{m})/5 = 0.16\,\text{m},$$

$x_{\rm cm} = y_{\rm cm} = 0.2\,\text{m}$.

E7-16 One piece moves off with momentum $m(31.4\,\text{m/s})\hat{\mathbf{i}}$, another moves off with momentum $2m(31.4\,\text{m/s})\hat{\mathbf{j}}$. The third piece must then have momentum $-m(31.4\,\text{m/s})\hat{\mathbf{i}} - 2m(31.4\,\text{m/s})\hat{\mathbf{j}}$ and velocity $-(1/3)(31.4\,\text{m/s})\hat{\mathbf{i}} - 2/3(31.4\,\text{m/s})\hat{\mathbf{j}} = -10.5\,\text{m/s}\,\hat{\mathbf{i}} - 20.9\,\text{m/s}\,\hat{\mathbf{j}}$. The magnitude of v_3 is $23.4\,\text{m/s}$ and direction $63.3°$ away from the lighter piece.

E7-17 It will take an impulse of $(84.7\,\text{kg})(3.87\,\text{m/s}) = 328\,\text{kg}\cdot\text{m/s}$ to stop the animal. This would come from firing n bullets where $n = (328\,\text{kg}\cdot\text{m/s})/[(0.0126\,\text{kg})(975\,\text{m/s}) = 27$.

E7-18 Conservation of momentum for firing one cannon ball of mass m with muzzle speed v_c forward out of a cannon on a trolley of original total mass M moving forward with original speed v_0 is

$$Mv_0 = (M - m)v_1 + m(v_c + v_1) = Mv_1 + mv_c,$$

where v_1 is the speed of the trolley after the cannonball is fired. Then to stop the trolley we require n cannonballs be fired so that

$$n = (Mv_0)/(mv_c) = [(3500\,\text{kg})(45\,\text{m/s})]/[(65\,\text{kg})(625\,\text{m/s})] = 3.88,$$

so $n = 4$.

$\boxed{\textbf{E7-19}}$ Label the velocities of the various containers as $\vec{\mathbf{v}}_k$ where k is an integer between one and twelve. The mass of each container is m. The subscript "g" refers to the goo; the subscript k refers to the kth container.

The total momentum before the collision is given by

$$\vec{\mathbf{P}} = \sum_k m\vec{\mathbf{v}}_{k,\rm i} + m_{\rm g}\vec{\mathbf{v}}_{\rm g,i} = 12m\vec{\mathbf{v}}_{\rm cont.,cm} + m_g\vec{\mathbf{v}}_{\rm g,i}.$$

We are told, however, that the initial velocity of the center of mass of the containers is at rest, so the initial momentum simplifies to $\vec{\mathbf{P}} = m_g \vec{\mathbf{v}}_{\mathrm{g,i}}$, and has a magnitude of 4000 kg·m/s.

(a) Then

$$v_{\mathrm{cm}} = \frac{P}{12m + m_{\mathrm{g}}} = \frac{(4000\,\mathrm{kg\cdot m/s})}{12(100.0\,\mathrm{kg}) + (50\,\mathrm{kg})} = 3.2\,\mathrm{m/s}.$$

(b) It doesn't matter if the cord breaks, we'll get the same answer for the motion of the center of mass.

E7-20 (a) $F = (3270\,\mathrm{m/s})(480\,\mathrm{kg/s}) = 1.57\times 10^6\mathrm{N}$.

(b) $m = 2.55\times 10^5\mathrm{kg} - (480\,\mathrm{kg}/s)(250\,\mathrm{s}) = 1.35\times 10^5\mathrm{kg}$.

(c) Eq. 7-32:

$$v_{\mathrm{f}} = (-3270\,\mathrm{m/s})\ln(1.35\times 10^5\mathrm{kg}/2.55\times 10^5\mathrm{kg}) = 2080\,\mathrm{m/s}.$$

$\boxed{\textbf{E7-21}}$ Use Eq. 7-32. The initial velocity of the rocket is 0. The mass ratio can then be found from a minor rearrangement;

$$\frac{M_{\mathrm{i}}}{M_{\mathrm{f}}} = e^{|v_{\mathrm{f}}/v_{\mathrm{rel}}|}$$

The "flipping" of the left hand side of this expression is possible because the exhaust velocity is *negative* with respect to the rocket. For part (a) $M_{\mathrm{i}}/M_{\mathrm{f}} = e = 2.72$. For part (b) $M_{\mathrm{i}}/M_{\mathrm{f}} = e^2 = 7.39$.

E7-22 Eq. 7-32 rearranged:

$$\frac{M_{\mathrm{f}}}{M_{\mathrm{i}}} = e^{-|\Delta v/v_{\mathrm{rel}}|} = e^{-(22.6\mathrm{m/s})/(1230\mathrm{m/s})} = 0.982.$$

The fraction of the initial mass which is discarded is 0.0182.

E7-23 The loaded rocket has a weight of $(1.11\times 10^5\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 1.09\times 10^6\mathrm{N}$; the thrust must be at least this large to get the rocket off the ground. Then $v \geq (1.09\times 10^6\mathrm{N})/(820\,\mathrm{kg/s}) = 1.33\times 10^3\mathrm{m/s}$ is the minimum exhaust speed.

E7-24 The acceleration down the incline is $(9.8\,\mathrm{m/s^2})\sin(26^\circ) = 4.3\,\mathrm{m/s^2}$. It will take $t = \sqrt{2(93\,\mathrm{m})/(4.3\,\mathrm{m/s^2})} = 6.6\,\mathrm{s}$. The sand doesn't affect the problem, so long as it only "leaks" out.

$\boxed{\textbf{E7-25}}$ We'll use Eq. 7-4 to solve this problem, but since we are given *weights* instead of *mass* we'll multiply the top and bottom by g like we did in Exercise 7-7. Then

$$\vec{\mathbf{v}}_{\mathrm{cm}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}\frac{g}{g} = \frac{W_1\vec{\mathbf{v}}_1 + W_2\vec{\mathbf{v}}_2}{W_1 + W_2}.$$

Now for the numbers

$$v_{\mathrm{cm}} = \frac{(9.75\ \mathrm{T})(1.36\,\mathrm{m/s}) + (0.50\ \mathrm{T})(0)}{(9.75\ \mathrm{T}) + (0.50\ \mathrm{T})} = 1.29\,\mathrm{m/s}.$$

P7-1 (a) The balloon moves down so that the center of mass is stationary;

$$0 = Mv_{\mathrm{b}} + mv_{\mathrm{m}} = Mv_{\mathrm{b}} + m(v + v_{\mathrm{b}}),$$

or $v_{\mathrm{b}} = -mv/(m + M)$.

(b) When the man stops so does the balloon.

P7-2 (a) The center of mass is midway between them.

(b) Measure from the heavier mass.

$$x_{\text{cm}} = (0.0560\,\text{m})(0.816\,\text{kg})/(1.700\,\text{kg}) = 0.0269\,\text{m},$$

which is 1.12 mm closer to the heavier mass than in part (a).

(c) Think Atwood's machine. The acceleration of the two masses is

$$a = 2\Delta m\, g/(m_1 + m_2) = 2(0.034\,\text{kg})g/(1.700\,\text{kg}) = 0.0400g,$$

the heavier going down while the lighter moves up. The acceleration of the center of mass is

$$a_{\text{cm}} = (am_1 - am_2)/(m_1 + m_2) = (0.0400g)2(0.034\,\text{kg})g/(1.700\,\text{kg}) = 0.00160g.$$

$\boxed{\textbf{P7-3}}$ This is a glorified Atwood's machine problem. The total mass on the right side is the mass per unit length times the length, $m_r = \lambda x$; similarly the mass on the left is given by $m_l = \lambda(L - x)$. Then

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = \frac{\lambda x - \lambda(L - x)}{\lambda x + \lambda(L - x)} g = \frac{2x - L}{L} g$$

which solves the problem. The acceleration is in the direction of the side of length x if $x > L/2$.

P7-4 (a) Assume the car is massless. Then moving the cannonballs is moving the center of mass, unless the cannonballs don't move but instead the car does. How far? L.

(b) Once the cannonballs stop moving so does the rail car.

$\boxed{\textbf{P7-5}}$ By symmetry, the center of mass of the empty storage tank should be in the very center, along the axis at a height $y_{\text{t,cm}} = H/2$. We can pretend that the entire mass of the tank, $m_{\text{t}} = M$, is located at this point.

The center of mass of the gasoline is also, by symmetry, located along the axis at half the height of the gasoline, $y_{\text{g,cm}} = x/2$. The mass, if the tank were filled to a height H, is m; assuming a uniform density for the gasoline, the mass present when the level of gas reaches a height x is $m_g = mx/H$.

(a) The center of mass of the entire system is at the center of the cylinder when the tank is full and when the tank is empty. When the tank is half full the center of mass is below the center. So as the tank changes from full to empty the center of mass drops, reaches some lowest point, and then rises back to the center of the tank.

(b) The center of mass of the entire system is found from

$$y_{\text{cm}} = \frac{m_{\text{g}} y_{\text{g,cm}} + m_{\text{t}} y_{\text{t,cm}}}{m_{\text{g}} + m_{\text{t}}} = \frac{(mx/H)(x/2) + (M)(H/2)}{(mx/H) + (M)} = \frac{mx^2 + MH^2}{2mx + 2MH}.$$

Take the derivative:

$$\frac{dy_{\text{cm}}}{dx} = \frac{m\left(mx^2 + 2xMH - MH^2\right)}{(mx + MH)^2}$$

Set this equal to zero to find the minimum; this means we want the numerator to vanish, or $mx^2 + 2xMH - MH^2 = 0$. Then

$$x = \frac{-M + \sqrt{M^2 + mM}}{m} H.$$

P7-6 The center of mass will be located along symmetry axis. Call this the x axis. Then

$$\begin{aligned} x_{\rm cm} &= \frac{1}{M}\int x dm, \\ &= \frac{4}{\pi R^2}\int_0^R \int_0^{\sqrt{R^2-x^2}} x\, dy\, dx, \\ &= \frac{4}{\pi R^2}\int_0^R x\sqrt{R^2-x^2}\, dx, \\ &= \frac{4}{\pi R^2} R^3/3 = \frac{4R}{3\pi}. \end{aligned}$$

P7-7 (a) The components of the shell velocity with respect to the cannon are

$$v'_x = (556\,\mathrm{m/s})\cos(39.0^\circ) = 432\,\mathrm{m/s} \text{ and } v'_y = (556\,\mathrm{m/s})\sin(39.0^\circ) = 350\,\mathrm{m/s}.$$

The vertical component with respect to the ground is the same, $v_y = v'_y$, but the horizontal component is found from conservation of momentum:

$$M(v_x - v'_x) + m(v_x) = 0,$$

so $v_x = (1400\,\mathrm{kg})(432\,\mathrm{m/s})(70.0\,\mathrm{kg} + 1400\,\mathrm{kg}) = 411\,\mathrm{m/s}$. The resulting speed is $v = 540\,\mathrm{m/s}$.

(b) The direction is $\theta = \arctan(350/411) = 40.4^\circ$.

P7-8 $v = (2870\,\mathrm{kg})(252\,\mathrm{m/s})/(2870\,\mathrm{kg} + 917\,\mathrm{kg}) = 191\,\mathrm{m/s}$.

P7-9 It takes $(1.5\,\mathrm{m/s})(20\,\mathrm{kg}) = 30\,\mathrm{N}$ to accelerate the luggage to the speed of the belt. The people when taking the luggage off will (on average) also need to exert a 30 N force to remove it; this force (because of friction) will be exerted on the belt. So the belt requires 60 N of additional force.

P7-10 (a) The thrust must be at least equal to the weight, so

$$dm/dt = (5860\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/(1170\,\mathrm{m/s}) = 49.1\,\mathrm{kg/s}.$$

(b) The net force on the rocket will need to be $F = (5860\,\mathrm{kg})(18.3\,\mathrm{m/s^2}) = 107000\,\mathrm{N}$. Add this to the weight to find the thrust, so

$$dm/dt = [107000\,\mathrm{N} + (5860\,\mathrm{kg})(9.81\,\mathrm{m/s^2})]/(1170\,\mathrm{m/s}) = 141\,\mathrm{kg/s}$$

$\boxed{\textbf{P7-11}}$ Consider Eq. 7-31. We want the barges to continue at constant speed, so the left hand side of that equation vanishes. Then

$$\sum \vec{\mathbf{F}}_{\rm ext} = -\vec{\mathbf{v}}_{\rm rel}\frac{dM}{dt}.$$

We are told that the frictional force is independent of the weight, since the speed doesn't change the frictional force should be constant and equal in magnitude to the force exerted by the engine *before* the shoveling happens. Then $\sum \vec{\mathbf{F}}_{\rm ext}$ is equal to the additional force required from the engines. We'll call it $\vec{\mathbf{P}}$.

The relative speed of the coal to the faster moving cart has magnitude: $21.2 - 9.65 = 11.6$ km/h= 3.22 m/s. The mass flux is 15.4 kg/s, so $P = (3.22\text{ m/s})(15.4\text{kg/s}) = 49.6$ N. The faster moving cart will need to *increase* the engine force by 49.6 N. The slower cart won't need to do anything, because the coal left the slower barge with a relative speed of *zero* according to our approximation.

P7-12 (a) Nothing is ejected from the string, so $v_{\mathrm{rel}} = 0$. Then Eq. 7-31 reduces to $m\, dv/dt = F_{\mathrm{ext}}$.

(b) Since F_{ext} is from the weight of the hanging string, and the fraction that is hanging is y/L, $F_{\mathrm{ext}} = mgy/L$. The equation of motion is then $d^2y/dt^2 = gy/L$.

(c) Take first derivative:

$$\frac{dy}{dt} = \frac{y_0}{2}(\sqrt{g/L})\left(e^{\sqrt{g/L}t} - e^{-\sqrt{g/L}t}\right),$$

and then second derivative,

$$\frac{d^2y}{dt^2} = \frac{y_0}{2}(\sqrt{g/L})^2\left(e^{\sqrt{g/L}t} + e^{-\sqrt{g/L}t}\right).$$

Substitute into equation of motion. It works! Note that when $t = 0$ we have $y = y_0$.

$\boxed{\textbf{E8-1}}$ An n-dimensional object can be oriented by stating the position of n different *carefully chosen* points P_i inside the body. Since each point has n coordinates, one might think there are n^2 coordinates required to completely specify the position of the body. But if the body is rigid then the distances between the points are fixed. There is a distance d_{ij} for every pair of points P_i and P_j. For each distance d_{ij} we need one fewer coordinate to specify the position of the body. There are $n(n-1)/2$ ways to connect n objects in pairs, so $n^2 - n(n-1)/2 = n(n+1)/2$ is the number of coordinates required.

E8-2 (1 rev/min)(2π rad/rev)/(60 s/min) = 0.105 rad/s.

E8-3 (a) $\omega = a + 3bt^2 - 4ct^3$.
(b) $\alpha = 6bt - 12t^2$.

E8-4 (a) The radius is $r = (2.3\times10^4\text{ly})(3.0\times10^8\text{m/s}) = 6.9\times10^{12}\text{m}\cdot\text{y/s}$. The time to make one revolution is $t = (2\pi 6.9\times10^{12}\text{m}\cdot\text{y/s})/(250\times10^3\text{m/s}) = 1.7\times10^8\text{y}$.
(b) The Sun has made $4.5\times10^9\text{y}/1.7\times10^8\text{y} = 26$ revolutions.

$\boxed{\textbf{E8-5}}$ (a) Integrate.

$$\omega_z = \omega_0 + \int_0^t \left(4at^3 - 3bt^2\right) dt = \omega_0 + at^4 - bt^3$$

(b) Integrate, again.

$$\Delta\theta = \int_0^t \omega_z dt = \int_0^t \left(\omega_0 + at^4 - bt^3\right) dt = \omega_0 t + \frac{1}{5}at^5 - \frac{1}{4}bt^4$$

E8-6 (a) (1 rev/min)(2π rad/rev)/(60 s/min) = 0.105 rad/s.
(b) (1 rev/h)(2π rad/rev)/(3600 s/h) = 1.75×10^{-3} rad/s.
(c) (1/12 rev/h)(2π rad/rev)/(3600 s/h) = 1.45×10^{-3} rad/s.

E8-7 85 mi/h = 125 ft/s. The ball takes t = (60 ft)/(125 ft/s) = 0.48 s to reach the plate. It makes (30 rev/s)(0.48 s) = 14 revolutions in that time.

E8-8 It takes $t = \sqrt{2(10\,\text{m})/(9.81\,\text{m/s}^2)} = 1.43\,\text{s}$ to fall 10 m. The average angular velocity is then $\omega = (2.5)(2\pi\text{ rad})/(1.43\,\text{s}) = 11$ rad/s.

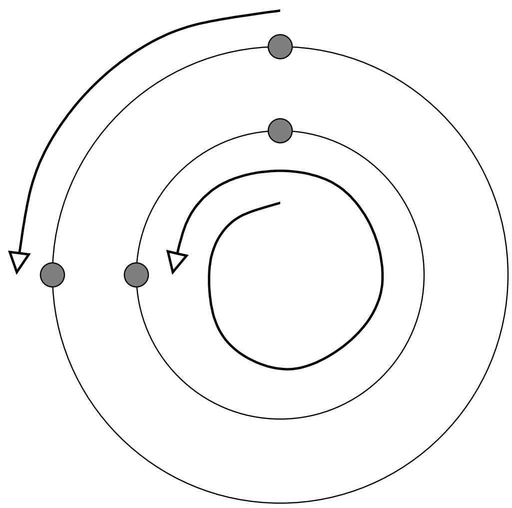
$\boxed{\textbf{E8-9}}$ (a) Since there are eight spokes, this means the wheel can make no more than 1/8 of a revolution while the arrow traverses the plane of the wheel. The wheel rotates at 2.5 rev/s; it makes one revolution every $1/2.5 = 0.4$ s; so the arrow must pass through the wheel in less than $0.4/8 = 0.05$ s.

The arrow is 0.24 m long, and it must move at least one arrow length in 0.05 s. The corresponding minimum speed is (0.24 m)/(0.05 s) = 4.8 m/s.

(b) It does not matter where you aim, because the wheel is rigid. It is the angle through which the spokes have turned, not the distance, which matters here.

E8-10 We look for the times when the Sun, the Earth, and the other planet are collinear in some specified order.

Since the outer planets revolve around the Sun more slowly than Earth, after one year the Earth has returned to the original position, but the outer planet has completed *less* than one revolution. The Earth will then "catch up" with the outer planet *before* the planet has completed a revolution. If θ_E is the angle through which Earth moved and θ_P is the angle through which the planet moved, then $\theta_E = \theta_P + 2\pi$, since the Earth completed one more revolution than the planet.



If ω_P is the angular velocity of the planet, then the angle through which it moves during the time T_S (the time for the planet to line up with the Earth). Then

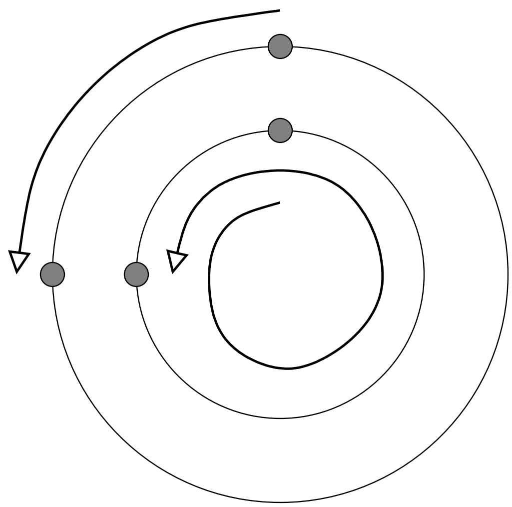
$$\begin{aligned}\theta_E &= \theta_P + 2\pi,\\ \omega_E T_S &= \omega_P T_S + 2\pi,\\ \omega_E &= \omega_P + 2\pi/T_S\end{aligned}$$

The angular velocity of a planet is $\omega = 2\pi/T$, where T is the period of revolution. Substituting this into the last equation above yields

$$1/T_E = 1/T_P + 1/T_S.$$

$\boxed{\textbf{E8-11}}$ We look for the times when the Sun, the Earth, and the other planet are collinear in some specified order.

Since the inner planets revolve around the Sun more quickly than Earth, after one year the Earth has returned to the original position, but the inner planet has completed *more* than one revolution. The inner planet must then have "caught-up" with the Earth *before* the Earth has completed a revolution. If θ_E is the angle through which Earth moved and θ_P is the angle through which the planet moved, then $\theta_P = \theta_E + 2\pi$, since the inner planet completed one more revolution than the Earth.



If ω_P is the angular velocity of the planet, then the angle through which it moves during the time T_S (the time for the planet to line up with the Earth). Then

$$\begin{aligned}\theta_P &= \theta_E + 2\pi,\\ \omega_P T_S &= \omega_E T_S + 2\pi,\\ \omega_P &= \omega_E + 2\pi/T_S\end{aligned}$$

The angular velocity of a planet is $\omega = 2\pi/T$, where T is the period of revolution. Substituting this into the last equation above yields

$$1/T_P = 1/T_E + 1/T_S.$$

E8-12 (a) $\alpha = (-78\text{ rev/min})/(0.533\text{ min}) = -150\text{ rev/min}^2$.

(b) Average angular speed while slowing down is 39 rev/min, so (39 rev/min)(0.533 min) = 21 rev.

E8-13 (a) $\alpha = (2880\text{ rev/min} - 1170\text{ rev/min})/(0.210\text{ min}) = 8140\text{ rev/min}^2$.

(b) Average angular speed while accelerating is 2030 rev/min, so (2030 rev/min)(0.210 min) = 425 rev.

E8-14 Find area under curve.

$$\frac{1}{2}(5\text{ min} + 2.5\text{ min})(3000\text{ rev/min}) = 1.13\times 10^4\text{ rev.}$$

$\boxed{\textbf{E8-15}}$ (a) $\omega_{0z} = 25.2$ rad/s; $\omega_z = 0$; $t = 19.7$ s; and α_z and ϕ are unknown. From Eq. 8-6,

$$\begin{aligned}\omega_z &= \omega_{0z} + \alpha_z t,\\ (0) &= (25.2\text{ rad/s}) + \alpha_z(19.7\text{ s}),\\ \alpha_z &= -1.28\text{ rad/s}^2\end{aligned}$$

(b) We use Eq. 8-7 to find the angle through which the wheel rotates.

$$\phi = \phi_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (0) + (25.2\text{ rad/s})(19.7\text{ s}) + \frac{1}{2}(-1.28\text{ rad/s}^2)(19.7\text{ s})^2 = 248\text{ rad}.$$

(c) $\phi = 248 \text{ rad}\frac{1 \text{ rev}}{2\pi \text{ rad}} = 39.5$ rev.

E8-16 (a) $\alpha = (225 \text{ rev/min} - 315 \text{ rev/min})/(1.00 \text{ min}) = -90.0 \text{ rev/min}^2$.

(b) $t = (0 - 225 \text{ rev/min})/(-90.0 \text{ rev/min}^2) = 2.50$ min.

(c) $(-90.0 \text{ rev/min}^2)(2.50 \text{ min})^2/2 + (225 \text{ rev/min})(2.50 \text{ min}) = 281$ rev.

E8-17 (a) The average angular speed was $(90 \text{ rev})/(15\,\text{s}) = 6.0$ rev/s. The angular speed at the beginning of the interval was then $2(6.0 \text{ rev/s}) - (10 \text{ rev/s}) = 2.0$ rev/s.

(b) The angular acceleration was $(10 \text{ rev/s} - 2.0 \text{ rev/s})/(15\,\text{s}) = 0.533 \text{ rev/s}^2$. The time required to get the wheel to 2.0 rev/s was $t = (2.0 \text{ rev/s})/(0.533 \text{ rev/s}^2) = 3.8\,\text{s}$.

E8-18 (a) The wheel will rotate through an angle ϕ where

$$\phi = (563 \text{ cm})/(8.14 \text{ cm}/2) = 138 \text{ rad}.$$

(b) $t = \sqrt{2(138 \text{ rad})/(1.47 \text{ rad/s}^2)} = 13.7\,\text{s}$.

$\boxed{\textbf{E8-19}}$ (a) We are given $\phi = 42.3$ rev$= 266$ rad, $\omega_{0z} = 1.44$ rad/s, and $\omega_z = 0$. Assuming a uniform deceleration, the average angular velocity during the interval is

$$\omega_{\text{av},z} = \frac{1}{2}(\omega_{0z} + \omega_z) = 0.72 \text{ rad/s}.$$

Then the time taken for deceleration is given by $\phi = \omega_{\text{av},z}t$, so $t = 369$ s.

(b) The angular acceleration can be found from Eq. 8-6,

$$\begin{aligned}
\omega_z &= \omega_{0z} + \alpha_z t, \\
(0) &= (1.44 \text{ rad/s}) + \alpha_z(369 \text{ s}), \\
\alpha_z &= -3.9 \times 10^{-3} \text{ rad/s}^2.
\end{aligned}$$

(c) We'll solve Eq. 8-7 for t,

$$\begin{aligned}
\phi &= \phi_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2, \\
(133 \text{ rad}) &= (0) + (1.44 \text{ rad/s})t + \frac{1}{2}(-3.9 \times 10^{-3} \text{ rad/s}^2)t^2, \\
0 &= -133 + (1.44\,\text{s}^{-1})t - (-1.95 \times 10^{-3}\text{s}^{-2})t^2.
\end{aligned}$$

Solving this quadratic expression yields two answers: $t = 108$ s and $t = 630$ s.

E8-20 The angular acceleration is $\alpha = (4.96 \text{ rad/s})/(2.33\,\text{s}) = 2.13 \text{ rad/s}^2$. The angle through which the wheel turned while accelerating is $\phi = (2.13 \text{ rad/s}^2)(23.0\,\text{s})^2/2 = 563$ rad. The angular speed at this time is $\omega = (2.13 \text{ rad/s}^2)(23.0\,\text{s}) = 49.0$ rad/s. The wheel spins through an additional angle of $(49.0 \text{ rad/s})(46\,\text{s} - 23\,\text{s}) = 1130$ rad, for a total angle of 1690 rad.

E8-21 $\omega = (14.6\,\text{m/s})/(110\,\text{m}) = 0.133$ rad/s.

E8-22 The linear acceleration is $(25\,\text{m/s} - 12\,\text{m/s})/(6.2\,\text{s}) = 2.1\,\text{m/s}^2$. The angular acceleration is $\alpha = (2.1\,\text{m/s}^2)/(0.75\,\text{m}/2) = 5.6$ rad/s.

$\boxed{\textbf{E8-23}}$ (a) The angular speed is given by $v_T = \omega r$. So $\omega = v_T/r = (28,700\ \text{km/hr})/(3220\ \text{km}) = 8.91$ rad/hr. That's the same thing as 2.48×10^{-3} rad/s.

(b) $a_R = \omega^2 r = (8.91\ \text{rad/h})^2(3220\ \text{km}) = 256000\ \text{km/h}^2$, or

$$a_R = 256000\ \text{km/h}^2(1/3600\ \text{h/s})^2(1000\ \text{m/km}) = 19.8\,\text{m/s}^2.$$

(c) If the speed is constant then the tangential acceleration is zero, regardless of the shape of the trajectory!

E8-24 The bar needs to make

$$(1.50\ \text{cm})(12.0\ \text{turns/cm}) = 18\ \text{turns}.$$

This will happen is (18 rev)/(237 rev/min) = 4.56 s.

E8-25 (a) The angular speed is $\omega = (2\pi\ \text{rad})/(86400\,\text{s}) = 7.27\times10^{-5}$ rad/s.

(b) The distance from the polar axis is $r = (6.37\times10^6\text{m})\cos(40^\circ) = 4.88\times10^6\text{m}$. The linear speed is then $v = (7.27\times10^{-5}\ \text{rad/s})(4.88\times10^6\text{m}) = 355\,\text{m/s}$.

(c) The angular speed is the same as part (a). The distance from the polar axis is $r = (6.37\times10^6\text{m})\cos(0^\circ) = 6.37\times10^6\text{m}$. The linear speed is then $v = (7.27\times10^{-5}\ \text{rad/s})(6.37\times10^6\text{m}) = 463\,\text{m/s}$.

E8-26 (a) $a_\text{T} = (14.2\ \text{rad/s}^2)(0.0283\,\text{m}) = 0.402\,\text{m/s}^2$.

(b) Full speed is $\omega = 289$ rad/s. $a_\text{R} = (289\ \text{rad/s})^2(0.0283\,\text{m}) = 2360\,\text{m/s}^2$.

(c) It takes

$$t = (289\ \text{rad/s})/(14.2\ \text{rad/s}^2) = 20.4\,\text{s}$$

to get up to full speed. Then $x = (0.402\,\text{m/s}^2)(20.4\,\text{s})^2/2 = 83.6\,\text{m}$ is the distance through which a point on the rim moves.

$\boxed{\textbf{E8-27}}$ (a) The pilot sees the propeller rotate, no more. So the tip of the propeller is moving with a tangential velocity of $v_T = \omega r = (2000\ \text{rev/min})(2\pi\ \text{rad/rev})(1.5\ \text{m}) = 18900$ m/min. This is the same thing as 315 m/s.

(b) The observer on the ground sees this tangential motion and sees the forward motion of the plane. These two velocity components are perpendicular, so the magnitude of the sum is $\sqrt{(315\,\text{m/s})^2 + (133\,\text{m/s})^2} = 342\,\text{m/s}$.

E8-28 $a_\text{T} = a_\text{R}$ when $r\alpha = r\omega^2 = r(\alpha t)^2$, or $t = \sqrt{1/(0.236\ \text{rad/s}^2)} = 2.06\,\text{s}$.

E8-29 (a) $a_\text{R} = r\omega^2 = r\alpha^2 t^2$.

(b) $a_\text{T} = r\alpha$.

(c) Since $a_\text{R} = a_\text{T}\tan(57.0^\circ)$, $t = \sqrt{\tan(57.0^\circ)/\alpha}$. Then

$$\phi = \frac{1}{2}\alpha t^2 = \frac{1}{2}\tan(57.0^\circ) = 0.77\ \text{rad} = 44.1^\circ.$$

E8-30 (a) The tangential speed of the edge of the wheel relative axle is $v = 27\,\text{m/s}$. $\omega = (27\,\text{m/s})/(0.38\,\text{m}) = 71$ rad/s.

(b) The average angular speed while slowing is 71 rad/s/2, the time required to stop is then $t = (30\times2\pi\ \text{rad})/(71\ \text{rad/s}/2) = 5.3\,\text{s}$. The angular acceleration is then $\alpha = (-71\ \text{rad/s})/(5.3\,\text{s}) = -13$ rad/s.

(c) The car moves forward $(27\,\text{m/s}/2)(5.3\,\text{s}) = 72\,\text{m}$.

E8-31 Yes, the speed would be wrong. The angular velocity of the small wheel would be $\omega = v_\mathrm{t}/r_\mathrm{s}$, but the reported velocity would be $v = \omega r_\mathrm{l} = v_\mathrm{t} r_\mathrm{l}/r_\mathrm{s}$. This would be in error by a fraction

$$\frac{\Delta v}{v_\mathrm{t}} = \frac{(72\text{ cm})}{(62\text{ cm})} - 1 = 0.16.$$

E8-32 (a) Square both equations and then add them:

$$x^2 + y^2 = (R\cos\omega t)^2 + (R\sin\omega t)^2 = R^2,$$

which is the equation for a circle of radius R.

(b) $v_x = -R\omega\sin\omega t = -\omega y$; $v_y = R\omega\cos\omega t = \omega x$. Square and add, $v = \omega R$. The direction is tangent to the circle.

(b) $a_x = -R\omega^2\cos\omega t = -\omega^2 x$; $a_y = -R\omega^2\sin\omega t = -\omega^2 y$. Square and add, $a = \omega^2 R$. The direction is toward the center.

$\boxed{\textbf{E8-33}}$ (a) The object is "slowing down", so $\vec{\alpha} = (-2.66\text{ rad/s}^2)\hat{\mathbf{k}}$. We know the direction because it is rotating about the z axis and we are given the direction of $\vec{\omega}$. Then from Eq. 8-19, $\vec{\mathbf{v}} = \vec{\omega}\times\vec{\mathbf{R}} = (14.3\text{ rad/s})\hat{\mathbf{k}}\times[(1.83\text{ m})\hat{\mathbf{j}} + (1.26\text{ m})\hat{\mathbf{k}}]$. But only the cross term $\hat{\mathbf{k}}\times\hat{\mathbf{j}}$ survives, so $\vec{\mathbf{v}} = (-26.2\,\mathrm{m/s})\hat{\mathbf{i}}$.

(b) We find the acceleration from Eq. 8-21,

$$\begin{aligned}
\vec{\mathbf{a}} &= \vec{\alpha}\times\vec{\mathbf{R}} + \vec{\omega}\times\vec{\mathbf{v}},\\
&= (-2.66\text{ rad/s}^2)\hat{\mathbf{k}}\times[(1.83\,\mathrm{m})\hat{\mathbf{j}} + (1.26\,\mathrm{m})\hat{\mathbf{k}}] + (14.3\text{ rad/s})\hat{\mathbf{k}}\times(-26.2\,\mathrm{m/s})\hat{\mathbf{i}},\\
&= (4.87\,\mathrm{m/s^2})\hat{\mathbf{i}} + (-375\,\mathrm{m/s^2})\hat{\mathbf{j}}.
\end{aligned}$$

E8-34 (a) $\vec{\mathbf{F}} = -2m\vec{\omega}\times\vec{\mathbf{v}} = -2m\omega v\cos\theta$, where θ is the latitude. Then

$$F = 2(12\,\mathrm{kg})(2\pi\text{ rad}/86400\,\mathrm{s})(35\,\mathrm{m/s})\cos(45^\circ) = 0.043\,\mathrm{N},$$

and is directed *west*.

(b) Reversing the velocity will reverse the direction, so *east*.

(c) No. The Coriolis force pushes it to the west on the way up and gives it a westerly velocity; on the way down the Coriolis force slows down the westerly motion, but does not push it back east. The object lands to the west of the starting point.

P8-1 (a) $\omega = (4.0\text{ rad/s}) - (6.0\text{ rad/s}^2)t + (3.0\text{ rad/s})t^2$. Then $\omega(2.0\,\mathrm{s}) = 4.0$ rad/s and $\omega(4.0\,\mathrm{s}) = 28.0$ rad/s.

(b) $\alpha_\mathrm{av} = (28.0\text{ rad/s} - 4.0\text{ rad/s})/(4.0\,\mathrm{s} - 2.0\,\mathrm{s}) = 12\text{ rad/s}^2$.

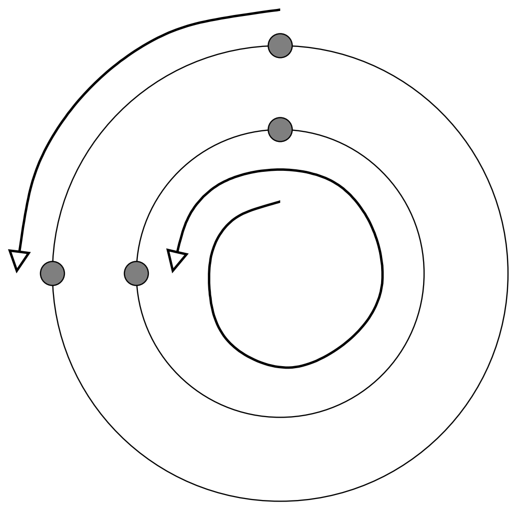
(c) $\alpha = -(6.0\text{ rad/s}^2) + (6.0\text{ rad/s})t$. Then $\alpha(2.0\,\mathrm{s}) = 6.0\text{ rad/s}^2$ and $\alpha(4.0\,\mathrm{s}) = 18.0\text{ rad/s}^2$.

P8-2 If the wheel really does move counterclockwise at 4.0 rev/min, then it turns through

$$(4.0\text{ rev/min})/[(60\text{ s/min})(24\text{ frames/s})] = 2.78\times10^{-3}\text{ rev/frame}.$$

This means that a spoke has moved 2.78×10^{-3} rev. There are 16 spokes each located 1/16 of a revolution around the wheel. If instead of moving counterclockwise the wheel was instead moving clockwise so that a different spoke had moved 1/16 rev $-$ 2.78×10^{-3} rev $= 0.0597$ rev, then the same effect would be present. The wheel then would be turning clockwise with a speed of $\omega = (0.0597\text{ rev})(60\text{ s/min})(24\text{ frames/s}) = 86$ rev/min.

P8-3 (a) In the diagram below the Earth is shown at two locations a day apart. The Earth rotates clockwise in this figure.



Note that the Earth rotates through 2π rad in order to be correctly oriented for a complete sidereal day, but because the Earth has moved in the orbit it needs to go farther through an angle θ in order to complete a solar day. By the time the Earth has gone all of the way around the sun the total angle θ will be 2π rad, which means that there was one more sidereal day than solar day.

(b) There are (365.25 d)(24.000 h/d) $= 8.7660\times10^3$ hours in a year with 265.25 solar days. But there are 366.25 sidereal days, so each one has a length of $8.7660\times10^3/366.25 = 23.934$ hours, or 23 hours and 56 minutes and 4 seconds.

P8-4 (a) The period is time per complete rotation, so $\omega = 2\pi/T$.

(b) $\alpha = \Delta\omega/\Delta t$, so

$$\begin{aligned}
\alpha &= \left(\frac{2\pi}{T_0+\Delta T} - \frac{2\pi}{T_0}\right)/(\Delta t),\\
&= \frac{2\pi}{\Delta t}\left(\frac{-\Delta T}{T_0(T_0+\Delta T)}\right),\\
&\approx \frac{2\pi}{\Delta t}\frac{-\Delta T}{T_0^2},\\
&= \frac{2\pi}{(3.16\times10^7\mathrm{s})}\frac{-(1.26\times10^{-5}\mathrm{s})}{(0.033\,\mathrm{s})^2} = -2.30\times10^{-9}\mathrm{rad/s}^2.
\end{aligned}$$

(c) $t = (2\pi/0.033\,\mathrm{s})/(2.30\times10^{-9}\mathrm{rad/s}^2) = 8.28\times10^{10}\mathrm{s}$, or 2600 years.

(d) $2\pi/T_0 = 2\pi/T - \alpha t$, or

$$T_0 = \Big(1/(0.033\,\mathrm{s}) - (-2.3\times10^{-9}\mathrm{rad/s}^2)(3.0\times10^{10}\,\mathrm{s})/(2\pi)\Big)^{-1} = 0.024\,\mathrm{s}.$$

$\boxed{\textbf{P8-5}}$ The final angular velocity during the acceleration phase is $\omega_z = \alpha_z t =$ (3.0 rad/s)(4.0 s) $=$ 12.0 rad/s. Since both the acceleration and deceleration phases are uniform with endpoints $\omega_z = 0$, the average angular velocity for both phases is the same, and given by half of the maximum: $\omega_{\mathrm{av},z} = 6.0$ rad/s.

The angle through which the wheel turns is then

$$\phi = \omega_{\mathrm{av},z} t = (6.0 \text{ rad/s})(4.1\,\text{s}) = 24.6 \text{ rad}.$$

The time is the total for *both* phases.

(a) The first student sees the wheel rotate through the smallest angle less than one revolution; this student would have no idea that the disk had rotated more than once. Since the disk moved through 3.92 revolutions, the first student will either assume the disk moved forward through 0.92 revolutions or backward through 0.08 revolutions.

(b) According to whom? We've already answered from the perspective of the second student.

P8-6 $\omega = (0.652 \text{ rad/s}^2)t$ and $\alpha = (0.652 \text{ rad/s}^2)$.

(a) $\omega = (0.652 \text{ rad/s}^2)(5.60\,\text{s}) = 3.65 \text{ rad/s}$

(b) $v_\mathrm{T} = \omega r = (3.65 \text{ rad/s})(10.4\,\text{m}) = 38\,\text{m/s}$.

(c) $a_\mathrm{T} = \alpha r = (0.652 \text{ rad/s}^2)(10.4\,\text{m}) = 6.78\,\text{m/s}^2$.

(d) $a_\mathrm{R} = \omega^2 r = (3.65 \text{ rad/s})^2(10.4\,\text{m}) = 139\,\text{m/s}^2$.

P8-7 (a) $\omega = (2\pi \text{ rad})/(3.16\times10^7\,\text{s}) = 1.99\times10^{-7} \text{ rad/s}$.

(b) $v_\mathrm{T} = \omega R = (1.99\times10^{-7} \text{ rad/s})(1.50\times10^{11}\text{m}) = 2.99\times10^4\text{m/s}$.

(c) $a_\mathrm{R} = \omega^2 R = (1.99\times10^{-7} \text{ rad/s})^2(1.50\times10^{11}\text{m}) = 5.94\times10^{-3}\text{m/s}^2$.

P8-8 (a) $\alpha = (-156 \text{ rev/min})/(2.2 \times 60 \text{ min}) = -1.18 \text{ rev/min}^2$.

(b) The average angular speed while slowing down is 78 rev/min, so the wheel turns through (78 rev/min)(2.2 × 60 min) = 10300 revolutions.

(c) $a_\mathrm{T} = (2\pi \text{ rad/rev})(-1.18 \text{ rev/min}^2)(0.524\,\text{m}) = -3.89 \text{ m/min}^2$. That's the same as $-1.08\times 10^{-3}\text{m/s}^2$.

(d) $a_\mathrm{R} = (2\pi \text{ rad/rev})(72.5 \text{ rev/min})^2(0.524\,\text{m}) = 1.73\times10^4 \text{ m/min}^2$. That's the same as 4.81m/s^2. This is so much larger than the a_T term that the magnitude of the total linear acceleration is simply 4.81m/s^2.

$\boxed{\textbf{P8-9}}$ (a) There are 500 teeth (and 500 spaces between these teeth); so disk rotates $2\pi/500$ rad between the outgoing light pulse and the incoming light pulse. The light traveled 1000 m, so the elapsed time is $t = (1000\,\text{m})/(3\times10^8 \text{ m/s}) = 3.33\times10^{-6}\text{s}$.

Then the angular speed of the disk is $\omega_z = \phi/t = 1.26\times10^{-2} \text{ rad})/(3.33\times10^{-6}\text{s}) = 3800 \text{ rad/s}$.

(b) The linear speed of a point on the edge of the would be

$$v_T = \omega R = (3800 \text{ rad/s})(0.05\,\text{m}) = 190\,\text{m/s}.$$

P8-10 The linear acceleration of the belt is $a = \alpha_A r_A$. The angular acceleration of C is $\alpha_C = a/r_C = \alpha_A(r_A/r_C)$. The time required for C to get up to speed is

$$t = \frac{(2\pi \text{ rad/rev})(100 \text{ rev/min})(1/60 \text{ min/s})}{(1.60 \text{ rad/s}^2)(10.0/25.0)} = 16.4\,\text{s}.$$

P8-11 (a) The final angular speed is $\omega_\mathrm{o} = (130 \text{ cm/s})/(5.80 \text{ cm}) = 22.4 \text{ rad/s}$.

(b) The recording area is $\pi({R_\mathrm{o}}^2 - {R_\mathrm{i}}^2)$, the recorded track has a length l and width w, so

$$l = \frac{\pi[(5.80 \text{ cm})^2 - (2.50 \text{ cm}^2)]}{(1.60\times10^{-4} \text{ cm})} = 5.38\times10^5 \text{ cm}.$$

(c) Playing time is $t = (5.38\times10^5 \text{ cm})/(130 \text{ cm/s}) = 4140\,\text{s}$, or 69 minutes.

P8-12 The angular position is given by $\phi = \arctan(vt/b)$. The derivative (Maple!) is

$$\omega = \frac{vb}{b^2 + v^2t^2},$$

and is directed *up*. Take the derivative again,

$$\alpha = \frac{2bv^3t}{(b^2 + v^2t^2)^2},$$

but is directed down.

$\boxed{\textbf{P8-13}}$ (a) Let the rocket sled move along the line $x = b$. The observer is at the origin and sees the rocket move with a constant angular speed, so the angle made with the x axis increases according to $\theta = \omega t$. The observer, rocket, and starting point form a right triangle; the position y of the rocket is the opposite side of this triangle, so

$$\tan\theta = y/b \text{ implies } y = b/\tan\omega t.$$

We want to take the derivative of this with respect to time and get

$$v(t) = \omega b/\cos^2(\omega t).$$

(b) The speed becomes infinite (which is clearly unphysical) when $t = \pi/2\omega$.

$\boxed{\textbf{E9-1}}$ (a) First, $\vec{\mathbf{F}} = (5.0 \text{ N})\hat{\mathbf{i}}$.

$$\begin{aligned}\vec{\tau} &= [yF_z - zF_y]\hat{\mathbf{i}} + [zF_x - xF_z]\hat{\mathbf{j}} + [xF_y - yF_x]\hat{\mathbf{k}},\\ &= [y(0) - (0)(0)]\hat{\mathbf{i}} + [(0)F_x - x(0)]\hat{\mathbf{j}} + [x(0) - yF_x]\hat{\mathbf{k}},\\ &= [-yF_x]\hat{\mathbf{k}} = -(3.0 \text{ m})(5.0 \text{ N})\hat{\mathbf{k}} = -(15.0 \text{ N}\cdot\text{m})\hat{\mathbf{k}}.\end{aligned}$$

(b) Now $\vec{\mathbf{F}} = (5.0 \text{ N})\hat{\mathbf{j}}$. Ignoring all zero terms,

$$\vec{\tau} = [xF_y]\hat{\mathbf{k}} = (2.0 \text{ m})(5.0 \text{ N})\hat{\mathbf{k}} = (10 \text{ N}\cdot\text{m})\hat{\mathbf{k}}.$$

(c) Finally, $\vec{\mathbf{F}} = (-5.0 \text{ N})\hat{\mathbf{i}}$.

$$\vec{\tau} = [-yF_x]\hat{\mathbf{k}} = -(3.0 \text{ m})(-5.0 \text{ N})\hat{\mathbf{k}} = (15.0 \text{ N}\cdot\text{m})\hat{\mathbf{k}}.$$

E9-2 (a) Everything is in the plane of the page, so the net torque will either be directed normal to the page. Let out be positive, then the net torque is $\tau = r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2$.

(b) $\tau = (1.30\,\text{m})(4.20\,\text{N})\sin(75.0^\circ) - (2.15\,\text{m})(4.90\,\text{N})\sin(58.0^\circ) = -3.66\text{N}\cdot\text{m}$.

E9-4 Everything is in the plane of the page, so the net torque will either be directed normal to the page. Let out be positive, then the net torque is

$$\tau = (8.0\,\text{m})(10\,\text{N})\sin(45^\circ) - (4.0\,\text{m})(16\,\text{N})\sin(90^\circ) + (3.0\,\text{m})(19\,\text{N})\sin(20^\circ) = 12\text{N}\cdot\text{m}.$$

$\boxed{\textbf{E9-5}}$ Since $\vec{\mathbf{r}}$ and $\vec{\mathbf{s}}$ lie in the xy plane, then $\vec{\mathbf{t}} = \vec{\mathbf{r}} \times \vec{\mathbf{s}}$ must be perpendicular to that plane, and can only point along the z axis.

The angle between $\vec{\mathbf{r}}$ and $\vec{\mathbf{s}}$ is $320^\circ - 85^\circ = 235^\circ$. So $|\vec{\mathbf{t}}| = rs|\sin\theta| = (4.5)(7.3)|\sin(235^\circ)| = 27$.

Now for the direction of $\vec{\mathbf{t}}$. The smaller rotation to bring $\vec{\mathbf{r}}$ into $\vec{\mathbf{s}}$ is through a counterclockwise rotation; the right hand rule would then show that the cross product points along the *positive* z direction.

E9-6 $\vec{\mathbf{a}} = (3.20)[\cos(63.0^\circ)\hat{\mathbf{j}} + \sin(63.0^\circ)\hat{\mathbf{k}}]$ and $\vec{\mathbf{b}} = (1.40)[\cos(48.0^\circ)\hat{\mathbf{i}} + \sin(48.0^\circ)\hat{\mathbf{k}}]$. Then

$$\begin{aligned}\vec{\mathbf{a}} \times \vec{\mathbf{b}} &= (3.20)\cos(63.0^\circ)(1.40)\sin(48.0^\circ)\hat{\mathbf{i}}\\ &\quad +(3.20)\sin(63.0^\circ)(1.40)\cos(48.0^\circ)\hat{\mathbf{j}}\\ &\quad -(3.20)\cos(63.0^\circ)(1.40)\cos(48.0^\circ)\hat{\mathbf{k}}\\ &= 1.51\hat{\mathbf{i}} + 2.67\hat{\mathbf{j}} - 1.36\hat{\mathbf{k}}.\end{aligned}$$

E9-7 $\vec{\mathbf{b}} \times \vec{\mathbf{a}}$ has magnitude $ab\sin\phi$ and points in the negative z direction. It is then perpendicular to $\vec{\mathbf{a}}$, so $\vec{\mathbf{c}}$ has magnitude $a^2b\sin\phi$. The direction of $\vec{\mathbf{c}}$ is perpendicular to $\vec{\mathbf{a}}$ but lies in the plane containing vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. Then it makes an angle $\pi/2 - \phi$ with $\vec{\mathbf{b}}$.

E9-8 (a) In unit vector notation,

$$\begin{aligned}\vec{\mathbf{c}} &= [(-3)(-3) - (-2)(1)]\hat{\mathbf{i}} + [(1)(4) - (2)(-3)]\hat{\mathbf{j}} + [(2)(-2) - (-3)(4)]\hat{\mathbf{k}},\\ &= 11\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 8\hat{\mathbf{k}}.\end{aligned}$$

(b) Evaluate $\arcsin[|\vec{\mathbf{a}} \times b|/(ab)]$, finding magnitudes with the Pythagoras relationship:

$$\phi = \arcsin{(16.8)}/[(3.74)(5.39)] = 56^\circ.$$

$\boxed{\textbf{E9-9}}$ This exercise is a three dimensional generalization of Ex. 9-1, except nothing is zero.

$$
\begin{aligned}
\vec{\tau} &= [yF_z - zF_y]\hat{\mathbf{i}} + [zF_x - xF_z]\hat{\mathbf{j}} + [xF_y - yF_x]\hat{\mathbf{k}}, \\
&= [(-2.0\text{ m})(4.3\text{ N}) - (1.6\text{ m})(-2.4\text{ N})]\hat{\mathbf{i}} + [(1.6\text{ m})(3.5\text{ N}) - (1.5\text{ m})(4.3\text{ N})]\hat{\mathbf{j}} \\
&\quad + [(1.5\text{ m})(-2.4\text{ N}) - (-2.0\text{ m})(3.5\text{ N})]\hat{\mathbf{k}}, \\
&= [-4.8\,\text{N}\cdot\text{m}]\hat{\mathbf{i}} + [-0.85\,\text{N}\cdot\text{m}]\hat{\mathbf{j}} + [3.4\,\text{N}\cdot\text{m}]\hat{\mathbf{k}}.
\end{aligned}
$$

E9-10 (a) $\vec{\mathbf{F}} = (2.6\,\text{N})\hat{\mathbf{i}}$, then $\vec{\tau} = (0.85\,\text{m})(2.6\,\text{N})\hat{\mathbf{j}} - (-0.36\,\text{m})(2.6\,\text{N})\hat{\mathbf{k}} = 2.2\,\text{N}\cdot\text{m}\hat{\mathbf{j}} + 0.94\,\text{N}\cdot\text{m}\hat{\mathbf{k}}$.
(b) $\vec{\mathbf{F}} = (-2.6\,\text{N})\hat{\mathbf{k}}$, then $\vec{\tau} = (-0.36\,\text{m})(-2.6\,\text{N})\hat{\mathbf{i}} - (0.54\,\text{m})(-2.6\,\text{N})\hat{\mathbf{j}} = 0.93\,\text{N}\cdot\text{m}\hat{\mathbf{i}} + 1.4\,\text{N}\cdot\text{m}\hat{\mathbf{j}}$.

E9-11 (a) The rotational inertia about an axis through the origin is

$$I = mr^2 = (0.025\,\text{kg})(0.74\,\text{m})^2 = 1.4\times10^{-2}\text{kg}\cdot\text{m}^2.$$

(b) $\alpha = (0.74\,\text{m})(22\,\text{N})\sin(50^\circ)/(1.4\times10^{-2}\text{kg}\cdot\text{m}^2) = 890$ rad/s.

E9-12 (a) $I_0 = (0.052\,\text{kg})(0.27\,\text{m})^2 + (0.035\,\text{kg})(0.45\,\text{m})^2 + (0.024\,\text{kg})(0.65\,\text{m})^2 = 2.1\times10^{-2}\text{kg}\cdot\text{m}^2$.
(b) The center of mass is located at

$$x_{\text{cm}} = \frac{(0.052\,\text{kg})(0.27\,\text{m}) + (0.035\,\text{kg})(0.45\,\text{m}) + (0.024\,\text{kg})(0.65\,\text{m})}{(0.052\,\text{kg}) + (0.035\,\text{kg}) + (0.024\,\text{kg})} = 0.41\,\text{m}.$$

Applying the parallel axis theorem yields $I_{\text{cm}} = 2.1\times10^{-2}\text{kg}\cdot\text{m}^2 - (0.11\,\text{kg})(0.41\,\text{m})^2 = 2.5\times10^{-3}\text{kg}\cdot\text{m}^2$.

$\boxed{\textbf{E9-13}}$ (a) Rotational inertia is additive so long as we consider the inertia about the same axis. We can use Eq. 9-10:

$$I = \sum m_n r_n^2 = (0.075\,\text{kg})(0.42\,\text{m})^2 + (0.030\,\text{kg})(0.65\,\text{m})^2 = 0.026\,\text{kg}\cdot\text{m}^2.$$

(b) No change.

E9-14 $\vec{\tau} = [(0.42\,\text{m})(2.5\,\text{N}) - (0.65\,\text{m})(3.6\,\text{N})]\hat{\mathbf{k}} = -1.29\,\text{N}\cdot\text{m}\,\hat{\mathbf{k}}$. Using the result from E9-13, $\vec{\alpha} = (-1.29\,\text{N}\cdot\text{m}\,\hat{\mathbf{k}})/(0.026\,\text{kg}\cdot\text{m}^2) = 50\text{ rad/s}^2\hat{\mathbf{k}}$. That's clockwise if viewed from above.

E9-15 (a) $F = m\omega^2 r = (110\,\text{kg})(33.5\text{ rad/s})^2(3.90\,\text{m}) = 4.81\times10^5\text{N}$.
(b) The angular acceleration is $\alpha = (33.5\text{ rad/s})/(6.70\,\text{s}) = 5.00\text{ rad/s}^2$. The rotational inertia about the axis of rotation is $I = (110\,\text{kg})(7.80\,\text{m})^2/3 = 2.23\times10^3\text{kg}\cdot\text{m}^2$. $\tau = I\alpha = (2.23\times10^3\text{kg}\cdot\text{m}^2)(5.00\text{ rad/s}^2) = 1.12\times10^4\text{N}\cdot\text{m}$.

E9-16 We can add the inertias for the three rods together,

$$I = 3\left(\frac{1}{3}ML^2\right) = (240\,\text{kg})(5.20\,\text{m})^2 = 6.49\times10^3\text{kg}\cdot\text{m}^2.$$

$\boxed{\textbf{E9-17}}$ The diagonal distance from the axis through the center of mass and the axis through the edge is $h = \sqrt{(a/2)^2 + (b/2)^2}$, so

$$I = I_{cm} + Mh^2 = \frac{1}{12}M\left(a^2 + b^2\right) + M\left((a/2)^2 + (b/2)^2\right) = \left(\frac{1}{12} + \frac{1}{4}\right)M\left(a^2 + b^2\right).$$

Simplifying, $I = \frac{1}{3}M\left(a^2 + b^2\right)$.

E9-18 $I = I_{cm} + Mh^2 = (0.56\,\text{kg})(1.0\,\text{m})^2/12 + (0.56)(0.30\,\text{m})^2 = 9.7\times 10^{-2}\text{kg}\cdot\text{m}^2$.

$\boxed{\textbf{E9-19}}$ For particle one $I_1 = mr^2 = mL^2$; for particle two $I_2 = mr^2 = m(2L)^2 = 4mL^2$. The rotational inertia of the rod is $I_{\text{rod}} = \frac{1}{3}(2M)(2L)^2 = \frac{8}{3}ML^2$. Add the three inertias:

$$I = \left(5m + \frac{8}{3}M\right)L^2.$$

E9-20 (a) $I = MR^2/2 = M(R/\sqrt{2})^2$.

(b) Let I be the rotational inertia. Assuming that k is the radius of a hoop with an equivalent rotational inertia, then $I = Mk^2$, or $k = \sqrt{I/M}$.

E9-21 Note the mistakes in the equation in the part (b) of the exercise text.

(a) $m_n = M/N$.

(b) Each piece has a thickness $t = L/N$, the distance from the end to the nth piece is $x_n = (n-1/2)t = (n-1/2)L/N$. The axis of rotation is the center, so the distance from the center is $r_n = x_n - L/2 = nL/N - (1 + 1/2N)L$.

(c) The rotational inertia is

$$\begin{aligned}
I &= \sum_{n=1}^{N} m_n r_n^2, \\
&= \frac{ML^2}{N^3}\sum_{n=1}^{N}(n - 1/2 - N)^2, \\
&= \frac{ML^2}{N^3}\sum_{n=1}^{N}\left(n^2 - (2N+1)n + (N+1/2)^2\right), \\
&= \frac{ML^2}{N^3}\left(\frac{N(N+1)(2N+1)}{6} - (2N+1)\frac{N(N+1)}{2} + (N+1/2)^2 N\right), \\
&\approx \frac{ML^2}{N^3}\left(\frac{2N^3}{6} - \frac{2N^3}{2} + N^3\right), \\
&= ML^2/3.
\end{aligned}$$

E9-22 $F = (46\,\text{N})(2.6\text{ cm})/(13\text{ cm}) = 9.2\,\text{N}$.

E9-23 Tower topples when center of gravity is no longer above base. Assuming center of gravity is located at the very center of the tower, then when the tower leans 7.0 m then the tower falls. This is 2.5 m farther than the present.

(b) $\theta = \arcsin(7.0\,\text{m}/55\,\text{m}) = 7.3^\circ$.

E9-24 If the torque from the force is sufficient to lift edge the cube then the cube will tip. The net torque about the edge which stays in contact with the ground will be $\tau = Fd - mgd/2$ if F is sufficiently large. Then $F \geq mg/2$ is the minimum force which will cause the cube to tip.

The minimum force to get the cube to slide is $F \geq \mu_s mg = (0.46)mg$. The cube will slide first.

$\boxed{\textbf{E9-25}}$ The ladder slips if the force of static friction required to keep the ladder up exceeds $\mu_s N$. Equations 9-31 give us the normal force in terms of the masses of the ladder and the firefighter, $N = (m + M)g$, and is independent of the location of the firefighter on the ladder. Also from Eq. 9-31 is the relationship between the force from the wall and the force of friction; the condition at which slipping occurs is $F_w \geq \mu_s(m + M)g$.

Now go straight to Eq. 9-32. The $a/2$ in the second term is the location of the firefighter, who in the example was halfway between the base of the ladder and the top of the ladder. In the exercise we don't know where the firefighter is, so we'll replace $a/2$ with x. Then

$$-F_w h + Mgx + \frac{mga}{3} = 0$$

is an expression for rotational equilibrium. Substitute in the condition of F_w when slipping just starts, and we get

$$-\left(\mu_s(m + M)g\right)h + Mgx + \frac{mga}{3} = 0.$$

Solve this for x,

$$x = \mu_s\left(\frac{m}{M} + 1\right)h - \frac{ma}{3M} = (0.54)\left(\frac{45\text{ kg}}{72\text{ kg}} + 1\right)(9.3\text{ m}) - \frac{(45\text{ kg})(7.6\text{ m})}{3(72\text{ kg})} = 6.6\text{ m}$$

This is the horizontal distance; the fraction of the total length along the ladder is then given by $x/a = (6.6\text{ m})/(7.6\text{ m}) = 0.87$. The firefighter can climb $(0.87)(12\,\text{m}) = 10.4\,\text{m}$ up the ladder.

E9-26 (a) The net torque about the rear axle is $(1360\,\text{kg})(9.8\,\text{m/s}^2)(3.05\,\text{m}-1.78\,\text{m})-F_\text{f}(3.05\,\text{m}) = 0$, which has solution $F_\text{f} = 5.55\times10^3\text{N}$. Each of the front tires support half of this, or $2.77\times10^3\text{N}$.

(b) The net torque about the front axle is $(1360\,\text{kg})(9.8\,\text{m/s}^2)(1.78\,\text{m}) - F_\text{f}(3.05\,\text{m}) = 0$, which has solution $F_\text{f} = 7.78\times10^3\text{N}$. Each of the front tires support half of this, or $3.89\times10^3\text{N}$.

E9-27 The net torque on the bridge about the end closest to the person is

$$(160\text{ lb})L/4 + (600\text{ lb})L/2 - F_\text{f}L = 0,$$

which has a solution for the supporting force on the far end of $F_\text{f} = 340$ lb.

The net force on the bridge is $(160\text{ lb})L/4 + (600\text{ lb})L/2 - (340\text{ lb}) - F_\text{c} = 0$, so the force on the close end of the bridge is $F_\text{c} = 420$ lb.

E9-28 The net torque on the board about the left end is

$$F_\text{r}(1.55\,\text{m}) - (142\,\text{N})(2.24\,\text{m}) - (582\,\text{N})(4.48\,\text{m}) = 0,$$

which has a solution for the supporting force for the right pedestal of $F_\text{r} = 1890\,\text{N}$. The force on the board from the pedestal is up, so the force on the pedestal from the board is down (compression).

The net force on the board is $F_\text{l} + (1890\,\text{N}) - (142\,\text{N}) - (582\,\text{N}) = 0$, so the force from the pedestal on the left is $F_\text{l} = -1170\,\text{N}$. The negative sign means up, so the pedestal is under tension.

$\boxed{\textbf{E9-29}}$ We can assume that both the force $\vec{\mathbf{F}}$ and the force of gravity $\vec{\mathbf{W}}$ act on the center of the wheel. Then the wheel will just start to lift when

$$\vec{\mathbf{W}} \times \vec{\mathbf{r}} + \vec{\mathbf{F}} \times \vec{\mathbf{r}} = 0,$$

or

$$W \sin\theta = F \cos\theta,$$

where θ is the angle between the vertical (pointing down) and the line between the center of the wheel and the point of contact with the step. The use of the sine on the left is a straightforward application of Eq. 9-2. Why the cosine on the right? Because

$$\sin(90^\circ - \theta) = \cos\theta.$$

Then $F = W\tan\theta$. We can express the angle θ in terms of trig functions, h, and r. $r\cos\theta$ is the vertical distance from the center of the wheel to the top of the step, or $r - h$. Then

$$\cos\theta = 1 - \frac{h}{r} \text{ and } \sin\theta = \sqrt{1 - \left(1 - \frac{h}{r}\right)^2}.$$

Finally by combining the above we get

$$F = W\frac{\sqrt{\frac{2h}{r} - \frac{h^2}{r^2}}}{1 - \frac{h}{r}} = W\frac{\sqrt{2hr - h^2}}{r - h}.$$

E9-30 (a) Assume that each of the two support points for the square sign experience the same tension, equal to half of the weight of the sign. The net torque on the rod about an axis through the hinge is

$$(52.3\,\text{kg}/2)(9.81\,\text{m/s}^2)(0.95\,\text{m}) + (52.3\,\text{kg}/2)(9.81\,\text{m/s}^2)(2.88\,\text{m}) - (2.88\,\text{m})T\sin\theta = 0,$$

where T is the tension in the cable and θ is the angle between the cable and the rod. The angle can be found from $\theta = \arctan(4.12\,\text{m}/2.88\,\text{m}) = 55.0^\circ$, so $T = 416\,\text{N}$.

(b) There are two components to the tension, one which is vertical, $(416\,\text{N})\sin(55.0^\circ) = 341\,\text{N}$, and another which is horizontal, $(416\,\text{N})\cos(55.0^\circ) = 239\,\text{N}$. The horizontal force exerted by the wall must then be $239\,\text{N}$. The net vertical force on the rod is $F + (341\,\text{N}) - (52.3\,\text{kg}/2)(9.81\,\text{m/s}^2) = 0$, which has solution $F = 172\,\text{N}$ as the vertical upward force of the wall on the rod.

E9-31 (a) The net torque on the rod about an axis through the hinge is

$$\tau = W(L/2)\cos(54.0^\circ) - TL\sin(153.0^\circ) = 0.$$

or $T = (52.7\text{ lb}/2)(\sin 54.0^\circ/\sin 153.0^\circ) = 47.0$ lb.

(b) The vertical upward force of the wire on the rod is $T_y = T\cos(27.0^\circ)$. The vertical upward force of the wall on the rod is $P_y = W - T\cos(27.0^\circ)$, where W is the weight of the rod. Then

$$P_y = (52.7\text{ lb}) - (47.0\text{ lb})\cos(27.0^\circ) = 10.8\text{ lb}$$

The horizontal force from the wall is balanced by the horizontal force from the wire. Then $P_x = (47.0\text{ lb})\sin(27.0^\circ) = 21.3$ lb.

E9-32 If the ladder is not slipping then the torque about an axis through the point of contact with the ground is

$$\tau = (WL/2)\cos\theta - Nh/\sin\theta = 0,$$

where N is the normal force of the edge on the ladder. Then $N = WL\cos\theta\,\sin\theta\,/(2h)$.

N has two components; one which is vertically up, $N_y = N\cos\theta$, and another which is horizontal, $N_x = N\sin\theta$. The horizontal force must be offset by the static friction.

The normal force on the ladder from the ground is given by

$$N_\text{g} = W - N\cos\theta = W[1 - L\cos^2\theta\,\sin\theta\,/(2h)].$$

The force of static friction can be as large as $f = \mu_s N_g$, so

$$\mu_s = \frac{WL\cos\theta\,\sin^2\theta\,/(2h)}{W[1 - L\cos^2\theta\,\sin\theta\,/(2h)]} = \frac{L\cos\theta\,\sin^2\theta}{2h - L\cos^2\theta\,\sin\theta}.$$

Put in the numbers and $\theta = 68.0°$. Then $\mu_s = 0.407$.

E9-33 Let out be positive. The net torque about the axis is then

$$\tau = (0.118\,\mathrm{m})(5.88\,\mathrm{N}) - (0.118\,\mathrm{m})(4.13\,\mathrm{m}) - (0.0493\,\mathrm{m})(2.12\,\mathrm{N}) = 0.102\,\mathrm{N}\cdot\mathrm{m}.$$

The rotational inertia of the disk is $I = (1.92\,\mathrm{kg})(0.118\,\mathrm{m})^2/2 = 1.34\times10^{-2}\mathrm{kg}\cdot\mathrm{m}^2$. Then $\alpha = (0.102\,\mathrm{N}\cdot\mathrm{m})/(1.34\times10^{-2}\mathrm{kg}\cdot\mathrm{m}^2) = 7.61\ \mathrm{rad/s}^2$.

E9-34 (a) $I = \tau/\alpha = (960\,\mathrm{N}\cdot\mathrm{m})/(6.23\ \mathrm{rad/s}^2) = 154\,\mathrm{kg}\cdot\mathrm{m}^2$.
(b) $m = (3/2)I/r^2 = (1.5)(154\,\mathrm{kg}\cdot\mathrm{m}^2)/(1.88\,\mathrm{m})^2 = 65.4\,\mathrm{kg}$.

E9-35 (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{6.20\ \mathrm{rad/s}}{0.22\ \mathrm{s}} = 28.2\ \mathrm{rad/s}^2$$

(b) From Eq. 9-11, $\tau = I\alpha = (12.0\,\mathrm{kg}\cdot\mathrm{m}^2)(28.2\ \mathrm{rad/s}^2) = 338\,\mathrm{N}\cdot\mathrm{m}$.

E9-36 The angular acceleration is $\alpha = 2(\pi/2\ \mathrm{rad})/(30\,\mathrm{s})^2 = 3.5\times10^{-3}\ \mathrm{rad/s}^2$. The required force is then

$$F = \tau/r = I\alpha/r = (8.7\times10^4\mathrm{kg}\cdot\mathrm{m}^2)(3.5\times10^{-3}\ \mathrm{rad/s}^2)/(2.4\,\mathrm{m}) = 127\,\mathrm{N}.$$

Don't let the door slam...

E9-37 The torque is $\tau = rF$, the angular acceleration is $\alpha = \tau/I = rF/I$. The angular velocity is

$$\omega = \int_0^t \alpha\,dt = \frac{rAt^2}{2I} + \frac{rBt^3}{3I},$$

so when $t = 3.60\,\mathrm{s}$,

$$\omega = \frac{(9.88\times10^{-2}\mathrm{m})(0.496\,\mathrm{N/s})(3.60\,\mathrm{s})^2}{2(1.14\times10^{-3}\mathrm{kg}\cdot\mathrm{m}^2)} + \frac{(9.88\times10^{-2}\mathrm{m})(0.305\,\mathrm{N/s}^2)(3.60\,\mathrm{s})^3}{3(1.14\times10^{-3}\mathrm{kg}\cdot\mathrm{m}^2)} = 690\ \mathrm{rad/s}.$$

E9-38 (a) $\alpha = 2\theta/t^2$.
(b) $a = \alpha R = 2\theta R/t^2$.
(c) T_1 and T_2 are *not* equal. Instead, $(T_1 - T_2)R = I\alpha$. For the hanging block $Mg - T_1 = Ma$. Then

$$T_1 = Mg - 2MR\theta/t^2,$$

and

$$T_2 = Mg - 2MR\theta/t^2 - 2(I/R)\theta/t^2.$$

$\boxed{\textbf{E9-39}}$ Apply a kinematic equation from chapter 2 to find the acceleration:

$$\begin{aligned} y &= v_{0y}t + \frac{1}{2}a_y t^2, \\ a_y &= \frac{2y}{t^2} = \frac{2(0.765 \text{ m})}{(5.11 \text{ s})^2} = 0.0586 \text{ m/s}^2 \end{aligned}$$

Closely follow the approach in Sample Problem 9-10. For the heavier block, $m_1 = 0.512$ kg, and Newton's second law gives

$$m_1 g - T_1 = m_1 a_y,$$

where a_y is positive and *down*. For the lighter block, $m_2 = 0.463$ kg, and Newton's second law gives

$$T_2 - m_2 g = m_2 a_y,$$

where a_y is positive and *up*. We do know that $T_1 > T_2$; the net force on the pulley creates a torque which results in the pulley rotating toward the heavier mass. That net force is $T_1 - T_2$; so the rotational form of Newton's second law gives

$$(T_1 - T_2)\,R = I\alpha_z = I a_T/R,$$

where $R = 0.049$ m is the radius of the pulley and a_T is the tangential acceleration. But this acceleration is equal to a_y, because everything— both blocks and the pulley— are moving together.

We then have *three* equations and *three* unknowns. We'll add the first two together,

$$\begin{aligned} m_1 g - T_1 + T_2 - m_2 g &= m_1 a_y + m_2 a_y, \\ T_1 - T_2 &= (g - a_y)m_1 - (g + a_y)m_2, \end{aligned}$$

and then combine this with the third equation by substituting for $T_1 - T_2$,

$$\begin{aligned} (g - a_y)m_1 - (g + a_y)m_2 &= I a_y/R^2, \\ \left[\left(\frac{g}{a_y} - 1\right)m_1 - \left(\frac{g}{a_y} + 1\right)m_2\right]R^2 &= I. \end{aligned}$$

Now for the numbers:

$$\begin{aligned} \left(\frac{(9.81\,\text{m/s}^2)}{(0.0586\,\text{m/s}^2)} - 1\right)(0.512\,\text{kg}) - \left(\frac{(9.81\,\text{m/s}^2)}{(0.0586\,\text{m/s}^2)} + 1\right)(0.463\,\text{kg}) &= 7.23\,\text{kg}, \\ (7.23\,\text{kg})(0.049\,\text{m})^2 &= 0.0174\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

E9-40 The wheel turns with an initial angular speed of $\omega_0 = 88.0$ rad/s. The average speed while decelerating is $\omega_{\text{av}} = \omega_0/2$. The wheel stops turning in a time $t = \phi/\omega_{\text{av}} = 2\phi/\omega_0$. The deceleration is then $\alpha = -\omega_0/t = -\omega_0^2/(2\phi)$.

The rotational inertia is $I = MR^2/2$, so the torque required to stop the disk is $\tau = I\alpha = -MR^2\omega_0^2/(4\phi)$. The force of friction on the disk is $f = \mu N$, so $\tau = Rf$. Then

$$\mu = \frac{MR\omega_0^2}{4N\phi} = \frac{(1.40\,\text{kg})(0.23\,\text{m})(88.0 \text{ rad/s})^2}{4(130\,\text{N})(17.6 \text{ rad})} = 0.272.$$

E9-41 (a) The automobile has an initial speed of $v_0 = 21.8\,\text{m/s}$. The angular speed is then $\omega_0 = (21.8\,\text{m/s})/(0.385\,\text{m}) = 56.6$ rad/s.

(b) The average speed while decelerating is $\omega_{\text{av}} = \omega_0/2$. The wheel stops turning in a time $t = \phi/\omega_{\text{av}} = 2\phi/\omega_0$. The deceleration is then

$$\alpha = -\omega_0/t = -\omega_0^2/(2\phi) = -(56.6 \text{ rad/s})^2/[2(180 \text{ rad})] = -8.90 \text{ rad/s}.$$

(c) The automobile traveled $x = \phi r = (180 \text{ rad})(0.385\,\text{m}) = 69.3\,\text{m}$.

E9-42 (a) The angular acceleration is derived in Sample Problem 9-13,

$$\alpha = \frac{g}{R_0}\frac{1}{1+I/(MR_0^2)} = \frac{(981\ \text{cm/s}^2)}{(0.320\ \text{cm})}\frac{1}{1+(0.950\,\text{kg}\cdot\text{cm}^2)/[(0.120\,\text{kg})(0.320\ \text{cm})^2]} = 39.1\ \text{rad/s}^2.$$

The acceleration is $a = \alpha R_0 = (39.1\ \text{rad/s}^2)(0.320\ \text{cm}) = 12.5\ \text{cm/s}^2$.

(b) Starting from rest, $t = \sqrt{2x/a} = \sqrt{2(134\ \text{cm})/(12.5\ \text{cm/s}^2)} = 4.63\,\text{s}$.

(c) $\omega = \alpha t = (39.1\ \text{rad/s}^2)(4.63\,\text{s}) = 181\ \text{rad/s}$. This is the same as 28.8 rev/s.

(d) The yo-yo accelerates toward the ground according to $y = at^2 + v_0 t$, where *down* is positive. The time required to move to the end of the string is found from

$$t = \frac{-v_0+\sqrt{v_0^2+4ay}}{2a} = \frac{-(1.30\,\text{m/s})+\sqrt{(1.30\,\text{m/s})^2+4(0.125\,\text{m/s}^2)(1.34\,\text{m})}}{2(0.125\,\text{m/s}^2)} = 0.945\,\text{s}$$

The initial rotational speed was $\omega_0 = (1.30\,\text{m/s})/(3.2\times 10^{-3}\text{m}) = 406\ \text{rad/s}$. Then

$$\omega = \omega_0 + \alpha t = (406\ \text{rad/s}) + (39.1\ \text{rad/s}^2)(0.945\,\text{s}) = 443\ \text{rad/s},$$

which is the same as 70.5 rev/s.

E9-43 (a) Assuming a perfect hinge at B, the only two vertical forces on the tire will be the normal force from the belt and the force of gravity. Then $N = W = mg$, or $N = (15.0\ \text{kg})(9.8\ \text{m/s}^2) = 147\ \text{N}$.

While the tire skids we have kinetic friction, so $f = \mu_k N = (0.600)(147\ \text{N}) = 88.2\ \text{N}$. The force of gravity and and the pull from the holding rod AB both act at the axis of rotation, so can't contribute to the net torque. The normal force acts at a point which is parallel to the displacement from the axis of rotation, so it doesn't contribute to the torque either (because the cross product would vanish); so the only contribution to the torque is from the frictional force.

The frictional force is perpendicular to the radial vector, so the magnitude of the torque is just $\tau = rf = (0.300\ \text{m})(88.2\ \text{N}) = 26.5\ \text{N·m}$. This means the angular acceleration will be $\alpha = \tau/I = (26.5\ \text{N·m})/(0.750\ \text{kg·m}^2) = 35.3\ \text{rad/s}^2$.

When $\omega R = v_T = 12.0\ \text{m/s}$ the tire is no longer slipping. We solve for ω and get $\omega = 40\ \text{rad/s}$.

Now we solve $\omega = \omega_0 + \alpha t$ for the time. The wheel started from rest, so $t = 1.13$ s.

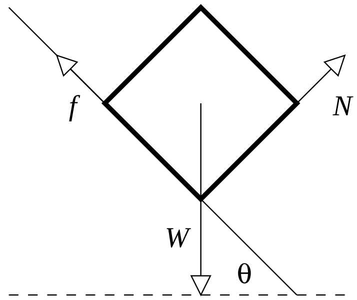
(b) The length of the skid is $x = vt = (12.0\ \text{m/s})(1.13\ \text{s}) = 13.6$ m long.

P9-1 The problem of sliding down the ramp has been solved (see Sample Problem 5-8); the critical angle θ_s is given by $\tan\theta_s = \mu_s$.

The problem of tipping is actually not that much harder: an object tips when the center of gravity is no longer over the base. The important angle for tipping is shown in the figure below; we can find that by trigonometry to be

$$\tan\theta_t = \frac{O}{A} = \frac{(0.56\ \text{m})}{(0.56\ \text{m})+(0.28\ \text{m})} = 0.67,$$

so $\theta_t = 34°$.



(a) If $\mu_s = 0.60$ then $\theta_s = 31°$ and the crate slides.
(b) If $\mu_s = 0.70$ then $\theta_s = 35°$ and the crate tips before sliding; it tips at 34°.

P9-2 (a) The total force up on the chain needs to be equal to the total force down; the force down is W. Assuming the tension at the end points is T then $T \sin\theta$ is the upward component, so $T = W/(2\sin\theta)$.

(b) There is a horizontal component to the tension $T\cos\theta$ at the wall; this *must* be the tension at the horizontal point at the bottom of the cable. Then $T_{\text{bottom}} = W/(2\tan\theta)$.

P9-3 (a) The rope exerts a force on the sphere which has horizontal $T\sin\theta$ and vertical $T\cos\theta$ components, where $\theta = \arctan(r/L)$. The weight of the sphere is balanced by the upward force from the rope, so $T\cos\theta = W$. But $\cos\theta = L/\sqrt{r^2 + L^2}$, so $T = W\sqrt{1 + r^2/L^2}$.

(b) The wall pushes outward against the sphere equal to the inward push on the sphere from the rope, or $P = T\sin\theta = W\tan\theta = Wr/L$.

P9-4 Treat the problem as having two forces: the man at one end lifting with force $F = W/3$ and the two men acting together a distance x away from the first man and lifting with a force $2F = 2W/3$. Then the torque about an axis through the end of the beam where the first man is lifting is $\tau = 2xW/3 - WL/2$, where L is the length of the beam. This expression equal zero when $x = 3L/4$.

$\boxed{\textbf{P9-5}}$ (a) We can solve this problem with Eq. 9-32 after a few modifications. We'll assume the center of mass of the ladder is at the center, then the third term of Eq. 9-32 is $mga/2$. The cleaner didn't climb half-way, he climbed 3.10/5.12 = 60.5% of the way, so the second term of Eq. 9-32 becomes $Mga(0.605)$. h, L, and a are related by $L^2 = a^2 + h^2$, so $h = \sqrt{(5.12\text{ m})^2 - (2.45\text{ m})^2} = 4.5$ m. Then, putting the correction into Eq. 9-32,

$$\begin{aligned}
F_w &= \frac{1}{h}\left[Mga(0.605) + \frac{mga}{2}\right], \\
&= \frac{1}{(4.5\text{ m})}\Big[(74.6\text{ kg})(9.81\text{ m/s}^2)(2.45\text{ m})(0.605)\,, \\
&\quad + (10.3\text{ kg})(9.81\text{ m/s}^2)(2.45\text{ m})/2\Big], \\
&= 269\text{ N}
\end{aligned}$$

(b) The vertical component of the force of the ground on the ground is the sum of the weight of the window cleaner and the weight of the ladder, or 833 N.

The horizontal component is equal in magnitude to the force of the ladder on the window. Then the net force of the ground on the ladder has magnitude

$$\sqrt{(269\,\mathrm{N})^2 + (833\,\mathrm{N})^2} = 875\,\mathrm{N}$$

and direction

$$\theta = \arctan(833/269) = 72^\circ \text{ above the horizontal.}$$

P9-6 (a) There are no upward forces other than the normal force on the bottom ball, so the force exerted on the bottom ball by the container is $2W$.

(c) The bottom ball must exert a force on the top ball which has a vertical component equal to the weight of the top ball. Then $W = N \sin\theta$ or the force of contact between the balls is $N = W/\sin\theta$.

(b) The force of contact between the balls has a horizontal component $P = N\cos\theta = W/\tan\theta$, this must also be the force of the walls on the balls.

P9-7 (a) There are three forces on the ball: weight $\vec{\mathbf{W}}$, the normal force from the lower plane $\vec{\mathbf{N}}_1$, and the normal force from the upper plane $\vec{\mathbf{N}}_2$. The force from the lower plane has components $N_{1,x} = -N_1 \sin\theta_1$ and $N_{1,y} = N_1 \cos\theta_1$. The force from the upper plane has components $N_{2,x} = N_2 \sin\theta_2$ and $N_{2,y} = -N_2 \cos\theta_2$. Then $N_1 \sin\theta_1 = N_2 \sin\theta_2$ and $N_1 \cos\theta_1 = W + N_2 \cos\theta_2$.

Solving for N_2 by dividing one expression by the other,

$$\frac{\cos\theta_1}{\sin\theta_1} = \frac{W}{N_2 \sin\theta_2} + \frac{\cos\theta_2}{\sin\theta_2},$$

or

$$\begin{aligned}
N_2 &= \frac{W}{\sin\theta_2}\left(\frac{\cos\theta_1}{\sin\theta_1} - \frac{\cos\theta_2}{\sin\theta_2}\right)^{-1}, \\
&= \frac{W}{\sin\theta_2}\frac{\cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1}{\sin\theta_1\,\sin\theta_2}, \\
&= \frac{W\sin\theta_1}{\sin(\theta_2 - \theta_1)}.
\end{aligned}$$

Then solve for N_1,

$$N_1 = \frac{W\sin\theta_2}{\sin(\theta_2 - \theta_1)}.$$

(b) Friction changes everything.

P9-8 (a) The net torque about a line through A is

$$\tau = Wx - TL\sin\theta = 0,$$

so $T = Wx/(L\sin\theta)$.

(b) The horizontal force on the pin is equal in magnitude to the horizontal component of the tension: $T\cos\theta = Wx/(L\tan\theta)$. The vertical component balances the weight: $W - Wx/L$.

(c) $x = (520\,\mathrm{N})(2.75\,\mathrm{m})\sin(32.0^\circ)/(315\,\mathrm{N}) = 2.41\,\mathrm{m}$.

P9-9 (a) As long as the center of gravity of an object (even if combined) is above the base, then the object will not tip.

Stack the bricks from the top down. The center of gravity of the top brick is $L/2$ from the edge of the top brick. This top brick can be offset nor more than $L/2$ from the one beneath. The center of gravity of the top two bricks is located at

$$x_{\mathrm{cm}} = [(L/2) + (L)]/2 = 3L/4.$$

These top two bricks can be offset no more than $L/4$ from the brick beneath. The center of gravity of the top three bricks is located at

$$x_{\text{cm}} = [(L/2) + 2(L)]/3 = 5L/6.$$

These top three bricks can be offset no more than $L/6$ from the brick beneath. The total offset is then $L/2 + L/4 + L/6 = 11L/12$.

(b) Actually, we never need to know the location of the center of gravity; we now realize that each brick is located with an offset $L/(2n)$ with the brick beneath, where n is the number of the brick counting from the top. The series is then of the form

$$(L/2)[(1/1) + (1/2) + (1/3) + (1/4) + \cdots],$$

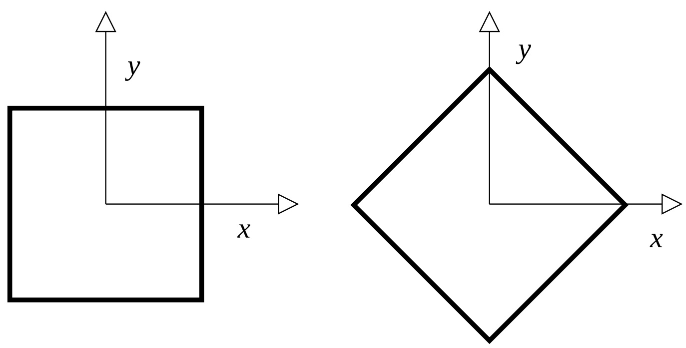
a series (harmonic, for those of you who care) which does not converge.

(c) The center of gravity would be half way between the ends of the two extreme bricks. This would be at NL/n; the pile will topple when this value exceeds L, or when $N = n$.

P9-10 (a) For a planar object which lies in the $x-y$ plane, $I_x = \int x^2 dm$ and $I_y = \int y^2 dm$. Then $I_x + I_y = \int (x^2 + y^2) dm = \int r^2\, dm$. But this is the rotational inertia about the z axis, sent r is the distance from the z axis.

(b) Since the rotational inertia about one diameter (I_x) should be the same as the rotational inertia about any other (I_y) then $I_x = I_y$ and $I_x = I_z/2 = MR^2/4$.

P9-11 Problem 9-10 says that $I_x + I_y = I_z$ for any thin, flat object which lies only in the $x - y$ plane.It doesn't matter in which direction the x and y axes are chosen, so long as they are perpendicular. We can then orient our square as in either of the pictures below:



By symmetry $I_x = I_y$ in either picture. Consequently, $I_x = I_y = I_z/2$ for either picture. It is the same square, so I_z is the same for both picture. Then I_x is also the same for both orientations.

P9-12 Let M_0 be the mass of the plate *before* the holes are cut out. Then $M_1 = M_0(a/L)^2$ is the mass of the part cut out of each hole and $M = M_0 - 9M_1$ is the mass of the plate. The rotational inertia (about an axis perpendicular to the plane through the center of the square) for the large uncut square is $M_0L^2/6$ and for each smaller cut out is $M_1a^2/6$.

From the large uncut square's inertia we need to remove $M_1a^2/6$ for the center cut-out, $M_1a^2/6 + M_1(L/3)^2$ for each of the four edge cut-outs, and $M_1a^2/6 + M_1(\sqrt{2}L/3)^2$ for each of the corner sections.

Then

$$\begin{aligned} I &= \frac{M_0L^2}{6} - 9\frac{M_1a^2}{6} - 4\frac{M_1L^2}{9} - 4\frac{2M_1L_2}{9}, \\ &= \frac{M_0L^2}{6} - 3\frac{M_0a^4}{2L^2} - 4\frac{M_0a^2}{3}. \end{aligned}$$

P9-13 (a) From Eq. 9-15, $I = \int r^2 dm$ about some axis of rotation when r is measured from that axis. If we consider the x axis as our axis of rotation, then $r = \sqrt{y^2 + z^2}$, since the distance to the x axis depends only on the y and z coordinates. We have similar equations for the y and z axes, so

$$\begin{aligned} I_x &= \int \left(y^2 + z^2\right) dm, \\ I_y &= \int \left(x^2 + z^2\right) dm, \\ I_z &= \int \left(x^2 + y^2\right) dm. \end{aligned}$$

These three equations can be added together to give

$$I_x + I_y + I_z = 2\int \left(x^2 + y^2 + z^2\right) dm,$$

so if we *now* define r to be measured from the origin (which is not the definition used above), then we end up with the answer in the text.

(b) The right hand side of the equation is integrated over the entire body, regardless of how the axes are defined. So the integral should be the same, no matter how the coordinate system is rotated.

P9-14 (a) Since the shell is spherically symmetric $I_x = I_y = I_z$, so $I_x = (2/3)\int r^2 dm = (2R^2/3)\int dm = 2MR^2/3$.

(b) Since the solid ball is spherically symmetric $I_x = I_y = I_z$, so

$$I_x = \frac{2}{3}\int r^2 \frac{3Mr^2\,dr}{R^3} = \frac{2}{5}MR^2.$$

P9-15 (a) A simple ratio will suffice:

$$\frac{dm}{2\pi r\,dr} = \frac{M}{\pi R^2} \text{ or } dm = \frac{2Mr}{R^2}dr.$$

(b) $dI = r^2 dm = (2Mr^3/R^2)dr$.

(c) $I = \int_0^R (2Mr^3/R^2)dr = MR^2/2$.

P9-16 (a) Another simple ratio will suffice:

$$\frac{dm}{\pi r^2 dz} = \frac{M}{(4/3)\pi R^3} \text{ or } dm = \frac{3M(R^2 - z^2)}{4R^3}dz.$$

(b) $dI = r^2 dm/2 = [3M(R^2 - z^2)^2/8R^3]dz$.

(c) There are a few steps to do here:

$$
\begin{aligned}
I &= \int_{-R}^{R} \frac{3M(R^2-z^2)^2}{8R^3} dz, \\
&= \frac{3M}{4R^3} \int_0^R (R^4 - 2R^2z^2 + z^4) dz, \\
&= \frac{3M}{4R^3}(R^5 - 2R^5/3 + R^5/5) = \frac{2}{5}MR^2.
\end{aligned}
$$

$\boxed{\textbf{P9-17}}$ The *rotational* acceleration will be given by $\alpha_z = \sum \tau / I$.

The torque about the pivot comes from the force of gravity on each block. This forces will both originally be at right angles to the pivot arm, so the net torque will be $\sum \tau = mgL_2 - mgL_1$, where clockwise as seen on the page is positive.

The rotational inertia about the pivot is given by $I = \sum m_n r_n^2 = m(L_2^2 + L_1^2)$. So we can now find the rotational acceleration,

$$
\alpha = \frac{\sum \tau}{I} = \frac{mgL_2 - mgL_1}{m(L_2^2 + L_1^2)} = g\frac{L_2 - L_1}{L_2^2 + L_1^2} = 8.66 \text{ rad/s}^2.
$$

The linear acceleration is the tangential acceleration, $a_T = \alpha R$. For the left block, $a_T = 1.73$ m/s^2; for the right block $a_T = 6.93$ m/s^2.

P9-18 (a) The force of friction on the hub is $\mu_k Mg$. The torque is $\tau = \mu_k Mga$. The angular acceleration has magnitude $\alpha = \tau/I = \mu_k ga/k^2$. The time it takes to stop the wheel will be $t = \omega_0/\alpha = \omega_0 k^2/(\mu_k ga)$.

(b) The average rotational speed while slowing is $\omega_0/2$. The angle through which it turns while slowing is $\omega_0 t/2$ radians, or $\omega_0 t/(4\pi) = \omega_0^2 k^2/(4\pi\mu_k ga)$

P9-19 (a) Consider a differential ring of the disk. The torque on the ring because of friction is

$$
d\tau = r\, dF = r\frac{\mu_k Mg}{\pi R^2} 2\pi r\, dr = \frac{2\mu_k Mgr^2}{R^2} dr.
$$

The net torque is then

$$
\tau = \int d\tau = \int_0^R \frac{2\mu_k Mgr^2}{R^2} dr = \frac{2}{3}\mu_k MgR.
$$

(b) The rotational acceleration has magnitude $\alpha = \tau/I = \frac{4}{3}\mu_k g/R$. Then it will take a time

$$
t = \omega_0/\alpha = \frac{3R\omega_0}{4\mu_k g}
$$

to stop.

P9-20 We need only show that the two objects have the same acceleration.

Consider first the hoop. There is a force $W_{||} = W \sin\theta = mg \sin\theta$ pulling it down the ramp and a frictional force f pulling it up the ramp. The frictional force is just large enough to cause a torque that will allow the hoop to roll without slipping. This means $a = \alpha R$; consequently, $fR = \alpha I = aI/R$. In this case $I = mR^2$.

The acceleration down the plane is

$$
ma = mg\sin\theta - f = mg\sin\theta - maI/R^2 = mg\sin\theta - ma.
$$

Then $a = g \sin\theta /2$. The mass and radius are irrelevant!

For a block sliding with friction there are also two forces: $W_{||} = W \sin\theta = mg \sin\theta$ and $f = \mu_k mg \cos\theta$. Then the acceleration down the plane will be given by

$$a = g \sin\theta - \mu_k g \cos\theta,$$

which will be equal to that of the hoop if

$$\mu_k = \frac{\sin\theta - \sin\theta/2}{\cos\theta} = \frac{1}{2} \tan\theta.$$

$\boxed{\textbf{P9-21}}$ This problem is equivalent to Sample Problem 9-11, except that we have a sphere instead of a cylinder. We'll have the same two equations for Newton's second law,

$$Mg \sin\theta - f = Ma_{cm} \text{ and } N - Mg \cos\theta = 0.$$

Newton's second law for rotation will look like

$$-fR = I_{cm}\alpha.$$

The conditions for accelerating without slipping are $a_{cm} = \alpha R$, rearrange the rotational equation to get

$$f = -\frac{I_{cm}\alpha}{R} = -\frac{I_{cm}(-a_{cm})}{R^2},$$

and then

$$Mg \sin\theta - \frac{I_{cm}(a_{cm})}{R^2} = Ma_{cm},$$

and solve for a_{cm}. For fun, let's write the rotational inertia as $I = \beta MR^2$, where $\beta = 2/5$ for the sphere. Then, upon some mild rearranging, we get

$$a_{cm} = g\frac{\sin\theta}{1+\beta}$$

For the sphere, $a_{cm} = 5/7g \sin\theta$.

(a) If $a_{cm} = 0.133g$, then $\sin\theta = 7/5(0.133) = 0.186$, and $\theta = 10.7°$.

(b) A frictionless block has no rotational properties; in this case $\beta = 0$! Then $a_{cm} = g \sin\theta = 0.186g$.

P9-22 (a) There are three forces on the cylinder: gravity W and the tension from each cable T. The downward acceleration of the cylinder is then given by $ma = W - 2T$.

The ropes unwind according to $\alpha = a/R$, but $\alpha = \tau/I$ and $I = mR^2/2$. Then

$$a = \tau R/I = (2TR)R/(mR^2/2) = 4T/m.$$

Combining the above, $4T = W - 2T$, or $T = W/6$.

(b) $a = 4(mg/6)/m = 2g/3$.

$\boxed{\textbf{P9-23}}$ The force of friction required to keep the cylinder rolling is given by

$$f = \frac{1}{3}Mg \sin\theta;$$

the normal force is given to be $N = Mg \cos\theta$; so the coefficient of static friction is given by

$$\mu_s \geq \frac{f}{N} = \frac{1}{3} \tan\theta.$$

P9-24 $a = F/M$, since F is the net force on the disk. The torque about the center of mass is FR, so the disk has an angular acceleration of

$$\alpha = \frac{FR}{I} = \frac{FR}{MR^2/2} = \frac{2F}{MR}.$$

P9-25 This problem is equivalent to Sample Problem 9-11, except that we have an unknown rolling object. We'll have the same two equations for Newton's second law,

$$Mg\sin\theta - f = Ma_{\rm cm} \text{ and } N - Mg\cos\theta = 0.$$

Newton's second law for rotation will look like

$$-fR = I_{\rm cm}\alpha.$$

The conditions for accelerating without slipping are $a_{\rm cm} = \alpha R$, rearrange the rotational equation to get

$$f = -\frac{I_{\rm cm}\alpha}{R} = -\frac{I_{\rm cm}(-a_{\rm cm})}{R^2},$$

and then

$$Mg\sin\theta - \frac{I_{\rm cm}(a_{\rm cm})}{R^2} = Ma_{\rm cm},$$

and solve for $a_{\rm cm}$. Write the rotational inertia as $I = \beta MR^2$, where $\beta = 2/5$ for a sphere, $\beta = 1/2$ for a cylinder, and $\beta = 1$ for a hoop. Then, upon some mild rearranging, we get

$$a_{\rm cm} = g\frac{\sin\theta}{1+\beta}$$

Note that a is largest when β is smallest; consequently the cylinder wins. Neither M nor R entered into the final equation.

E10-1 $l = rp = mvr = (13.7\times10^{-3}\mathrm{kg})(380\,\mathrm{m/s})(0.12\,\mathrm{m}) = 0.62\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$.

E10-2 (a) $\vec{\mathbf{l}} = m\vec{\mathbf{r}}\times\vec{\mathbf{v}}$, or

$$\vec{\mathbf{l}} = m(yv_z - zv_y)\hat{\mathbf{i}} + m(zv_x - xv_z)\hat{\mathbf{j}} + m(xv_y - yv_x)\hat{\mathbf{k}}.$$

(b) If $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ exist only in the xy plane then $z = v_z = 0$, so only the uk term survives.

$\boxed{\textbf{E10-3}}$ If the angular momentum $\vec{\mathbf{l}}$ is constant in time, then $d\vec{\mathbf{l}}/dt = 0$. Trying this on Eq. 10-1,

$$\begin{aligned}
\frac{d\vec{\mathbf{l}}}{dt} &= \frac{d}{dt}\left(\vec{\mathbf{r}}\times\vec{\mathbf{p}}\right),\\
&= \frac{d}{dt}\left(\vec{\mathbf{r}}\times m\vec{\mathbf{v}}\right),\\
&= m\frac{d\vec{\mathbf{r}}}{dt}\times\vec{\mathbf{v}} + m\vec{\mathbf{r}}\times\frac{d\vec{\mathbf{v}}}{dt},\\
&= m\vec{\mathbf{v}}\times\vec{\mathbf{v}} + m\vec{\mathbf{r}}\times\vec{\mathbf{a}}.
\end{aligned}$$

Now the cross product of a vector with itself is zero, so the first term vanishes. But in the exercise we are told the particle has constant velocity, so $\vec{\mathbf{a}} = 0$, and consequently the second term vanishes. Hence, $\vec{\mathbf{l}}$ is constant for a single particle if $\vec{\mathbf{v}}$ is constant.

E10-4 (a) $L = \sum l_i$; $l_i = r_i m_i v_i$. Putting the numbers in for each planet and then summing (I won't bore you with the arithmetic details) yields $L = 3.15\times10^{43}\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$.

(b) Jupiter has $l = 1.94\times10^{43}\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$, which is 61.6% of the total.

E10-5 $l = mvr = m(2\pi r/T)r = 2\pi(84.3\,\mathrm{kg})(6.37\times10^6\mathrm{m})^2/(86400\,\mathrm{s}) = 2.49\times10^{11}\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$.

E10-6 (a) Substitute and expand:

$$\begin{aligned}
\vec{\mathbf{L}} &= \sum(\vec{\mathbf{r}}_{\mathrm{cm}} + \vec{\mathbf{r}}_i')\times(m_i\vec{\mathbf{v}}_{\mathrm{cm}} + \vec{\mathbf{p}}_i'),\\
&= \sum(m_i\vec{\mathbf{r}}_{\mathrm{cm}}\times\vec{\mathbf{v}}_{\mathrm{cm}} + \vec{\mathbf{r}}_{\mathrm{cm}}\times\vec{\mathbf{p}}_i' + m_i\vec{\mathbf{r}}_i'\times\vec{\mathbf{v}}_{\mathrm{cm}} + \vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i'),\\
&= M\vec{\mathbf{r}}_{\mathrm{cm}}\times\vec{\mathbf{v}}_{\mathrm{cm}} + \vec{\mathbf{r}}_{\mathrm{cm}}\times(\sum\vec{\mathbf{p}}_i') + (\sum m_i\vec{\mathbf{r}}_i')\times\vec{\mathbf{v}}_{\mathrm{cm}} + \sum\vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i'.
\end{aligned}$$

(b) But $\sum\vec{\mathbf{p}}_i' = 0$ and $\sum m_i\vec{\mathbf{r}}_i' = 0$, because these two quantities are *in* the center of momentum and center of mass. Then

$$\vec{\mathbf{L}} = M\vec{\mathbf{r}}_{\mathrm{cm}}\times\vec{\mathbf{v}}_{\mathrm{cm}} + \sum\vec{\mathbf{r}}_i'\times\vec{\mathbf{p}}_i' = \vec{\mathbf{L}}' + M\vec{\mathbf{r}}_{\mathrm{cm}}\times\vec{\mathbf{v}}_{\mathrm{cm}}.$$

E10-7 (a) Substitute and expand:

$$\vec{\mathbf{p}}_i' = m_i\frac{d\vec{\mathbf{r}}_i'}{dt} = m_i\frac{d\vec{\mathbf{r}}_i}{dt} - m_i\frac{d\vec{\mathbf{r}}_{\mathrm{cm}}}{dt} = \vec{\mathbf{p}}_i - m_i\vec{\mathbf{v}}_{\mathrm{cm}}.$$

(b) Substitute and expand:

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum\frac{d\vec{\mathbf{r}}_i'}{dt}\times\vec{\mathbf{p}}_i' + \sum\vec{\mathbf{r}}_i'\times\frac{d\vec{\mathbf{p}}_i'}{dt} = \sum\vec{\mathbf{r}}_i'\times\frac{d\vec{\mathbf{p}}_i'}{dt}.$$

The first term vanished because $\vec{\mathbf{v}}_i'$ is parallel to $\vec{\mathbf{p}}_i'$.

(c) Substitute and expand:

$$\begin{aligned}
\frac{d\vec{\mathbf{L}}'}{dt} &= \sum \vec{\mathbf{r}}_i' \times \frac{d(\vec{\mathbf{p}}_i - m_i\vec{\mathbf{v}}_{\rm cm})}{dt}, \\
&= \sum \vec{\mathbf{r}}_i' \times (m_i\vec{\mathbf{a}}_i - m_i\vec{\mathbf{a}}_{\rm c}{\rm m}), \\
&= \sum \vec{\mathbf{r}}_i' \times m_i\vec{\mathbf{a}}_i + \left(\sum m_i\vec{\mathbf{r}}_i'\right) \times \vec{\mathbf{a}}_{\rm c}{\rm m})
\end{aligned}$$

The second term vanishes because of the definition of the center of mass. Then

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum \vec{\mathbf{r}}_i' \times \vec{\mathbf{F}}_i,$$

where $\vec{\mathbf{F}}_i$ is the net force on the ith particle. The force $\vec{\mathbf{F}}_i$ may include both internal and external components. If there is an internal component, say between the ith and jth particles, then the torques from these two third law components will cancel out. Consequently,

$$\frac{d\vec{\mathbf{L}}'}{dt} = \sum \vec{\tau}_i = \vec{\tau}_{\rm ext}.$$

E10-8 (a) Integrate.

$$\int \vec{\tau}\,dt = \int \frac{d\vec{\mathbf{L}}}{dt}dt = \int d\vec{\mathbf{L}} = \Delta\vec{\mathbf{L}}.$$

(b) If I is fixed, $\Delta L = I\Delta\omega$. Not only that,

$$\int \tau dt = \int Fr\,dt = r\int F\,dt = rF_{\rm av}\Delta t,$$

where we use the definition of average that depends on time.

$\boxed{\textbf{E10-9}}$ (a) $\vec{\tau}\Delta t = \Delta\vec{\mathbf{l}}$. The disk starts from rest, so $\Delta\vec{\mathbf{l}} = \vec{\mathbf{l}} - \vec{\mathbf{l}}_0 = \vec{\mathbf{l}}$. We need only concern ourselves with the magnitudes, so

$$l = \Delta l = \tau\Delta t = (15.8\,{\rm N\cdot m})(0.033\,{\rm s}) = 0.521\,{\rm kg\cdot m^2/s}.$$

(b) $\omega = l/I = (0.521\,{\rm kg\cdot m^2/s})/(1.22\times10^{-3}{\rm kg\cdot m^2}) = 427$ rad/s.

E10-10 (a) Let v_0 be the initial speed; the average speed while slowing to a stop is $v_0/2$; the time required to stop is $t = 2x/v_0$; the acceleration is $a = -v_0/t = -v_0^2/(2x)$. Then

$$a = -(43.3\,{\rm m/s})^2/[2(225\,{\rm m/s})] = -4.17\,{\rm m/s^2}.$$

(b) $\alpha = a/r = (-4.17\,{\rm m/s^2})/(0.247\,{\rm m}) = -16.9\ {\rm rad/s^2}$.
(c) $\tau = I\alpha = (0.155\,{\rm kg\cdot m^2})(-16.9\ {\rm rad/s^2}) = -2.62\,{\rm N\cdot m}$.

E10-11 Let $\vec{\mathbf{r}}_i = \vec{\mathbf{z}} + \vec{\mathbf{r}}_i'$. From the figure, $\vec{\mathbf{p}}_1 = -\vec{\mathbf{p}}_2$ and $\vec{\mathbf{r}}_1' = -\vec{\mathbf{r}}_2'$. Then

$$\begin{aligned}
\vec{\mathbf{L}} &= \vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 = \vec{\mathbf{r}}_1 \times \vec{\mathbf{p}}_1 + \vec{\mathbf{r}}_2 \times \vec{\mathbf{p}}_2, \\
&= (\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) \times \vec{\mathbf{p}}_1, \\
&= (\vec{\mathbf{r}}_1' - \vec{\mathbf{r}}_2') \times \vec{\mathbf{p}}_1, \\
&= 2\vec{\mathbf{r}}_1' \times \vec{\mathbf{p}}_1.
\end{aligned}$$

Since $\vec{\mathbf{r}}_1'$ and $\vec{\mathbf{p}}_1$ both lie in the xy plane then $\vec{\mathbf{L}}$ must be along the z axis.

E10-12 Expand:

$$\begin{aligned}
\vec{\mathbf{L}} &= \sum \vec{\mathbf{l}}_i = \sum \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i, \\
&= \sum m_i \vec{\mathbf{r}}_i \times \vec{\mathbf{v}}_i = \sum m_i \vec{\mathbf{r}}_i \times (\vec{\omega} \times \vec{\mathbf{r}}_i) \\
&= \sum m_i [(\vec{\mathbf{r}}_i \cdot \vec{\mathbf{r}}_i)\vec{\omega} - (\vec{\mathbf{r}}_i \cdot \vec{\omega})\vec{\mathbf{r}})i], \\
&= \sum m_i [r_i^2 \vec{\omega} - (z_i^2 \omega)\hat{\mathbf{k}} - (z_i x_i \omega)\hat{\mathbf{i}} - (z_i y_i \omega)\hat{\mathbf{j}}],
\end{aligned}$$

but if the body is symmetric about the z axis then the last two terms vanish, leaving

$$\vec{\mathbf{L}} = \sum m_i [r_i^2 \vec{\omega} - (z_i^2 \omega)\hat{\mathbf{k}}] = \sum m_i (x_i^2 + y_i^2)\vec{\omega} = I\vec{\omega}.$$

$\boxed{\textbf{E10-13}}$ An impulse of 12.8 N·s will change the linear momentum by 12.8 N·s; the stick starts from rest, so the final momentum *must* be 12.8 N·s. Since $p = mv$, we then can find $v = p/m = (12.8$ N·s)/(4.42 kg) = 2.90 m/s.

Impulse is a vector, given by $\int \vec{\mathbf{F}} dt$. We can take the cross product of the impulse with the displacement vector $\vec{\mathbf{r}}$ (measured from the axis of rotation to the point where the force is applied) and get

$$\vec{\mathbf{r}} \times \int \vec{\mathbf{F}} dt \approx \int \vec{\mathbf{r}} \times \vec{\mathbf{F}} dt,$$

The two sides of the above expression are only equal if $\vec{\mathbf{r}}$ has a constant magnitude *and* direction. This won't be true, but if the force is of sufficiently short duration then it hopefully won't change much. The right hand side is an integral over a torque, and will equal the change in angular momentum of the stick.

The exercise states that the force is perpendicular to the stick, then $|\vec{\mathbf{r}} \times \vec{\mathbf{F}}| = rF$, and the "torque impulse" is then (0.464 m)(12.8 N·s) = 5.94 kg·m/s. This "torque impulse" is equal to the change in the angular momentum, but the stick started from rest, so the final angular momentum of the stick is 5.94 kg·m/s.

But how fast is it rotating? We can use Fig. 9-15 to find the rotational inertia about the center of the stick: $I = \frac{1}{12}ML^2 = \frac{1}{12}(4.42\text{ kg})(1.23\text{ m})^2 = 0.557\text{ kg·m}^2$. The angular velocity of the stick is $\omega = l/I = (5.94\text{ kg·m/s})/(0.557\text{ kg·m}^2) = 10.7$ rad/s.

E10-14 The point of rotation is the point of contact with the plane; the torque about that point is $\tau = rmg \sin\theta$. The angular momentum is $I\omega$, so $\tau = I\alpha$. In this case $I = mr^2/2 + mr^2$, the second term from the parallel axis theorem. Then

$$a = r\alpha = r\tau/I = mr^2 g \sin\theta/(3mr^2/2) = \frac{2}{3} g \sin\theta.$$

E10-15 From Exercise 8 we can immediately write

$$I_1(\omega_1 - \omega_0)/r_1 = I_2(\omega_2 - 0)/r_2,$$

but we also have $r_1\omega_1 = -r_2\omega_2$. Then

$$\omega_2 = -\frac{r_1 r_2 I_1 \omega_0}{r_1^2 I_2 - r_2^2 I_1}.$$

E10-16 (a) $\Delta\omega/\omega = (1/T_1 - 1/T_2)/(1/T_1) = -(T_2 - T_1)/T_2 = -\Delta T/T$, which in this case is $-(6.0\times10^{-3}\text{s})(8.64\times10^{4}\text{s}) = -6.9\times10^{-8}$.

(b) Assuming conservation of angular momentum, $\Delta I/I = -\Delta\omega/\omega$. Then the fractional change would be 6.9×10^{-8}.

$\boxed{\textbf{E10-17}}$ The rotational inertia of a solid sphere is $I = \frac{2}{5}MR^2$; so as the sun collapses

$$\begin{aligned} \vec{\mathbf{L}}_\text{i} &= \vec{\mathbf{L}}_\text{f}, \\ I_\text{i}\vec{\omega}_\text{i} &= I_\text{f}\vec{\omega}_\text{f}, \\ \frac{2}{5}M{R_\text{i}}^2\vec{\omega}_\text{i} &= \frac{2}{5}M{R_\text{f}}^2\vec{\omega}_\text{f}, \\ {R_\text{i}}^2\vec{\omega}_i &= {R_\text{f}}^2\vec{\omega}_f. \end{aligned}$$

The angular frequency is inversely proportional to the period of rotation, so

$$T_\text{f} = T_\text{i}\frac{{R_\text{f}}^2}{{R_\text{i}}^2} = (3.6\times10^4\text{ min})\left(\frac{(6.37\times10^6\text{m})}{(6.96\times10^8\text{m})}\right)^2 = 3.0\text{ min}.$$

E10-18 The final angular velocity of the train with respect to the tracks is $\omega_\text{tt} = Rv$. The conservation of angular momentum implies

$$0 = MR^2\omega + mR^2(\omega_\text{tt} + \omega),$$

or

$$\omega = \frac{-mv}{(m+M)R}.$$

E10-19 This is much like a center of mass problem.

$$0 = I_\text{p}\phi_\text{p} + I_\text{m}(\phi_\text{mp} + \phi_\text{p}),$$

or

$$\phi_\text{mp} = -\frac{(I_\text{p} + I_\text{m})\phi_\text{p}}{I_\text{m}} \approx -\frac{(12.6\,\text{kg}\cdot\text{m}^2)(25^\circ)}{(2.47\times10^{-3}\text{kg}\cdot\text{m}^2)} = 1.28\times10^{5\circ}.$$

That's 354 rotations!

E10-20 $\omega_\text{f} = (I_\text{i}/I_\text{f})\omega_\text{i} = [(6.13\,\text{kg}\cdot\text{m}^2)/(1.97\,\text{kg}\cdot\text{m}^2)](1.22\text{ rev/s}) = 3.80\text{ rev/s}.$

$\boxed{\textbf{E10-21}}$ We have two disks which are originally not in contact which then come into contact; there are no external torques. We can write

$$\begin{aligned} \vec{\mathbf{l}}_{1,\text{i}} + \vec{\mathbf{l}}_{2,\text{i}} &= \vec{\mathbf{l}}_{1,\text{f}} + \vec{\mathbf{l}}_{2,\text{f}}, \\ I_1\vec{\omega}_{1,\text{i}} + I_2\vec{\omega}_{2,\text{i}} &= I_1\vec{\omega}_{1,\text{f}} + I_2\vec{\omega}_{2,\text{f}}. \end{aligned}$$

The final angular velocities of the two disks will be equal, so the above equation can be simplified and rearranged to yield

$$\omega_\text{f} = \frac{I_1}{I_1 + I_2}\omega_{1,\text{i}} = \frac{(1.27\text{ kg}\cdot\text{m}^2)}{(1.27\text{ kg}\cdot\text{m}^2) + (4.85\text{ kg}\cdot\text{m}^2)}(824\text{ rev/min}) = 171\text{ rev/min}$$

E10-22 $l_\perp = l\cos\theta = mvr\cos\theta = mvh.$

E10-23 (a) $\omega_\mathrm{f} = (I_1/I_2)\omega_\mathrm{i}$, $I_1 = (3.66\,\mathrm{kg})(0.363\,\mathrm{m})^2 = 0.482\,\mathrm{kg}\cdot\mathrm{m}^2$. Then

$$\omega_\mathrm{f} = [(0.482\,\mathrm{kg}\cdot\mathrm{m}^2)/(2.88\,\mathrm{kg}\cdot\mathrm{m}^2)](57.7\ \mathrm{rad/s}) = 9.66\ \mathrm{rad/s},$$

with the same rotational sense as the original wheel.

(b) Same answer, since friction is an internal force internal here.

E10-24 (a) Assume the merry-go-round is a disk. Then conservation of angular momentum yields

$$(\frac{1}{2}m_\mathrm{m}R^2 + m_\mathrm{g}R^2)\omega + (m_\mathrm{r}R^2)(v/R) = 0,$$

or

$$\omega = -\frac{(1.13\,\mathrm{kg})(7.82\,\mathrm{m/s})/(3.72\,\mathrm{m})}{(827\,\mathrm{kg})/2 + (50.6\,\mathrm{kg})} = -5.12\times10^{-3}\mathrm{rad/s}.$$

(b) $v = \omega R = (-5.12\times10^{-3}\mathrm{rad/s})(3.72\,\mathrm{m}) = -1.90\times10^{-2}\mathrm{m/s}$.

E10-25 Conservation of angular momentum:

$$(m_\mathrm{m}k^2 + m_\mathrm{g}R^2)\omega = m_\mathrm{g}R^2(v/R),$$

so

$$\omega = \frac{(44.3\,\mathrm{kg})(2.92\,\mathrm{m/s})/(1.22\,\mathrm{m})}{(176\,\mathrm{kg})(0.916\,\mathrm{m})^2 + (44.3\,\mathrm{kg})(1.22\,\mathrm{m})^2} = 0.496\ \mathrm{rad/s}.$$

E10-26 Use Eq. 10-22:

$$\omega_P = \frac{Mgr}{I\omega} = \frac{(0.492\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(3.88\times10^{-2}\mathrm{m})}{(5.12\times10^{-4}\mathrm{kg}\cdot\mathrm{m}^2)(2\pi 28.6\ \mathrm{rad/s})} = 2.04\ \mathrm{rad/s} = 0.324\ \mathrm{rev/s}.$$

$\boxed{\textbf{E10-27}}$ The relevant precession expression is Eq. 10-22.

The rotational inertia will be a sum of the contributions from both the disk and the axle, but the radius of the axle is probably very small compared to the disk, probably as small as 0.5 cm. Since I is proportional to the radius squared, we expect contributions from the axle to be less than $(1/100)^2$ of the value for the disk. For the disk only we use

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(1.14\ \mathrm{kg})(0.487\ \mathrm{m})^2 = 0.135\ \mathrm{kg}\cdot\mathrm{m}^2.$$

Now for ω,

$$\omega = 975\ \mathrm{rev/min}\left(\frac{2\pi\ \mathrm{rad}}{1\ \mathrm{rev}}\right)\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right) = 102\ \mathrm{rad/s}.$$

Then $L = I\omega = 13.8\ \mathrm{kg{\cdot}m^2/s}$.

Back to Eq. 10-22,

$$\omega_p = \frac{Mgr}{L} = \frac{(1.27\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(0.0610\ \mathrm{m})}{13.8\ \mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}} = 0.0551\ \mathrm{rad/s}.$$

The *time* for one precession is

$$t = \frac{1\mathrm{rev}}{\omega_p} = \frac{2\pi\ \mathrm{rad}}{(0.0551\ \mathrm{rad/s})} = 114\ \mathrm{s}.$$

P10-1 Positive z is out of the page.

(a) $\vec{\mathbf{l}} = rmv\sin\theta\hat{\mathbf{k}} = (2.91\,\mathrm{m})(2.13\,\mathrm{kg})(4.18\,\mathrm{m})\sin(147°)\hat{\mathbf{k}} = 14.1\,\mathrm{kg}\cdot\mathrm{m}^2/s\hat{\mathbf{k}}$.

(b) $\vec{\tau} = rF\sin\theta\hat{\mathbf{k}} = (2.91\,\mathrm{m})(1.88\,\mathrm{N})\sin(26°)\hat{\mathbf{k}} = 2.40\,\mathrm{N}\cdot\mathrm{m}\hat{\mathbf{k}}$.

P10-2 Regardless of where the origin is located one can orient the coordinate system so that the two paths lie in the xy plane and are both parallel to the y axis. The one of the particles travels along the path $x = vt$, $y = a$, $z = b$; the momentum of this particle is $\vec{\mathbf{p}}_1 = mv\hat{\mathbf{i}}$. The other particle will then travel along a path $x = c - vt$, $y = a + d$, $z = b$; the momentum of this particle is $\vec{\mathbf{p}}_2 = -mv\hat{\mathbf{i}}$. The angular momentum of the first particle is

$$\vec{\mathbf{l}}_1 = mvb\hat{\mathbf{j}} - mva\hat{\mathbf{k}},$$

while that of the second is

$$\vec{\mathbf{l}}_2 = -mvb\hat{\mathbf{j}} + mv(a+d)\hat{\mathbf{k}},$$

so the total is $\vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 = mvd\hat{\mathbf{k}}$.

$\boxed{\textbf{P10-3}}$ Assume that the cue stick strikes the ball horizontally with a force of constant magnitude F for a time Δt. Then the magnitude of the change in linear momentum of the ball is given by $F\Delta t = \Delta p = p$, since the initial momentum is zero.

If the force is applied a distance x above the center of the ball, then the magnitude of the torque about a horizontal axis through the center of the ball is $\tau = xF$. The change in angular momentum of the ball is given by $\tau\Delta t = \Delta l = l$, since initially the ball is not rotating.

For the ball to roll without slipping we need $v = \omega R$. We can start with this:

$$\begin{aligned} v &= \omega R, \\ \frac{p}{m} &= \frac{lR}{I}, \\ \frac{F\Delta t}{m} &= \frac{\tau\Delta t R}{I}, \\ \frac{F}{m} &= \frac{xFR}{I}. \end{aligned}$$

Then $x = I/mR$ is the condition for rolling without sliding from the start. For a solid sphere, $I = \frac{2}{5}mR^2$, so $x = \frac{2}{5}R$.

P10-4 The change in momentum of the block is $M(v_2 - v_1)$, this is equal to the magnitude the impulse delivered to the cylinder. According to E10-8 we can write $M(v_2 - v_1)R = I\omega_\mathrm{f}$. But in the end the box isn't slipping, so $\omega_f = v_2/R$. Then

$$Mv_2 - Mv_1 = (I/R^2)v_2,$$

or

$$v_2 = v_1/(1 + I/MR^2).$$

P10-5 Assume that the cue stick strikes the ball horizontally with a force of constant magnitude F for a time Δt. Then the magnitude of the change in linear momentum of the ball is given by $F\Delta t = \Delta p = p$, since the initial momentum is zero. Consequently, $F\Delta t = mv_0$.

If the force is applied a distance h above the center of the ball, then the magnitude of the torque about a horizontal axis through the center of the ball is $\tau = hF$. The change in angular momentum of the ball is given by $\tau\Delta t = \Delta l = l_0$, since initially the ball is not rotating. Consequently, the initial angular momentum of the ball is $l_0 = hmv_0 = I\omega_0$.

The ball originally slips while moving, but eventually it rolls. When it has begun to roll without slipping we have $v = R\omega$. Applying the results from E10-8,

$$m(v - v_0)R + I(\omega - \omega_0) = 0,$$

or

$$m(v - v_0)R + \frac{2}{5}mR^2\frac{v}{R} - hmv_0 = 0,$$

then, if $v = 9v_0/7$,

$$h = \left(\frac{9}{7} - 1\right)R + \frac{2}{5}R\left(\frac{9}{7}\right) = \frac{4}{5}R.$$

P10-6 (a) Refer to the previous answer. We now want $v = \omega = 0$, so

$$m(v - v_0)R + \frac{2}{5}mR^2\frac{v}{R} - hmv_0 = 0,$$

becomes

$$-v_0R - hv_0 = 0,$$

or $h = -R$. That'll scratch the felt.

(b) Assuming only a horizontal force then

$$v = \frac{(h + R)v_0}{R(1 + 2/5)},$$

which can only be negative if $h < -R$, which means hitting below the ball. Can't happen. If instead we allow for a downward component, then we can increase the "reverse English" as much as we want without increasing the initial forward velocity, and as such it would be possible to get the ball to move backwards.

$\boxed{\textbf{P10-7}}$ We assume the bowling ball is solid, so the rotational inertia will be $I = (2/5)MR^2$ (see Figure 9-15).

The normal force on the bowling ball will be $N = Mg$, where M is the mass of the bowling ball. The kinetic friction on the bowling ball is $F_f = \mu_k N = \mu_k Mg$. The magnitude of the net torque on the bowling ball while skidding is then $\tau = \mu_k MgR$.

Originally the angular momentum of the ball is zero; the final angular momentum will have magnitude $l = I\omega = Iv/R$, where v is the final translational speed of the ball.

(a) The time requires for the ball to stop skidding is the time required to change the angular momentum to l, so

$$\Delta t = \frac{\Delta l}{\tau} = \frac{(2/5)MR^2v/R}{\mu_k MgR} = \frac{2v}{5\mu_k g}.$$

Since we don't know v, we can't solve this for Δt. But the same time through which the angular momentum of the ball is increasing the linear momentum of the ball is decreasing, so we also have

$$\Delta t = \frac{\Delta p}{-F_f} = \frac{Mv - Mv_0}{-\mu_k Mg} = \frac{v_0 - v}{\mu_k g}.$$

Combining,

$$\begin{aligned}\Delta t &= \frac{v_0 - v}{\mu_k g},\\ &= \frac{v_0 - 5\mu_k g\Delta t/2}{\mu_k g},\end{aligned}$$

$$\begin{aligned}
2\mu_k g\Delta t &= 2v_0 - 5\mu_k g\Delta t,\\
\Delta t &= \frac{2v_0}{7\mu_k g},\\
&= \frac{2(8.50\,\mathrm{m/s})}{7(0.210)(9.81\,\mathrm{m/s^2})} = 1.18\,\mathrm{s}.
\end{aligned}$$

(d) Use the expression for angular momentum and torque,

$$v = 5\mu_k g\Delta t/2 = 5(0.210)(9.81\,\mathrm{m/s^2})(1.18\,\mathrm{s})/2 = 6.08\,\mathrm{m/s}.$$

(b) The acceleration of the ball is $F/M = -\mu g$. The distance traveled is then given by

$$\begin{aligned}
x &= \frac{1}{2}at^2 + v_0 t,\\
&= -\frac{1}{2}(0.210)(9.81\,\mathrm{m/s^2})(1.18\,\mathrm{s})^2 + (8.50\,\mathrm{m/s})(1.18\,\mathrm{s}) = 8.6\,\mathrm{m},
\end{aligned}$$

(c) The angular acceleration is $\tau/I = 5\mu_k g/(2R)$. Then

$$\begin{aligned}
\theta &= \frac{1}{2}\alpha t^2 + \omega_0 t,\\
&= \frac{5(0.210)(9.81\,\mathrm{m/s^2})}{4(0.11\,\mathrm{m})}(1.18\,\mathrm{s})^2 = 32.6 \text{ rad} = 5.19 \text{ revolutions}.
\end{aligned}$$

P10-8 (a) $l = I\omega_0 = (1/2)MR^2\omega_0$.

(b) The initial speed is $v_0 = R\omega_0$. The chip decelerates in a time $t = v_0/g$, and during this time the chip travels with an average speed of $v_0/2$ through a distance of

$$y = v_{\mathrm{av}}t = \frac{v_0}{2}\frac{v_0}{g} = \frac{R^2\omega^2}{2g}.$$

(c) Loosing the chip won't change the angular velocity of the wheel.

P10-9 Since $L = I\omega = 2\pi I/T$ and L is constant, then $I \propto T$. But $I \propto R^2$, so $R^2 \propto T$ and

$$\frac{\Delta T}{T} = \frac{2R\Delta R}{R^2} = \frac{2\Delta R}{R}.$$

Then

$$\Delta T = (86400\,\mathrm{s})\frac{2(30\,\mathrm{m})}{(6.37\times10^6\mathrm{m})} \approx 0.8\,\mathrm{s}.$$

P10-10 Originally the rotational inertia was

$$I_{\mathrm{i}} = \frac{2}{5}MR^2 = \frac{8\pi}{15}\rho_0 R^5.$$

The average density can be found from Appendix C. Now the rotational inertia is

$$I_{\mathrm{f}} = \frac{8\pi}{15}(\rho_1 - \rho_2)R_1^5 + \frac{8\pi}{15}\rho_2 R^5,$$

where ρ_1 is the density of the core, R_1 is the radius of the core, and ρ_2 is the density of the mantle. Since the angular momentum is constant we have $\Delta T/T = \Delta I/I$. Then

$$\frac{\Delta T}{T} = \frac{\rho_1 - \rho_2}{\rho_0}\frac{R_1^5}{R^5} + \frac{\rho_2}{\rho_0} - 1 = \frac{10.3 - 4.50}{5.52}\frac{3570^5}{6370^5} + \frac{4.50}{5.52} - 1 = -0.127,$$

so the day is getting longer.

$\boxed{\textbf{P10-11}}$ The cockroach initially has an angular speed of $\omega_{\mathrm{c,i}} = -v/r$. The rotational inertia of the cockroach about the axis of the turntable is $I_{\mathrm{c}} = mR^2$. Then conservation of angular momentum gives

$$\begin{aligned} l_{\mathrm{c,i}} + l_{\mathrm{s,i}} &= l_{\mathrm{c,f}} + l_{\mathrm{s,f}}, \\ I_{\mathrm{c}}\omega_{\mathrm{c,i}} + I_{\mathrm{s}}\omega_{\mathrm{s,i}} &= I_{\mathrm{c}}\omega_{\mathrm{c,f}} + I_{\mathrm{s}}\omega_{\mathrm{s,f}}, \\ -mR^2v/r + I\omega &= (mR^2 + I)\omega_{\mathrm{f}}, \\ \omega_{\mathrm{f}} &= \frac{I\omega - mvR}{I + mR^2}. \end{aligned}$$

P10-12 (a) The skaters move in a circle of radius $R = (2.92\,\mathrm{m})/2 = 1.46\,\mathrm{m}$ centered midway between the skaters. The angular velocity of the system will be $\omega_{\mathrm{i}} = v/R = (1.38\,\mathrm{m/s})/(1.46\,\mathrm{m}) = 0.945$ rad/s.

(b) Moving closer will decrease the rotational inertia, so

$$\omega_{\mathrm{f}} = \frac{2MR_{\mathrm{i}}{}^2}{2MR_{\mathrm{f}}{}^2}\omega_{\mathrm{i}} = \frac{(1.46\,\mathrm{m})^2}{(0.470\,\mathrm{m})^2}(0.945 \text{ rad/s}) = 9.12 \text{ rad/s}.$$

$\boxed{\textbf{E11-1}}$ (a) Apply Eq. 11-2, $W = Fs\cos\phi = (190\,\mathrm{N})(3.3\,\mathrm{m})\cos(22°) = 580\,\mathrm{J}$.

(b) The force of gravity is perpendicular to the displacement of the crate, so there is no work done by the force of gravity.

(c) The normal force is perpendicular to the displacement of the crate, so there is no work done by the normal force.

E11-2 (a) The force required is $F = ma = (106\,\mathrm{kg})(1.97\,\mathrm{m/s^2}) = 209\,\mathrm{N}$. The object moves with an average velocity $v_{\mathrm{av}} = v_0/2$ in a time $t = v_0/a$ through a distance $x = v_{\mathrm{av}}t = v_0^2/(2a)$. So

$$x = (51.3\,\mathrm{m/s})^2/[2(1.97\,\mathrm{m/s^2})] = 668\,\mathrm{m}.$$

The work done is $W = Fx = (-209\,\mathrm{N})(668\,\mathrm{m}) = 1.40\times10^5\mathrm{J}$.

(b) The force required is

$$F = ma = (106\,\mathrm{kg})(4.82\,\mathrm{m/s^2}) = 511\,\mathrm{N}.$$

$x = (51.3\,\mathrm{m/s})^2/[2(4.82\,\mathrm{m/s^2})] = 273\,\mathrm{m}$. The work done is $W = Fx = (-511\,\mathrm{N})(273\,\mathrm{m}) = 1.40{\times}10^5\mathrm{J}$.

E11-3 (a) $W = Fx = (120\,\mathrm{N})(3.6\,\mathrm{m}) = 430\,\mathrm{J}$.

(b) $W = Fx\cos\theta = mgx\cos\theta = (25\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(3.6\,\mathrm{m})\cos(117°) = 400\,\mathrm{N}$.

(c) $W = Fx\cos\theta$, but $\theta = 90°$, so $W = 0$.

E11-4 The worker pushes with a force $\vec{\mathbf{P}}$; this force has components $P_x = P\cos\theta$ and $P_y = P\sin\theta$, where $\theta = -32.0°$. The normal force of the ground on the crate is $N = mg - P_y$, so the force of friction is $f = \mu_{\mathrm{k}}N = \mu_{\mathrm{k}}(mg - P_y)$. The crate moves at constant speed, so $P_x = f$. Then

$$\begin{aligned} P\cos\theta &= \mu_{\mathrm{k}}(mg - P\sin\theta), \\ P &= \frac{\mu_{\mathrm{k}}mg}{\cos\theta + \mu_{\mathrm{k}}\sin\theta}. \end{aligned}$$

The work done on the crate is

$$\begin{aligned} W &= \vec{\mathbf{P}}\cdot\vec{\mathbf{x}} = Px\cos\theta = \frac{\mu_{\mathrm{k}}xmg}{1 + \mu_{\mathrm{k}}\tan\theta}, \\ &= \frac{(0.21)(31.3\text{ ft})(58.7\text{ lb})}{1 + (0.21)\tan(-32.0°)} = 444\text{ ft}\cdot\text{lb}. \end{aligned}$$

E11-5 The components of the weight are $W_{||} = mg\sin\theta$ and $W_\perp = mg\cos\theta$. The push $\vec{\mathbf{P}}$ has components $P_{||} = P\cos\theta$ and $P_\perp = P\sin\theta$.

The normal force on the trunk is $N = W_\perp + P\perp$ so the force of friction is $f = \mu_{\mathrm{k}}(mg\cos\theta + P\sin\theta)$. The push required to move the trunk up at constant speed is then found by noting that $P_{||} = W_{||} + f$.

Then

$$P = \frac{mg(\tan\theta + \mu_{\mathrm{k}})}{1 - \mu_{\mathrm{k}}\tan\theta}.$$

(a) The work done by the applied force is

$$W = Px\cos\theta = \frac{(52.3\,\mathrm{kg})(9.81\,\mathrm{m/s^2})[\sin(28.0°) + (0.19)\cos(28.0°)](5.95\,\mathrm{m})}{1 - (0.19)\tan(28.0°)} = 2160\,\mathrm{J}.$$

(b) The work done by the force of gravity is

$$W = mgx\cos(\theta + 90°) = (52.3\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(5.95\,\mathrm{m})\cos(118°) = -1430\,\mathrm{J}.$$

E11-6 $\theta = \arcsin(0.902\,\text{m}/1.62\,\text{m}) = 33.8°$.

The components of the weight are $W_{||} = mg\sin\theta$ and $W_\perp = mg\cos\theta$.

The normal force on the ice is $N = W_\perp$ so the force of friction is $f = \mu_k mg\cos\theta$. The push required to allow the ice to slide down at constant speed is then found by noting that $P = W_{||} - f$. Then $P = mg(\sin\theta - \mu_k\cos\theta)$.

(a) $P = (47.2\,\text{kg})(9.81\,\text{m/s}^2)[\sin(33.8°) - (0.110)\cos(33.8°)] = 215\,\text{N}$.

(b) The work done by the applied force is $W = Px = (215\,\text{N})(-1.62\,\text{m}) = -348\,\text{J}$.

(c) The work done by the force of gravity is

$$W = mgx\cos(90° - \theta) = (47.2\,\text{kg})(9.81\,\text{m/s}^2)(1.62\,\text{m})\cos(56.2°) = 417\,\text{J}.$$

$\boxed{\textbf{E11-7}}$ Equation 11-5 describes how to find the dot product of two vectors from components,

$$\begin{aligned}\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} &= a_xb_x + a_yb_y + a_zb_z,\\ &= (3)(2) + (3)(1) + (3)(3) = 18.\end{aligned}$$

Equation 11-3 can be used to find the angle between the vectors,

$$\begin{aligned}a &= \sqrt{(3)^2 + (3)^2 + (3)^2} = 5.19,\\ b &= \sqrt{(2)^2 + (1)^2 + (3)^2} = 3.74.\end{aligned}$$

Now use Eq. 11-3,

$$\cos\phi = \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{ab} = \frac{(18)}{(5.19)(3.74)} = 0.927,$$

and then $\phi = 22.0°$.

E11-8 $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = (12)(5.8)\cos(55°) = 40.$

E11-9 $\vec{\mathbf{r}}\cdot\vec{\mathbf{s}} = (4.5)(7.3)\cos(320° - 85°) = -19.$

E11-10 (a) Add the components individually:

$$\vec{\mathbf{r}} = (5 + 2 + 4)\hat{\mathbf{i}} + (4 - 2 + 3)\hat{\mathbf{j}} + (-6 - 3 + 2)\hat{\mathbf{k}} = 11\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 7\hat{\mathbf{k}}.$$

(b) $\theta = \arccos(-7/\sqrt{11^2 + 5^2 + 7^2}) = 120°$.

(c) $\theta = \arccos(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}/\sqrt{ab})$, or

$$\theta = \frac{(5)(-2) + (4)(2) + (-6)(3)}{\sqrt{(5^2 + 4^2 + 6^2)(2^2 + 2^2 + 3^2)}} = 124°.$$

$\boxed{\textbf{E11-11}}$ There are two forces on the woman, the force of gravity directed down and the normal force of the floor directed up. These will be effectively equal, so $N = W = mg$. Consequently, the 57 kg woman must exert a force of $F = (57\text{kg})(9.8\text{m/s}^2) = 560\text{N}$ to propel herself up the stairs.

From the reference frame of the woman the stairs are moving down, and she is exerting a force down, so the work done by the woman is given by

$$W = Fs = (560\text{ N})(4.5\text{ m}) = 2500\text{ J},$$

this work is positive because the force is in the same direction as the displacement.

The average power supplied by the woman is given by Eq. 11-7,

$$P = W/t = (2500\text{ J})/(3.5\text{ s}) = 710\text{ W}.$$

E11-12 $P = W/t = mgy/t = (100 \times 667\,\text{N})(152\,\text{m})/(55.0\,\text{s}) = 1.84\times10^5\text{W}$.

E11-13 $P = Fv = (110\,\text{N})(0.22\,\text{m/s}) = 24\,\text{W}$.

E11-14 $F = P/v$, but the units are awkward.

$$F = \frac{(4800\text{ hp})}{(77\text{ knots})}\frac{1\text{ knot}}{1.688\text{ ft/s}}\frac{1\text{ ft/s}}{0.3048\text{ m/s}}\frac{745.7\,\text{W}}{1\text{ hp}} = 9.0\times10^4\text{N}.$$

$\boxed{\textbf{E11-15}}$ $P = Fv = (720\text{ N})(26\text{ m/s}) = 19000\text{ W}$; in horsepower, $P = 19000\,\text{W}(1/745.7\text{ hp/W}) = 25\text{ hp}$.

E11-16 Change to metric units! Then $P = 4920\,\text{W}$, and the flow rate is $Q = 13.9\text{ L/s}$. The density of water is approximately 1.00 kg/L , so the mass flow rate is $R = 13.9$ kg/s.

$$y = \frac{P}{gR} = \frac{(4920\,\text{kg})}{(9.81\,\text{m/s}^2)(13.9\text{ kg/s})} = 36.1\,\text{m},$$

which is the same as approximately 120 feet.

$\boxed{\textbf{E11-17}}$ (a) Start by converting kilowatt-hours to Joules:

$$1\text{ kW}\cdot\text{h} = (1000\text{ W})(3600\text{ s}) = 3.6\times10^6\text{ J}.$$

The car gets 30 mi/gal, and one gallon of gas produces 140 MJ of energy. The gas required to produce 3.6×10^6J is

$$3.6\times10^6\text{J}\left(\frac{1\text{ gal}}{140\times10^6\text{J}}\right) = 0.026\text{ gal}.$$

The distance traveled on this much gasoline is

$$0.026\text{ gal}\left(\frac{30\text{ mi}}{1\text{ gal}}\right) = 0.78\text{ mi}.$$

(b) At 55 mi/h, it will take

$$0.78\text{ mi}\left(\frac{1\text{ hr}}{55\text{ mi}}\right) = 0.014\text{ h} = 51\text{ s}.$$

The rate of energy expenditure is then $(3.6\times10^6\text{ J})/(51\text{ s}) = 71000\text{ W}$.

E11-18 The linear speed is $v = 2\pi(0.207\,\text{m})(2.53\text{ rev/s}) = 3.29\,\text{m/s}$. The frictional force is $f = \mu_\text{k}N = (0.32)(180\,\text{N}) = 57.6\,\text{N}$. The power developed is $P = Fv = (57.6\,\text{N})(3.29\,\text{m}) = 190\,\text{W}$.

E11-19 The net force required will be $(1380\,\text{kg} - 1220\,\text{kg})(9.81\,\text{m/s}^2) = 1570\,\text{N}$. The work is $W = Fy$, the power output is $P = W/t = (1570\,\text{N})(54.5\,\text{m})/(43.0\,\text{s}) = 1990\,\text{W}$, or $P = 2.67$ hp.

E11-20 (a) The momentum change of the ejected material in one second is

$$\Delta p = (70.2\,\text{kg})(497\,\text{m/s} - 184\,\text{m/s}) + (2.92\,\text{kg})(497\,\text{m/s}) = 2.34\times10^4\text{kg}\cdot\text{m/s}.$$

The thrust is then $F = \Delta p/\Delta t = 2.34\times10^4$N.

(b) The power is $P = Fv = (2.34\times10^4\text{ N})(184\,\text{m/s}) = 4.31\times10^6$W. That's 5780 hp.

$\boxed{\textbf{E11-21}}$ The acceleration on the object as a function of position is given by

$$a = \frac{20 \text{ m/s}^2}{8 \text{ m}} x,$$

The work done on the object is given by Eq. 11-14,

$$W = \int_0^8 F_x dx = \int_0^8 (10 \text{ kg}) \frac{20 \text{ m/s}^2}{8 \text{ m}} x\, dx = 800 \text{ J}.$$

E11-22 Work is area between the curve and the line $F = 0$. Then

$$W = (10\,\text{N})(2\,\text{s}) + \frac{1}{2}(10\,\text{N})(2\,\text{s}) + \frac{1}{2}(-5\,\text{N})(2\,\text{s}) = 25\,\text{J}.$$

$\boxed{\textbf{E11-23}}$ (a) For a spring, $F = -kx$, and $\Delta F = -k\Delta x$.

$$k = -\frac{\Delta F}{\Delta x} = -\frac{(-240 \text{ N}) - (-110 \text{ N})}{(0.060 \text{ m}) - (0.040 \text{ m})} = 6500 \text{ N/m}.$$

With no force on the spring,

$$\Delta x = -\frac{\Delta F}{k} = -\frac{(0) - (-110 \text{ N})}{(6500 \text{ N/m})} = -0.017 \text{ m}.$$

This is the amount *less* than the 40 mm mark, so the position of the spring with no force on it is 23 mm.

(b) $\Delta x = -10$ mm compared to the 100 N picture, so

$$\Delta F = -k\Delta x = -(6500\,\text{N/m})(-0.010\,\text{m}) = 65\,\text{N}.$$

The weight of the last object is $110\,\text{N} - 65\,\text{N} = 45\,\text{N}$.

E11-24 (a) $W = \frac{1}{2}k({x_\text{f}}^2 - {x_\text{i}}^2) = \frac{1}{2}(1500\,\text{N/m})(7.60\times10^{-3}\text{m})^2 = 4.33\times10^{-2}\text{J}$.

(b) $W = \frac{1}{2}(1500\,\text{N/m})[(1.52\times10^{-2}\text{m})^2 - (7.60\times10^{-3}\text{m})^2 = 1.30\times10^{-1}\text{J}$.

E11-25 Start with Eq. 11-20, and let $F_x = 0$ while $F_y = -mg$:

$$W = \int_i^f (F_x dx + F_y dy) = -mg \int_i^f dy = -mgh.$$

E11-26 (a) $F_0 = mv_0^2/r_0 = (0.675\,\text{kg})(10.0\,\text{m/s})^2/(0.500\,\text{m}) = 135\,\text{N}$.

(b) Angular momentum is conserved, so $v = v_0(r_0/r)$. The force then varies as $F = mv^2/r = mv_0^2 r_0^2/r^3 = F_0(r_0/r)^3$. The work done is

$$W = \int \vec{\mathbf{F}} \cdot \vec{\mathbf{dr}} = \frac{(-135\,\text{N})(0.500\,\text{m})^3}{-2}\left((0.300\,\text{m})^{-2} - (0.500\,\text{m})^{-2}\right) = 60.0\,\text{J}.$$

$\boxed{\textbf{E11-27}}$ The kinetic energy of the electron is

$$4.2 \text{ eV}\left(\frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right) = 6.7 \times 10^{-19} \text{ J}.$$

Then

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(6.7\times10^{-19}\text{J})}{(9.1\times10^{-31}\text{kg})}} = 1.2\times10^6\text{m/s}.$$

E11-28 (a) $K = \frac{1}{2}(110\,\text{kg})(8.1\,\text{m/s})^2 = 3600\,\text{J}$.

(b) $K = \frac{1}{2}(4.2\times10^{-3}\text{kg})(950\,\text{m/s})^2 = 1900\,\text{J}$.

(c) $m = 91,400$ tons$(907.2\,\text{kg/ton}) = 8.29\times10^7\text{kg}$;

$$v = 32.0 \text{ knots}(1.688 \text{ ft/s/knot})(0.3048 \text{ m/ft}) = 16.5\,\text{m/s}.$$

$$K = \frac{1}{2}(8.29\times10^7\text{kg})(16.5\,\text{m/s})^2 = 1.13\times10^{10}\text{J}.$$

E11-29 (b) $\Delta K = W = Fx = (1.67\times10^{-27}\text{kg})(3.60\times10^{15}\text{m/s}^2)(0.0350\,\text{m}) = 2.10\times10^{-13}\text{J}$. That's $2.10\times10^{-13}\text{J}/(1.60\times10^{-19}\text{J/eV}) = 1.31\times10^6\text{eV}$.

(a) $K_\text{f} = 2.10\times10^{-13}\text{J} + \frac{1}{2}(1.67\times10^{-27}\text{kg})(2.40\times10^7\text{m/s}^2) = 6.91\times10^{-13}\text{J}$. Then

$$v_\text{f} = \sqrt{2K/m} = \sqrt{2(6.91\times10^{-13}\text{J})/(1.67\times10^{-27}\text{kg})} = 2.88\times10^7\text{m/s}.$$

E11-30 Work is negative if kinetic energy is decreasing. This happens only in region CD. The work is zero in region BC. Otherwise it is positive.

$\boxed{\textbf{E11-31}}$ (a) Find the velocity of the particle by taking the time derivative of the position:

$$v = \frac{dx}{dt} = (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)t + (3.0\,\text{m/s}^3)t^2.$$

Find v at two times: $t = 0$ and $t = 4\,\text{s}$.

$$\begin{aligned} v(0) &= (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)(0) + (3.0\,\text{m/s}^3)(0)^2 = 3.0\,\text{m/s}, \\ v(4) &= (3.0\,\text{m/s}) - (8.0\,\text{m/s}^2)(4.0\,\text{s}) + (3.0\,\text{m/s}^3)(4.0\,\text{s})^2 = 19.0\,\text{m/s} \end{aligned}$$

The initial kinetic energy is $K_\text{i} = \frac{1}{2}(2.80\,\text{kg})(3.0\,\text{m/s})^2 = 13\,\text{J}$, while the final kinetic energy is $K_\text{f} = \frac{1}{2}(2.80\,\text{kg})(19.0\,\text{m/s})^2 = 505\,\text{J}$.

The work done by the force is given by Eq. 11-24,

$$W = K_\text{f} - K_\text{i} = 505\,\text{J} - 13\,\text{J} = 492\,\text{J}.$$

(b) This question is asking for the instantaneous power when $t = 3.0\,\text{s}$. $P = Fv$, so first find a;

$$a = \frac{dv}{dt} = -(8.0\,\text{m/s}^2) + (6.0\,\text{m/s}^3)t.$$

Then the power is given by $P = mav$, and when $t = 3\,\text{s}$ this gives

$$P = mav = (2.80\,\text{kg})(10\,\text{m/s}^2)(6\,\text{m/s}) = 168\,\text{W}.$$

E11-32 $W = \Delta K = -K_\text{i}$. Then

$$W = -\frac{1}{2}(5.98\times10^{24}\text{kg})(29.8\times10^3\text{m/s})^2 = 2.66\times10^{33}\text{J}.$$

E11-33 (a) $K = \frac{1}{2}(1600\,\text{kg})(20\,\text{m/s})^2 = 3.2\times10^5\text{J}$.

(b) $P = W/t = (3.2\times10^5\text{J})/(33\,\text{s}) = 9.7\times10^3\text{W}$.

(c) $P = Fv = mav = (1600\,\text{kg})(20\,\text{m/s}/33.0\,\text{s})(20\,\text{m/s}) = 1.9\times10^4\text{W}$.

E11-34 (a) $I = 1.40\times10^4\text{u}\cdot\text{pm}^2(1.66\times10^{-27}\text{kg}/mboxu) = 2.32\times10^{-47}\text{kg}\cdot\text{m}^2$.

(b) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(2.32\times10^{-47}\text{kg}\cdot\text{m}^2)(4.30\times10^{12}\text{ rad/s})^2 = 2.14\times10^{-22}\text{J}$. That's 1.34 meV.

E11-35 The translational kinetic energy is $K_\mathrm{t} = \frac{1}{2}mv^2$, the rotational kinetic energy is $K_\mathrm{r} = \frac{1}{2}I\omega^2 = \frac{2}{3}K_\mathrm{t}$. Then

$$\omega = \sqrt{\frac{2m}{3I}}v = \sqrt{\frac{2(5.30\times10^{-26}\mathrm{kg})}{3(1.94\times10^{-46}\mathrm{kg}\cdot\mathrm{m}^2)}}(500\,\mathrm{m/s}) = 6.75\times10^{12}\ \mathrm{rad/s}.$$

E11-36 $K_\mathrm{r} = \frac{1}{2}I\omega^2 = \frac{1}{4}(512\,\mathrm{kg})(0.976\,\mathrm{m})^2(624\ \mathrm{rad/s})^2 = 4.75\times10^7\mathrm{J}$.

(b) $t = W/P = (4.75\times10^7\mathrm{J})/(8130\,\mathrm{W}) = 5840\,\mathrm{s}$, or 97.4 minutes.

$\boxed{\textbf{E11-37}}$ From Eq. 11-29, $K_\mathrm{i} = \frac{1}{2}I{\omega_\mathrm{i}}^2$. The object is a hoop, so $I = MR^2$. Then

$$K_\mathrm{i} = \frac{1}{2}MR^2\omega^2 = \frac{1}{2}(31.4\,\mathrm{kg})(1.21\,\mathrm{m})^2(29.6\,\mathrm{rad/s})^2 = 2.01\times10^4\ \mathrm{J}.$$

Finally, the average power required to stop the wheel is

$$P = \frac{W}{t} = \frac{K_\mathrm{f} - K_\mathrm{i}}{t} = \frac{(0) - (2.01\times10^4\ \mathrm{J})}{(14.8\,\mathrm{s})} = -1360\,\mathrm{W}.$$

E11-38 The wheels are connected by a belt, so $r_A\omega_A = r_B\omega_B$, or $\omega_A = 3\omega_B$.

(a) If $l_A = l_B$ then

$$\frac{I_A}{I_B} = \frac{l_A/\omega_A}{l_B/\omega_B} = \frac{\omega_B}{\omega_A} = \frac{1}{3}.$$

(b) If instead $K_A = K_B$ then

$$\frac{I_A}{I_B} = \frac{2K_A/{\omega_A}^2}{2K_B/{\omega_B}^2} = \frac{{\omega_B}^2}{{\omega_A}^2} = \frac{1}{9}.$$

E11-39 (a) $\omega = 2\pi/T$, so

$$K = \frac{1}{2}I\omega^2 = \frac{4\pi^2}{5}\frac{(5.98\times10^{24}\mathrm{kg})(6.37\times10^6\mathrm{m})^2}{(86,400\,\mathrm{s})^2} = 2.57\times10^{29}\mathrm{J}$$

(b) $t = (2.57\times10^{29}\mathrm{J})/(6.17\times10^{12}\mathrm{W}) = 4.17\times10^{16}\mathrm{s}$, or 1.3 billion years.

E11-40 (a) The velocities relative to the center of mass are $m_1v_1 = m_2v_2$; combine with $v_1 + v_2 = 910.0\,\mathrm{m/s}$ and get

$$(290.0\,\mathrm{kg})v_1 = (150.0\,\mathrm{kg})(910\,\mathrm{m/s} - v_1),$$

or

$$v_1 = (150\,\mathrm{kg})(910\,\mathrm{m/s})/(290\,\mathrm{kg} + 150\,\mathrm{kg}) = 310\,\mathrm{m/s}$$

and $v_2 = 600\,\mathrm{m/s}$. The rocket case was sent back, so $v_\mathrm{c} = 7600\,\mathrm{m/s} - 310\,\mathrm{m/s} = 7290\,\mathrm{m/s}$. The payload capsule was sent forward, so $v_\mathrm{p} = 7600\,\mathrm{m/s} + 600\,\mathrm{m/s} = 8200\,\mathrm{m/s}$.

(b) Before,

$$K_\mathrm{i} = \frac{1}{2}(290\,\mathrm{kg} + 150\,\mathrm{kg})(7600\,\mathrm{m/s})^2 = 1.271\times10^{10}\mathrm{J}.$$

After,

$$K_\mathrm{f} = \frac{1}{2}(290\,\mathrm{kg})(7290\,\mathrm{m/s})^2 + \frac{1}{2}(150\,\mathrm{kg})(8200\,\mathrm{m/s})^2 = 1.275\times10^{10}\mathrm{J}.$$

The "extra" energy came from the spring.

$\boxed{\textbf{E11-41}}$ Let the mass of the freight car be M and the initial speed be v_i. Let the mass of the caboose be m and the final speed of the coupled cars be v_f. The caboose is originally at rest, so the expression of momentum conservation is

$$Mv_\mathrm{i} = Mv_\mathrm{f} + mv_\mathrm{f} = (M+m)v_\mathrm{f}$$

The decrease in kinetic energy is given by

$$\begin{aligned} K_\mathrm{i} - K_\mathrm{f} &= \frac{1}{2}M{v_\mathrm{i}}^2 - \left(\frac{1}{2}M{v_\mathrm{f}}^2 + \frac{1}{2}m{v_\mathrm{f}}^2\right), \\ &= \frac{1}{2}\left(M{v_\mathrm{i}}^2 - (M+m){v_\mathrm{f}}^2\right) \end{aligned}$$

What we really want is $(K_\mathrm{i} - K_\mathrm{f})/K_\mathrm{i}$, so

$$\begin{aligned} \frac{K_\mathrm{i} - K_\mathrm{f}}{K_\mathrm{i}} &= \frac{M{v_\mathrm{i}}^2 - (M+m){v_\mathrm{f}}^2}{M{v_\mathrm{i}}^2}, \\ &= 1 - \frac{M+m}{M}\left(\frac{v_\mathrm{f}}{v_\mathrm{i}}\right)^2, \\ &= 1 - \frac{M+m}{M}\left(\frac{M}{M+m}\right)^2, \end{aligned}$$

where in the last line we substituted from the momentum conservation expression.

Then

$$\frac{K_\mathrm{i} - K_\mathrm{f}}{K_\mathrm{i}} = 1 - \frac{M}{M+m} = 1 - \frac{Mg}{Mg+mg}.$$

The left hand side is 27%. We want to solve this for mg, the weight of the caboose. Upon rearranging,

$$mg = \frac{Mg}{1-0.27} - Mg = \frac{(35.0\text{ ton})}{(0.73)} - (35.0\text{ ton}) = 12.9\text{ ton}.$$

E11-42 Since the body splits into two parts with equal mass then the velocity gained by one is identical to the velocity "lost" by the other. The initial kinetic energy is

$$K_\mathrm{i} = \frac{1}{2}(8.0\,\mathrm{kg})(2.0\,\mathrm{m/s})^2 = 16\,\mathrm{J}.$$

The final kinetic energy is 16 J greater than this, so

$$\begin{aligned} K_\mathrm{f} &= 32\,\mathrm{J} = \frac{1}{2}(4.0\,\mathrm{kg})(2.0\,\mathrm{m/s}+v)^2 + \frac{1}{2}(4.0\,\mathrm{kg})(2.0\,\mathrm{m/s}-v)^2, \\ &= \frac{1}{2}(8.0\,\mathrm{kg})[(2.0\,\mathrm{m/s})^2 + v^2], \end{aligned}$$

so $16.0\,\mathrm{J} = (4.0\,\mathrm{kg})v^2$. Then $v = 2.0\,\mathrm{m/s}$; one chunk comes to a rest while the other moves off at a speed of 4.0 m/s.

E11-43 The initial velocity of the neutron is $v_0\hat{\mathbf{i}}$, the final velocity is $v_1\hat{\mathbf{j}}$. By momentum conservation the final momentum of the deuteron is $m_\mathrm{n}(v_0\hat{\mathbf{i}} - v_1\hat{\mathbf{j}})$. Then $m_\mathrm{d}v_2 = m_\mathrm{n}\sqrt{v_0^2 + v_1^2}$.

There is also conservation of kinetic energy:

$$\frac{1}{2}m_\mathrm{n}v_0^2 = \frac{1}{2}m_\mathrm{n}v_1^2 + \frac{1}{2}m_\mathrm{d}v_2^2.$$

Rounding the numbers slightly we have $m_\mathrm{d} = 2m_\mathrm{n}$, then $4v_2^2 = v_0^2 + v_1^2$ is the momentum expression and $v_0^2 = v_1^2 + 2v_2^2$ is the energy expression. Combining,

$$2v_0^2 = 2v_1^2 + (v_0^2 + v_1^2),$$

or $v_1^2 = v_0^2/3$. So the neutron is left with 1/3 of its original kinetic energy.

E11-44 (a) The third particle must have a momentum

$$\begin{aligned}\vec{\mathbf{p}}_3 &= -(16.7\times 10^{-27}\mathrm{kg})(6.22\times 10^{6}\mathrm{m/s})\hat{\mathbf{i}} + (8.35\times 10^{-27}\mathrm{kg})(7.85\times 10^{6}\mathrm{m/s})\hat{\mathbf{j}} \\ &= (-1.04\hat{\mathbf{i}} + 0.655\hat{\mathbf{j}})\times 10^{-19}\mathrm{kg}\cdot\mathrm{m/s}.\end{aligned}$$

(b) The kinetic energy can also be written as $K = \frac{1}{2}mv^2 = \frac{1}{2}m(p/m)^2 = p^2/2m$. Then the kinetic energy appearing in this process is

$$\begin{aligned}K &= \frac{1}{2}(16.7\times 10^{-27}\mathrm{kg})(6.22\times 10^{6}\mathrm{m/s})^2 + \frac{1}{2}(8.35\times 10^{-27}\mathrm{kg})(7.85\times 10^{6}\mathrm{m/s})^2 \\ &\quad + \frac{1}{2(11.7\times 10^{-27}\mathrm{kg})}(1.23\times 10^{-19}\mathrm{kg}\cdot\mathrm{m/s})^2 = 1.23\times 10^{-12}\mathrm{J}.\end{aligned}$$

This is the same as 7.66 MeV.

$\boxed{\textbf{P11-1}}$ Change your units! Then

$$F = \frac{W}{s} = \frac{(4.5\ \mathrm{eV})(1.6\times 10^{-19}\ \mathrm{J/eV})}{(3.4\times 10^{-9}\ \mathrm{m})} = 2.1\times 10^{-10}\ \mathrm{N}.$$

P11-2 (a) If the acceleration is $-g/4$ the the net force on the block is $-Mg/4$, so the tension in the cord must be $T = 3Mg/4$.

(a) The work done by the cord is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (3Mg/4)(-d) = -(3/4)Mgd.$

(b) The work done by gravity is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (-Mg)(-d) = Mgd.$

P11-3 (a) There are *four* cords which are attached to the bottom load L. Each supports a tension F, so to lift the load $4F = (840\ \mathrm{lb}) + (20.0\ \mathrm{lb})$, or $F = 215$ lb.

(b) The work done against gravity is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (840\ \mathrm{lb})(12.0\ \mathrm{ft}) = 10100\ \mathrm{ft}\cdot\mathrm{lb}$.

(c) To lift the load 12 feet each segment of the cord must be shortened by 12 ft; there are four segments, so the end of the cord must be pulled through a distance of 48.0 ft.

(d) The work done by the applied force is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (215\ \mathrm{lb})(48.0\ \mathrm{ft}) = 10300\ \mathrm{ft}\cdot\mathrm{lb}$.

P11-4 The incline has a height h where $h = W/mg = (680\,\mathrm{J})/[(75\,\mathrm{kg})(9.81\,\mathrm{m/s^2})]$. The work required to lift the block is the same regardless of the path, so the length of the incline l is $l = W/F = (680\,\mathrm{J})/(320\,\mathrm{N})$. The angle of the incline is then

$$\theta = \arcsin\frac{h}{l} = \arcsin\frac{F}{mg} = \arcsin\frac{(320\,\mathrm{N})}{(75\,\mathrm{kg})(9.81\,\mathrm{m/s^2})} = 25.8^\circ.$$

P11-5 (a) In 12 minutes the horse moves $x = (5280\ \mathrm{ft/mi})(6.20\ \mathrm{mi/h})(0.200\ \mathrm{h}) = 6550$ ft. The work done by the horse in that time is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{s}} = (42.0\ \mathrm{lb})(6550\ \mathrm{ft})\cos(27.0^\circ) = 2.45\times 10^{5}\ \mathrm{ft}\cdot\mathrm{lb}$.

(b) The power output of the horse is

$$P = \frac{(2.45\times 10^{5}\ \mathrm{ft}\cdot\mathrm{lb})}{(720\,\mathrm{s})} = 340\ \mathrm{ft}\cdot\mathrm{lb/s}\frac{1\ \mathrm{hp}}{550\ \mathrm{ft}\cdot\mathrm{lb/s}} = 0.618\ \mathrm{hp}.$$

P11-6 In this problem $\theta = \arctan(28.2/39.4) = 35.5^\circ$.

The weight of the block has components $W_{||} = mg\sin\theta$ and $W_\perp = mg\cos\theta$. The force of friction on the block is $f = \mu_\mathrm{k}N = \mu_\mathrm{k}mg\cos\theta$. The tension in the rope must then be

$$T = mg(\sin\theta + \mu_\mathrm{k}\cos\theta)$$

in order to move the block up the ramp. The power supplied by the winch will be $P = Tv$, so

$$P = (1380\,\mathrm{kg})(9.81\,\mathrm{m/s^2})[\sin(35.5^\circ) + (0.41)\cos(35.5^\circ)](1.34\,\mathrm{m/s}) = 1.66\times 10^{4}\mathrm{W}.$$

$\boxed{\textbf{P11-7}}$ If the power is constant then the force on the car is given by $F = P/v$. But the force is related to the acceleration by $F = ma$ and to the speed by $F = m\frac{dv}{dt}$ for motion in one dimension. Then

$$\begin{aligned}
F &= \frac{P}{v}, \\
m\frac{dv}{dt} &= \frac{P}{v}, \\
m\frac{dx}{dt}\frac{dv}{dx} &= \frac{P}{v}, \\
mv\frac{dv}{dx} &= \frac{P}{v}, \\
\int_0^v mv^2 dv &= \int_0^x P dx, \\
\frac{1}{3}mv^3 &= Px.
\end{aligned}$$

We can rearrange this final expression to get v as a function of x, $v = (3xP/m)^{1/3}$.

P11-8 (a) If the drag is $D = bv^2$, then the force required to move the plane forward at constant speed is $F = D = bv^2$, so the power required is $P = Fv = bv^3$.

(b) $P \propto v^3$, so if the speed increases to 125% then P increases by a factor of $1.25^3 = 1.953$, or increases by 95.3%.

P11-9 (a) $P = mgh/t$, but m/t is the persons per minute times the average mass, so

$$P = (100\ \text{people/min})(75.0\text{kg})(9.81\,\text{m/s}^2)(8.20\,\text{m}) = 1.01\times10^4\text{W}.$$

(b) In 9.50 s the Escalator has moved $(0.620\,\text{m/s})(9.50\,\text{s}) = 5.89\,\text{m}$; so the Escalator has "lifted" the man through a distance of $(5.89\,\text{m})(8.20\,\text{m}/13.3\,\text{m}) = 3.63\,\text{m}$. The man did the rest himself.

The work done by the Escalator is then $W = (83.5\,\text{kg})(9.81\,\text{m/s}^2)(3.63\,\text{m}) = 2970\,\text{J}$.

(c) Yes, because the point of contact is moving in a direction with at least some non-zero component of the force. The power is

$$P = (83.5\,\text{m/s}^2)(9.81\,\text{m/s}^2)(0.620\,\text{m/s})(8.20\,\text{m}/13.3\,\text{m}) = 313\,\text{W}.$$

(d) If there is a force of contact between the man and the Escalator then the Escalator is doing work on the man.

P11-10 (a) $dP/dv = ab - 3av^2$, so P_{max} occurs when $3v^2 = b$, or $v = \sqrt{b/3}$.

(b) $F = P/v$, so $dF/dv = -2v$, which means F is a maximum when $v = 0$.

(c) No; $P = 0$, but $F = ab$.

$\boxed{\textbf{P11-11}}$ (b) Integrate,

$$W = \int_0^{3x_0} \vec{\mathbf{F}}\cdot d\vec{\mathbf{s}} = \frac{F_0}{x_0}\int_0^{3x_0}(x - x_0)dx = F_0x_0\left(\frac{9}{2} - 3\right),$$

or $W = 3F_0x_0/2$.

P11-12 (a) Simpson's rule gives

$$W = \frac{1}{3}\left[(10\,\mathrm{N}) + 4(2.4\,\mathrm{N}) + (0.8\,\mathrm{N})\right](1.0\,\mathrm{m}) = 6.8\,\mathrm{J}.$$

(b) $W = \int F\,ds = \int (A/x^2)dx = -A/x$, evaluating this between the limits of integration gives $W = (9\,\mathrm{N}\cdot\mathrm{m}^2)(1/1\,\mathrm{m} - 1/3\,\mathrm{m}) = 6\,\mathrm{J}$.

$\boxed{\textbf{P11-13}}$ The work required to stretch the spring from x_i to x_f is given by

$$W = \int_{x_\mathrm{i}}^{x_\mathrm{f}} kx^3 dx = \frac{k}{4}{x_\mathrm{f}}^4 - \frac{k}{4}{x_\mathrm{i}}^4.$$

The problem gives

$$W_0 = \frac{k}{4}(l)^4 - \frac{k}{4}(0)^4 = \frac{k}{4}l^4.$$

We then want to find the work required to stretch from $x = l$ to $x = 2l$, so

$$\begin{aligned} W_{l\to 2l} &= \frac{k}{4}(2l)^4 - \frac{k}{4}(l)^4, \\ &= 16\frac{k}{4}l^4 - \frac{k}{4}l^4, \\ &= 15\frac{k}{4}l^4 = 15W_0. \end{aligned}$$

P11-14 (a) The spring extension is $\delta l = \sqrt{l_0^2 + x^2} - l_0$. The force from one spring has magnitude $k\delta l$, but only the x component contributes to the problem, so

$$F = 2k\left(\sqrt{l_0^2 + x^2} - l_0\right)\frac{x}{\sqrt{l_0^2 + x^2}}$$

is the force required to move the point.

The work required is the integral, $W = \int_0^x F\,dx$, which is

$$W = kx^2 - 2kl_0\sqrt{l_0^2 + x^2} + 2kl_0^2$$

Note that it does reduce to the expected behavior for $x \gg l_0$.

(b) Binomial expansion of square root gives

$$\sqrt{l_0^2 + x^2} = l_0\left(1 + \frac{1}{2}\frac{x^2}{l_0^2} - \frac{1}{8}\frac{x^4}{l_0^4}\cdots\right),$$

so the first term in the above expansion cancels with the last term in W; the second term cancels with the first term in W, leaving

$$W = \frac{1}{4}k\frac{x^4}{l_0^2}.$$

P11-15 Number the springs clockwise from the top of the picture. Then the four forces *on* each spring are

$$\begin{aligned} F_1 &= k(l_0 - \sqrt{x^2 + (l_0 - y)^2}), \\ F_2 &= k(l_0 - \sqrt{(l_0 - x)^2 + y^2}), \\ F_3 &= k(l_0 - \sqrt{x^2 + (l_0 + y)^2}), \\ F_4 &= k(l_0 - \sqrt{(l_0 + x)^2 + y^2}). \end{aligned}$$

The directions are *much* harder to work out, but for small x and y we can assume that

$$\begin{aligned}
\vec{\mathbf{F_1}} &= k(l_0 - \sqrt{x^2 + (l_0 - y)^2})\hat{\mathbf{j}}, \\
\vec{\mathbf{F_2}} &= k(l_0 - \sqrt{(l_0 - x)^2 + y^2})\hat{\mathbf{i}}, \\
\vec{\mathbf{F_3}} &= k(l_0 - \sqrt{x^2 + (l_0 + y)^2})\hat{\mathbf{j}}, \\
\vec{\mathbf{F_4}} &= k(l_0 - \sqrt{(l_0 + x)^2 + y^2})\hat{\mathbf{i}}.
\end{aligned}$$

Then

$$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = \int (F_1 + F_3)dy + \int (F_2 + F_4)dx,$$

Since x and y are small, expand the force(s) in a binomial expansion:

$$F_1(x, y) \approx F_1(0, 0) + \left.\frac{\partial F_1}{\partial x}\right|_{x,y=0} x + \left.\frac{\partial F_1}{\partial y}\right|_{x,y=0} y = ky;$$

there will be similar expression for the other four forces. Then

$$W = \int 2ky\,dy + \int 2kx\,dx = k(x^2 + y^2) = kd^2.$$

P11-16 (a) $K_\mathrm{i} = \frac{1}{2}(1100\,\mathrm{kg})(12.8\,\mathrm{m/s})^2 = 9.0\times 10^4\mathrm{J}$. Removing 51 kJ leaves 39 kJ behind, so

$$v_\mathrm{f} = \sqrt{2K_\mathrm{f}/m} = \sqrt{2(3.9\times 10^4\mathrm{J})/(1100\,\mathrm{kg})} = 8.4\,\mathrm{m/s},$$

or 30 km/h.

(b) 39 kJ, as was found above.

P11-17 Let M be the mass of the man and m be the mass of the boy. Let v_M be the original speed of the man and v_m be the original speed of the boy. Then

$$\frac{1}{2}Mv_M^2 = \frac{1}{2}\left(\frac{1}{2}mv_m^2\right)$$

and

$$\frac{1}{2}M(v_M + 1.0\,\mathrm{m/s})^2 = \frac{1}{2}mv_m^2.$$

Combine these two expressions and solve for v_M,

$$\begin{aligned}
\frac{1}{2}Mv_M^2 &= \frac{1}{2}\left(\frac{1}{2}M(v_M + 1.0\,\mathrm{m/s})^2\right), \\
v_M^2 &= \frac{1}{2}(v_M + 1.0\,\mathrm{m/s})^2, \\
0 &= -v_M^2 + (2.0\,\mathrm{m/s})v_M + (1.0\,\mathrm{m/s})^2.
\end{aligned}$$

The last line can be solved as a quadratic, and $v_M = (1.0\,\mathrm{m/s}) \pm (1.41\,\mathrm{m/s})$. Now we use the very first equation to find the speed of the boy,

$$\begin{aligned}
\frac{1}{2}Mv_M^2 &= \frac{1}{2}\left(\frac{1}{2}mv_m^2\right), \\
v_M^2 &= \frac{1}{4}v_m^2, \\
2v_M &= v_m.
\end{aligned}$$

P11-18 (a) The work done by gravity on the projectile as it is raised up to 140 m is $W = -mgy = -(0.550\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(140\,\mathrm{m}) = -755\,\mathrm{J}$. Then the kinetic energy at the highest point is $1550\,\mathrm{J} - 755\,\mathrm{J} = 795\,\mathrm{J}$. Since the projectile must be moving horizontally at the highest point, the horizontal velocity is $v_x = \sqrt{2(795\,\mathrm{J})/(0.550\,\mathrm{kg})} = 53.8\,\mathrm{m/s}$.

(b) The magnitude of the velocity at launch is $v = \sqrt{2(1550\,\mathrm{J})/(0.550\,\mathrm{kg})} = 75.1\,\mathrm{m/s}$. Then $v_y = \sqrt{(75.1\,\mathrm{m/s})^2 - (53.8\,\mathrm{m/s})^2} = 52.4\,\mathrm{m/s}$.

(c) The kinetic energy at that point is $\frac{1}{2}(0.550\,\mathrm{kg})[(65.0\,\mathrm{m/s})^2 + (53.8\,\mathrm{m/s})^2] = 1960\,\mathrm{J}$. Since it has extra kinetic energy, it must be below the launch point, and gravity has done 410 J of work on it. That means it is $y = (410\,\mathrm{J})/[(0.550\,\mathrm{kg})(9.81\,\mathrm{m/s^2})] = 76.0\,\mathrm{m}$ below the launch point.

P11-19 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(8.38\times10^{11}\mathrm{kg})(3.0\times10^4\mathrm{m/s})^2 = 3.77\times10^{20}\mathrm{J}$. In terms of TNT, $K = 9.0\times10^4$ megatons.

(b) The diameter will be $\sqrt[3]{8.98\times10^4} = 45$ km.

P11-20 (a) $W_\mathrm{g} = -(0.263\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(-0.118\,\mathrm{m}) = 0.304\,\mathrm{J}$.

(b) $W_\mathrm{s} = -\frac{1}{2}(252\,\mathrm{N/m})(-0.118\,\mathrm{m})^2 = -1.75\,\mathrm{J}$.

(c) The kinetic energy just before hitting the block would be $1.75\,\mathrm{J} - 0.304\,\mathrm{J} = 1.45\,\mathrm{J}$. The speed is then $v = \sqrt{2(1.45\,\mathrm{J})/(0.263\,\mathrm{kg})} = 3.32\,\mathrm{m/s}$.

(d) Doubling the speed quadruples the initial kinetic energy to 5.78 J. The compression will then be given by

$$-5.78\,\mathrm{J} = -\frac{1}{2}(252\,\mathrm{N/m})y^2 - (0.263\,\mathrm{kg})(9.81\,\mathrm{m/s^2})y,$$

with solution $y = 0.225\,\mathrm{m}$.

$\boxed{\textbf{P11-21}}$ (a) We can solve this with a trick of integration.

$$\begin{aligned}
W &= \int_0^x F\,dx, \\
&= \int_0^x ma_x \frac{dt}{dt}\,dx = ma_x \int_0^t \frac{dx}{dt}dt \\
&= ma_x \int_0^t v_x\,dt = ma_x \int_0^t at\,dt, \\
&= \frac{1}{2}ma_x^2 t^2.
\end{aligned}$$

Basically, we changed the variable of integration from x to t, and then used the fact the the acceleration was constant so $v_x = v_{0x} + a_x t$. The object started at rest so $v_{0x} = 0$, and we are given in the problem that $v_\mathrm{f} = at_\mathrm{f}$. Combining,

$$W = \frac{1}{2}ma_x^2 t^2 = \frac{1}{2}m\left(\frac{v_\mathrm{f}}{t_\mathrm{f}}\right)^2 t^2.$$

(b) Instantaneous power will be the derivative of this, so

$$P = \frac{dW}{dt} = m\left(\frac{v_\mathrm{f}}{t_\mathrm{f}}\right)^2 t.$$

P11-22 (a) $\alpha = (-39.0 \text{ rev/s})(2\pi \text{ rad/rev})/(32.0\,\text{s}) = -7.66 \text{ rad/s}^2$.

(b) The total rotational inertia of the system about the axis of rotation is

$$I = (6.40\,\text{kg})(1.20\,\text{m})^2/12 + 2(1.06\,\text{kg})(1.20\,\text{m}/2)^2 = 1.53\,\text{kg}\cdot\text{m}^2.$$

The torque is then $\tau = (1.53\,\text{kg}\cdot\text{m}^2)(7.66 \text{ rad/s}^2) = 11.7\,\text{N}\cdot\text{m}$.

(c) $K = \frac{1}{2}(1.53\,\text{kg}\cdot\text{m}^2)(245 \text{ rad/s})^2 = 4.59\times10^4\text{J}$.

(d) $\theta = \omega_{\text{av}} t = (39.0 \text{ rev/s}/2)(32.0\,\text{s}) = 624$ rev.

(e) Only the loss in kinetic energy is independent of the behavior of the frictional torque.

P11-23 The wheel turn with angular speed $\omega = v/r$, where r is the radius of the wheel and v the speed of the car. The total rotational kinetic energy in the four wheels is then

$$K\text{r} = 4\frac{1}{2}I\omega^2 = 2\left[\frac{1}{2}(11.3\,\text{kg})r^2\right]\left[\frac{v}{r}\right]^2 = (11.3\,\text{kg})v^2.$$

The translational kinetic energy is $K_{\text{t}} = \frac{1}{2}(1040\,\text{kg})v^2$, so the fraction of the total which is due to the rotation of the wheels is

$$\frac{11.3}{520 + 11.3} = 0.0213 \text{ or } 2.13\%.$$

P11-24 (a) Conservation of angular momentum: $\omega_{\text{f}} = (6.13\,\text{kg}\cdot\text{m}^2/1.97\,\text{kg}\cdot\text{m}^2)(1.22 \text{ rev/s}) = 3.80$ rev/s.

(b) $K_{\text{r}} \propto I\omega^2 \propto l^2/I$, so

$$K_{\text{f}}/K_{\text{i}} = I_{\text{i}}/I_{\text{f}} = (6.13\,\text{kg}\cdot\text{m}^2)/(1.97\,\text{kg}\cdot\text{m}^2) = 3.11.$$

$\boxed{\textbf{P11-25}}$ We did the first part of the solution in Ex. 10-21. The initial kinetic energy is (again, ignoring the shaft),

$$K_{\text{i}} = \frac{1}{2}I_1{\vec{\omega}_{1,\text{i}}}^2,$$

since the second wheel was originally at rest. The final kinetic energy is

$$K_{\text{f}} = \frac{1}{2}(I_1 + I_2){\vec{\omega}_{\text{f}}}^2,$$

since the two wheels moved as one. Then

$$\begin{aligned}
\frac{K_{\text{i}} - K_{\text{f}}}{K_{\text{i}}} &= \frac{\frac{1}{2}I_1{\vec{\omega}_{1,\text{i}}}^2 - \frac{1}{2}(I_1 + I_2){\vec{\omega}_{\text{f}}}^2}{\frac{1}{2}I_1{\vec{\omega}_{1,\text{i}}}^2}, \\
&= 1 - \frac{(I_1 + I_2){\vec{\omega}_{\text{f}}}^2}{I_1{\vec{\omega}_{1,\text{i}}}^2}, \\
&= 1 - \frac{I_1}{I_1 + I_2},
\end{aligned}$$

where in the last line we substituted from the results of Ex. 10-21.

Using the numbers from Ex. 10-21,

$$\frac{K_{\text{i}} - K_{\text{f}}}{K_{\text{i}}} = 1 - \frac{(1.27\,\text{kg}\cdot\text{m}^2)}{(1.27\,\text{kg}\cdot\text{m}^2) + (4.85\,\text{kg}\cdot\text{m}^2)} = 79.2\%.$$

P11-26 See the solution to P10-11.

$$K_\mathrm{i} = \frac{I}{2}\omega^2 + \frac{m}{2}v^2$$

while

$$K_\mathrm{f} = \frac{1}{2}(I + mR^2){\omega_\mathrm{f}}^2$$

according to P10-11,

$$\omega_\mathrm{f} = \frac{I\omega - mvR}{I + mR^2}.$$

Then

$$K_\mathrm{f} = \frac{1}{2}\frac{(I\omega - mvR)^2}{I + mR^2}.$$

Finally,

$$\begin{aligned}
\Delta K &= \frac{1}{2}\frac{(I\omega - mvR)^2 - (I + mR^2)(I\omega^2 + mv^2)}{I + mR^2}, \\
&= -\frac{1}{2}\frac{ImR^2\omega^2 + 2mvRI\omega + Imv^2}{I + mR^2}, \\
&= -\frac{Im}{2}\frac{(R\omega + v)^2}{I + mR^2}.
\end{aligned}$$

P11-27 See the solution to P10-12.

(a) $K_\mathrm{i} = \frac{1}{2}I_\mathrm{i}{\omega_\mathrm{i}}^2$, so

$$K_\mathrm{i} = \frac{1}{2}\left(2(51.2\,\mathrm{kg})(1.46\,\mathrm{m})^2\right)(0.945\ \mathrm{rad/s})^2 = 97.5\,\mathrm{J}.$$

(b) $K_\mathrm{f} = \frac{1}{2}\left(2(51.2\,\mathrm{kg})(0.470\,\mathrm{m})^2\right)(9.12\ \mathrm{rad/s})^2 = 941\,\mathrm{J}$. The energy comes from the work they do while pulling themselves closer together.

P11-28 $K = \frac{1}{2}mv^2 = \frac{1}{2m}p^2 = \frac{1}{2m}\vec{\mathbf{p}}\cdot\vec{\mathbf{p}}$. Then

$$\begin{aligned}
K_\mathrm{f} &= \frac{1}{2m}(\vec{\mathbf{p}}_\mathrm{i} + \Delta\vec{\mathbf{p}})\cdot(\vec{\mathbf{p}}_\mathrm{i} + \Delta\vec{\mathbf{p}}), \\
&= \frac{1}{2m}\left({p_\mathrm{i}}^2 + 2\vec{\mathbf{p}}_\mathrm{i}\cdot\Delta\vec{\mathbf{p}} + (\Delta p)^2\right), \\
\Delta K &= \frac{1}{2m}\left(2\vec{\mathbf{p}}_\mathrm{i}\cdot\Delta\vec{\mathbf{p}} + (\Delta p)^2\right).
\end{aligned}$$

In all three cases $\Delta p = (3000\,\mathrm{N})(65.0\,\mathrm{s}) = 1.95\times10^5\mathrm{N}\cdot\mathrm{s}$ and $p_\mathrm{i} = (2500\,\mathrm{kg})(300\,\mathrm{m/s}) = 7.50\times10^5\mathrm{kg}\cdot\mathrm{m/s}$.

(a) If the thrust is backward (pushing rocket forward),

$$\Delta K = \frac{+2(7.50\times10^5\mathrm{kg}\cdot\mathrm{m/s})(1.95\times10^5\mathrm{N}\cdot\mathrm{s}) + (1.95\times10^5\mathrm{N}\cdot\mathrm{s})^2}{2(2500\,\mathrm{kg})} = +6.61\times10^7\mathrm{J}.$$

(b) If the thrust is forward,

$$\Delta K = \frac{-2(7.50\times10^5\mathrm{kg}\cdot\mathrm{m/s})(1.95\times10^5\mathrm{N}\cdot\mathrm{s}) + (1.95\times10^5\mathrm{N}\cdot\mathrm{s})^2}{2(2500\,\mathrm{kg})} = -5.09\times10^7\mathrm{J}.$$

(c) If the thrust is sideways the first term vanishes,

$$\Delta K = \frac{+(1.95\times10^5\mathrm{N}\cdot\mathrm{s})^2}{2(2500\,\mathrm{kg})} = 7.61\times10^6\mathrm{J}.$$

$\boxed{\textbf{P11-29}}$ There's nothing to integrate here! Start with the work-energy theorem

$$\begin{aligned} W &= K_\mathrm{f} - K_\mathrm{i} = \frac{1}{2}m{v_\mathrm{f}}^2 - \frac{1}{2}m{v_\mathrm{i}}^2, \\ &= \frac{1}{2}m\left({v_\mathrm{f}}^2 - {v_\mathrm{i}}^2\right), \\ &= \frac{1}{2}m\left(v_\mathrm{f} - v_\mathrm{i}\right)\left(v_\mathrm{f} + v_\mathrm{i}\right), \end{aligned}$$

where in the last line we factored the difference of two squares. Continuing,

$$\begin{aligned} W &= \frac{1}{2}\left(mv_\mathrm{f} - mv_\mathrm{i}\right)\left(v_\mathrm{f} + v_\mathrm{i}\right), \\ &= \frac{1}{2}\left(\Delta p\right)\left(v_\mathrm{f} + v_\mathrm{i}\right), \end{aligned}$$

but $\Delta p = J$, the impulse. That finishes this problem.

P11-30 Let M be the mass of the helicopter. It will take a force Mg to keep the helicopter airborne. This force comes from pushing the air down at a rate $\Delta m/\Delta t$ with a speed of v; so $Mg = v\Delta m/\Delta t$. The blades sweep out a cylinder of cross sectional area A, so $\Delta m/\Delta t = \rho A v$. The force is then $Mg = \rho A v^2$; the speed that the air must be pushed down is $v = \sqrt{Mg/\rho A}$. The minimum power is then

$$P = Fv = Mg\sqrt{\frac{Mg}{\rho A}} = \sqrt{\frac{(1820\,\mathrm{kg})^3(9.81\,\mathrm{m/s^2})^3}{(1.23\,\mathrm{kg/m^3})\pi(4.88\,\mathrm{m})^2}} = 2.49\times 10^5\,\mathrm{W}.$$

P11-31 (a) Inelastic collision, so $v_\mathrm{f} = mv_\mathrm{i}/(m+M)$.
(b) $K = \frac{1}{2}mv^2 = p^2/2m$, so

$$\frac{\Delta K}{K_\mathrm{i}} = \frac{1/m - 1/(m+M)}{1/m} = \frac{M}{m+M}.$$

P11-32 Inelastic collision, so

$$v_\mathrm{f} = \frac{(1.88\,\mathrm{kg})(10.3\,\mathrm{m/s}) + (4.92\,\mathrm{kg})(3.27\,\mathrm{m/s})}{(1.88\,\mathrm{kg}) + (4.92\,\mathrm{kg})} = 5.21\,\mathrm{m/s}.$$

The loss in kinetic energy is

$$\Delta K = \frac{(1.88\,\mathrm{kg})(10.3\,\mathrm{m/s})^2}{2} + \frac{(4.92\,\mathrm{kg})(3.27\,\mathrm{m/s})^2}{2} - \frac{(1.88\,\mathrm{kg} + 4.92\,\mathrm{kg})(5.21\,\mathrm{m/s})^2}{2} = 33.7\,\mathrm{J}.$$

This change is because of work done on the spring, so

$$x = \sqrt{2(33.7\,\mathrm{J})/(1120\,\mathrm{N/m})} = 0.245\,\mathrm{m}$$

P11-33 $\vec{\mathbf{p}}_{\mathrm{f},B} = \vec{\mathbf{p}}_{\mathrm{i},A} + \vec{\mathbf{p}}_{\mathrm{i},B} - \vec{\mathbf{p}}_{\mathrm{f},A}$, so

$$\begin{aligned} \vec{\mathbf{p}}_{\mathrm{f},B} &= [(2.0\,\mathrm{kg})(15\,\mathrm{m/s}) + (3.0\,\mathrm{kg})(-10\,\mathrm{m/s}) - (2.0\,\mathrm{kg})(-6.0\,\mathrm{m/s})]\hat{\mathbf{i}} \\ &\quad +[(2.0\,\mathrm{kg})(30\,\mathrm{m/s}) + (3.0\,\mathrm{kg})(5.0\,\mathrm{m/s}) - (2.0\,\mathrm{kg})(30\,\mathrm{m/s})]\hat{\mathbf{j}}, \\ &= (12\,\mathrm{kg\cdot m/s})\hat{\mathbf{i}} + (15\,\mathrm{kg\cdot m/s})\hat{\mathbf{j}}. \end{aligned}$$

Then $\vec{\mathbf{v}}_{f,B} = (4.0\,\mathrm{m/s})\hat{\mathbf{i}} + (5.0\,\mathrm{m/s})\hat{\mathbf{j}}$. Since $K = \frac{m}{2}(v_x^2 + v_y^2)$, the change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{(2.0\,\mathrm{kg})[(-6.0\,\mathrm{m/s})^2 + (30\,\mathrm{m/s})^2 - (15\,\mathrm{m/s})^2 - (30\,\mathrm{m/s})^2]}{2} \\ &\quad + \frac{(3.0\,\mathrm{kg})[(4.0\,\mathrm{m/s})^2 + (5\,\mathrm{m/s})^2 - (-10\,\mathrm{m/s})^2 - (5.0\,\mathrm{m/s})^2]}{2} \\ &= -315\,\mathrm{J}.\end{aligned}$$

P11-34 For the observer on the train the acceleration of the particle is a, the distance traveled is $x_\mathrm{t} = \frac{1}{2}at^2$, so the work done as measured by the train is $W_\mathrm{t} = max_\mathrm{t} = \frac{1}{2}a^2t^2$. The final speed of the particle as measured by the train is $v_\mathrm{t} = at$, so the kinetic energy as measured by the train is $K = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2$. The particle started from rest, so $\Delta K_\mathrm{t} = W_\mathrm{t}$.

For the observer on the ground the acceleration of the particle is a, the distance traveled is $x_\mathrm{g} = \frac{1}{2}at^2 + ut$, so the work done as measured by the ground is $W_\mathrm{g} = max_\mathrm{g} = \frac{1}{2}a^2t^2 + maut$. The final speed of the particle as measured by the ground is $v_\mathrm{g} = at + u$, so the kinetic energy as measured by the ground is

$$K_\mathrm{g} = \frac{1}{2}mv^2 = \frac{1}{2}m(at+u)^2 = \frac{1}{2}a^2t^2 + maut + \frac{1}{2}mu^2.$$

But the initial kinetic energy as measured by the ground is $\frac{1}{2}mu^2$, so $W_\mathrm{g} = \Delta K_\mathrm{g}$.

P11-35 (a) $K_\mathrm{i} = \frac{1}{2}m_1 {v_{1,\mathrm{i}}}^2$.

(b) After collision $v_\mathrm{f} = m_1 v_{1,\mathrm{i}}/(m_1 + m_2)$, so

$$K_\mathrm{f} = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 v_{1,\mathrm{i}}}{m_1 + m_2}\right)^2 = \frac{1}{2}m_1 {v_{1,\mathrm{i}}}^2\left(\frac{m_1}{m_1 + m_2}\right).$$

(c) The fraction lost was

$$1 - \frac{m_1}{m_1 + m_2} = \frac{m_2}{m_1 + m_2}.$$

(d) Note that $v_\mathrm{cm} = v_\mathrm{f}$. The initial kinetic energy of the system is

$$K_\mathrm{i} = \frac{1}{2}m_1 {v'_{1,\mathrm{i}}}^2 + \frac{1}{2}m_2 {v'_{2,\mathrm{i}}}^2.$$

The final kinetic energy is zero (they stick together!), so the fraction lost is 1. The *amount* lost, however, is the same.

P11-36 Only consider the first two collisions, the one between m and m', and then the one between m' and M.

Momentum conservation applied to the first collision means the speed of m' will be between $v' = mv_0/(m + m')$ (completely inelastic)and $v' = 2mv_0/(m + m')$ (completely elastic). Momentum conservation applied to the second collision means the speed of M will be between $V = m'v'/(m'+M)$ and $V = 2m'v'/(m'+M)$. The largest kinetic energy for M will occur when it is moving the fastest, so

$$v' = \frac{2mv_0}{m + m'} \text{ and } V = \frac{2m'v'}{m' + M} = \frac{4m'mv_0}{(m + m')(m' + M)}.$$

We want to maximize V as a function of m', so take the derivative:

$$\frac{dV}{dm'} = \frac{4mv_0(mM - m'^2)}{(m' + M)^2(m + m')^2}.$$

This vanishes when $m' = \sqrt{mM}$.

E12-1 (a) Integrate.

$$U(x) = -\int_\infty^x G\frac{m_1 m_2}{x^2}dx = -G\frac{m_1 m_2}{x}.$$

(b) $W = U(x) - U(x+d)$, so

$$W = Gm_1m_2\left(\frac{1}{x} - \frac{1}{x+d}\right) = Gm_1m_2\frac{d}{x(x+d)}.$$

E12-2 If $d << x$ then $x(x+d) \approx x^2$, so

$$W \approx G\frac{m_1 m_2}{x^2}d.$$

$\boxed{\textbf{E12-3}}$ Start with Eq. 12-6.

$$\begin{aligned} U(x) - U(x_0) &= -\int_{x_0}^x F_x(x)dx, \\ &= -\int_{x_0}^x \left(-\alpha x e^{-\beta x^2}\right)dx, \\ &= \left.\frac{-\alpha}{2\beta}e^{-\beta x^2}\right|_{x_0}^x. \end{aligned}$$

Finishing the integration,

$$U(x) = U(x_0) + \frac{\alpha}{2\beta}\left(e^{-\beta x_0^2} - e^{-\beta x^2}\right).$$

If we *choose* $x_0 = \infty$ *and* $U(x_0) = 0$ we would be left with

$$U(x) = -\frac{\alpha}{2\beta}e^{-\beta x^2}.$$

E12-4 $\Delta K = -\Delta U$ so $\Delta K = mg\Delta y$. The power output is then

$$P = (58\%)\frac{(1000\,\mathrm{kg/m^3})(73,800\,\mathrm{m^3})}{(60\,\mathrm{s})}(9.81\,\mathrm{m/s^2})(96.3\,\mathrm{m}) = 6.74\times10^8\mathrm{W}.$$

E12-5 $\Delta U = -\Delta K$, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$. Then

$$k = \frac{mv^2}{x^2} = \frac{(2.38\,\mathrm{kg})(10.0\times10^3\,/\mathrm{s})}{(1.47\,\mathrm{m})^2} = 1.10\times10^8\mathrm{N/m}.$$

Wow.

E12-6 $\Delta U_\mathrm{g} + \Delta U_\mathrm{s} = 0$, since $K = 0$ when the man jumps and when the man stops. Then $\Delta U_\mathrm{s} = -mg\Delta y = (220\text{ lb})(40.4\text{ ft}) = 8900\text{ ft}\cdot\text{lb}$.

$\boxed{\textbf{E12-7}}$ Apply Eq. 12-15,

$$\begin{aligned} K_\mathrm{f} + U_\mathrm{f} &= K_\mathrm{i} + U_\mathrm{i}, \\ \frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f} &= \frac{1}{2}m{v_\mathrm{i}}^2 + mgy_\mathrm{i}, \\ \frac{1}{2}{v_\mathrm{f}}^2 + g(-r) &= \frac{1}{2}(0)^2 + g(0). \end{aligned}$$

Rearranging,

$$v_\mathrm{f} = \sqrt{-2g(-r)} = \sqrt{-2(9.81\,\mathrm{m/s^2})(-0.236\,\mathrm{m})} = 2.15\,\mathrm{m/s}.$$

E12-8 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(2.40\,\mathrm{kg})(150\,\mathrm{m/s})^2 = 2.70\times10^4\,\mathrm{J}$.

(b) Assuming that the ground is zero, $U = mgy = (2.40\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(125\,\mathrm{m}) = 2.94\times10^3\,\mathrm{J}$.

(c) $K_\mathrm{f} = K_\mathrm{i} + U_\mathrm{i}$ since $U_\mathrm{f} = 0$. Then

$$v_\mathrm{f} = \sqrt{2\frac{(2.70\times10^4\,\mathrm{J}) + (2.94\times10^3\,\mathrm{J})}{(2.40\,\mathrm{kg})}} = 158\,\mathrm{m/s}.$$

Only (a) and (b) depend on the mass.

E12-9 (a) Since $\Delta y = 0$, then $\Delta U = 0$ and $\Delta K = 0$. Consequently, at B, $v = v_0$.

(b) At C $K_C = K_A + U_A - U_C$, or

$$\frac{1}{2}m{v_C}^2 = \frac{1}{2}m{v_0}^2 + mgh - mg\frac{h}{2},$$

or

$$v_B = \sqrt{{v_0}^2 + 2g\frac{h}{2}} = \sqrt{{v_0}^2 + gh}.$$

(c) At D $K_D = K_A + U_A - U_D$, or

$$\frac{1}{2}m{v_D}^2 = \frac{1}{2}m{v_0}^2 + mgh - mg(0),$$

or

$$v_B = \sqrt{{v_0}^2 + 2gh}.$$

E12-10 From the slope of the graph, $k = (0.4\,\mathrm{N})/(0.04\,\mathrm{m}) = 10\,\mathrm{N/m}$.

(a) $\Delta K = -\Delta U$, so $\frac{1}{2}m{v_\mathrm{f}}^2 = \frac{1}{2}k{x_\mathrm{i}}^2$, or

$$v_\mathrm{f} = \sqrt{\frac{(10\,\mathrm{N/m})}{(0.00380\,\mathrm{kg})}}(0.0550\,\mathrm{m}) = 2.82\,\mathrm{m/s}.$$

(b) $\Delta K = -\Delta U$, so $\frac{1}{2}m{v_\mathrm{f}}^2 = \frac{1}{2}k({x_\mathrm{i}}^2 - {x_\mathrm{f}}^2)$, or

$$v_\mathrm{f} = \sqrt{\frac{(10\,\mathrm{N/m})}{(0.00380\,\mathrm{kg})}\left[(0.0550\,\mathrm{m})^2 - (0.0150\,\mathrm{m})^2\right]} = 2.71\,\mathrm{m/s}.$$

$\boxed{\textbf{E12-11}}$ (a) The force constant of the spring is

$$k = F/x = mg/x = (7.94\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/(0.102\,\mathrm{m}) = 764\,\mathrm{N/m}.$$

(b) The potential energy stored in the spring is given by Eq. 12-8,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(764\,\mathrm{N/m})(0.286\,\mathrm{m} + 0.102\,\mathrm{m})^2 = 57.5\,\mathrm{J}.$$

(c) Conservation of energy,

$$\begin{aligned} K_\mathrm{f} + U_\mathrm{f} &= K_\mathrm{i} + U_\mathrm{i}, \\ \frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f} + \frac{1}{2}k{x_\mathrm{f}}^2 &= \frac{1}{2}m{v_\mathrm{i}}^2 + mgy_\mathrm{i} + \frac{1}{2}k{x_\mathrm{i}}^2, \\ \frac{1}{2}(0)^2 + mgh + \frac{1}{2}k(0)^2 &= \frac{1}{2}(0)^2 + mg(0) + \frac{1}{2}k{x_\mathrm{i}}^2. \end{aligned}$$

Rearranging,

$$h = \frac{k}{2mg}{x_\mathrm{i}}^2 = \frac{(764\,\mathrm{N/m})}{2(7.94\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}(0.388\,\mathrm{m})^2 = 0.738\,\mathrm{m}.$$

E12-12 The annual mass of water is $m = (1000\,\mathrm{kg/m^3})(8\times10^{12}\mathrm{m^2})(0.75\,\mathrm{m})$. The change in potential energy each year is then $\Delta U = -mgy$, where $y = -500\,\mathrm{m}$. The power available is then

$$P = \frac{1}{3}(1000\,\mathrm{kg/m^3})(8\times10^{12}\mathrm{m^2})\frac{(0.75\,\mathrm{m})}{(3.15\times10^7\mathrm{m})}(500\,\mathrm{m}) = 3.2\times10^7\mathrm{W}.$$

E12-13 (a) From kinematics, $v = -gt$, so $K = \frac{1}{2}mg^2t^2$ and $U = U_0 - K = mgh - \frac{1}{2}mg^2t^2$.
(b) $U = mgy$ so $K = U_0 - U = mg(h-y)$.

E12-14 The potential energy is the same in both cases. Consequently, $mg_\mathrm{E}\Delta y_\mathrm{E} = mg_\mathrm{M}\Delta y_\mathrm{M}$, and then

$$y_\mathrm{M} = (2.05\,\mathrm{m} - 1.10\,\mathrm{m})(9.81\,\mathrm{m/s^2})/(1.67\,\mathrm{m/s^2}) + 1.10\,\mathrm{m} = 6.68\,\mathrm{m}.$$

$\boxed{\textbf{E12-15}}$ The working is identical to Ex. 12-11,

$$\begin{aligned} K_\mathrm{f} + U_\mathrm{f} &= K_\mathrm{i} + U_\mathrm{i}, \\ \frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f} + \frac{1}{2}k{x_\mathrm{f}}^2 &= \frac{1}{2}m{v_\mathrm{i}}^2 + mgy_\mathrm{i} + \frac{1}{2}k{x_\mathrm{i}}^2, \\ \frac{1}{2}(0)^2 + mgh + \frac{1}{2}k(0)^2 &= \frac{1}{2}(0)^2 + mg(0) + \frac{1}{2}k{x_\mathrm{i}}^2, \end{aligned}$$

so

$$h = \frac{k}{2mg}{x_\mathrm{i}}^2 = \frac{(2080\,\mathrm{N/m})}{2(1.93\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}(0.187\,\mathrm{m})^2 = 1.92\,\mathrm{m}.$$

The distance up the incline is given by a trig relation,

$$d = h/\sin\theta = (1.92\,\mathrm{m})/\sin(27^\circ) = 4.23\,\mathrm{m}.$$

E12-16 The vertical position of the pendulum is $y = -l\cos\theta$, where θ is measured from the downward vertical and l is the length of the string. The total mechanical energy of the pendulum is

$$E = \frac{1}{2}m{v_\mathrm{b}}^2$$

if we set $U = 0$ at the bottom of the path and v_b is the speed at the bottom. In this case $U = mg(l+y)$.
(a) $K = E - U = \frac{1}{2}m{v_\mathrm{b}}^2 - mgl(1-\cos\theta)$. Then

$$v = \sqrt{(8.12\,\mathrm{m/s})^2 - 2(9.81\,\mathrm{m/s^2})(3.82\,\mathrm{kg})(1-\cos 58.0^\circ)} = 5.54\,\mathrm{m/s}.$$

(b) $U = E - K$, but at highest point $K = 0$. Then

$$\theta = \arccos\left(1 - \frac{1}{2}\frac{(8.12\,\mathrm{m/s})^2}{(9.81\,\mathrm{m/s^2})(3.82\,\mathrm{kg})}\right) = 83.1^\circ.$$

(c) $E = \frac{1}{2}(1.33\,\mathrm{kg})(8.12\,\mathrm{m/s})^2 = 43.8\,\mathrm{J}$.

E12-17 The equilibrium position is when $F = ky = mg$. Then $\Delta U_\mathrm{g} = -mgy$ and $\Delta U_\mathrm{s} = \frac{1}{2}(ky)y = \frac{1}{2}mgy$. So $2\Delta U_\mathrm{s} = -\Delta U_\mathrm{g}$.

E12-18 Let the spring get compressed a distance x. If the object fell from a height $h = 0.436\,\mathrm{m}$, then conservation of energy gives $\frac{1}{2}kx^2 = mg(x + h)$. Solving for x,

$$x = \frac{mg}{k} \pm \sqrt{\left(\frac{mg}{k}\right)^2 + 2\frac{mg}{k}h}$$

only the positive answer is of interest, so

$$x = \frac{(2.14\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(1860\,\mathrm{N/m})} \pm \sqrt{\left(\frac{(2.14\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(1860\,\mathrm{N/m})}\right)^2 + 2\frac{(2.14\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(1860\,\mathrm{N/m})}(0.436\,\mathrm{m})} = 0.111\,\mathrm{m}.$$

$\boxed{\textbf{E12-19}}$ The horizontal distance traveled by the marble is $R = vt_\mathrm{f}$, where t_f is the time of flight and v is the speed of the marble when it leaves the gun. We find *that* speed using energy conservation principles applied to the spring just before it is released and just after the marble leaves the gun.

$$\begin{aligned} K_\mathrm{i} + U_\mathrm{i} &= K_\mathrm{f} + U_\mathrm{f}, \\ 0 + \frac{1}{2}kx^2 &= \frac{1}{2}mv^2 + 0. \end{aligned}$$

$K_\mathrm{i} = 0$ because the marble isn't moving originally, and $U_\mathrm{f} = 0$ because the spring is no longer compressed. Substituting R into this,

$$\frac{1}{2}kx^2 = \frac{1}{2}m\left(\frac{R}{t_\mathrm{f}}\right)^2.$$

We have two values for the compression, x_1 and x_2, and two ranges, R_1 and R_2. We can put both pairs into the above equation and get two expressions; if we divide one expression by the other we get

$$\left(\frac{x_2}{x_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^2.$$

We can easily take the square root of both sides, then

$$\frac{x_2}{x_1} = \frac{R_2}{R_1}.$$

R_1 was Bobby's try, and was equal to $2.20 - 0.27 = 1.93\,\mathrm{m}$. $x_1 = 1.1$ cm was his compression. If Rhoda wants to score, she wants $R_2 = 2.2\,\mathrm{m}$, then

$$x_2 = \frac{2.2\,\mathrm{m}}{1.93\,\mathrm{m}}1.1\text{ cm} = 1.25\text{ cm}.$$

E12-20 Conservation of energy— $U_1 + K_1 = U_2 + K_2$— but $U_1 = mgh$, $K_1 = 0$, and $U_2 = 0$, so $K_2 = \frac{1}{2}mv^2 = mgh$ at the bottom of the swing.

The net force on Tarzan at the bottom of the swing is $F = mv^2/r$, but this net force is equal to the tension T minus the weight $W = mg$. Then $2mgh/r = T - mg$. Rearranging,

$$T = (180\text{ lb})\left(\frac{2(8.5\text{ ft})}{(50\text{ ft})} + 1\right) = 241\text{ lb}.$$

This isn't enough to break the vine, but it is close.

E12-21 Let point 1 be the start position of the first mass, point 2 be the collision point, and point 3 be the highest point in the swing after the collision. Then $U_1 = K_2$, or $\frac{1}{2}m_1{v_1}^2 = m_1 gd$, where v_1 is the speed of m_1 just before it collides with m_2. Then $v_1 = \sqrt{2gd}$.

After the collision the speed of both objects is, by momentum conservation, $v_2 = m_1 v_1/(m_1+m_2)$.

Then, by energy conservation, $U_3 = K_2'$, or $\frac{1}{2}(m_1+m_2){v_2}^2 = (m_1+m_2)gy$, where y is the height to which the combined masses rise.

Combining,

$$y = \frac{{v_2}^2}{2g} = \frac{{m_1}^2{v_1}^2}{2(m_1+m_2)^2 g} = \left(\frac{m_1}{m_1+m_2}\right)^2 d.$$

E12-22 $\Delta K = -\Delta U$, so

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh,$$

where $I = \frac{1}{2}MR^2$ and $\omega = v/R$. Combining,

$$\frac{1}{2}mv^2 + \frac{1}{4}Mv^2 = mgh,$$

so

$$v = \sqrt{\frac{4mgh}{2m+M}} = \sqrt{\frac{4(0.0487\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(0.540\,\mathrm{m})}{2(0.0487\,\mathrm{kg}) + (0.396\,\mathrm{kg})}} = 1.45\,\mathrm{m/s}.$$

$\boxed{\textbf{E12-23}}$ There are *three* contributions to the kinetic energy: rotational kinetic energy of the shell (K_s), rotational kinetic energy of the pulley (K_p), and translational kinetic energy of the block (K_b). The conservation of energy statement is then

$$\begin{aligned} K_\mathrm{s,i} + K_\mathrm{p,i} + K_\mathrm{b,i} + U_\mathrm{i} &= K_\mathrm{s,f} + K_\mathrm{p,f} + K_\mathrm{b,f} + U_\mathrm{f}, \\ (0) + (0) + (0) + (0) &= \frac{1}{2}I_\mathrm{s}{\omega_\mathrm{s}}^2 + \frac{1}{2}I_\mathrm{p}{\omega_\mathrm{p}}^2 + \frac{1}{2}m{v_\mathrm{b}}^2 + mgy. \end{aligned}$$

Finally, $y = -h$ and

$$\omega_\mathrm{s} R = \omega_\mathrm{p} r = v_\mathrm{b}.$$

Combine all of this together, and our energy conservation statement will look like this:

$$0 = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v_\mathrm{b}}{R}\right)^2 + \frac{1}{2}I_\mathrm{p}\left(\frac{v_\mathrm{b}}{r}\right)^2 + \frac{1}{2}m{v_\mathrm{b}}^2 - mgh$$

which can be fairly easily rearranged into

$${v_\mathrm{b}}^2 = \frac{2mgh}{2M/3 + I_\mathrm{P}/r^2 + m}.$$

E12-24 The angular speed of the flywheel and the speed of the car are related by

$$k = \frac{\omega}{v} = \frac{(1490\ \mathrm{rad/s})}{(24.0\,\mathrm{m/s})} = 62.1\ \mathrm{rad/m}.$$

The height of the slope is $h = (1500\,\mathrm{m})\sin(5.00^\circ) = 131\,\mathrm{m}$. The rotational inertia of the flywheel is

$$I = \frac{1}{2}\frac{(194\,\mathrm{N})}{(9.81\,\mathrm{m/s^2})}(0.54\,\mathrm{m})^2 = 2.88\,\mathrm{kg\cdot m^2}.$$

(a) Energy is conserved as the car moves down the slope: $U_{\rm i} = K_{\rm f}$, or

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}Ik^2v^2,$$

or

$$v = \sqrt{\frac{2mgh}{m + Ik^2}} = \sqrt{\frac{2(822\,\text{kg})(9.81\,\text{m/s}^2)(131\,\text{m})}{(822\,\text{kg}) + (2.88\,\text{kg}\cdot\text{m}^2)(62.1\text{ rad/m})^2}} = 13.3\,\text{m/s},$$

or 47.9 m/s.

(b) The average speed down the slope is $13.3\,\text{m/s}/2 = 6.65\,\text{m/s}$. The time to get to the bottom is $t = (1500\,\text{m})/(6.65\,\text{m/s}) = 226\,\text{s}$. The angular acceleration of the disk is

$$\alpha = \frac{\omega}{t} = \frac{(13.3\,\text{m/s})(62.1\text{ rad/m})}{(226\,\text{s})} = 3.65\text{ rad/s}^2.$$

(c) $P = \tau\omega = I\alpha\omega$, so

$$P = (2.88\,\text{kg}\cdot\text{m}^2)(3.65\text{ rad/s}^2)(13.3\,\text{m/s})(62.1\text{ rad/m}) = 8680\,\text{W}.$$

E12-25 (a) For the solid sphere $I = \frac{2}{5}mr^2$; if it rolls without slipping $\omega = v/r$; conservation of energy means $K_{\rm i} = U_{\rm f}$. Then

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = mgh.$$

or

$$h = \frac{(5.18\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} + \frac{(5.18\,\text{m/s})^2}{5(9.81\,\text{m/s}^2)} = 1.91\,\text{m}.$$

The distance up the incline is $(1.91\,\text{m})/\sin(34.0^\circ) = 3.42\,\text{m}$.

(b) The sphere will travel a distance of 3.42 m with an average speed of $5.18\,\text{m}/2$, so $t = (3.42\,\text{m})/(2.59\,\text{m/s}) = 1.32\,\text{s}$. But wait, it goes up then comes back down, so double this time to get 2.64 s.

(c) The total distance is 6.84 m, so the number of rotations is $(6.84\,\text{m})/(0.0472\,\text{m})/(2\pi) = 23.1$.

E12-26 Conservation of energy means $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$. But $\omega = v/r$ and we are told $h = 3v^2/4g$, so

$$\frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} = mg\frac{3v^2}{4g},$$

or

$$I = 2r^2\left(\frac{3}{4}m - \frac{1}{2}m\right) = \frac{1}{2}mr^2,$$

which could be a solid disk or cylinder.

$\boxed{\textbf{E12-27}}$ We assume the cannon ball is solid, so the rotational inertia will be $I = (2/5)MR^2$

The normal force on the cannon ball will be $N = Mg$, where M is the mass of the bowling ball. The kinetic friction on the cannon ball is $F_f = \mu_k N = \mu_k Mg$. The magnitude of the net torque on the bowling ball while skidding is then $\tau = \mu_k MgR$.

Originally the angular momentum of the cannon ball is zero; the final angular momentum will have magnitude $l = I\omega = Iv/R$, where v is the final translational speed of the ball.

The time requires for the cannon ball to stop skidding is the time required to change the angular momentum to l, so

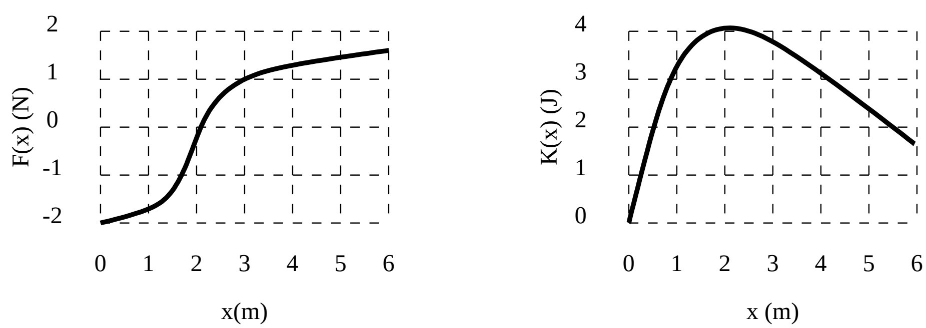
$$\Delta t = \frac{\Delta l}{\tau} = \frac{(2/5)MR^2v/R}{\mu_k MgR} = \frac{2v}{5\mu_k g}.$$

Since we don't know Δt, we can't solve this for v. But the same time through which the angular momentum of the ball is increasing the linear momentum of the ball is decreasing, so we also have

$$\Delta t = \frac{\Delta p}{-F_f} = \frac{Mv - Mv_0}{-\mu_k M g} = \frac{v_0 - v}{\mu_k g}.$$

Combining,

$$\begin{aligned} \frac{2v}{5\mu_k g} &= \frac{v_0 - v}{\mu_k g}, \\ 2v &= 5(v_0 - v), \\ v &= 5v_0/7 \end{aligned}$$



E12-28

E12-29 (a) $F = -\Delta U/\Delta x = -[(-17\,\text{J}) - (-3\,\text{J})]/[(4\,\text{m}) - (1\,\text{m})] = 4.7\,\text{N}$.

(b) The total energy is $\frac{1}{2}(2.0\,\text{kg})(-2.0\,\text{m/s})^2 + (-7\,\text{J})$, or $-3\,\text{J}$. The particle is constrained to move between $x = 1\,\text{m}$ and $x = 14\,\text{m}$.

(c) When $x = 7\,\text{m}$ $K = (-3\,\text{J}) - (-17\,\text{J}) = 14\,\text{J}$. The speed is $v = \sqrt{2(14\,\text{J})/(2.0\,\text{kg})} = 3.7\,\text{m/s}$.

E12-30 Energy is conserved, so

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgy,$$

or

$$v = \sqrt{v_0^2 - 2gy},$$

which depends only on y.

$\boxed{\textbf{E12-31}}$ (a) We can find F_x and F_y from the appropriate derivatives of the potential,

$$\begin{aligned} F_x &= -\frac{\partial U}{\partial x} = -kx, \\ F_y &= -\frac{\partial U}{\partial y} = -ky. \end{aligned}$$

The force at point (x, y) is then

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = -kx\hat{\mathbf{i}} - ky\hat{\mathbf{j}}.$$

(b) Since the force vector points directly toward the origin there is *no* angular component, and $F_\theta = 0$. Then $F_r = -kr$ where r is the distance from the origin.

(c) A spring which is attached to a point; the spring is free to rotate, perhaps?

E12-32 (a) By symmetry we expect F_x, F_y, and F_z to all have the same form.

$$F_x = -\frac{\partial U}{\partial x} = \frac{-kx}{(x^2+y^2+z^2)^{3/2}},$$

with similar expressions for F_y and F_z. Then

$$\vec{\mathbf{F}} = \frac{-k}{(x^2+y^2+z^2)^{3/2}}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}).$$

(b) In spherical polar coordinates $r^2 = x^2+y^2+z^2$. Then $U = -k/r$ and

$$F_r = -\frac{\partial U}{\partial r} = -\frac{k}{r^2}.$$

E12-33 We'll just do the paths, showing only non-zero terms.
Path 1: $W = \int_0^b (-k_2 a)\,dy) = -k_2 ab$.
Path 2: $W = \int_0^a (-k_1 b)\,dx) = -k_1 ab$.
Path 3: $W = (\cos\phi\,\sin\phi)\int_0^d (-k_1 - k_2) r\,dr = -(k_1+k_2)ab/2$.
These three are only equal if $k_1 = k_2$.

P12-1 (a) We need to integrate an expression like

$$-\int_\infty^z \frac{k}{(z+l)^2}\,dz = \frac{k}{z+l}.$$

The second half is dealt with in a similar manner, yielding

$$U(z) = \frac{k}{z+l} - \frac{k}{z-l}.$$

(b) If $z \gg l$ then we can expand the denominators, then

$$\begin{aligned} U(z) &= \frac{k}{z+l} - \frac{k}{z-l}, \\ &\approx \left(\frac{k}{z} - \frac{kl}{z^2}\right) - \left(\frac{k}{z} + \frac{kl}{z^2}\right), \\ &= -\frac{2kl}{z^2}. \end{aligned}$$

P12-2 The ball just reaches the top, so $K_2 = 0$. Then $K_1 = U_2 - U_1 = mgL$, so $v_1 = \sqrt{2(mgL)/m} = \sqrt{2gL}$.

P12-3 Measure distances along the incline by x, where $x = 0$ is measured from the maximally compressed spring. The vertical position of the mass is given by $x\sin\theta$. For the spring $k = (268\,\mathrm{N})/(0.0233\,\mathrm{m}) = 1.15\times10^4\,\mathrm{N/m}$. The total energy of the system is

$$\frac{1}{2}(1.15\times10^4\,\mathrm{N/m})(0.0548\,\mathrm{m})^2 = 17.3\,\mathrm{J}.$$

(a) The block needs to have moved a vertical distance $x\sin(32.0^\circ)$, where

$$17.3\,\mathrm{J} = (3.18\,\mathrm{kg})(9.81\,\mathrm{m/s^2})x\sin(32.0^\circ),$$

or $x = 1.05\,\mathrm{m}$.

(b) When the block hits the top of the spring the gravitational potential energy has changed by

$$\Delta U = -(3.18\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.05\,\mathrm{m} - 0.0548\,\mathrm{m})\sin(32.0^\circ) = 16.5,\mathrm{J};$$

hence the speed is $v = \sqrt{2(16.5\,\mathrm{J})/(3.18\,\mathrm{kg})} = 3.22\,\mathrm{m/s}$.

P12-4 The potential energy associated with the hanging part is

$$U = \int_{-L/4}^{0} \frac{M}{L} gy\,dy = \frac{Mg}{2L} y^2 \Big|_{-L/4}^{0} = -\frac{MgL}{32},$$

so the work required is $W = MgL/32$.

$\boxed{\textbf{P12-5}}$ (a) Considering points P and Q we have

$$\begin{aligned} K_P + U_P &= K_Q + U_Q, \\ (0) + mg(5R) &= \frac{1}{2}mv^2 + mg(R), \\ 4mgR &= \frac{1}{2}mv^2, \\ \sqrt{8gR} &= v. \end{aligned}$$

There are two forces on the block, the normal force from the track,

$$N = \frac{mv^2}{R} = \frac{m(8gR)}{R} = 8mg,$$

and the force of gravity $W = mg$. They are orthogonal so

$$F_{\text{net}} = \sqrt{(8mg)^2 + (mg)^2} = \sqrt{65}\, mg$$

and the angle from the horizontal by

$$\tan\theta = \frac{-mg}{8mg} = -\frac{1}{8},$$

or $\theta = 7.13^\circ$ below the horizontal.

(b) If the block *barely* makes it over the top of the track then the speed at the top of the loop (point S, perhaps?) is just fast enough so that the centripetal force is equal in magnitude to the weight,

$$m{v_{\text{S}}}^2/R = mg.$$

Assume the block was released from point T. The energy conservation problem is then

$$\begin{aligned} K_T + U_T &= K_S + U_S, \\ (0) + mgy_T &= \frac{1}{2}mv_S^2 + mgy_S, \\ y_T &= \frac{1}{2}(R) + m(2R), \\ &= 5R/2. \end{aligned}$$

P12-6 The wedge slides to the left, the block to the right. Conservation of momentum requires $Mv_{\text{w}} + mv_{\text{b},x} = 0$. The block is constrained to move on the surface of the wedge, so

$$\tan\alpha = \frac{v_{\text{b},y}}{v_{\text{b},x} - v_{\text{w}}},$$

or

$$v_{\text{b},y} = v_{\text{b},x} \tan\alpha(1 + m/M).$$

Conservation of energy requires

$$\frac{1}{2}m{v_\mathrm{b}}^2 + \frac{1}{2}M{v_\mathrm{w}}^2 = mgh.$$

Combining,

$$\begin{aligned}
\frac{1}{2}m\left({v_{\mathrm{b},x}}^2 + {v_{\mathrm{b},y}}^2\right) + \frac{1}{2}M\left(\frac{m}{M}v_{\mathrm{b},x}\right)^2 &= mgh,\\
\left(\tan^2\alpha(1+m/M)^2 + 1 + \frac{m}{M}\right){v_{\mathrm{b},x}}^2 &= 2gh,\\
\left(\sin^2\alpha(M+m)^2 + M^2\cos^2\alpha + mM\cos^2\alpha\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha,\\
\left(M^2 + mM + mM\sin^2\alpha + m^2\sin^2\alpha\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha,\\
\left((M+m)(M+m\sin^2\alpha)\right){v_{\mathrm{b},x}}^2 &= 2M^2gh\cos^2\alpha,
\end{aligned}$$

or

$$v_{\mathrm{b},x} = M\cos\alpha\sqrt{\frac{2gh}{(M+m)(M+m\sin^2\alpha)}}.$$

Then

$$v_\mathrm{w} = -m\cos\alpha\sqrt{\frac{2gh}{(M+m)(M+m\sin^2\alpha)}}.$$

P12-7 $U(x) = -\int F_x\,dx = -Ax^2/2 - Bx^3/3.$

(a) $U = -(-3.00\,\mathrm{N/m})(2.26\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(2.26\,\mathrm{m})^3/3 = 26.9\,\mathrm{J}.$

(b) There are two points to consider:

$$\begin{aligned}
U_1 &= -(-3.00\,\mathrm{N/m})(4.91\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(4.91\,\mathrm{m})^3/3 = 233\,\mathrm{J},\\
U_2 &= -(-3.00\,\mathrm{N/m})(1.77\,\mathrm{m})^2/2 - (-5.00\,\mathrm{N/m^2})(1.77\,\mathrm{m})^3/3 = 13.9\,\mathrm{J},\\
K_1 &= \frac{1}{2}(1.18\mathrm{kg})(4.13\,\mathrm{m/s})^2 = 10.1\,\mathrm{J}.
\end{aligned}$$

Then

$$v_2 = \sqrt{\frac{2(10.1\,\mathrm{J} + 233\,\mathrm{J} - 13.9\,\mathrm{J})}{(1.18\,\mathrm{kg})}} = 19.7\,\mathrm{m/s}.$$

P12-8 Assume that $U_0 = K_0 = 0$. Then conservation of energy requires $K = -U$; consequently, $v = \sqrt{2g(-y)}$.

(a) $v = \sqrt{2(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m})} = 4.85\,\mathrm{m/s}.$

(b) $v = \sqrt{2(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m} - 0.45\,\mathrm{m} - 0.45\,\mathrm{m})} = 2.43\,\mathrm{m/s}.$

$\boxed{\textbf{P12-9}}$ Assume that $U_0 = K_0 = 0$. Then conservation of energy requires $K = -U$; consequently, $v = \sqrt{2g(-y)}$. If the ball *barely* swings around the top of the peg then the speed at the top of the loop is just fast enough so that the centripetal force is equal in magnitude to the weight,

$$mv^2/R = mg.$$

The energy conservation problem is then

$$\begin{aligned}
mv^2 &= 2mg(L - 2(L-d)) = 2mg(2d - L)\\
mg(L-d) &= 2mg(2d-L),\\
d &= 3L/5.
\end{aligned}$$

P12-10 The speed at the top and the speed at the bottom are related by

$$\frac{1}{2}m{v_\mathrm{b}}^2 = \frac{1}{2}m{v_\mathrm{t}}^2 + 2mgR.$$

The magnitude of the net force is $F = mv^2/R$, the tension at the top is

$$T_\mathrm{t} = m{v_\mathrm{t}}^2/R - mg,$$

while tension at the bottom is

$$T_\mathrm{b} = m{v_\mathrm{b}}^2/R + mg,$$

The difference is

$$\Delta T = 2mg + m({v_\mathrm{b}}^2 - {v_\mathrm{t}}^2)/R = 2mg + 4mg = 6mg.$$

$\boxed{\textbf{P12-11}}$ Let the angle θ be measured from the horizontal to the point on the hemisphere where the boy is located. There are then two components to the force of gravity— a component tangent to the hemisphere, $W_{||} = mg\cos\theta$, and a component directed radially toward the center of the hemisphere, $W_\perp = mg\sin\theta$.

While the boy is in contact with the hemisphere the motion is circular so

$$mv^2/R = W_\perp - N.$$

When the boy leaves the surface we have $mv^2/R = W_\perp$, or $mv^2 = mgR\sin\theta$. Now for energy conservation,

$$\begin{aligned} K + U &= K_0 + U_0, \\ \frac{1}{2}mv^2 + mgy &= \frac{1}{2}m(0)^2 + mgR, \\ \frac{1}{2}gR\sin\theta + mgy &= mgR, \\ \frac{1}{2}y + y &= R, \\ y &= 2R/3. \end{aligned}$$

P12-12 (a) To be in contact at the top requires $m{v_\mathrm{t}}^2/R = mg$. The speed at the bottom would be given by energy conservation

$$\frac{1}{2}m{v_\mathrm{b}}^2 = \frac{1}{2}m{v_\mathrm{t}}^2 + 2mgR,$$

so $v_\mathrm{b} = \sqrt{5gR}$ is the speed at the bottom that will allow the object to make it around the circle without loosing contact.

(b) The particle will lose contact with the track if $mv^2/R \le mg\sin\theta$. Energy conservation gives

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mgR(1 + \sin\theta)$$

for points above the half-way point. Then the condition for "sticking" to the track is

$$\frac{1}{R}v_0^2 - 2g(1 + \sin\theta) \le g\sin\theta,$$

or, if $v_0 = 0.775v_\mathrm{m}$,

$$5(0.775)^2 - 2 \le 3\sin\theta,$$

or $\theta = \arcsin(1/3)$.

P12-13 The rotational inertia is

$$I = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2.$$

Conservation of energy is

$$\frac{1}{2}I\omega^2 = 3Mg(L/2),$$

so $\omega = \sqrt{9g/(4L)}$.

P12-14 The rotational speed of the sphere is $\omega = v/r$; the rotational kinetic energy is $K_\mathrm{r} = \frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$.

(a) For the marble to stay on the track $mv^2/R = mg$ at the top of the track. Then the marble needs to be released from a point

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + 2mgR,$$

or $h = R/2 + R/5 + 2R = 2.7R$.

(b) Energy conservation gives

$$6mgR = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + mgR,$$

or $mv^2/R = 50mg/7$. This corresponds to the horizontal force acting on the marble.

P12-15 $\frac{1}{2}mv_0^2 + mgy = 0$, where y is the distance beneath the rim, or $y = -r\cos\theta_0$. Then

$$v_0 = \sqrt{-2gy} = \sqrt{2gr\cos\theta_0}.$$

P12-16 (a) For E_1 the atoms will eventually move apart completely.

(b) For E_2 the moving atom will bounce back and forth between a closest point and a farthest point.

(c) $U \approx -1.2\times10^{-19}\mathrm{J}$.

(d) $K = E_1 - U \approx 2.2\times10^{-19}\mathrm{J}$.

(e) Find the slope of the curve, so

$$F \approx -\frac{(-1\times10^{-19}\mathrm{J}) - (-2\times10^{-19}\mathrm{J})}{(0.3\times10^{-9}\mathrm{m}) - (0.2\times10^{-9}\mathrm{m})} = -1\times10^{-9}\mathrm{N},$$

which would point toward the larger mass.

$\boxed{\textbf{P12-17}}$ The function needs to fall off at infinity in both directions; an exponential envelope would work, but it will need to have an $-x^2$ term to force the potential to zero on *both* sides. So we propose something of the form

$$U(x) = P(x)e^{-\beta x^2}$$

where $P(x)$ is a polynomial in x and β is a positive constant.

We proposed the *polynomial* because we need a symmetric function which has two zeroes. A quadratic of the form $\alpha x^2 - U_0$ would work, it has two zeroes, a minimum at $x = 0$, and is symmetric.

So our *trial* function is

$$U(x) = \left(\alpha x^2 - U_0\right)e^{-\beta x^2}.$$

This function should have *three* extrema. Take the derivative, and then we'll set it equal to zero,

$$\frac{dU}{dx} = 2\alpha x e^{-\beta x^2} - 2\left(\alpha x^2 - U_0\right)\beta x e^{-\beta x^2}.$$

Setting this equal to zero leaves two possibilities,

$$\begin{aligned} x &= 0, \\ 2\alpha - 2\left(\alpha x^2 - U_0\right)\beta &= 0. \end{aligned}$$

The first equation is trivial, the second is easily rearranged to give

$$x = \pm\sqrt{\frac{\alpha + \beta U_0}{\beta\alpha}}$$

These are the points $\pm x_1$. We can, if we wanted, try to find α and β from the picture, but you might notice we have one equation, $U(x_1) = U_1$ and two unknowns. It really isn't very illuminating to take this problem much farther, but we could.

(b) The force is the derivative of the potential; this expression was found above.

(c) As long as the energy is *less* than the two peaks, then the motion would be oscillatory, trapped in the well.

P12-18 (a) $F = -\partial U/\partial r$, or

$$F = -U_0\left(\frac{r_0}{r^2} + \frac{1}{r}\right)e^{-r/r_0}.$$

(b) Evaluate the force at the four points:

$$\begin{aligned} F(r_0) &= -2(U_0/r_0)e^{-1}, \\ F(2r_0) &= -(3/4)(U_0/r_0)e^{-2}, \\ F(4r_0) &= -(5/16)(U_0/r_0)e^{-4}, \\ F(10r_0) &= -(11/100)(U_0/r_0)e^{-10}. \end{aligned}$$

The ratios are then

$$\begin{aligned} F(2r_0)/F(r_0) &= (3/8)e^{-1} = 0.14, \\ F(4r_0)/F(r_0) &= (5/32)e^{-3} = 7.8\times10^{-3}, \\ F(10r_0)/F(r_0) &= (11/200)e^{-9} = 6.8\times10^{-6}. \end{aligned}$$

$\boxed{\textbf{E13-1}}$ If the projectile had *not* experienced air drag it would have risen to a height y_2, but because of air drag 68 kJ of mechanical energy was dissipated so it only rose to a height y_1. In either case the initial velocity, and hence initial kinetic energy, was the same; and the velocity at the highest point was zero. Then $W = \Delta U$, so the potential energy would have been 68 kJ greater, and

$$\Delta y = \Delta U/mg = (68\times 10^3\text{J})/(9.4\,\text{kg})(9.81\,\text{m/s}^2) = 740\,\text{m}$$

is how much higher it would have gone without air friction.

E13-2 (a) The road incline is $\theta = \arctan(0.08) = 4.57°$. The frictional forces are the same; the car is now moving with a vertical upward speed of $(15\,\text{m/s})\sin(4.57°) = 1.20\,\text{m/s}$. The additional power required to drive up the hill is then $\Delta P = mgv_y = (1700\,\text{kg})(9.81\,\text{m/s}^2)(1.20\,\text{m/s}) = 20000\,\text{W}$. The total power required is 36000 W.

(b) The car will "coast" if the power generated by rolling downhill is equal to 16000 W, or

$$v_y = (16000\,\text{W})/[(1700\,\text{kg})(9.81\,\text{m/s}^2)] = 0.959\,\text{m/s},$$

down. Then the incline is

$$\theta = \arcsin(0.959\,\text{m/s}/15\,\text{m/s}) = 3.67°.$$

This corresponds to a downward grade of $\tan(3.67°) = 6.4\,\%$.

E13-3 Apply energy conservation:

$$\frac{1}{2}mv^2 + mgy + \frac{1}{2}ky^2 = 0,$$

so

$$v = \sqrt{-2(9.81\,\text{m/s}^2)(-0.084\,\text{m}) - (262\,\text{N/m})(-0.084\,\text{m})^2/(1.25\,\text{kg})} = 0.41\,\text{m/s}.$$

E13-4 The car climbs a vertical distance of $(225\,\text{m})\sin(10°) = 39.1\,\text{m}$ in coming to a stop. The change in energy of the car is then

$$\Delta E = -\frac{1}{2}\frac{(16400\,\text{N})}{(9.81\,\text{m/s}^2)}(31.4\,\text{m/s})^2 + (16400\,\text{N})(39.1\,\text{m}) = -1.83\times 10^5\text{J}.$$

$\boxed{\textbf{E13-5}}$ (a) Applying conservation of energy to the points where the ball was dropped and where it entered the oil,

$$\begin{aligned}
\frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} &= \frac{1}{2}m{v_\text{i}}^2 + mgy_\text{i},\\
\frac{1}{2}{v_\text{f}}^2 + g(0) &= \frac{1}{2}(0)^2 + gy_\text{i},\\
v_\text{f} &= \sqrt{2gy_\text{i}},\\
&= \sqrt{2(9.81\,\text{m/s}^2)(0.76\,\text{m})} = 3.9\,\text{m/s}.
\end{aligned}$$

(b) The change in internal energy of the ball + oil can be found by considering the points where the ball was released and where the ball reached the bottom of the container.

$$\begin{aligned}
\Delta E &= K_\text{f} + U_\text{f} - K_\text{i} - U_\text{i},\\
&= \frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} - \frac{1}{2}m(0)^2 - mgy_\text{i},\\
&= \frac{1}{2}(12.2\times 10^{-3}\text{kg})(1.48\text{m/s})^2 - (12.2\times 10^{-3}\text{kg})(9.81\text{m/s}^2)(-0.55\text{m} - 0.76\text{m}),\\
&= -0.143\,\text{J}
\end{aligned}$$

E13-6 (a) $U_\mathrm{i} = (25.3\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(12.2\,\mathrm{m}) = 3030\,\mathrm{J}$.
(b) $K_\mathrm{f} = \frac{1}{2}(25.3\,\mathrm{kg})(5.56\,\mathrm{m/s})^2 = 391\,\mathrm{J}$.
(c) $\Delta E_\mathrm{int} = 3030\,\mathrm{J} - 391\,\mathrm{J} = 2640\,\mathrm{J}$.

E13-7 (a) At atmospheric entry the kinetic energy is

$$K = \frac{1}{2}(7.9\times10^4\mathrm{kg})(8.0\times10^3\mathrm{m/s})^2 = 2.5\times10^{12}\mathrm{J}.$$

The gravitational potential energy is

$$U = (7.9\times10^4\mathrm{kg})(9.8\,\mathrm{m/s^2})(1.6\times10^5\mathrm{m}) = 1.2\times10^{11}\mathrm{J}.$$

The total energy is 2.6×10^{12}J.
(b) At touch down the kinetic energy is

$$K = \frac{1}{2}(7.9\times10^4\mathrm{kg})(9.8\times10^1\mathrm{m/s})^2 = 3.8\times10^8\mathrm{J}.$$

E13-8 $\Delta E/\Delta t = (68\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(59\,\mathrm{m/s}) = 39000\,\mathrm{J/s}$.

$\boxed{\textbf{E13-9}}$ Let m be the mass of the water under consideration. Then the percentage of the potential energy "lost" which appears as kinetic energy is

$$\frac{K_\mathrm{f} - K_\mathrm{i}}{U_\mathrm{i} - U_\mathrm{f}}.$$

Then

$$\begin{aligned}
\frac{K_\mathrm{f} - K_\mathrm{i}}{U_\mathrm{i} - U_\mathrm{f}} &= \frac{1}{2}m\left({v_\mathrm{f}}^2 - {v_\mathrm{i}}^2\right)/\left(mgy_\mathrm{i} - mgy_\mathrm{f}\right), \\
&= \frac{{v_\mathrm{f}}^2 - {v_\mathrm{i}}^2}{-2g\Delta y}, \\
&= \frac{(13\,\mathrm{m/s})^2 - (3.2\,\mathrm{m/s})^2}{-2(9.81\,\mathrm{m/s^2})(-15\,\mathrm{m})}, \\
&= 54\,\%.
\end{aligned}$$

The rest of the energy would have been converted to sound and thermal energy.

E13-10 The change in energy is

$$\Delta E = \frac{1}{2}(524\,\mathrm{kg})(62.6\,\mathrm{m/s})^2 - (524\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(292\,\mathrm{m}) = 4.74\times10^5\mathrm{J}.$$

E13-11 $U_\mathrm{f} = K_\mathrm{i} - (34.6\,\mathrm{J})$. Then

$$h = \frac{1}{2}\frac{(7.81\,\mathrm{m/s})^2}{(9.81\,\mathrm{m/s^2})} - \frac{(34.6\,\mathrm{J})}{(4.26\,\mathrm{kg})(9.81\,\mathrm{m/s^2})} = 2.28\,\mathrm{m};$$

which means the distance along the incline is $(2.28\,\mathrm{m})/\sin(33.0^\circ) = 4.19\,\mathrm{m}$.

E13-12 (a) $K_\text{f} = U_\text{i} - U_\text{f}$, so

$$v_\text{f} = \sqrt{2(9.81\,\text{m/s}^2)[(862\,\text{m}) - (741\,\text{m})]} = 48.7\,\text{m/s}.$$

That's a quick 175 km/h; but the speed at the bottom of the valley is 40% of the speed of sound!

(b) $\Delta E = U_\text{f} - U_\text{i}$, so

$$\Delta E = (54.4\,\text{kg})(9.81\,\text{m/s}^2)[(862\,\text{m}) - (741\,\text{m})] = -6.46\times10^4\text{J};$$

which means the internal energy of the snow and skis increased by 6.46×10^4J.

$\boxed{\textbf{E13-13}}$ The final potential energy is 15% less than the initial kinetic plus potential energy of the ball, so

$$0.85(K_\text{i} + U_\text{i}) = U_\text{f}.$$

But $U_\text{i} = U_\text{f}$, so $K_\text{i} = 0.15U_\text{f}/0.85$, and then

$$v_\text{i} = \sqrt{\frac{0.15}{0.85}2gh} = \sqrt{2(0.176)(9.81\,\text{m/s}^2)(12.4\,\text{m})} = 6.54\,\text{m/s}.$$

E13-14 Focus on the potential energy. After the nth bounce the ball will have a potential energy at the top of the bounce of $U_n = 0.9U_{n-1}$. Since $U \propto h$, one can write $h_n = (0.9)^n h_0$. Solving for n,

$$n = \log(h_n/h_0)/\log(0.9) = \log(3\text{ ft}/6\text{ ft})/\log(0.9) = 6.58,$$

which must be rounded up to 7.

E13-15 Let m be the mass of the ball and M be the mass of the block.

The kinetic energy of the ball just before colliding with the block is given by $K_1 = U_0$, so $v_1 = \sqrt{2(9.81\,\text{m/s}^2)(0.687\,\text{m})} = 3.67\,\text{m/s}$.

Momentum is conserved, so if v_2 and v_3 are velocities of the ball and block after the collision then $mv_1 = mv_2 + Mv_3$. Kinetic energy is not conserved, instead

$$\frac{1}{2}\left(\frac{1}{2}mv_1^2\right) = \frac{1}{2}mv_2^2 + \frac{1}{2}Mv_3^2.$$

Combine the energy and momentum expressions to eliminate v_3:

$$\begin{aligned} mv_1^2 &= 2mv_2^2 + 2M\left(\frac{m}{M}(v_1 - v_2)\right)^2, \\ Mv_1^2 &= 2Mv_2^2 + 2mv_1^2 - 4mv_1v_2 + 2mv_2^2, \end{aligned}$$

which can be formed into a quadratic. The solution for v_2 is

$$v_2 = \frac{2m \pm \sqrt{2(M^2 - mM)}}{2(M+m)}v_1 = (0.600 \pm 1.95)\,\text{m/s}.$$

The corresponding solutions for v_3 are then found from the momentum expression to be $v_3 = 0.981\,\text{m/s}$ and $v_3 = 0.219$. Since it is unlikely that the ball passed through the block we can toss out the second set of answers.

E13-16 $E_\text{f} = K_\text{f} + U_\text{f} = 3mgh$, or

$$v_\text{f} = \sqrt{2(9.81\,\text{m/s}^2)2(0.18\,\text{m})} = 2.66\,\text{m/s}.$$

$\boxed{\textbf{E13-17}}$ We can find the kinetic energy of the center of mass of the woman when her feet leave the ground by considering energy conservation and her highest point. Then

$$\begin{aligned}
\frac{1}{2}m{v_\mathrm{i}}^2 + mgy_\mathrm{i} &= \frac{1}{2}m{v_\mathrm{f}}^2 + mgy_\mathrm{f},\\
\frac{1}{2}mv_\mathrm{i} &= mg\Delta y,\\
&= (55.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.20\,\mathrm{m} - 0.90\,\mathrm{m}) = 162\,\mathrm{J}.
\end{aligned}$$

(a) During the jumping phase her potential energy changed by

$$\Delta U = mg\Delta y = (55.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(0.50\,\mathrm{m}) = 270\,\mathrm{J}$$

while she was moving up. Then

$$F_\mathrm{ext} = \frac{\Delta K + \Delta U}{\Delta s} = \frac{(162\,\mathrm{J}) + (270\,\mathrm{J})}{(0.5\,\mathrm{m})} = 864\,\mathrm{N}.$$

(b) Her fastest speed was when her feet left the ground,

$$v = \frac{2K}{m} = \frac{2(162\,\mathrm{J})}{(55.0\,\mathrm{kg})} = 2.42\,\mathrm{m/s}.$$

E13-18 (b) The ice skater needs to lose $\frac{1}{2}(116\,\mathrm{kg})(3.24\,\mathrm{m/s})^2 = 609\,\mathrm{J}$ of internal energy.
(a) The average force exerted on the rail is $F = (609\,\mathrm{J})/(0.340\,\mathrm{m}) = 1790\,\mathrm{N}$.

E13-19 12.6 km/h is equal to 3.50 m/s; the initial kinetic energy of the car is

$$\frac{1}{2}(2340\,\mathrm{kg})(3.50\,\mathrm{m/s})^2 = 1.43\times10^4\mathrm{J}.$$

(a) The force exerted on the car is $F = (1.43\times10^4\mathrm{J})/(0.64\,\mathrm{m}) = 2.24\times10^4\mathrm{N}$.
(b) The change increase in internal energy of the car is

$$\Delta E_\mathrm{int} = (2.24\times10^4\mathrm{N})(0.640\,\mathrm{m} - 0.083\,\mathrm{m}) = 1.25\times10^4\mathrm{J}.$$

E13-20 Note that $v_n^2 = {v'_n}^2 - 2\vec{\mathbf{v}}'_n\cdot\vec{\mathbf{v}}_\mathrm{cm} + {v_\mathrm{cm}}^2$. Then

$$\begin{aligned}
K &= \sum_n \frac{1}{2}\left(m_n{v'_n}^2 - 2m_n\vec{\mathbf{v}}'_n\cdot\vec{\mathbf{v}}_\mathrm{cm} + m_n{v_\mathrm{cm}}^2\right),\\
&= \sum_n \frac{1}{2}m_n{v'_n}^2 - \left(\sum_n m_n\vec{\mathbf{v}}'_n\right)\cdot\vec{\mathbf{v}}_\mathrm{cm} + \left(\sum_n \frac{1}{2}m_n\right){v_\mathrm{cm}}^2,\\
&= K_\mathrm{int} - \left(\sum_n m_n\vec{\mathbf{v}}'_n\right)\cdot\vec{\mathbf{v}}_\mathrm{cm} + K_\mathrm{cm}.
\end{aligned}$$

The middle term vanishes because of the definition of velocities relative to the center of mass.

$\boxed{\textbf{E13-21}}$ Momentum conservation requires $mv_0 = mv+MV$, where the sign indicates the direction. We are assuming one dimensional collisions. Energy conservation requires

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 + E.$$

Combining,

$$\begin{aligned}
\frac{1}{2}mv_0^2 &= \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{m}{M}v_0 - \frac{m}{M}v\right)^2 + E, \\
Mv_0^2 &= Mv^2 + m\,(v_0 - v)^2 + 2(M/m)E.
\end{aligned}$$

Arrange this as a quadratic in v,

$$(M+m)\,v^2 - (2mv_0)\,v + \left(2(M/m)E + mv_0^2 - Mv_0^2\right) = 0.$$

This will only have real solutions if the discriminant ($b^2 - 4ac$) is greater than or equal to zero. Then

$$(2mv_0)^2 \geq 4\,(M+m)\left(2(M/m)E + mv_0^2 - Mv_0^2\right)$$

is the condition for the minimum v_0. Solving the equality condition,

$$4m^2v_0^2 = 4(M+m)\left(2(M/m)E + (m-M)v_0^2\right),$$

or $M^2v_0^2 = 2(M+m)(M/m)E$. One last rearrangement, and $v_0 = \sqrt{2(M+m)E/(mM)}$.

$\boxed{\textbf{P13-1}}$ (a) The initial kinetic energy will equal the potential energy at the highest point *plus* the amount of energy which is dissipated because of air drag.

$$\begin{aligned}
mgh + fh &= \frac{1}{2}mv_0^2, \\
h &= \frac{v_0^2}{2(g+f/m)} = \frac{v_0^2}{2g(1+f/w)}.
\end{aligned}$$

(b) The final kinetic energy when the stone lands will be equal to the initial kinetic energy *minus* twice the energy dissipated on the way up, so

$$\begin{aligned}
\frac{1}{2}mv^2 &= \frac{1}{2}mv_0^2 - 2fh, \\
&= \frac{1}{2}mv_0^2 - 2f\frac{v_0^2}{2g(1+f/w)}, \\
&= \left(\frac{m}{2} - \frac{f}{g(1+f/w)}\right)v_0^2, \\
v^2 &= \left(1 - \frac{2f}{w+f}\right)v_0^2, \\
v &= \left(\frac{w-f}{w+f}\right)^{1/2} v_0.
\end{aligned}$$

P13-2 The object starts with $U = (0.234\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(1.05\,\mathrm{m}) = 2.41\,\mathrm{J}$. It will move back and forth across the flat portion $(2.41\,\mathrm{J})/(0.688\,\mathrm{J}) = 3.50$ times, which means it will come to a rest at the center of the flat part while attempting one last right to left journey.

P13-3 (a) The work done on the block block because of friction is

$$(0.210)(2.41\,\text{kg})(9.81\,\text{m/s}^2)(1.83\,\text{m}) = 9.09\,\text{J}.$$

The energy dissipated because of friction is $(9.09\,\text{J})/0.83 = 11.0\,\text{J}$.

The speed of the block just after the bullet comes to a rest is

$$v = \sqrt{2K/m} = \sqrt{2(1.10\,\text{J})/(2.41\,\text{kg})} = 3.02\,\text{m/s}.$$

(b) The initial speed of the bullet is

$$v_0 = \frac{M+m}{m}v = \frac{(2.41\,\text{kg}) + (0.00454\,\text{kg})}{(0.00454\,\text{kg})}(3.02\,\text{m/s}) = 1.60\times 10^3\,\text{m/s}.$$

P13-4 The energy stored in the spring after compression is $\frac{1}{2}(193\,\text{N/m})(0.0416\,\text{m})^2 = 0.167\,\text{J}$. Since 117 mJ was dissipated by friction, the kinetic energy of the block before colliding with the spring was $0.284\,\text{J}$. The speed of the block was then

$$v = \sqrt{2(0.284\,\text{J})/(1.34\,\text{kg})} = 0.651\,\text{m/s}.$$

P13-5 (a) Using Newton's second law, $F = ma$, so for circular motion around the proton

$$\frac{mv^2}{r} = F = k\frac{e^2}{r^2}.$$

The kinetic energy of the electron in an orbit is then

$$K = \frac{1}{2}mv^2 = \frac{1}{2}k\frac{e^2}{r}.$$

The change in kinetic energy is

$$\Delta K = \frac{1}{2}ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

(b) The potential energy difference is

$$\Delta U = -\int_{r_1}^{r_2}\frac{ke^2}{r^2}dr = -ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

(c) The total energy change is

$$\Delta E = \Delta K + \Delta U = -\frac{1}{2}ke^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

P13-6 (a) The initial energy of the system is $(4000\text{ lb})(12ft) = 48,000\text{ ft}\cdot\text{lb}$. The safety device removes $(1000\text{ lb})(12ft) = 12,000\text{ ft}\cdot\text{lb}$ before the elevator hits the spring, so the elevator has a kinetic energy of $36,000\text{ ft}\cdot\text{lb}$ when it hits the spring. The speed of the elevator when it hits the spring is

$$v = \sqrt{\frac{2(36,000\text{ ft}\cdot\text{lb})(32.0\text{ ft/s}^2)}{(4000\text{ lb})}} = 24.0\text{ ft/s}.$$

(b) Assuming the safety clamp remains in effect while the elevator is in contact with the spring then the distance compressed will be found from

$$36,000\text{ ft}\cdot\text{lb} = \frac{1}{2}(10,000\text{ lb/ft})y^2 - (4000\text{ lb})y + (1000\text{ lb})y.$$

This is a quadratic expression in y which can be simplified to look like

$$5y^2 - 3y - 36 = 0,$$

which has solutions $y = (0.3 \pm 2.7)$ ft. Only $y = 3.00$ ft makes sense here.

(c) The distance through which the elevator will bounce back up is found from

$$33,000 \text{ ft} = (4000 \text{ lb})y - (1000 \text{ lb})y,$$

where y is measured from the most compressed point of the spring. Then $y = 11$ ft, or the elevator bounces back up 8 feet.

(d) The elevator will bounce until it has traveled a total distance so that the safety device dissipates all of the original energy, or 48 ft.

P13-7 The net force on the top block while it is being pulled is

$$11.0\,\mathrm{N} - F_f = 11.0\,\mathrm{N} - (0.35)(2.5\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 2.42\,\mathrm{N}.$$

This means it is accelerating at $(2.42\,\mathrm{N})/(2.5\,\mathrm{kg}) = 0.968\,\mathrm{m/s^2}$. That acceleration will last a time $t = \sqrt{2(0.30\,\mathrm{m})/(0.968\,\mathrm{m/s^2})} = 0.787\,\mathrm{s}$. The speed of the top block after the force stops pulling is then $(0.968\,\mathrm{m/s^2})(0.787\,\mathrm{s}) = 0.762\,\mathrm{m/s}$. The force on the bottom block is F_f, so the acceleration of the bottom block is

$$(0.35)(2.5\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/(10.0\,\mathrm{kg}) = 0.858\,\mathrm{m/s^2},$$

and the speed after the force stops pulling on the top block is $(0.858\,\mathrm{m/s^2})(0.787\,\mathrm{s}) = 0.675\,\mathrm{m/s}$.

(a) $W = Fs = (11.0\,\mathrm{N})(0.30\,\mathrm{m}) = 3.3\,\mathrm{J}$ of energy were delivered to the system, but after the force stops pulling only

$$\frac{1}{2}(2.5\,\mathrm{kg})(0.762\,\mathrm{m/s})^2 + \frac{1}{2}(10.0\,\mathrm{kg})(0.675\,\mathrm{m/s})^2 = 3.004\,\mathrm{J}$$

were present as kinetic energy. So $0.296\,\mathrm{J}$ is "missing" and would be now present as internal energy.

(b) The impulse received by the two block system is then $J = (11.0\,\mathrm{N})(0.787\,\mathrm{s}) = 8.66\,\mathrm{N{\cdot}s}$. This impulse causes a change in momentum, so the speed of the two block system after the external force stops pulling and both blocks move as one is $(8.66\,\mathrm{N{\cdot}s})(12.5\,\mathrm{kg}) = 0.693\,\mathrm{m/s}$. The final kinetic energy is

$$\frac{1}{2}(12.5\,\mathrm{kg})(0.693\,\mathrm{m/s})^2 = 3.002\,\mathrm{J};$$

this means that $0.002\,\mathrm{J}$ are dissipated.

P13-8 Hmm.

E14-1 $F_\mathrm{S}/F_\mathrm{E} = M_\mathrm{S}{r_\mathrm{E}}^2/M_\mathrm{E}{r_\mathrm{S}}^2$, since everything else cancels out in the expression. Then

$$\frac{F_\mathrm{S}}{F_\mathrm{E}} = \frac{(1.99\times10^{30}\mathrm{kg})(3.84\times10^{8}\mathrm{m})^2}{(5.98\times10^{24})(1.50\times10^{11}\mathrm{m})^2} = 2.18$$

E14-2 Consider the force from the Sun and the force from the Earth. $F_\mathrm{S}/F_\mathrm{E} = M_\mathrm{S}{r_\mathrm{E}}^2/M_\mathrm{E}{r_\mathrm{S}}^2$, since everything else cancels out in the expression. We want the ratio to be one; we are also constrained because $r_\mathrm{E} + r_\mathrm{S} = R$ is the distance from the Sun to the Earth. Then

$$\begin{aligned} M_\mathrm{E}\left(R - r_\mathrm{E}\right)^2 &= M_\mathrm{S}{r_\mathrm{E}}^2, \\ R - r_\mathrm{E} &= \sqrt{\frac{M_\mathrm{S}}{M_\mathrm{E}}}r_\mathrm{E}, \\ r_\mathrm{E} &= (1.50\times10^{11}\mathrm{m})/\left(1 + \sqrt{\frac{(1.99\times10^{30}\mathrm{kg})}{(5.98\times10^{24})}}\right) = 2.6\times10^{8}\mathrm{m}. \end{aligned}$$

$\boxed{\textbf{E14-3}}$ The masses of each object are $m_1 = 20.0\,\mathrm{kg}$ and $m_2 = 7.0\,\mathrm{kg}$; the distance between the centers of the two objects is $15 + 3 = 18\,\mathrm{m}$.

The magnitude of the force from Newton's law of gravitation is then

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(20.0\,\mathrm{kg})(7.0\,\mathrm{kg})}{(18\,\mathrm{m})^2} = 2.9\times10^{-11}\mathrm{N}.$$

E14-4 (a) The magnitude of the force from Newton's law of gravitation is

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(12.7\,\mathrm{kg})(9.85\times10^{-3}\,\mathrm{kg})}{(0.108\,\mathrm{m})^2} = 7.15\times10^{-10}\mathrm{N}.$$

(b) The torque is $\tau = 2(0.262\,\mathrm{m})(7.15\times10^{-10}\mathrm{N}) = 3.75\times10^{-10}\mathrm{N}\cdot\mathrm{m}$.

$\boxed{\textbf{E14-5}}$ The force of gravity on an object near the surface of the earth is given by

$$F = \frac{GMm}{(r_e + y)^2},$$

where M is the mass of the Earth, m is the mass of the object, r_e is the radius of the Earth, and y is the height above the surface of the Earth. Expand the expression since $y \ll r_e$. We'll use a Taylor expansion, where $F(r_e + y) \approx F(r_e) + y\partial F/\partial r_e$;

$$F \approx \frac{GMm}{r_e^2} - 2y\frac{GMm}{r_e^3}$$

Since we are interested in the difference between the force at the top and the bottom, we really want

$$\Delta F = 2y\frac{GMm}{r_e^3} = 2\frac{y}{r_e}\frac{GMm}{r_e^2} = 2\frac{y}{r_e}W,$$

where in the last part we substituted for the weight, which is the same as the force of gravity,

$$W = \frac{GMm}{r_e^2}.$$

Finally,

$$\Delta F = 2(411\,\mathrm{m})/(6.37\times10^{6}\mathrm{m})(120\ \mathrm{lb}) = 0.015\ \mathrm{lb}.$$

E14-6 $g \propto 1/r^2$, so $g_1/g_2 = r_2^2/r_1^2$. Then

$$r_2 = \sqrt{(9.81\,\mathrm{m/s^2})/(7.35\,\mathrm{m/s^2})}(6.37\times10^6\mathrm{m}) = 7.36\times10^6\,\mathrm{m}.$$

That's 990 kilometers above the surface of the Earth.

E14-7 (a) $a = GM/r^2$, or

$$a = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\mathrm{kg})}{(10.0\times10^3\mathrm{m})^2} = 1.33\times10^{12}\mathrm{m/s^2}.$$

(b) $v = \sqrt{2ax} = \sqrt{2(1.33\times10^{12}\mathrm{m/s^2})(1.2\,\mathrm{m/s})} = 1.79\times10^6\mathrm{m/s}.$

E14-8 (a) $g_0 = GM/r^2$, or

$$g_0 = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(7.36\times10^{22}\mathrm{kg})}{(1.74\times10^6\mathrm{m})^2} = 1.62\,\mathrm{m/s^2}.$$

(b) $W_\mathrm{m} = W_\mathrm{e}(g_\mathrm{m}/g_\mathrm{e})$ so

$$W_\mathrm{m} = (100\,\mathrm{N})(1.62\,\mathrm{m/s^2}/9.81\,\mathrm{m/s^2}) = 16.5\,\mathrm{N}.$$

(c) Invert $g = GM/r^2$;

$$r = \sqrt{GM/g} = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})/(1.62\,\mathrm{m/s^2})} = 1.57\times10^7\mathrm{m}.$$

That's 2.46 Earth radii, or 1.46 Earth radii above the surface of the Earth.

$\boxed{\textbf{E14-9}}$ The object fell through $y = -10.0\,\mathrm{m}$; the time required to fall would then be

$$t = \sqrt{-2y/g} = \sqrt{-2(-10.0\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 1.43\,\mathrm{s}.$$

We are interested in the *error*, that means taking the total derivative of $y = -\frac{1}{2}gt^2$. and getting

$$\delta y = -\frac{1}{2}\delta g\, t^2 - gt\,\delta t.$$

$\delta y = 0$ so $-\frac{1}{2}\delta g\, t = g\delta\, t$, which can be rearranged as

$$\delta t = -\frac{\delta g\, t}{2g}$$

The percentage error in t needs to be $\delta t/t = 0.1\,\%/2 = 0.05\,\%$. The absolute error is then $\delta t = (0.05\,\%)(1.43\,\mathrm{s}) = 0.7$ ms.

E14-10 Treat mass which is inside a spherical shell as being located at the center of that shell. Ignore any contributions from shells farther away from the center than the point in question.

(a) $F = G(M_1 + M_2)m/a^2$.
(b) $F = G(M_1)m/b^2$.
(c) $F = 0$.

$\boxed{\textbf{E14-11}}$ For a sphere of uniform density and radius $R > r$,

$$\frac{M(r)}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3},$$

where M is the total mass.

The force of gravity on the object of mass m is then

$$F = \frac{GMm}{r^2}\frac{r^3}{R^3} = \frac{GMmr}{R^3}.$$

g is the free-fall acceleration of the object, and is the gravitational force divided by the mass, so

$$g = \frac{GMr}{R^3} = \frac{GM}{R^2}\frac{r}{R} = \frac{GM}{R^2}\frac{R-D}{R}.$$

Since R is the distance from the center to the surface, and D is the distance of the object beneath the surface, then $r = R - D$ is the distance from the center to the object. The first fraction is the free-fall acceleration on the surface, so

$$g = \frac{GM}{R^2}\frac{R-D}{R} = g_s\frac{R-D}{R} = g_s\left(1 - \frac{D}{R}\right)$$

E14-12 The work required to move the object is $GM_{\mathrm{S}}m/r$, where r is the gravitational radius. But if this equals mc^2 we can write

$$\begin{aligned} mc^2 &= GM_{\mathrm{S}}m/r, \\ r &= GM_{\mathrm{S}}/c^2. \end{aligned}$$

For the sun, $r = (6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(1.99\times10^{30}\mathrm{kg})/(3.00\times10^8\mathrm{m/s})^2 = 1.47\times10^3\mathrm{m}$. That's $2.1\times10^{-6}R_{\mathrm{S}}$.

E14-13 The distance from the center is

$$r = (80000)(3.00\times10^8\mathrm{m/s})(3.16\times10^7\mathrm{s}) = 7.6\times10^{20}\mathrm{m}.$$

The mass of the galaxy is

$$M = (1.4\times10^{11})(1.99\times10^{30}\mathrm{kg}) = 2.8\times10^{41}\mathrm{kg}.$$

The escape velocity is

$$v = \sqrt{2GM/r} = \sqrt{2(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(2.8\times10^{41}\mathrm{kg})/(7.6\times10^{20}\mathrm{m})} = 2.2\times10^5\mathrm{m/s}.$$

E14-14 Staying in a circular orbit requires the centripetal force be equal to the gravitational force, so

$$m{v_{\mathrm{orb}}}^2/r = GMm/r^2,$$

or $m{v_{\mathrm{orb}}}^2 = GMm/r$. But $-GMm/r$ is the gravitational potential energy; to escape one requires a kinetic energy

$$m{v_{\mathrm{esc}}}^2/2 = GMm/r = m{v_{\mathrm{orb}}}^2,$$

which has solution $v_{\mathrm{esc}} = \sqrt{2}v_{\mathrm{orb}}$.

$\boxed{\textbf{E14-15}}$ (a) Near the surface of the Earth the total energy is

$$E = K + U = \frac{1}{2}m\left(2\sqrt{gR_\mathrm{E}}\right)^2 - \frac{GM_\mathrm{E}m}{R_\mathrm{E}}$$

but

$$g = \frac{GM}{{R_\mathrm{E}}^2},$$

so the total energy is

$$\begin{aligned} E &= 2mgR_\mathrm{E} - \frac{GM_\mathrm{E}m}{R_\mathrm{E}}, \\ &= 2m\left(\frac{GM}{{R_\mathrm{E}}^2}\right)R_\mathrm{E} - \frac{GM_\mathrm{E}m}{R_\mathrm{E}}, \\ &= \frac{GM_\mathrm{E}m}{R_\mathrm{E}} \end{aligned}$$

This is a positive number, so the rocket will escape.

(b) Far from earth there is no gravitational potential energy, so

$$\frac{1}{2}mv^2 = \frac{GM_\mathrm{E}m}{R_\mathrm{E}} = \frac{GM_\mathrm{E}}{{R_\mathrm{E}}^2}mR_\mathrm{E} = gmR\mathrm{E},$$

with solution $v = \sqrt{2gR_\mathrm{E}}$.

E14-16 The rotational acceleration of the sun is related to the galactic acceleration of free fall by

$$4\pi^2 mr/T^2 = GNm^2/r^2,$$

where N is the number of "sun" sized stars of mass m, r is the size of the galaxy, T is period of revolution of the sun. Then

$$N = \frac{4\pi^2 r^3}{GmT^2} = \frac{4\pi^2(2.2\times10^{20}\mathrm{m})^3}{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(2.0\times10^{30}\mathrm{kg})(7.9\times10^{15}\mathrm{s})^2} = 5.1\times10^{10}.$$

$\boxed{\textbf{E14-17}}$ Energy conservation is $K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}$, but at the highest point $K_\mathrm{f} = 0$, so

$$\begin{aligned} U_\mathrm{f} &= K_\mathrm{i} + U_\mathrm{i}, \\ -\frac{GM_\mathrm{E}m}{R} &= \frac{1}{2}mv_0^2 - \frac{GM_\mathrm{E}m}{R_\mathrm{E}}, \\ \frac{1}{R} &= \frac{1}{R_\mathrm{E}} - \frac{1}{2GM_\mathrm{E}}v_0^2, \\ \frac{1}{R} &= \frac{1}{(6.37\times10^6\mathrm{m})} - \frac{(9.42\times10^3\mathrm{m/s})^2)}{2(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})}, \\ R &= 2.19\times10^7\ \mathrm{m}. \end{aligned}$$

The distance above the Earth's surface is $2.19\times10^7\ \mathrm{m} - 6.37\times10^6\mathrm{m} = 1.55\times10^6\ \mathrm{m}$.

E14-18 (a) Free-fall acceleration is $g = GM/r^2$. Escape speed is $v = \sqrt{2GM/r}$. Then $v = \sqrt{2gr} = \sqrt{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})} = 2.02\times10^3\mathrm{m/s}$.

(b) $U_\mathrm{f} = K_\mathrm{i} + U_\mathrm{i}$. But $U/m = -g_0 r_0^2/r$, so

$$\frac{1}{r_\mathrm{f}} = \frac{1}{(1.569\times10^6\mathrm{m})} - \frac{(1.01\times10^3\mathrm{m/s})^2}{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2} = \frac{1}{2.09\times10^6\mathrm{m}}.$$

That's 523 km above the surface.

(c) $K_\mathrm{f} = U_\mathrm{i} - U_\mathrm{f}$. But $U/m = -g_0 r_0^2/r$, so

$$v = \sqrt{2(1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2[1/(1.569\times10^6\mathrm{m}) - 1/(2.569\times10^6\mathrm{m})]} = 1260\,\mathrm{m/s}.$$

(d) $M = gr^2/G$, or

$$M = (1.30\,\mathrm{m/s^2})(1.569\times10^6\mathrm{m})^2/(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2}) = 4.8\times10^{22}\mathrm{kg}.$$

E14-19 (a) Apply $\Delta K = -\Delta U$. Then $mv^2 = Gm^2(1/r_2 - 1/r_1)$, so

$$v = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.56\times10^{30}\mathrm{kg})\left(\frac{1}{(4.67\times10^4\mathrm{m})} - \frac{1}{(9.34\times10^4\mathrm{m})}\right)} = 3.34\times10^7\mathrm{m/s}.$$

(b) Apply $\Delta K = -\Delta U$. Then $mv^2 = Gm^2(1/r_2 - 1/r_1)$, so

$$v = \sqrt{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.56\times10^{30}\mathrm{kg})\left(\frac{1}{(1.26\times10^4\mathrm{m})} - \frac{1}{(9.34\times10^4\mathrm{m})}\right)} = 5.49\times10^7\mathrm{m/s}.$$

E14-20 Call the particles 1 and 2. Then conservation of momentum requires the particle to have the same momentum of the same magnitude, $p = mv_1 = Mv_2$. The momentum of the particles is given by

$$\begin{aligned}
\frac{1}{2m}p^2 + \frac{1}{2M}p^2 &= \frac{GMm}{d}, \\
\frac{m+M}{mM}p^2 &= 2GMm/d, \\
p &= mM\sqrt{2G/d(m+M)}.
\end{aligned}$$

Then $v_\mathrm{rel} = |v_1| + |v_2|$ is equal to

$$\begin{aligned}
v_\mathrm{rel} &= mM\sqrt{2G/d(m+M)}\left(\frac{1}{m} + \frac{1}{M}\right), \\
&= mM\sqrt{2G/d(m+M)}\left(\frac{m+M}{mM}\right), \\
&= \sqrt{2G(m+M)/d}
\end{aligned}$$

E14-21 The maximum speed is $mv^2 = Gm^2/d$, or $v = \sqrt{Gm/d}$.

E14-22 $T_1^2/r_1^3 = T_2^2/r_2^3$, or

$$T_2 = T_1(r_2/r_1)^{3/2} = (1.00\text{ y})(1.52)^{3/2} = 1.87\text{ y}.$$

$\boxed{\textbf{E14-23}}$ We can use Eq. 14-23 to find the mass of Mars; all we need to do is rearrange to solve for M—

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(9.4\times10^6\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2.75\times10^4\text{s})^2} = 6.5\times10^{23}\text{kg}.$$

E14-24 Use $GM/r^2 = 4\pi^2 r/T^2$, so $M = 4\pi^2 r^3/GT^2$, and

$$M = \frac{4\pi^2(3.82\times10^8\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(27.3\times86400\,\text{s})^2} = 5.93\times10^{24}\text{kg}.$$

E14-25 $T_1^2/r_1^3 = T_2^2/r_2^3$, or

$$T_2 = T_1(r_2/r_1)^{3/2} = (1.00\text{ month})(1/2)^{3/2} = 0.354\text{ month}.$$

E14-26 Geosynchronous orbit was found in Sample Problem 14-8 to be 4.22×10^7m. The latitude is given by

$$\theta = \arccos(6.37\times10^6\text{m}/4.22\times10^7\text{m}) = 81.3^\circ.$$

$\boxed{\textbf{E14-27}}$ (b) Make the assumption that the altitude of the satellite is so low that the radius of the orbit is effectively the radius of the moon. Then

$$\begin{aligned} T^2 &= \left(\frac{4\pi^2}{GM}\right) r^3, \\ &= \left(\frac{4\pi^2}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22}\text{kg})}\right)(1.74\times10^6\text{m})^3 = 4.24\times10^7\text{s}^2. \end{aligned}$$

So $T = 6.5\times10^3$s.

(a) The speed of the satellite is the circumference divided by the period, or

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.74\times10^6\text{m})}{(6.5\times10^3\text{s})} = 1.68\times10^3\text{m/s}.$$

E14-28 The total energy is $-GMm/2a$. Then

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a},$$

so

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right).$$

E14-29 $r_\text{a} = a(1+e)$, so from Ex. 14-28,

$$v_\text{a} = \sqrt{GM\left(\frac{2}{a(1+e)} - \frac{1}{a}\right)};$$

$r_\text{p} = a(1-e)$, so from Ex. 14-28,

$$v_\text{p} = \sqrt{GM\left(\frac{2}{a(1-e)} - \frac{1}{a}\right)};$$

Dividing one expression by the other,

$$v_\text{p} = v_\text{a}\sqrt{\frac{2/(1-e)-1}{2/(1+e)-1}} = (3.72\text{ km/s})\sqrt{\frac{2/0.12-1}{2/1.88-1}} = 58.3\text{ km/s}.$$

E14-30 (a) Convert.

$$G = \left(6.67\times10^{-11}\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)\left(\frac{1.99\times10^{30}\text{kg}}{M_\text{S}}\right)\left(\frac{3.156\times10^{7}\text{s}}{\text{y}}\right)^2\left(\frac{\text{AU}}{1.496\times10^{11}\text{m}}\right)^3,$$

which is $G = 39.49\,\text{AU}^3/{M_\text{S}}^2\cdot\text{y}^2$.

(b) Here is a hint: $4\pi^2 = 39.48$. Kepler's law then looks like

$$T^2 = \left(\frac{{M_\text{S}}^2\cdot\text{y}^2}{\text{AU}^3}\right)\frac{r^3}{M}.$$

$\boxed{\textbf{E14-31}}$ Kepler's third law states $T^2 \propto r^3$, where r is the mean distance from the *Sun* and T is the period of revolution. Newton was in a position to find the acceleration of the Moon toward the Earth by assuming the Moon moved in a circular orbit, since $a_c = v^2/r = 4\pi^2 r/T^2$. But this means that, because of Kepler's law, $a_c \propto r/T^2 \propto 1/r^2$.

E14-32 (a) The force of attraction between the two bodies is

$$F = \frac{GMm}{(r+R)^2}.$$

The centripetal acceleration for the body of mass m is

$$\begin{aligned} r\omega^2 &= \frac{GM}{(r+R)^2}, \\ \omega^2 &= \frac{GM}{r^3(1+R/r)^2}, \\ T^2 &= \frac{4\pi^2}{GM}r^3(1+R/r)^2. \end{aligned}$$

(b) Note that $r = Md/(m+M)$ and $R = md/(m+M)$. Then $R/r = m/M$, so the correction is $(1 + 5.94\times10^{24}/1.99\times10^{30})^2 = 1.000006$ for the Earth/Sun system and 1.025 for the Earth/Moon system.

E14-33 (a) Use the results of Exercise 14-32. The center of mass is located a distance $r = 2md/(m+2m) = 2d/3$ from the star of mass m and a distance $R = d/3$ from the star of mass $2m$. The period of revolution is then given by

$$T^2 = \frac{4\pi^2}{G(2m)}\left(\frac{2}{3}d\right)^3\left(1+\frac{d/3}{2d/3}\right)^2 = \frac{4\pi^2}{3Gm}d^3.$$

(b) Use $L_m = mr^2\omega$, then

$$\frac{L_m}{L_M} = \frac{mr^2}{MR^2} = \frac{m(2d/3)^2}{(2m)(d/3)^2} = 2.$$

(c) Use $K = I\omega^2/2 = mr^2\omega^2/2$. Then

$$\frac{K_m}{K_M} = \frac{mr^2}{MR^2} = \frac{m(2d/3)^2}{(2m)(d/3)^2} = 2.$$

E14-34 Since we don't know which direction the orbit will be, we will assume that the satellite on the surface of the Earth starts with zero kinetic energy. Then $E_\mathrm{i} = U_\mathrm{i}$.

$\Delta U = U_\mathrm{f} - U_\mathrm{i}$ to get the satellite up to the specified altitude. $\Delta K = K_\mathrm{f} = -U_\mathrm{f}/2$. We want to know if $\Delta U - \Delta K$ is positive (more energy to get it up) or negative (more energy to put it in orbit). Then we are interested in

$$\Delta U - \Delta K = 3U_\mathrm{f}/2 - U_\mathrm{i} = GMm\left(\frac{1}{r_\mathrm{i}} - \frac{3}{2r_\mathrm{f}}\right).$$

The "break-even" point is when $r_\mathrm{f} = 3r_\mathrm{i}/2 = 3(6400\text{ km})/2 = 9600\text{ km}$, which is 3200 km above the Earth.

(a) More energy to put it in orbit.
(b) Same energy for both.
(c) More energy to get it up.

E14-35 (a) The approximate force of gravity on a 2000 kg pickup truck on Eros will be

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.0\times10^{15}\text{ kg})(2000\,\mathrm{kg})}{(7000\,\mathrm{m})^2} = 13.6\,\mathrm{N}.$$

(b) Use

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.0\times10^{15}\text{ kg})}{(7000\,\mathrm{m})}} = 6.9\,\mathrm{m/s}.$$

E14-36 (a) $U = -GMm/r$. The variation is then

$$\begin{aligned}\Delta U &= (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1.99\times10^{30}\mathrm{kg})(5.98\times10^{24}\mathrm{kg})\left(\frac{1}{(1.47\times10^{11}\mathrm{m})} - \frac{1}{(1.52\times10^{11}\mathrm{m})}\right)\\ &= 1.78\times10^{32}\mathrm{J}.\end{aligned}$$

(b) $\Delta K + \Delta U = \Delta E = 0$, so $|\Delta K| = 1.78\times10^{32}\mathrm{J}$.
(c) $\Delta E = 0$.
(d) Since $\Delta l = 0$ and $l = mvr$, we have

$$v_\mathrm{p} - v_\mathrm{a} = v_\mathrm{p}\left(1 - \frac{r_\mathrm{p}}{r_\mathrm{a}}\right) = v_\mathrm{p}\left(1 - \frac{(1.47\times10^{11}\mathrm{m})}{(1.52\times10^{11}\mathrm{m})}\right) = 3.29\times10^{-2}v_\mathrm{p}.$$

But $v_\mathrm{p} \approx v_\mathrm{av} = 2\pi(1.5\times10^{11}\mathrm{m})/(3.16\times10^{7}\mathrm{s}) = 2.98\times10^{4}\mathrm{m/s}$. Then $\Delta v = 981\,\mathrm{m/s}$.

E14-37 Draw a triangle. The angle made by Chicago, Earth center, satellite is 47.5°. The distance from Earth center to satellite is 4.22×10^7m. The distance from Earth center to Chicago is 6.37×10^6m. Applying the cosine law we find the distance from Chicago to the satellite is

$$\sqrt{(4.22\times10^7\mathrm{m})^2 + (6.37\times10^6\mathrm{m})^2 - 2(4.22\times10^7\mathrm{m})(6.37\times10^6\mathrm{m})\cos(47.5^\circ)} = 3.82\times10^7\mathrm{m}.$$

Applying the sine law we find the angle made by Earth center, Chicago, satellite to be

$$\arcsin\left(\frac{(4.22\times10^7\mathrm{m})}{(3.82\times10^7\mathrm{m})}\sin(47.5^\circ)\right) = 126^\circ.$$

That's 36° above the horizontal.

E14-38 (a) The new orbit is an ellipse with eccentricity given by $r = a'(1+e)$. Then

$$e = r/a' - 1 = (6.64\times 10^6\text{m})/(6.52\times 10^6\text{m}) - 1 = 0.0184.$$

The distance at P' is given by $r_{P'} = a'(1-e)$. The potential energy at P' is

$$U_{P'} = U_P \frac{1+e}{1-e} = 2(-9.76\times 10^{10}\text{J})\frac{1+0.0184}{1-0.0184} = -2.03\times 10^{11}\text{J}.$$

The kinetic energy at P' is then

$$K_{P'} = (-9.94\times 10^{10}\text{J}) - (-2.03\times 10^{11}\text{J}) = 1.04\times 10^{11}\text{J}.$$

That would mean $v = \sqrt{2(1.04\times 10^{11}\text{J})/(3250\text{kg})} = 8000\,\text{m/s}$.

(b) The average speed is

$$v = \frac{2\pi(6.52\times 10^6\text{m})}{(5240\,\text{s})} = 7820\,\text{m/s}.$$

$\boxed{\textbf{E14-39}}$ (a) The Starshine satellite was approximately 275 km above the surface of the Earth on 1 January 2000. We can find the orbital period from Eq. 14-23,

$$\begin{aligned} T^2 &= \left(\frac{4\pi^2}{GM}\right) r^3, \\ &= \left(\frac{4\pi^2}{(6.67\times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24}\text{kg})}\right)(6.65\times 10^6\text{m})^3 = 2.91\times 10^7\text{s}^2, \end{aligned}$$

so $T = 5.39\times 10^3$s.

(b) Equation 14-25 gives the total energy of the system of a satellite of mass m in a circular orbit of radius r around a stationary body of mass $M \gg m$,

$$E = -\frac{GMm}{2r}.$$

We want the rate of change of this with respect to time, so

$$\frac{dE}{dt} = \frac{GMm}{2r^2}\frac{dr}{dt}$$

We can estimate the value of dr/dt from the diagram. I'll choose February 1 and December 1 as my two reference points.

$$\left.\frac{dr}{dt}\right|_{t=t_0} \approx \frac{\Delta r}{\Delta t} = \frac{(240\text{ km}) - (300\text{ km})}{(62\text{ days})} \approx -1\text{ km/day}$$

The rate of energy loss is then

$$\frac{dE}{dt} = \frac{(6.67\times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24}\text{kg})(39\,\text{kg})}{2(6.65\times 10^6\text{m})^2}\frac{-1000\,\text{m}}{8.64\times 10^4\,\text{s}} = -2.0\,\text{J/s}.$$

$\boxed{\textbf{P14-1}}$ The object on the top experiences a force down from gravity W_1 and a force down from the tension in the rope T. The object on the bottom experiences a force down from gravity W_2 and a force up from the tension in the rope.

In either case, the magnitude of W_i is

$$W_i = \frac{GMm}{r_i^2}$$

where r_i is the distance of the ith object from the center of the Earth. While the objects fall they have the same acceleration, and since they have the same mass we can quickly write

$$\frac{GMm}{r_1^2} + T = \frac{GMm}{r_2^2} - T,$$

or

$$\begin{aligned} T &= \frac{GMm}{2r_2^2} - \frac{GMm}{2r_1^2}, \\ &= \frac{GMm}{2}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right), \\ &= \frac{GMm}{2}\frac{r_2^2 - r_1^2}{r_1^2 r_2^2}. \end{aligned}$$

Now $r_1 \approx r_2 \approx R$ in the denominator, but $r_2 = r_1 + l$, so $r_2^2 - r_1^2 \approx 2Rl$ in the numerator. Then

$$T \approx \frac{GMml}{R^3}.$$

P14-2 For a planet of uniform density, $g = GM/r^2 = G(4\pi\rho r^3/3)/r^2 = 4\pi G\rho r/3$. Then if ρ is doubled while r is halved we find that g will be unchanged.

P14-3 (a) $F = GMm/r^2$, $a = F/m = GM/r$.

(b) The acceleration of the Earth toward the center of mass is $a_{\mathrm{E}} = F/M = Gm/r^2$. The relative acceleration is then $GM/r + Gm/r = G(m + M)/r$. Only if $M \gg m$ can we assume that a is independent of m relative to the Earth.

P14-4 (a) $g = GM/r^2$, $\delta g = -(2GM/r^3)\delta r$. In this case $\delta r = h$ and $M = 4\pi\rho r^3/3$. Then

$$\delta W = m\,\delta g = 8\pi G\rho m h/3.$$

(b) $\Delta W/W = \Delta g/g = 2h/r$. Then an error of one part in a million will occur when h is one part in two million of r, or 3.2 meters.

$\boxed{\textbf{P14-5}}$ (a) The magnitude of the gravitational force from the Moon on a particle at A is

$$F_A = \frac{GMm}{(r-R)^2},$$

where the denominator is the distance from the center of the moon to point A.

(b) At the center of the Earth the gravitational force of the moon on a particle of mass m is $F_C = GMm/r^2$.

(c) Now we want to know the difference between these two expressions:

$$\begin{aligned} F_A - F_C &= \frac{GMm}{(r-R)^2} - \frac{GMm}{r^2}, \\ &= GMm\left(\frac{r^2}{r^2(r-R)^2} - \frac{(r-R)^2}{r^2(r-R)^2}\right), \\ &= GMm\left(\frac{r^2 - (r-R)^2}{r^2(r-R)^2}\right), \\ &= GMm\left(\frac{R(2r-R)}{r^2(r-R)^2}\right). \end{aligned}$$

To simplify assume $R \ll r$ and then substitute $(r - R) \approx r$. The force difference simplifies to

$$F_T = GMm \frac{R(2r)}{r^2(r)^2} = \frac{2GMmR}{r^3}$$

(d) Repeat part (c) except we want $r + R$ instead of $r - R$. Then

$$\begin{aligned}
F_A - F_C &= \frac{GMm}{(r+R)^2} - \frac{GMm}{r^2}, \\
&= GMm\left(\frac{r^2}{r^2(r+R)^2} - \frac{(r+R)^2}{r^2(r+R)^2}\right), \\
&= GMm\left(\frac{r^2-(r+R)^2}{r^2(r+R)^2}\right), \\
&= GMm\left(\frac{-R(2r+R)}{r^2(r+R)^2}\right).
\end{aligned}$$

To simplify assume $R \ll r$ and then substitute $(r + R) \approx r$. The force difference simplifies to

$$F_T = GMm \frac{-R(2r)}{r^2(r)^2} = -\frac{2GMmR}{r^3}$$

The negative sign indicates that this "apparent" force points *away* from the moon, not toward it.

(e) Consider the directions: the water is effectively attracted to the moon when closer, but repelled when farther.

P14-6 $F_{\text{net}} = mr{\omega_{\text{s}}}^2$, where ω_{s} is the rotational speed of the ship. But since the ship is moving relative to the earth with a speed v, we can write $\omega_{\text{s}} = \omega \pm v/r$, where the sign is positive if the ship is sailing east. Then $F_{\text{net}} = mr(\omega \pm v/r)^2$.

The scale measures a force W which is given by $mg - F_{\text{net}}$, or

$$W = mg - mr(\omega \pm v/r)^2.$$

Note that $W_0 = m(g - r\omega^2)$. Then

$$\begin{aligned}
W &= W_0 \frac{g - r(\omega \pm v/r)^2}{g - r\omega^2}, \\
&\approx W_0\left(1 \pm \frac{2\omega v}{1 - r\omega^2}\right), \\
&\approx W_0(1 \pm 2v\omega/g).
\end{aligned}$$

P14-7 (a) $a = GM/r^2 - r\omega^2$. ω is the rotational speed of the Earth. Since Frank observes $a = g/2$ we have

$$\begin{aligned}
g/2 &= GM/r^2 - r\omega^2, \\
r^2 &= (2GM - 2r^3\omega^2)/g, \\
r &= \sqrt{2(GM - r^3\omega^2)/g}
\end{aligned}$$

Note that

$$GM = (6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{kg}) = 3.99\times10^{14}\text{m}^3/\text{s}^2$$

while

$$r^3\omega^2 = (6.37\times10^6\text{m})^3(2\pi/86400\,\text{s})^2 = 1.37\times10^{12}\text{m}^3/\text{s}^2.$$

Consequently, $r^3\omega^2$ can be treated as a perturbation of GM bear the Earth. Solving iteratively,

$$\begin{aligned}
r_0 &= \sqrt{2[(3.99\times10^{14}\mathrm{m}^3/\mathrm{s}^2) - (6.37\times10^6\mathrm{m})^3(2\pi/86400\,\mathrm{s})^2]/(9.81\,\mathrm{m/s}^2)} = 9.00\times10^6\mathrm{m},\\
r_1 &= \sqrt{2[(3.99\times10^{14}\mathrm{m}^3/\mathrm{s}^2) - (9.00\times10^6\mathrm{m})^3(2\pi/86400\,\mathrm{s})^2]/(9.81\,\mathrm{m/s}^2)} = 8.98\times10^6\mathrm{m},
\end{aligned}$$

which is close enough for me. Then $h = 8.98\times10^6\mathrm{m} - 6.37\times10^6\mathrm{m} = 2610$ km.

(b) $\Delta E = E_\mathrm{f} - E_\mathrm{i} = U_\mathrm{f}/2 - U_\mathrm{i}$. Then

$$\Delta E = (6.67\times10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\mathrm{kg})(100\,\mathrm{kg})\left(\frac{1}{(6.37\times10^6\mathrm{m})} - \frac{1}{2(8.98\times10^6\mathrm{m})}\right) = 4.0\times10^9\mathrm{J}.$$

P14-8 (a) Equate centripetal force with the force of gravity.

$$\begin{aligned}
\frac{4\pi^2 mr}{T^2} &= \frac{GMm}{r^2},\\
\frac{4\pi^2}{T^2} &= \frac{G(4/3)\pi r^3\rho}{r^3},\\
T &= \sqrt{\frac{3\pi}{G\rho}}
\end{aligned}$$

(b) $T = \sqrt{3\pi/(6.7\times10^{-11}\mathrm{N\cdot m^2/kg^2})(3.0\times10^3\mathrm{kg/m^3})} = 6800\,\mathrm{s}$.

P14-9 (a) One can find δg by pretending the Earth is not there, but the material in the hole is. Concentrate on the vertical component of the resulting force of attraction. Then

$$\delta g = \frac{GM}{r^2}\frac{d}{r},$$

where r is the straight line distance from the prospector to the center of the hole and M is the mass of material that would fill the hole. A few substitutions later,

$$\delta g = \frac{4\pi G\rho R^3 d}{3(\sqrt{d^2+x^2})^3}.$$

(b) Directly above the hole $x = 0$, so a ratio of the two readings gives

$$\frac{(10.0\text{ milligals})}{(14.0\text{ milligals})} = \left(\frac{d^2}{d^2+(150\,\mathrm{m})^2}\right)^{3/2}$$

or

$$(0.800)(d^2 + 2.25\times10^4\mathrm{m}^2) = d^2,$$

which has solution $d = 300\,\mathrm{m}$. Then

$$R^3 = \frac{3(14.0\times10^{-5}\mathrm{m/s^2})(300\,\mathrm{m})^2}{4\pi(6.7\times10^{-11}\mathrm{N\cdot m^2/kg^2})(2800\,\mathrm{kg/m^3})},$$

so $R = 250\,\mathrm{m}$. The top of the cave is then $300\,\mathrm{m} - 250\,\mathrm{m} = 50\,\mathrm{m}$ beneath the surface.

(b) All of the formulae stay the same *except* replace ρ with the difference between rock and water. d doesn't change, but R will now be given by

$$R^3 = \frac{3(14.0\times10^{-5}\mathrm{m/s^2})(300\,\mathrm{m})^2}{4\pi(6.7\times10^{-11}\mathrm{N\cdot m^2/kg^2})(1800\,\mathrm{kg/m^3})},$$

so $R = 292\,\mathrm{m}$, and then the cave is located $300\,\mathrm{m} - 292\,\mathrm{m} = 7\,\mathrm{m}$ beneath the surface.

P14-10 $g = GM/r^2$, where M is the mass enclosed in within the sphere of radius r. Then $dg = (G/r^2)dM - 2(GM/r^3)dr$, so that g is locally constant if $dM/dr = 2M/r$. Expanding,

$$\begin{aligned} 4\pi r^2 \rho_l &= 8\pi r^2 \rho/3, \\ \rho_l &= 2\rho/3. \end{aligned}$$

$\boxed{\textbf{P14-11}}$ The force of gravity on the small sphere of mass m is equal to the force of gravity from a solid lead sphere minus the force which would have been contributed by the smaller lead sphere which would have filled the hole. So we need to know about the size and mass of the lead which was removed to make the hole.

The density of the lead is given by

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

The hole has a radius of $R/2$, so if the density is constant the mass of the hole will be

$$M_h = \rho V = \left(\frac{M}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

The "hole" is closer to the small sphere; the center of the hole is $d - R/2$ away. The force of the whole lead sphere minus the force of the "hole" lead sphere is

$$\frac{GMm}{d^2} - \frac{G(M/8)m}{(d - R/2)^2}$$

P14-12 (a) Use $v = \omega\sqrt{R^2 - r^2}$, where $\omega = \sqrt{GM_E/R^3}$. Then

$$\begin{aligned} T &= \int_0^T dt = \int_R^0 \frac{dr}{dr/dt} = \int_R^0 \frac{dr}{v}, \\ &= \int_R^0 \frac{dr}{\omega\sqrt{R^2 - r^2}}, \\ &= \frac{\pi}{2\omega} \end{aligned}$$

Knowing that

$$\omega = \sqrt{\frac{(6.67\times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times 10^{24}\mathrm{kg})}{(6.37\times 10^6\mathrm{m})^3}} = 1.24\times 10^{-3}/\mathrm{s},$$

we can find $T = 1260\,\mathrm{s} = 21$ min.

(b) Same time, 21 minutes. To do a complete journey would require four times this, or $2\pi/\omega$. That's 84 minutes!

(c) The answers are the same.

P14-13 (a) $g = GM/r^2$ and $M = 1.93\times 10^{24}\mathrm{kg} + 4.01\times 10^{24}\mathrm{kg} + 3.94\times 10^{22}\mathrm{kg} = 5.98\times 10^{24}\mathrm{kg}$ so

$$g = (6.67\times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times 10^{24}\mathrm{kg})/(6.37\times 10^6\mathrm{m})^2 = 9.83\,\mathrm{m/s}^2.$$

(b) Now $M = 1.93\times 10^{24}\mathrm{kg} + 4.01\times 10^{24}\mathrm{kg} = 5.94\times 10^{24}\mathrm{kg}$ so

$$g = (6.67\times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.94\times 10^{24}\mathrm{kg})/(6.345\times 10^6\mathrm{m})^2 = 9.84\,\mathrm{m/s}^2.$$

(c) For a uniform body, $g = 4\pi G\rho r/3 = GMr/R^3$, so

$$g = (6.67\times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(5.98\times 10^{24}\mathrm{kg})(6.345\times 10^6\mathrm{m})/(6.37\times 10^6\mathrm{m})^3 = 9.79\,\mathrm{m/s}^2.$$

P14-14 (a) Use $g = GM/r^2$, then

$$g = (6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(1.93\times10^{24}\mathrm{kg})/(3.490\times10^6\mathrm{m})^2 = 106\,\mathrm{m/s}^2.$$

The variation with depth is linear if core has uniform density.

(b) In the mantle we have $g = G(M_\mathrm{c}+M)/r^2$, where M is the amount of the mass of the mantle which is enclosed in the sphere of radius r. The density of the core is

$$\rho_\mathrm{c} = \frac{3(1.93\times10^{24}\mathrm{kg})}{4\pi(3.490\times10^6\mathrm{m})^3} = 1.084\times10^4\mathrm{kg/m}^3.$$

The density of the mantle is harder to find,

$$\rho_\mathrm{c} = \frac{3(4.01\times10^{24}\mathrm{kg})}{4\pi[(6.345\times10^6\mathrm{m})^3 - (3.490\times10^6\mathrm{m})^3} = 4.496\times10^3\mathrm{kg/m}^3.$$

We can pretend that the core is made up of a point mass at the center and the rest has a density equal to that of the mantle. That point mass would be

$$M_\mathrm{p} = \frac{4\pi(3.490\times10^6\mathrm{m})^3(1.084\times10^4\mathrm{kg/m}^3 - 4.496\times10^3\mathrm{kg/m}^3)}{3} = 1.130\times10^{24}\mathrm{kg}.$$

Then

$$g = GM_\mathrm{p}/r^2 + 4\pi G\rho_\mathrm{m}r/3.$$

Find dg/dr, and set equal to zero. This happens when

$$2M_\mathrm{p}/r^3 = 4\pi\rho_\mathrm{m}/3,$$

or $r = 4.93\times10^6\mathrm{m}$. Then $g = 9.29\,\mathrm{m/s}^2$. Since this is less than the value at the end points it must be a minimum.

$\boxed{\textbf{P14-15}}$ (a) We will use part of the hint, but we will integrate instead of assuming the bit about g_av; doing it this way will become important for later chapters. Consider a small horizontal slice of the column of thickness dr. The weight of the material above the slice exerts a force $F(r)$ on the top of the slice; there is a force of gravity on the slice given by

$$dF = \frac{GM(r)\,dm}{r^2},$$

where $M(r)$ is the mass contained in the sphere of radius r,

$$M(r) = \frac{4}{3}\pi r^3\rho.$$

Lastly, the mass of the slice dm is related to the thickness and cross sectional area by $dm = \rho A\,dr$. Then

$$dF = \frac{4\pi GA\rho^2}{3}r\,dr.$$

Integrate both sides of this expression. On the left the limits are 0 to F_center, on the right the limits are R to 0; we need to throw in an extra negative sign because the force increases as r decreases. Then

$$F = \frac{2}{3}\pi GA\rho^2R^2.$$

Divide both sides by A to get the compressive stress.

(b) Put in the numbers!

$$S = \frac{2}{3}\pi(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(4000\,\mathrm{kg/m}^3)^2(3.0\times10^5\mathrm{m})^2 = 2.0\times10^8\mathrm{N/m}^2.$$

(c) Rearrange, and then put in numbers;

$$R = \sqrt{\frac{3(4.0\times10^7\mathrm{N/m}^2)}{2\pi(6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(3000\,\mathrm{kg/m}^3)^2}} = 1.8\times10^5\,\mathrm{m}.$$

P14-16 The two mass increments each exert a vertical and a horizontal force on the particle, but the horizontal components will cancel. The vertical component is proportional to the sine of the angle, so that

$$dF = \frac{2Gm\,dm}{r^2}\frac{y}{r} = \frac{2Gm\lambda\,dx}{r^2}\frac{y}{r},$$

where $r^2 = x^2 + y^2$. We will eventually integrate from 0 to ∞, so

$$\begin{aligned} F &= \int_0^\infty \frac{2Gm\lambda\,dx}{r^2}\frac{y}{r}, \\ &= 2Gm\lambda y\int_0^\infty \frac{dx}{(x^2+y^2)^{3/2}}, \\ &= \frac{2Gm\lambda}{y}. \end{aligned}$$

P14-17 For any arbitrary point P the cross sectional area which is perpendicular to the axis $dA' = r^2 d\Omega$ is not equal to the projection dA onto the surface of the sphere. It depends on the angle that the axis makes with the normal, according to $dA' = \cos\theta dA$. Fortunately, the angle made at point 1 is identical to the angle made at point 2, so we can write

$$\begin{aligned} d\Omega_1 &= d\Omega_2, \\ dA_1/r_1^2 &= dA_2/r_2^2 \end{aligned}$$

But the mass of the shell contained in dA is proportional to dA, so

$$\begin{aligned} r_1^2 dm_1 &= r_2^2 dm_2, \\ Gm\,dm_1/r_1^2 &= Gm\,dm_2/r_2^2. \end{aligned}$$

Consequently, the force on an object at point P is balanced by both cones.

(b) Evaluate $\int d\Omega$ for the top and bottom halves of the sphere. Since every $d\Omega$ on the top is balanced by one on the bottom, the net force is zero.

P14-18

P14-19 Assume that the small sphere is always between the two spheres. Then

$$\begin{aligned} W &= \Delta U_1 + \Delta U_2, \\ &= (6.67\times10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2)(0.212\,\mathrm{kg})\,[(7.16\,\mathrm{kg}) - (2.53\,\mathrm{kg})]\left[\frac{1}{(0.420\,\mathrm{m})} - \frac{1}{(1.14\,\mathrm{m})}\right], \\ &= 9.85\times10^{-11}\mathrm{J}. \end{aligned}$$

P14-20 Note that $\frac{1}{2}m{v_{\text{esc}}}^2 = -U_0$, where U_0 is the potential energy at the burn-out height. Energy conservation gives

$$\begin{aligned} K &= K_0 + U_0, \\ \frac{1}{2}mv^2 &= \frac{1}{2}mv_0^2 - \frac{1}{2}m{v_{\text{esc}}}^2, \\ v &= \sqrt{v_0^2 - {v_{\text{esc}}}^2}. \end{aligned}$$

$\boxed{\textbf{P14-21}}$ (a) The force of one star on the other is given by $F = Gm^2/d^2$, where d is the distance between the stars. The stars revolve around the center of mass, which is halfway between the stars so $r = d/2$ is the radius of the orbit of the stars. If a is the centripetal acceleration of the stars, the period of revolution is then

$$T = \sqrt{\frac{4\pi^2 r}{a}} = \sqrt{\frac{4m\pi^2 r}{F}} = \sqrt{\frac{16\pi^2 r^3}{Gm}}.$$

The numerical value is

$$T = \sqrt{\frac{16\pi^2(1.12\times10^{11}\text{m})^3}{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(3.22\times10^{30}\text{kg})}} = 3.21\times10^7\text{s} = 1.02\text{ y}.$$

(b) The gravitational potential energy per kilogram midway between the stars is

$$-2\frac{Gm}{r} = -2\frac{(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(3.22\times10^{30}\text{kg})}{(1.12\times10^{11}\text{m})} = -3.84\times10^9\text{J/kg}.$$

An object of mass M at the center between the stars would need $(3.84\times10^9\text{J/kg})M$ kinetic energy to escape, this corresponds to a speed of

$$v = \sqrt{2K/M} = \sqrt{2(3.84\times10^9\text{J/kg})} = 8.76\times10^4\text{m/s}.$$

P14-22 (a) Each differential mass segment on the ring contributes the same amount to the force on the particle,

$$dF = \frac{Gm\,dm}{r^2}\frac{x}{r},$$

where $r^2 = x^2 + R^2$. Since the differential mass segments are all equal distance, the integration is trivial, and the net force is

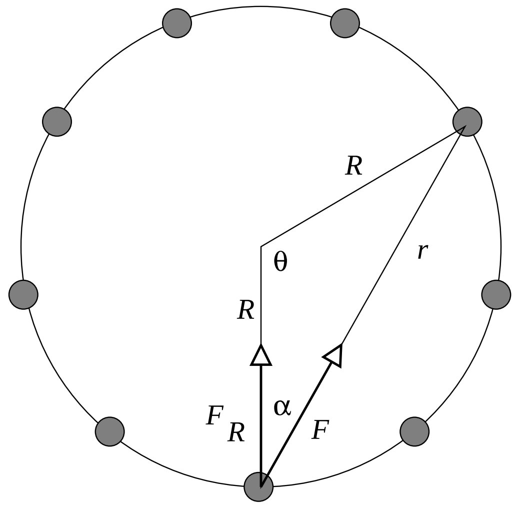
$$F = \frac{GMmx}{(x^2 + R^2)^{3/2}}.$$

(b) The potential energy can be found by integrating with respect to x,

$$\Delta U = \int_0^\infty F\,dx = \int_0^\infty \frac{GMmx}{(x^2 + R^2)^{3/2}}\,dx = \frac{GMm}{R}.$$

Then the particle of mass m will pass through the center of the ring with a speed $v = \sqrt{2\Delta U/m} = \sqrt{2GM/R}$.

$\boxed{\textbf{P14-23}}$ (a) Consider the following diagram.



The distance r is given by the cosine law to be

$$r^2 = R^2 + R^2 - 2R^2 \cos\theta = 2R^2(1 - \cos\theta).$$

The force between two particles is then $F = Gm^2/r^2$. Each particle has a symmetric partner, so only the force component directed toward the center contributes. If we call this the R component we have

$$F_R = F\cos\alpha = F\cos(90^\circ - \theta/2) = F\sin(\theta/2).$$

Combining,

$$F_R = \frac{Gm^2}{2R^2}\frac{\sin(\theta/2)}{1 - \cos\theta}.$$

But *each* of the other particles contributes to this force, so

$$F_{\text{net}} = \frac{Gm^2}{2R^2}\sum_i \frac{\sin(\theta_i/2)}{1 - \cos\theta_i}$$

When there are only 9 particles the angles are in steps of 40°; the θ_i are then 40°, 80°, 120°, 160°, 200°, 240°, 280°, and 320°. With a little patience you will find

$$\sum_i \frac{\sin(\theta_i/2)}{1 - \cos\theta_i} = 6.649655,$$

using these angles. Then $F_{\text{net}} = 3.32Gm^2/R^2$.

(b) The rotational period of the ring would need to be

$$T = \sqrt{\frac{4\pi^2 R}{a}} = \sqrt{\frac{4m\pi^2 R}{F}} = \sqrt{\frac{16\pi^2 R^3}{3.32Gm}}.$$

P14-24 The potential energy of the system is $U = -Gm^2/r$. The kinetic energy is mv^2. The total energy is $E = -Gm^2/d$. Then

$$\frac{dr}{dt} = 2\sqrt{Gm(1/r - 1/d)},$$

so the time to come together is

$$T = \int_d^0 \frac{dr}{2\sqrt{Gm(1/r - 1/d)}} = \sqrt{\frac{d^3}{4Gm}} \int_0^1 \sqrt{\frac{x}{1-x}} dx = \frac{\pi}{4}\sqrt{\frac{d^3}{Gm}}.$$

P14-25 (a) $E = U/2$ for each satellite, so the total mechanical energy is $-GMm/r$.

(b) Now there is no K, so the total mechanical energy is simply $U = -2GMm/r$. The factor of 2 is because there are two satellites.

(c) The wreckage falls vertically onto the Earth.

P14-26 Let $r_\mathrm{a} = a(1+e)$ and $r_\mathrm{p} = a(1-e)$. Then $r_\mathrm{a} + r_\mathrm{e} = 2a$ and $r_\mathrm{a} - r_\mathrm{p} = 2ae$. So the answer is

$$2(0.0167)(1.50\times 10^{11}\mathrm{m}) = 5.01\times 10^9\mathrm{m},$$

or 7.20 solar radii.

P14-27

P14-28 The net force on an orbiting star is

$$F = Gm\left(\frac{M}{r^2} + m4r^2\right).$$

This is the centripetal force, and is equal to $4\pi^2 mr/T^2$. Combining,

$$\frac{4\pi^2}{T^2} = \frac{G}{4r^3}(4M+m),$$

so $T = 4\pi\sqrt{r^3/[G(4M+m)]}$.

P14-29 (a) $v = \sqrt{GM/r}$, so

$$v = \sqrt{(6.67\times 10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times 10^{24}\mathrm{kg})/(7.01\times 10^6\mathrm{m})} = 7.54\times 10^3\mathrm{m/s}.$$

(b) $T = 2\pi(7.01\times 10^6\mathrm{m})/(7.54\times 10^3\mathrm{m/s}) = 5.84\times 10^3\mathrm{s}$.

(c) Originally $E_0 = U/2$, or

$$E = -\frac{(6.67\times 10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times 10^{24}\mathrm{kg})(220\mathrm{kg})}{2(7.01\times 10^6\mathrm{m})} = -6.25\times 10^9\mathrm{J}.$$

After 1500 orbits the energy is now $-6.25\times 10^9\mathrm{J} - (1500)(1.40\times 10^5\mathrm{J}) = -6.46\times 10^9\mathrm{J}$. The new distance from the Earth is then

$$r = -\frac{(6.67\times 10^{-11}\mathrm{N\cdot m^2/kg^2})(5.98\times 10^{24}\mathrm{kg})(220\mathrm{kg})}{2(-6.46\times 10^9\mathrm{J})} = 6.79\times 10^6\mathrm{m}.$$

The altitude is now $6790 - 6370 = 420$ km.

(d) $F = (1.40\times 10^5\mathrm{J})/(2\pi 7.01\times 10^6\mathrm{m}) = 3.2\times 10^{-3}\mathrm{N}$.

(e) No.

P14-30 Let the satellite S be directly overhead at some time. The magnitude of the speed is equal to that of a geosynchronous satellite T whose orbit is not inclined, but since there are both parallel and perpendicular components to the motion of S it will appear to move north while "losing ground" compared to T. Eventually, though, it must pass overhead again in 12 hours. When S is as far north as it will go (6 hours) it has a velocity which is parallel to T, but it is located in a region where the required speed to appear fixed is slower. Hence, it will appear to be "gaining ground" against the background stars. Consequently, the motion against the background stars appears to be a figure 8.

P14-31 The net force of gravity on one star because of the other two is

$$F = \frac{2GM^2}{L^2}\cos(30^\circ).$$

The stars orbit about a point $r = L/2\cos(30^\circ)$ from any star. The orbital speed is then found from

$$\frac{Mv^2}{r} = \frac{Mv^2}{L/2\cos(30^\circ)} = \frac{2GM^2}{L^2}\cos(30^\circ),$$

or $v = \sqrt{GM/L}$.

P14-32 A parabolic path will eventually escape; this means that the speed of the comet at any distance is the escape speed for that distance, or $v = \sqrt{2GM/r}$. The angular momentum is constant, and is equal to

$$l = mv_\mathrm{A}r_\mathrm{A} = m\sqrt{2GMr_\mathrm{A}}.$$

For a parabolic path, $r = 2r_\mathrm{A}/(1+\cos\theta)$. Combining with Eq. 14-21 and the equation before that one we get

$$\frac{d\theta}{dt} = \frac{\sqrt{2GMr_\mathrm{A}}}{4{r_\mathrm{A}}^2}(1+\cos\theta)^2.$$

The time required is the integral

$$T = \sqrt{\frac{8{r_\mathrm{A}}^3}{GM}}\int_0^{\pi/2}\frac{d\theta}{(1+\cos\theta)^2} = \sqrt{\frac{8{r_\mathrm{A}}^3}{GM}}\left(\frac{2}{3}\right).$$

Note that $\sqrt{{r_\mathrm{A}}^3/GM}$ is equal to $1/2\pi$ years. Then the time for the comet to move is

$$T = \frac{1}{2\pi}\sqrt{8}\frac{2}{3}\text{ y} = 0.300\text{ y}.$$

$\boxed{\textbf{P14-33}}$ There are three forces on loose matter (of mass m_0) sitting on the moon: the force of gravity toward the moon, $F_m = Gmm_0/a^2$, the force of gravity toward the planet, $F_M = GMm_0/(r-a)^2$, and the normal force N of the moon pushing the loose matter away from the center of the moon.

The net force on this loose matter is $F_M + N - F_m$, this value is *exactly* equal to the centripetal force necessary to keep the loose matter moving in a uniform circle. The period of revolution of the loose matter is identical to that of the moon,

$$T = 2\pi\sqrt{r^3/GM},$$

but since the loose matter is actually revolving at a radial distance $r-a$ the centripetal force is

$$F_\mathrm{c} = \frac{4\pi^2 m_0(r-a)}{T^2} = \frac{GMm_0(r-a)}{r^3}.$$

Only if the normal force is zero can the loose matter can lift off, and this will happen when $F_{\mathrm{c}} = F_M - F_m$, or

$$\begin{aligned}
\frac{M(r-a)}{r^3} &= \frac{M}{(r-a)^2} - \frac{m}{a^2}, \\
&= \frac{Ma^2 - m(r-a)^2}{a^2(r-a)^2}, \\
Ma^2(r-a)^3 &= Mr^3a^2 - mr^3(r-a)^2, \\
-3r^2a^3 + 3ra^4 - a^4 &= \frac{m}{M}\left(-r^5 + 2r^4a - r^3a^2\right)
\end{aligned}$$

Let $r = ax$, then x is dimensionless; let $\beta = m/M$, then β is dimensionless. The expression then simplifies to

$$-3x^2 + 3x - 1 = \beta(-x^5 + 2x^4 - x^3).$$

If we assume than x is very large ($r \gg a$) then only the largest term on each side survives. This means $3x^2 \approx \beta x^5$, or $x = (3/\beta)^{1/3}$. In that case, $r = a(3M/m)^{1/3}$. For the Earth's moon $r_{\mathrm{c}} = 1.1\times10^7$m, which is only 4,500 km away from the surface of the Earth. It is somewhat interesting to note that the radius r is actually independent of both a and m if the moon has a uniform density!

$\boxed{\textbf{E15-1}}$ The pressure in the syringe is

$$p = \frac{(42.3\,\text{N})}{\pi(1.12\times 10^{-2}\text{m}/s)^2} = 4.29\times 10^5\text{Pa}.$$

E15-2 The total mass of fluid is

$$m = (0.5{\times}10^{-3}\text{m}^3)(2600\,\text{kg/m}^3)+(0.25{\times}10^{-3}\text{m}^3)(1000\,\text{kg/m}^3)+(0.4{\times}10^{-3}\text{m}^3)(800\,\text{kg/m}^3) = 1.87\,\text{kg}.$$

The weight is $(18.7\,\text{kg})(9.8\,\text{m/s}^2) = 18\,\text{N}$.

E15-3 $F = A\Delta p$, so

$$F = (3.43\,\text{m})(2.08\,\text{m})(1.00\text{ atm} - 0.962\text{ atm})(1.01\times 10^5\text{Pa/ atm}) = 2.74\times 10^4\text{N}.$$

E15-4 $B\Delta V/V = -\Delta p$; $V = L^3$; $\Delta V \approx L^2\Delta L/3$. Then

$$\Delta p = (140\times 10^9\text{Pa})\frac{(5\times 10^{-3}\text{m})}{3(0.85\,\text{m})} = 2.74\times 10^9\text{Pa}.$$

$\boxed{\textbf{E15-5}}$ There is an inward force F_1 pushing the lid closed from the pressure of the air outside the box; there is an outward force F_2 pushing the lid open from the pressure of the air inside the box. To lift the lid we need to exert an additional outward force F_3 to get a net force of zero.

The magnitude of the inward force is $F_1 = P_{\text{out}}A$, where A is the area of the lid and P_{out} is the pressure outside the box. The magnitude of the outward force F_2 is $F_2 = P_{\text{in}}A$. We are told $F_3 = 108$ lb. Combining,

$$\begin{aligned} F_2 &= F_1 - F_3, \\ P_{\text{in}}A &= P_{\text{out}}A - F_3, \\ P_{\text{in}} &= P_{\text{out}} - F_3/A, \end{aligned}$$

so $P_{\text{in}} = (15\text{ lb/in}^2 - (108\text{ lb})/(12\text{ in}^2) = 6.0\text{ lb/in}^2$.

E15-6 $h = \Delta p/\rho g$, so

$$h = \frac{(0.05\text{ atm})(1.01\times 10^5\text{Pa/ atm})}{(1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)} = 0.52\,\text{m}.$$

E15-7 $\Delta p = (1060\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(1.83\,\text{m}) = 1.90\times 10^4\text{Pa}$.

E15-8 $\Delta p = (1024\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(118\,\text{m}) = 1.19\times 10^6\text{Pa}$. Add this to p_0; the total pressure is then $1.29\times 10^6\text{Pa}$.

$\boxed{\textbf{E15-9}}$ The pressure differential assuming we don't have a sewage pump:

$$p_2 - p_1 = -\rho g\,(y_2 - y_1) = (926\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(8.16\,\text{m} - 2.08\,\text{m}) = 5.52\times 10^4\text{Pa}.$$

We need to overcome this pressure difference with the pump.

E15-10 (a) $p = (1.00\text{ atm})e^{-5.00/8.55} = 0.557$ atm.
(b) $h = (8.55\text{ km})\ln(1.00/0.500) = 5.93$ km.

E15-11 The mercury will rise a distance a on one side and fall a distance a on the other so that the difference in mercury height will be $2a$. Since the masses of the "excess" mercury and the water will be proportional, we have $2a\rho_\text{m} = d\rho_\text{w}$, so

$$a = \frac{(0.112\text{m})(1000\,\text{kg/m}^3)}{2(13600\,\text{kg/m}^3)} = 4.12\times 10^{-3}\text{m}.$$

E15-12 (a) The pressure (due to the water alone) at the bottom of the pool is

$$P = (62.45 \text{ lb/ft}^3)(8.0 \text{ ft}) = 500 \text{ lb/ft}^2.$$

The force on the bottom is

$$F = (500 \text{ lb/ft}^2)(80 \text{ ft})(30 \text{ ft}) = 1.2\times 10^6 \text{ lb}.$$

The average pressure on the side is half the pressure on the bottom, so

$$F = (250 \text{ lb/ft}^2)(80 \text{ ft})(8.0 \text{ ft}) = 1.6\times 10^5 \text{ lb}.$$

The average pressure on the end is half the pressure on the bottom, so

$$F = (250 \text{ lb/ft}^2)(30 \text{ ft})(8.0 \text{ ft}) = 6.0\times 10^4 \text{ lb}.$$

(b) No, since that additional pressure acts on both sides.

$\boxed{\textbf{E15-13}}$ (a) Equation 15-8 can be used to find the height y_2 of the atmosphere if the density is constant. The pressure at the top of the atmosphere would be $p_2 = 0$, and the height of the bottom y_1 would be zero. Then

$$y_2 = (1.01\times 10^5\text{Pa})/\left[(1.21\,\text{kg/m}^3)(9.81\,\text{m/s}^2)\right] = 8.51\times 10^3\text{m}.$$

(b) We have to go back to Eq. 15-7 for an atmosphere which has a density which varies linearly with altitude. Linear variation of density means

$$\rho = \rho_0\left(1 - \frac{y}{y_\text{max}}\right)$$

Substitute this into Eq. 15-7,

$$\begin{aligned}
p_2 - p_1 &= -\int_0^{y_\text{max}} \rho g\,dy, \\
&= -\int_0^{y_\text{max}} \rho_0 g\left(1 - \frac{y}{y_\text{max}}\right)\,dy, \\
&= -\rho_0 g\left(y - \frac{y^2}{2y_\text{max}}\right)\Bigg|_0^{y_\text{max}}, \\
&= -\rho g y_\text{max}/2.
\end{aligned}$$

In this case we have $y_\text{max} = 2p_1/(\rho g)$, so the answer is twice that in part (a), or 17 km.

E15-14 $\Delta P = (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(112\,\text{m}) = 1.1\times 10^6\text{Pa}$. The force required is then $F = (1.1\times 10^6\text{Pa})(1.22\,\text{m})(0.590\,\text{m}) = 7.9\times 10^5\text{N}$.

E15-15 (a) Choose *any* infinitesimally small spherical region where equal volumes of the two fluids are in contact. The denser fluid will have the larger mass. We can treat the system as being a sphere of uniform mass with a hemisphere of additional mass being superimposed in the region of higher density. The orientation of this hemisphere is the only variable when calculating the potential energy. The center of mass of this hemisphere will be a low as possible only when the surface is horizontal. So all two-fluid interfaces will be horizontal.

(b) If there exists a region where the interface is not horizontal then there will be two different values for $\Delta p = \rho g h$, depending on the path taken. This means that there will be a horizontal pressure gradient, and the fluid will flow along that gradient until the horizontal pressure gradient is equalized.

E15-16 The mass of liquid originally in the first vessel is $m_1 = \rho A h_1$; the center of gravity is at $h_1/2$, so the potential energy of the liquid in the first vessel is originally $U_1 = \rho g A h_1^2/2$. A similar expression exists for the liquid in the second vessel. Since the two vessels have the same cross sectional area the final height in both containers will be $h_\mathrm{f} = (h_1 + h_2)/2$. The final potential energy of the liquid in *each* container will be $U_\mathrm{f} = \rho g A (h_1 + h_2)^2/8$. The work done by gravity is then

$$\begin{aligned} W &= U_1 + U_2 - 2U_\mathrm{f}, \\ &= \frac{\rho g A}{4}\left[2h_1^2 + 2h_2^2 - (h_1^2 + 2h_1h_2 + h_2^2)\right], \\ &= \frac{\rho g A}{4}(h_1 - h_2)^2. \end{aligned}$$

$\boxed{\textbf{E15-17}}$ There are *three* force on the block: gravity ($W = mg$), a buoyant force $B_0 = m_\mathrm{w} g$, and a tension T_0. When the container is at rest all three forces balance, so $B_0 - W - T_0 = 0$. The tension in this case is $T_0 = (m_\mathrm{w} - m)g$.

When the container accelerates upward we now have $B - W - T = ma$. Note that neither the tension *nor* the buoyant force stay the same; the buoyant force increases according to $B = m_\mathrm{w}(g+a)$. The new tension is then

$$T = m_\mathrm{w}(g + a) - mg - ma = (m_\mathrm{w} - m)(g + a) = T_0(1 + a/g).$$

E15-18 (a) $F_1/d_1^2 = F_2/d_2^2$, so

$$F_2 = (18.6 \text{ kN})(3.72 \text{ cm})^2/(51.3 \text{ cm})^2 = 97.8\,\mathrm{N}.$$

(b) $F_2 h_2 = F_1 h_1$, so

$$h_2 = (1.65\,\mathrm{m})(18.6 \text{ kN})/(97.8\,\mathrm{N}) = 314\,\mathrm{m}.$$

E15-19 (a) 35.6 kN; the boat doesn't get heavier or lighter just because it is in different water!

(b) Yes.

$$\Delta V = \frac{(35.6\times10^3\mathrm{N})}{(9.81\,m/\mathrm{s}^2)}\left(\frac{1}{(1024\,\mathrm{kg/m^3})} - \frac{1}{(1000\,\mathrm{kg/m^3})}\right) = -8.51\times10^{-2}\mathrm{m}^3.$$

E15-20 (a) $\rho_2 = \rho_1(V_1/V_2) = (1000\,\mathrm{kg/m^3})(0.646) = 646\,\mathrm{kg/m^3}$.

(b) $\rho_2 = \rho_1(V_1/V_2) = (646\,\mathrm{kg/m^3})(0.918)^{-1} = 704\,\mathrm{kg/m^3}$.

$\boxed{\textbf{E15-21}}$ The can has a volume of 1200 cm^3, so it can displace that much water. This would provide a buoyant force of

$$B = \rho V g = (998\text{kg/m}^3)(1200\times10^{-6}\text{m}^3)(9.81\,\text{m/s}^2) = 11.7\,\text{N}.$$

This force can then support a total mass of $(11.7\,\text{N})/(9.81\,\text{m/s}^2) = 1.20\,\text{kg}$. If 130 g belong to the can, then the can will be able to carry 1.07 kg of lead.

E15-22 $\rho_2 = \rho_1(V_1/V_2) = (0.98\text{ g/cm}^3)(2/3)^{-1} = 1.47\text{ g/cm}^3$.

E15-23 Let the object have a mass m. The buoyant force of the air on the object is then

$$B_\text{o} = \frac{\rho_\text{a}}{\rho_\text{o}} mg.$$

There is also a buoyant force on the brass, equal to

$$B_\text{b} = \frac{\rho_\text{a}}{\rho_\text{b}} mg.$$

The fractional error in the weighing is then

$$\frac{B_\text{o} - B_\text{b}}{mg} = \frac{(0.0012\text{ g/cm}^3)}{(3.4\text{ g/cm}^3)} - \frac{(0.0012\text{ g/cm}^3)}{(8.0\text{ g/cm}^3)} = 2.0\times10^{-4}$$

E15-24 The volume of iron is

$$V_\text{i} = (6130\,\text{N})/(9.81\,\text{m/s}^2)(7870\,\text{kg/m}^3) = 7.94\times10^{-2}\text{m}^3.$$

The buoyant force of water is $6130\,\text{N} - 3970\,\text{N} = 2160\,\text{N}$. This corresponds to a volume of

$$V_\text{w} = (2160\,\text{N})/(9.81\,\text{m/s}^2)(1000\,\text{kg/m}^3) = 2.20\times10^{-1}\text{m}^3.$$

The volume of air is then $2.20\times10^{-1}\text{m}^3 - 7.94\times10^{-2}\text{m}^3 = 1.41\times10^{-1}\text{m}^3$.

E15-25 (a) The pressure on the top surface is $p = p_0 + \rho g L/2$. The downward force is

$$\begin{aligned} F_\text{t} &= (p_0 + \rho g L/2)L^2, \\ &= \left[(1.01\times10^5\text{Pa}) + (944\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.608\,\text{m})/2\right](0.608\,\text{m})^2 = 3.84\times10^4\text{N}. \end{aligned}$$

(b) The pressure on the bottom surface is $p = p_0 + 3\rho g L/2$. The upward force is

$$\begin{aligned} F_\text{b} &= (p_0 + 3\rho g L/2)L^2, \\ &= \left[(1.01\times10^5\text{Pa}) + 3(944\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.608\,\text{m})/2\right](0.608\,\text{m})^2 = 4.05\times10^4\text{N}. \end{aligned}$$

(c) The tension in the wire is given by $T = W + F_\text{t} - F_\text{b}$, or

$$T = (4450\,\text{N}) + (3.84\times10^4\text{N}) - (4.05\times10^4\text{N}) = 2350\,\text{N}.$$

(d) The buoyant force is

$$B = L^3 \rho g = (0.608^3)(944\,\text{kg/m}^3)(9.81\,\text{m/s}^2) = 2080\,\text{N}.$$

E15-26 The fish has the (average) density of water if

$$\rho_{\rm w} = \frac{m_{\rm f}}{V_{\rm c} + V_{\rm a}}$$

or

$$V_{\rm a} = \frac{m_{\rm f}}{\rho_{\rm w}} - V_{\rm c}.$$

We want the fraction $V_{\rm a}/(V_{\rm c} + V_{\rm a})$, so

$$\begin{aligned} \frac{V_{\rm a}}{V_{\rm c} + V_{\rm a}} &= 1 - \rho_{\rm w}\frac{V_{\rm c}}{m_{\rm f}}, \\ &= 1 - \rho_{\rm w}/\rho_{\rm c} = 1 - (1.024\ {\rm g/cm^3})/(1.08\ {\rm g/cm^3}) = 5.19\times10^{-2}. \end{aligned}$$

$\boxed{\textbf{E15-27}}$ There are *three* force on the dirigible: gravity ($W = m_{\rm g}g$), a buoyant force $B = m_{\rm a}g$, and a tension T. Since these forces must balance we have $T = B - W$. The masses are related to the densities, so we can write

$$T = (\rho_{\rm a} - \rho_{\rm g})Vg = (1.21\,{\rm kg/m^3} - 0.796\,{\rm kg/m^3})(1.17\times10^6{\rm m^3})(9.81{\rm m/s^2}) = 4.75\times10^6\,{\rm N}.$$

E15-28 $\Delta m = \Delta\rho V$, so

$$\Delta m = [(0.160\,kg/{\rm m^3}) - (0.0810\,{\rm kg/m^3})](5000\,{\rm m^3}) = 395\,{\rm kg}.$$

E15-29 The volume of one log is $\pi(1.05/2\ {\rm ft})^2(5.80\ {\rm ft}) = 5.02\ {\rm ft}^3$. The weight of the log is $(47.3\ {\rm lb/ft^3})(5.02\ {\rm ft^3}) = 237$ lb. Each log if completely submerged will displace a weight of water $(62.4\ {\rm lb/ft^3})(5.02\ {\rm ft^3}) = 313$ lb. So each log can support at most 313 lb − 237 lb = 76 lb. The three children have a total weight of 247 lb, so that will require 3.25 logs. Round up to four.

E15-30 (a) The ice will hold up the automobile if

$$\rho_{\rm w} > \frac{m_{\rm a} + m_{\rm i}}{V_{\rm i}} = \frac{m_{\rm a}}{At} + \rho_{\rm i}.$$

Then

$$A = \frac{(1120\,{\rm kg})}{(0.305\,{\rm m})[(1000\,{\rm kg/m^3}) - (917\,{\rm kg/m^3})]} = 44.2\,{\rm m^2}.$$

$\boxed{\textbf{E15-31}}$ If there were *no* water vapor pressure above the barometer then the height of the water would be $y_1 = p/(\rho g)$, where $p = p_0$ is the atmospheric pressure. If there is water vapor where there should be a vacuum, then p is the difference, and we would have $y_2 = (p_0 - p_{\rm v})/(\rho g)$. The relative error is

$$\begin{aligned} (y_1 - y_2)/y_1 &= \left[p_0/(\rho g) - (p_0 - p_{\rm v})/(\rho g)\right] / \left[p_0/(\rho g)\right], \\ &= p_{\rm v}/p_0 = (3169\,{\rm Pa})/(1.01\times10^5{\rm Pa}) = 3.14\,\%. \end{aligned}$$

E15-32 $\rho = (1.01\times10^5{\rm Pa})/(9.81\,{\rm m/s^2})(14\,{\rm m}) = 740\,{\rm kg/m^3}$.

E15-33 $h = (90)(1.01\times10^5{\rm Pa})/(8.60\,{\rm m/s^2})(1.36\times10^4{\rm kg/m^3}) = 78\,{\rm m}$.

E15-34 $\Delta U = 2(4.5\times10^{-2}{\rm N/m})4\pi(2.1\times10^{-2}{\rm m})^2 = 5.0\times10^{-4}{\rm J}$.

$\boxed{\textbf{E15-35}}$ The force required is just the surface tension times the circumference of the circular patch. Then

$$F = (0./072\,\text{N/m})2\pi(0.12\,\text{m}) = 5.43\times10^{-2}\text{N}.$$

E15-36 $\Delta U = 2(2.5\times10^{-2}\text{N/m})4\pi(1.4\times10^{-2}\text{m})^2 = 1.23\times10^{-4}\text{J}.$

P15-1 (a) One can replace the two hemispheres with an open flat end with two hemispheres with a closed flat end. Then the area of relevance is the area of the flat end, or πR^2. The net force from the pressure difference is $\Delta pA = \Delta p\pi R^2$; this much force must be applied to pull the hemispheres apart.

(b) $F = \pi(0.9)(1.01\times10^5\text{Pa})(0.305\,\text{m})^2 = 2.6\times10^4\text{N}.$

P15-2 The pressure required is 4×10^9Pa. This will happen at a depth

$$h = \frac{(4\times10^9\text{Pa})}{(9.8\,\text{m/s}^2)(3100\,\text{kg/m}^3)} = 1.3\times10^5\text{m}.$$

$\boxed{\textbf{P15-3}}$ (a) The resultant force on the wall will be

$$\begin{aligned} F &= \int\int P\,dx\,dy, \\ &= \int(-\rho gy)W\,dy, \\ &= \rho gD^2W/2. \end{aligned}$$

(b) The torque will is given by $\tau = F(D-y)$ (the distance is from the bottom) so if we generalize,

$$\begin{aligned} \tau &= \int\int Py\,dx\,dy, \\ &= \int(-\rho g(D-y))\,yW\,dy, \\ &= \rho gD^3W/6. \end{aligned}$$

(c) Dividing to find the location of the equivalent resultant force,

$$d = \tau/F = (\rho gD^3W/6)/(\rho gD^2W/2) = D/3,$$

this distance being measured from the bottom.

P15-4 $p = \rho gy = \rho g(3.6\,\text{m})$; the force on the bottom is $F = pA = \rho g(3.6\,\text{m})\pi(0.60\,\text{m})^2 = 1.296\pi\rho g$. The volume of liquid is

$$V = (1.8\,\text{m})\left[\pi(0.60\,\text{m}) + 4.6\times10^{-4}\text{m}^2\right] = 2.037\,\text{m}^3$$

The weight is $W = \rho g(2.037\,\text{m}^3)$. The ratio is 2.000.

P15-5 The pressure at b is $\rho_\text{c}(3.2\times10^4\text{m})+\rho_\text{m}y$. The pressure at a is $\rho_\text{c}(3.8\times10^4\text{m}+d)+\rho_\text{m}(y-d)$. Set these quantities equal to each other:

$$\begin{aligned} \rho_\text{c}(3.8\times10^4\text{m}+d)+\rho_\text{m}(y-d) &= \rho_\text{c}(3.2\times10^4\text{m})+\rho_\text{m}y, \\ \rho_\text{c}(6\times10^3\text{m}+d) &= \rho_\text{m}d, \\ d &= \rho_\text{c}(6\times10^3\text{m})/(\rho_\text{m}-\rho_\text{c}), \\ &= (2900\,\text{kg/m}^3)(6\times10^3\text{m})/(400\,\text{kg/m}^3) = 4.35\times10^4\text{m}. \end{aligned}$$

P15-6 (a) The pressure (difference) at a depth y is $\Delta p = \rho g y$. Since $\rho = m/V$, then

$$\Delta\rho \approx -\frac{m}{V}\frac{\Delta V}{V} = \rho_{\rm s}\frac{\Delta p}{B}.$$

Then

$$\rho \approx \rho_{\rm s} + \Delta\rho = \rho_{\rm s} + \frac{{\rho_{\rm s}}^2 g y}{B}.$$

(b) $\Delta\rho/\rho_{\rm s} = \rho_{\rm s} g y/B$, so

$$\Delta\rho/\rho \approx (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(4200\,\text{m})/(2.2\times10^9\text{Pa}) = 1.9\,\%.$$

$\boxed{\textbf{P15-7}}$ (a) Use Eq. 15-10, $p = (p_0/\rho_0)\rho$, then Eq. 15-13 will look like

$$(p_0/\rho_0)\rho = (p_0/\rho_0)\rho_0 e^{-h/a}.$$

(b) The upward velocity of the rocket as a function of time is given by $v = a_r t$. The height of the rocket above the ground is given by $y = \frac{1}{2}a_r t^2$. Combining,

$$v = a_r\sqrt{\frac{2y}{a_r}} = \sqrt{2ya_r}.$$

Put this into the expression for drag, along with the equation for density variation with altitude;

$$D = CA\rho v^2 = CA\rho_0 e^{-y/a}2ya_r.$$

Now take the derivative with respect to y,

$$dD/dy = (-1/a)CA\rho_0 e^{-y/a}(2ya_r) + CA\rho_0 e^{-y/a}(2a_r).$$

This will vanish when $y = a$, regardless of the acceleration a_r.

P15-8 (a) Consider a slice of cross section A a depth h beneath the surface. The net force on the fluid above the slice will be

$$F_{\rm net} = ma = \rho h A g,$$

Since the weight of the fluid above the slice is

$$W = mg = \rho h A g,$$

then the upward force on the bottom of the fluid at the slice must be

$$W + F_{\rm net} = \rho h A(g + a),$$

so the pressure is $p = F/A = \rho h(g + a)$.
(b) Change a to $-a$.
(c) The pressure is zero (ignores atmospheric contributions.)

P15-9 (a) Consider a portion of the liquid at the surface. The net force on this portion is $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = ma\hat{\mathbf{i}}$. The force of gravity on this portion is $\vec{\mathbf{W}} = -mg\hat{\mathbf{j}}$. There must then be a buoyant force on the portion with direction $\vec{\mathbf{B}} = \vec{\mathbf{F}} - \vec{\mathbf{W}} = m(a\hat{\mathbf{i}} + g\hat{\mathbf{j}})$. The buoyant force makes an angle $\theta = \arctan(a/g)$ with the vertical. The buoyant force must be perpendicular to the surface of the fluid; there are no pressure-related forces which are parallel to the surface. Consequently, the surface must make an angle $\theta = \arctan(a/g)$ with the *horizontal*.
(b) It will still vary as $\rho g h$; the derivation on page 334 is still valid for vertical displacements.

P15-10 $dp/dr = -\rho g$, but now $g = 4\pi G\rho r/3$. Then

$$\begin{aligned}\int_0^p dp &= -\frac{4}{3}\pi G\rho^2 \int_R^r r\,dr,\\ p &= \frac{2}{3}\pi G\rho^2 \left(R^2 - r^2\right).\end{aligned}$$

$\boxed{\textbf{P15-11}}$ We can start with Eq. 15-11, except that we'll write our distance in terms of r instead if y. Into this we can substitute our expression for g,

$$g = g_0\frac{R^2}{r^2}.$$

Substituting, then integrating,

$$\begin{aligned}\frac{dp}{p} &= -\frac{g\rho_0}{p_0}dr,\\ \frac{dp}{p} &= -\frac{g_0\rho_0 R^2}{p_0}\frac{dr}{r^2},\\ \int_{p_0}^{p}\frac{dp}{p} &= -\int_R^r \frac{g_0\rho_0 R^2}{p_0}\frac{dr}{r^2},\\ \ln\frac{p}{p_0} &= \frac{g_0\rho_0 R^2}{p_0}\left(\frac{1}{r} - \frac{1}{R}\right)\end{aligned}$$

If $k = g_0\rho_0 R^2/p_0$, then

$$p = p_0 e^{k(1/r - 1/R)}.$$

P15-12 (a) The net force on a small volume of the fluid is $dF = r\omega^2\,dm$ directed toward the center. For radial displacements, then, $dF/dr = -r\omega^2 dm/dr$ or $dp/dr = -r\omega^2\rho$.

(b) Integrating outward,

$$p = p_c + \int_0^r \rho\omega^2 r\,dr = p_c + \frac{1}{2}\rho r^2\omega^2.$$

(c) Do part (d) first.

(d) It will still vary as $\rho g h$; the derivation on page 334 is still valid for vertical displacements.

(c) The pressure anywhere in the liquid is then given by

$$p = p_0 + \frac{1}{2}\rho r^2\omega^2 - \rho g y,$$

where p_0 is the pressure on the surface, y is measured from the bottom of the paraboloid, and r is measured from the center. The surface is defined by $p = p_0$, so

$$\frac{1}{2}\rho r^2\omega^2 - \rho g y = 0,$$

or $y = r^2\omega^2/2g$.

P15-13 The total mass of the shell is $m = \rho_w \pi {d_o}^3/3$, or it wouldn't barely float. The mass of iron in the shell is $m = \rho_i\pi({d_o}^3 - {d_i}^3)/3$, so

$${d_i}^3 = \frac{\rho_i - \rho_w}{\rho_i}{d_o}^3,$$

so

$$d_i = \sqrt[3]{\frac{(7870\,\text{kg/m}^3) - (1000\,\text{kg/m}^3)}{(7870\,\text{kg/m}^3)}}(0.587\,\text{m}) = 0.561\,\text{m}.$$

P15-14 The wood will displace a volume of water equal to $(3.67\,\mathrm{kg})/(594\,\mathrm{kg/m^3})(0.883) = 5.45\times 10^{-3}\mathrm{m}^3$ in either case. That corresponds to a mass of $(1000\,\mathrm{kg/m^3})(5.45\times10^{-3}\mathrm{m}^3) = 5.45\,\mathrm{kg}$ that can be supported.

(a) The mass of lead is $5.45\,\mathrm{kg} - 3.67\,\mathrm{kg} = 1.78\,\mathrm{kg}$.

(b) When the lead is submerged beneath the water it displaces water, which affects the "apparent" mass of the lead. The true weight of the lead is mg, the buoyant force is $(\rho_\mathrm{w}/\rho_\mathrm{l})mg$, so the apparent weight is $(1-\rho_\mathrm{w}/\rho_\mathrm{l})mg$. This means the apparent mass of the submerged lead is $(1-\rho_\mathrm{w}/\rho_\mathrm{l})m$. This apparent mass is 1.78 kg, so the true mass is

$$m = \frac{(11400\,\mathrm{kg/m^3})}{(11400\,\mathrm{kg/m^3}) - (1000\,\mathrm{kg})}(1.78\,\mathrm{kg}) = 1.95\,\mathrm{kg}.$$

P15-15 We initially have

$$\frac{1}{4} = \frac{\rho_\mathrm{o}}{\rho_\mathrm{mercury}}.$$

When water is poured over the object the simple relation no longer works.

Once the water is over the object there are two buoyant forces: one from mercury, F_1, and one from the water, F_2. Following a derivation which is similar to Sample Problem 15-3, we have

$$F_1 = \rho_1 V_1 g \text{ and } F_2 = \rho_2 V_2 g$$

where ρ_1 is the density of mercury, V_1 the volume of the object which is in the mercury, ρ_2 is the density of water, and V_2 is the volume of the object which is in the water. We also have

$$F_1 + F_2 = \rho_\mathrm{o} V_\mathrm{o} g \text{ and } V_1 + V_2 = V_\mathrm{o}$$

as expressions for the net force on the object (zero) and the total volume of the object. Combining these four expressions,

$$\rho_1 V_1 + \rho_2 V_2 = \rho_\mathrm{o} V_\mathrm{o},$$

or

$$\begin{aligned} \rho_1 V_1 + \rho_2\,(V_\mathrm{o} - V_1) &= \rho_\mathrm{o} V_\mathrm{o}, \\ (\rho_1 - \rho_2)\,V_1 &= (\rho_\mathrm{o} - \rho_2)\,V_\mathrm{o}, \\ \frac{V_1}{V_\mathrm{o}} &= \frac{\rho_\mathrm{o} - \rho_2}{\rho_1 - \rho_2}. \end{aligned}$$

The left hand side is the fraction that is submerged in the mercury, so we just need to substitute our result for the density of the material from the beginning to solve the problem. The fraction submerged after adding water is then

$$\begin{aligned} \frac{V_1}{V_\mathrm{o}} &= \frac{\rho_\mathrm{o} - \rho_2}{\rho_1 - \rho_2}, \\ &= \frac{\rho_1/4 - \rho_2}{\rho_1 - \rho_2}, \\ &= \frac{(13600\,\mathrm{kg/m^3})/4 - (998\,\mathrm{kg/m^3})}{(13600\,\mathrm{kg/m^3}) - (998\,\mathrm{kg/m^3})} = 0.191. \end{aligned}$$

P15-16 (a) The car floats if it displaces a mass of water equal to the mass of the car. Then $V = (1820\,\mathrm{kg})/(1000\,\mathrm{kg/m^3}) = 1.82\,\mathrm{m}^3$.

(b) The car has a total volume of $4.87\,\mathrm{m}^3 + 0.750\,\mathrm{m}^3 + 0.810\,\mathrm{m}^3 = 6.43\,\mathrm{m}^3$. It will sink if the total mass inside the car (car + water) is then $(6.43\,\mathrm{m}^3)(1000\,\mathrm{kg/m^3}) = 6430\,\mathrm{kg}$. So the mass of the water in the car is $6430\,\mathrm{kg} - 1820\,\mathrm{kg} = 4610\,\mathrm{kg}$ when it sinks. That's a volume of $(4610\,\mathrm{kg})/(1000\,\mathrm{kg/m^3}) = 4.16\,\mathrm{m}^3$.

P15-17 When the beaker is half filled with water it has a total mass exactly equal to the maximum amount of water it can displace. The total mass of the beaker is the mass of the beaker plus the mass of the water inside the beaker. Then

$$\rho_{\mathrm{w}}(m_{\mathrm{g}}/\rho_{\mathrm{g}} + V_{\mathrm{b}}) = m_{\mathrm{g}} + \rho_{\mathrm{w}} V_{\mathrm{b}}/2,$$

where $m_{\mathrm{g}}/\rho_{\mathrm{g}}$ is the volume of the glass which makes up the beaker. Rearrange,

$$\rho_{\mathrm{g}} = \frac{m_{\mathrm{g}}}{m_{\mathrm{g}}/\rho_{\mathrm{w}} - V_{\mathrm{b}}/2} = \frac{(0.390\,\mathrm{kg})}{(0.390\,\mathrm{kg})/(1000\,\mathrm{kg/m^3}) - (5.00\times10^{-4}\,\mathrm{m^3})/2} = 2790\,\mathrm{kg/m^3}.$$

P15-18 (a) If each atom is a cube then the cube has a side of length

$$l = \sqrt[3]{(6.64\times10^{-27}\,\mathrm{kg})/(145\,\mathrm{kg/m^3})} = 3.58\times10^{-10}\mathrm{m}.$$

Then the atomic surface density is $l^{-2} = (3.58\times10^{-10}\mathrm{m})^{-2} = 7.8\times10^{18}/\mathrm{m}^2$.

(b) The bond surface density is *twice* the atomic surface density. Show this by drawing a square array of atoms and then joining each adjacent pair with a bond. You will need twice as many bonds as there are atoms. Then the energy per bond is

$$\frac{(3.5\times10^{-4}\mathrm{N/m})}{2(7.8\times10^{18}/\mathrm{m}^2)(1.6\times10^{-19}\ \mathrm{J/eV})} = 1.4\times10^{-4}\ \mathrm{eV}.$$

$\boxed{\textbf{P15-19}}$ Pretend the bubble consists of two hemispheres. The force from surface tension holding the hemispheres together is $F = 2\gamma L = 4\pi r\gamma$. The "extra" factor of two occurs because *each* hemisphere has a circumference which "touches" the boundary that is held together by the surface tension of the liquid. The pressure difference between the inside and outside is $\Delta p = F/A$, where A is the area of the flat side of one of the hemispheres, so $\Delta p = (4\pi r\gamma)/(\pi r^2) = 4\gamma/r$.

P15-20 Use the results of Problem 15-19. To get a numerical answer you need to know the surface tension; try $\gamma = 2.5\times10^{-2}\mathrm{N/m}$. The initial pressure inside the bubble is $p_{\mathrm{i}} = p_0 + 4\gamma/r_{\mathrm{i}}$. The final pressure inside the bell jar is $p = p_{\mathrm{f}} - 4\gamma/r_{\mathrm{f}}$. The initial and final pressure inside the bubble are related by $p_{\mathrm{i}} r_{\mathrm{i}}{}^3 = p_{\mathrm{f}} r_{\mathrm{f}}{}^3$. Now for numbers:

$$p_{\mathrm{i}} = (1.00\times10^5\mathrm{Pa}) + 4(2.5\times10^{-2}\mathrm{N/m})/(1.0\times10^{-3}\mathrm{m}) = 1.001\times10^5\mathrm{Pa}.$$

$$p_{\mathrm{f}} = (1.0\times10^{-3}\mathrm{m}/1.0\times10^{-2}\mathrm{m})^3(1.001\times10^5\mathrm{Pa}) = 1.001\times10^2\mathrm{Pa}.$$

$$p = (1.001\times10^2\mathrm{Pa}) - 4(2.5\times10^{-2}\mathrm{N/m})/(1.0\times10^{-2}\mathrm{m}) = 90.1\mathrm{Pa}.$$

P15-21 The force on the liquid in the space between the rod and the cylinder is $F = \gamma L = 2\pi\gamma(R + r)$. This force can support a mass of water $m = F/g$. This mass has a volume $V = m/\rho$. The cross sectional area is $\pi(R^2 - r^2)$, so the height h to which the water rises is

$$\begin{aligned} h &= \frac{2\pi\gamma(R + r)}{\rho g\pi(R^2 - r^2)} = \frac{2\gamma}{\rho g(R - r)}, \\ &= \frac{2(72.8\times10^{-3}\mathrm{N/m})}{(1000\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})(4.0\times10^{-3}\mathrm{m})} = 3.71\times10^{-3}\mathrm{m}. \end{aligned}$$

P15-22 (a) Refer to Problem 15-19. The initial pressure difference is

$$4(2.6\times10^{-2}\text{N/m})/(3.20\times10^{-2}\text{m}) = 3.25\text{Pa}.$$

(b) The final pressure difference is

$$4(2.6\times10^{-2}\text{N/m})/(5.80\times10^{-2}\text{m}) = 1.79\text{Pa}.$$

(c) The work done against the atmosphere is $p\Delta V$, or

$$(1.01\times10^{5}\text{Pa})\frac{4\pi}{3}[(5.80\times10^{-2}\text{m})^3 - (3.20\times10^{-2}\text{m})^3] = 68.7\,\text{J}.$$

(d) The work done in stretching the bubble surface is $\gamma\Delta A$, or

$$(2.60\times10^{-2}\text{N/m})4\pi[(5.80\times10^{-2}\text{m})^2 - (3.20\times10^{-2}\text{m})^2] = 7.65\times10^{-4}\text{J}.$$

E16-1 $R = Av = \pi d^2 v/4$ and $V = Rt$, so

$$t = \frac{4(1600\,\mathrm{m}^3)}{\pi(0.345\,\mathrm{m})^2(2.62\,\mathrm{m})} = 6530\,\mathrm{s}$$

E16-2 $A_1 v_1 = A_2 v_2$, $A_1 = \pi d_1^2/4$ for the hose, and $A_2 = N\pi d_2^2$ for the sprinkler, where $N = 24$. Then

$$v_2 = \frac{(0.75\text{ in})^2}{(24)(0.050\text{ in})^2}(3.5\text{ ft/s}) = 33\text{ ft/s}.$$

$\boxed{\textbf{E16-3}}$ We'll assume that each river has a rectangular cross section, despite what the picture implies. The cross section area of the two streams is then

$$A_1 = (8.2\,\mathrm{m})(3.4\,\mathrm{m}) = 28\,\mathrm{m}^2 \text{ and } A_2 = (6.8\,\mathrm{m})(3.2\,\mathrm{m}) = 22\,\mathrm{m}^2.$$

The volume flow rate in the first stream is

$$R_1 = A_1 v_1 = (28\,\mathrm{m}^2)(2.3\,\mathrm{m/s}) = 64\,\mathrm{m}^3/\mathrm{s},$$

while the volume flow rate in the second stream is

$$R_2 = A_2 v_2 = (22\,\mathrm{m}^2)(2.6\,\mathrm{m/s}) = 57\,\mathrm{m}^3/\mathrm{s}.$$

The amount of fluid in the stream/river system is conserved, so

$$R_3 = R_1 + R_2 = (64\,\mathrm{m}^3/\mathrm{s}) + (57\,\mathrm{m}^3/\mathrm{s}) = 121\,\mathrm{m}^3/\mathrm{s}.$$

where R_3 is the volume flow rate in the river. Then

$$D_3 = R_3/(v_3 W_3) = (121\,\mathrm{m}^3/\mathrm{s})/[(10.7\,\mathrm{m})(2.9\,\mathrm{m/s})] = 3.9\,\mathrm{m}.$$

E16-4 The speed of the water is originally zero so both the kinetic and potential energy is zero. When it leaves the pipe at the top it has a kinetic energy of $\frac{1}{2}(5.30\,\mathrm{m/s})^2 = 14.0\,\mathrm{J/kg}$ and a potential energy of $(9.81\,\mathrm{m/s^2})(2.90\,\mathrm{m}) = 28.4\,\mathrm{J/kg}$. The water is flowing out at a volume rate of $R = (5.30\,\mathrm{m/s})\pi(9.70\times10^{-3}\mathrm{m})^2 = 1.57\times10^{-3}\,\mathrm{m}^3/\mathrm{s}$. The mass rate is $\rho R = (1000\,\mathrm{kg/m^3})(1.57\times 10^{-3}\,\mathrm{m}^3/\mathrm{s}) = 1.57\,\mathrm{kg/s}$.

The power supplied by the pump is $(42.8\,\mathrm{J/kg})(1.57\,\mathrm{kg/s}) = 67.2\mathrm{W}$.

$\boxed{\textbf{E16-5}}$ There are 8500 km^2 which collects an average of (0.75)(0.48 m/y), where the 0.75 reflects the fact that 1/4 of the water evaporates, so

$$R = \left[8500(10^3\,\mathrm{m})^2\right](0.75)(0.48\,\mathrm{m/y})\left(\frac{1\text{ y}}{365\times24\times60\times60\,\mathrm{s}}\right) = 97\,\mathrm{m}^3/\mathrm{s}.$$

Then the speed of the water in the river is

$$v = R/A = (97\,\mathrm{m}^3/\mathrm{s})/[(21\,\mathrm{m})(4.3\,\mathrm{m})] = 1.1\,\mathrm{m/s}.$$

E16-7 (a) $\Delta p = \rho g(y_1 - y_2) + \rho(v_1^2 - v_2^2)/2$. Then

$$\Delta p = (62.4\text{ lb/ft}^3)(572\text{ ft}) + [(62.4\text{ lb/ft}^3)/(32\text{ ft/s}^2)][(1.33\text{ ft/s})^2 - (31.0\text{ ft/s})^2]/2 = 3.48\times10^4\text{lb/ft}^2.$$

(b) $A_2 v_2 = A_1 v_1$, so

$$A_2 = (7.60\text{ ft}^2)(1.33\text{ ft/s})/(31.0\text{ ft/s}) = 0.326\text{ ft}^2.$$

E16-8 (a) $A_2 v_2 = A_1 v_1$, so

$$v_2 = (2.76\,\text{m/s})[(0.255\,\text{m})^2 - (0.0480\,\text{m})^2]/(0.255\,\text{m})^2 = 2.66\,\text{m/s}.$$

(b) $\Delta p = \rho(v_1^2 - v_2^2)/2$,

$$\Delta p = (1000\,\text{kg/m}^3)[(2.66\,\text{m/s})^2 - (2.76\,\text{m/s})^2]/2 = -271\,\text{Pa}$$

$\boxed{\textbf{E16-9}}$ (b) We will do part (b) first.

$$R = (100\,\text{m}^2)(1.6\,\text{m/y})\left(\frac{1\text{ y}}{365 \times 24 \times 60 \times 60\,\text{s}}\right) = 5.1\times10^{-6}\,\text{m}^3/\text{s}.$$

(b) The speed of the flow R through a hole of cross sectional area a will be $v = R/a$. $p = p_0 + \rho g h$, where $h = 2.0\,\text{m}$ is the depth of the hole. Bernoulli's equation can be applied to find the speed of the water as it travels a horizontal stream line out the hole,

$$p_0 + \frac{1}{2}\rho v^2 = p,$$

where we drop any terms which are either zero or the same on both sides. Then

$$v = \sqrt{2(p - p_0)/\rho} = \sqrt{2gh} = \sqrt{2(9.81\,\text{m/s}^2)(2.0\,\text{m})} = 6.3\,\text{m/s}.$$

Finally, $a = (5.1\times10^{-6}\,\text{m}^3/\text{s})/(6.3\,\text{m/s}) = 8.1\times10^{-7}\text{m}^2$, or about 0.81 mm^2.

E16-10 (a) $v_2 = (A_1/A_2)v_1 = (4.20\text{ cm}^2)(5.18\,\text{m/s})/(7.60\text{ cm}^2) = 2.86\,\text{m/s}$.
(b) Use Bernoulli's equation:

$$p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2.$$

Then

$$\begin{aligned} p_2 &= (1.52\times10^5\text{Pa}) + (1000\,\text{kg/m}^3)\left[(9.81\,\text{m/s}^2)(9.66\,\text{m}) + \frac{1}{2}(5.18\,\text{m/s})^2 - \frac{1}{2}(2.86\,\text{m/s})^2\right], \\ &= 2.56\times10^5\text{Pa}. \end{aligned}$$

E16-11 (a) The wind speed is (110 km/h)(1000 m/km)/(3600 s/h) = $30.6\,\text{m/s}$. The pressure difference is then

$$\Delta p = \frac{1}{2}(1.2\,\text{kg/m}^3)(30.6\,\text{m/s})^2 = 562\,\text{Pa}.$$

(b) The lifting force would be $F = (562\,\text{Pa})(93\,\text{m}^2) = 52000\,\text{N}$.

E16-12 The pressure difference is

$$\Delta p = \frac{1}{2}(1.23\,\text{kg/m}^3)(28.0\,\text{m/s})^2 = 482\,\text{N}.$$

The net force is then $F = (482\,\text{N})(4.26\,\text{m})(5.26\,\text{m}) = 10800\,\text{N}$.

$\boxed{\textbf{E16-13}}$ The lower pipe has a radius $r_1 = 2.52$ cm, and a cross sectional area of $A_1 = \pi r_1^2$. The speed of the fluid flow at this point is v_1. The higher pipe has a radius of $r_2 = 6.14$ cm, a cross sectional area of $A_2 = \pi r_2^2$, and a fluid speed of v_2. Then

$$A_1 v_1 = A_2 v_2 \text{ or } r_1^2 v_1 = r_2^2 v_2.$$

Set $y_1 = 0$ for the lower pipe. The problem specifies that the pressures in the two pipes are the same, so

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ \frac{1}{2}v_1^2 &= \frac{1}{2}v_2^2 + g y_2, \end{aligned}$$

We can combine the results of the equation of continuity with this and get

$$\begin{aligned} v_1^2 &= v_2^2 + 2gy_2, \\ v_1^2 &= \left(v_1 r_1^2/r_2^2\right)^2 + 2gy_2, \\ v_1^2\left(1 - r_1^4/r_2^4\right) &= 2gy_2, \\ v_1^2 &= 2gy_2/\left(1 - r_1^4/r_2^4\right). \end{aligned}$$

Then

$$v_1^2 = 2(9.81\,\text{m/s}^2)(11.5\,\text{m})/\left(1 - (0.0252\,\text{m})^4/(0.0614\,\text{m})^4\right) = 232\,\text{m}^2/\text{s}^2$$

The volume flow rate in the bottom (and top) pipe is

$$R = \pi r_1^2 v_1 = \pi(0.0252\,\text{m})^2(15.2\,\text{m/s}) = 0.0303\,\text{m}^3/\text{s}.$$

E16-14 (a) As instructed,

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_3^2 + \rho g y_3, \\ 0 &= \frac{1}{2}v_3^2 + g(y_3 - y_1), \end{aligned}$$

But $y_3 - y_1 = -h$, so $v_3 = \sqrt{2gh}$.

(b) h above the hole. Just reverse your streamline!

(c) It won't come out as fast and it won't rise as high.

$\boxed{\textbf{E16-15}}$ Sea level will be defined as $y = 0$, and at that point the fluid is assumed to be at rest. Then

$$\begin{aligned} p_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_0 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ 0 &= \frac{1}{2}v_2^2 + g y_2, \end{aligned}$$

where $y_2 = -200$ m. Then

$$v_2 = \sqrt{-2gy_2} = \sqrt{-2(9.81\,\text{m/s}^2)(-200\,\text{m})} = 63\,\text{m/s}.$$

E16-16 Assume streamlined flow, then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ (p_1 - p_2)/\rho + g(y_1 - y_2) &= \frac{1}{2}v_2^2. \end{aligned}$$

Then upon rearranging

$$v_2 = \sqrt{2\left[(2.1)(1.01\times10^5\mathrm{Pa})/(1000\,\mathrm{kg/m^3}) + (9.81\,\mathrm{m/s^2})(53.0\,\mathrm{m})\right]} = 38.3\,\mathrm{m/s}.$$

E16-17 (a) Points 1 and 3 are both at atmospheric pressure, and both will move at the same speed. But since they are at different heights, Bernoulli's equation will be violated.

(b) The flow isn't steady.

E16-18 The atmospheric pressure difference between the two sides will be $\Delta p = \frac{1}{2}\rho_\mathrm{a} v^2$. The height difference in the U-tube is given by $\Delta p = \rho_\mathrm{w} g h$. Then

$$h = \frac{(1.20\,\mathrm{kg/m^3})(15.0\,\mathrm{m/s})^2}{2(1000\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})} = 1.38\times10^{-2}\mathrm{m}.$$

$\boxed{\textbf{E16-19}}$ (a) There are three forces on the plug. The force from the pressure of the water, $F_1 = P_1 A$, the force from the pressure of the air, $F_2 = P_2 A$, and the force of friction, F_3. These three forces must balance, so $F_3 = F_1 - F_2$, or $F_3 = P_1 A - P_2 A$. But $P_1 - P_2$ is the pressure difference between the surface and the water 6.15 m below the surface, so

$$\begin{aligned} F_3 &= \Delta P A = -\rho g y A, \\ &= -(998\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})(-6.15\,\mathrm{m})\pi(0.0215\,\mathrm{m})^2, \\ &= 87.4\,\mathrm{N} \end{aligned}$$

(b) To find the volume of water which flows out in three hours we need to know the volume flow rate, and for that we need both the cross section area of the hole and the speed of the flow. The speed of the flow can be found by an application of Bernoulli's equation. We'll consider the horizontal motion only— a point just inside the hole, and a point just outside the hole. These points are at the same level, so

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

Combine this with the results of Pascal's principle above, and

$$v_2 = \sqrt{2(p_1 - p_2)/\rho} = \sqrt{-2gy} = \sqrt{-2(9.81\,\mathrm{m/s^2})(-6.15\,\mathrm{m})} = 11.0\,\mathrm{m/s}.$$

The volume of water which flows out in three hours is

$$V = Rt = (11.0\,\mathrm{m/s})\pi(0.0215\,\mathrm{m})^2(3\times3600\,\mathrm{s}) = 173\,\mathrm{m^3}.$$

E16-20 Apply Eq. 16-12:

$$v_1 = \sqrt{2(9.81\,\mathrm{m/s^2})(0.262\,\mathrm{m})(810\,\mathrm{kg/m^3})/(1.03\,\mathrm{kg/m^3})} = 63.6\,\mathrm{m/s}.$$

$\boxed{\textbf{E16-21}}$ We'll assume that the central column of air down the pipe exerts minimal force on the card when it is deflected to the sides. Then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

The resultant upward force on the card is the area of the card times the pressure difference, or

$$F = (p_1 - p_2)A = \frac{1}{2}\rho A v^2.$$

E16-22 If the air blows uniformly over the surface of the plate then there can be no torque about any axis through the center of mass of the plate. Since the weight also doesn't introduce a torque, then the hinge can't exert a force on the plate, because any such force would produce an unbalanced torque. Consequently $mg = \Delta p A$. $\Delta p = \rho v^2/2$, so

$$v = \sqrt{\frac{2mg}{\rho A}} = \sqrt{\frac{2(0.488\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(1.21\,\mathrm{kg/m^3})(9.10\times10^{-2}\mathrm{m})^2}} = 30.9\,\mathrm{m/s}.$$

E16-23 Consider a streamline which passes above the wing and a streamline which passes beneath the wing. Far from the wing the two streamlines are close together, move with zero relative velocity, and are effectively at the same pressure. So we can pretend they are actually one streamline. Then, since the altitude difference between the two points above and below the wing (on this new, single streamline) is so small we can write

$$\Delta p = \frac{1}{2}\rho({v_\mathrm{t}}^2 - {v_\mathrm{u}}^2)$$

The lift force is then

$$L = \Delta p\, A = \frac{1}{2}\rho A({v_\mathrm{t}}^2 - {v_\mathrm{u}}^2)$$

E16-24 (a) From Exercise 16-23,

$$L = \frac{1}{2}(1.17\,\mathrm{kg/m^3})(2)(12.5\,\mathrm{m^2})\left[(49.8\,\mathrm{m/s})^2 - (38.2\,\mathrm{m/s})^2\right] = 1.49\times10^4\mathrm{N}.$$

The mass of the plane must be $m = L/g = (1.49\times10^4\mathrm{N})/(9.81\,\mathrm{m/s^2}) = 1520\,\mathrm{kg}$.

(b) The lift is directed straight up.

(c) The lift is directed 15° off the vertical toward the rear of the plane.

(d) The lift is directed 15° off the vertical toward the front of the plane.

$\boxed{\textbf{E16-25}}$ The larger pipe has a radius $r_1 = 12.7$ cm, and a cross sectional area of $A_1 = \pi r_1^2$. The speed of the fluid flow at this point is v_1. The smaller pipe has a radius of $r_2 = 5.65$ cm, a cross sectional area of $A_2 = \pi r_2^2$, and a fluid speed of v_2. Then

$$A_1 v_1 = A_2 v_2 \text{ or } r_1^2 v_1 = r_2^2 v_2.$$

Now Bernoulli's equation. The two pipes are at the same level, so $y_1 = y_2$. Then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

Combining this with the results from the equation of continuity,

$$
\begin{aligned}
p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\
v_1^2 &= v_2^2 + \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2 &= \left(v_1 \frac{r_1^2}{r_2^2}\right)^2 + \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2\left(1 - \frac{r_1^4}{r_2^4}\right) &= \frac{2}{\rho}\left(p_2 - p_1\right), \\
v_1^2 &= \frac{2\left(p_2 - p_1\right)}{\rho\left(1 - r_1^4/r_2^4\right)}.
\end{aligned}
$$

It may look a mess, but we can solve it to find v_1,

$$v_1 = \sqrt{\frac{2(32.6\times10^3\text{Pa} - 57.1\times10^3\text{Pa})}{(998\,\text{kg/m}^3)(1-(0.127\,\text{m})^4/(0.0565\,\text{m})^4)}} = 1.41\,\text{m/s}.$$

The volume flow rate is then

$$R = Av = \pi(0.127\,\text{m})^2(1.41\,\text{m/s}) = 7.14\times10^{-3}\text{m}^3/\text{s}.$$

That's about 71 liters/second.

E16-26 The lines are parallel and equally spaced, so the velocity is everywhere the same. We can transform to a reference frame where the liquid appears to be at rest, so the Pascal's equation would apply, and $p + \rho g y$ would be a constant. Hence,

$$p_0 = p + \rho g y + \frac{1}{2}\rho v^2$$

is the same for all streamlines.

E16-27 (a) The "particles" of fluid in a whirlpool would obey conservation of angular momentum, meaning a particle off mass m would have $l = mvr$ be constant, so the speed of the fluid as a function of radial distance from the center would be given by $k = vr$, where k is some constant representing the angular momentum per mass. Then $v = k/r$.

(b) Since $v = k/r$ and $v = 2\pi r/T$, the period would be $T \propto r^2$.

(c) Kepler's third law says $T \propto r^{3/2}$.

E16-28 $R_c = 2000$. Then

$$v < \frac{R_c\eta}{\rho D} = \frac{(2000)(4.0\times10^{-3}\text{N}\cdot\text{s/m}^2)}{(1060\,\text{kg/m}^3)(3.8\times10^{-3}\text{m})} = 2.0\,\text{m/s}.$$

$\boxed{\textbf{E16-29}}$ (a) The volume flux is given; from that we can find the average speed of the fluid in the pipe.

$$v = \frac{5.35\times10^{-2}\ \text{L/min}}{\pi(1.88\ \text{cm})^2} = 4.81\times10^{-3}\ \text{L/cm}^2\cdot\text{min}.$$

But 1 L is the same as 1000 cm^3 and 1 min is equal to 60 seconds, so $v = 8.03\times10^{-4}$ m/s.

Reynold's number from Eq. 16-22 is then

$$R = \frac{\rho D v}{\eta} = \frac{(13600\,\mathrm{kg/m^3})(0.0376\,\mathrm{m})(8.03\times10^{-4}\,\mathrm{m/s})}{(1.55\times10^{-3}\mathrm{N\cdot s/m^2})} = 265.$$

This is well below the critical value of 2000.

(b) Poiseuille's Law, Eq. 16-20, can be used to find the pressure difference between the ends of the pipe. But first, note that the mass flux dm/dt is equal to the volume rate times the density when the density is constant. Then $\rho\, dV/dt = dm/dt$, and Poiseuille's Law can be written as

$$\delta p = \frac{8\eta L}{\pi R^4}\frac{dV}{dt} = \frac{8(1.55\times10^{-3}\mathrm{N\cdot s/m^2})(1.26\,\mathrm{m})}{\pi(1.88\times10^{-2}\ \mathrm{m})^4}(8.92\times10^{-7}\mathrm{m^3/s}) = 0.0355\,\mathrm{Pa}.$$

P16-1 The volume of water which needs to flow out of the bay is

$$V = (6100\,\mathrm{m})(5200\,\mathrm{m})(3\,\mathrm{m}) = 9.5\times10^7\mathrm{m^3}$$

during a 6.25 hour (22500 s) period. The average speed through the channel must be

$$v = \frac{(9.5\times10^7\mathrm{m^3})}{(22500\,\mathrm{s})(190\,\mathrm{m})(6.5\,\mathrm{m})} = 3.4\,\mathrm{m/s}.$$

P16-2 (a) The speed of the fluid through either hole is $v = \sqrt{2gh}$. The mass flux through a hole is $Q = \rho A v$, so $\rho_1 A_1 = \rho_2 A_2$. Then $\rho_1/\rho_2 = A_2/A_1 = 2$.

(b) $R = Av$, so $R_1/R_2 = A_1/A_2 = 1/2$.

(c) If $R_1 = R_2$ then $A_1\sqrt{2gh_1} = A_2\sqrt{2gh_2}$. Then

$$h_2/h_1 = (A_1/A_2)^2 = (1/2)^2 = 1/4.$$

So $h_2 = h_1/4$.

$\boxed{\textbf{P16-3}}$ (a) Apply Torricelli's law (Exercise 16-14): $v = \sqrt{2gh}$. The speed v is a horizontal velocity, and serves as the initial horizontal velocity of the fluid "projectile" after it leaves the tank. There is no initial vertical velocity.

This fluid "projectile" falls through a vertical distance $H-h$ before splashing against the ground. The equation governing the time t for it to fall is

$$-(H-h) = -\frac{1}{2}gt^2,$$

Solve this for the time, and $t = \sqrt{2(H-h)/g}$. The equation which governs the horizontal distance traveled during the fall is $x = v_x t$, but $v_x = v$ and we just found t, so

$$x = v_x t = \sqrt{2gh}\,\sqrt{2(H-h)/g} = 2\sqrt{h(H-h)}.$$

(b) How many values of h will lead to a distance of x? We need to invert the expression, and we'll start by squaring both sides

$$x^2 = 4h(H-h) = 4hH - 4h^2,$$

and then solving the resulting quadratic expression for h,

$$h = \frac{4H \pm \sqrt{16H^2 - 16x^2}}{8} = \frac{1}{2}\left(H \pm \sqrt{H^2 - x^2}\right).$$

For values of x between 0 and H there are two real solutions, if $x = H$ there is one real solution, and if $x > H$ there are no real solutions.

If h_1 is a solution, then we can write $h_1 = (H + \Delta)/2$, where $\Delta = 2h_1 - H$ could be positive or negative. Then $h_2 = (H + \Delta)/2$ is also a solution, and

$$h_2 = (H + 2h_1 - 2H)/2 = h_1 - H$$

is also a solution.

(c) The farthest distance is $x = H$, and this happens when $h = H/2$, as we can see from the previous section.

P16-4 (a) Apply Torricelli's law (Exercise 16-14): $v = \sqrt{2g(d + h_2)}$, assuming that the liquid remains in contact with the walls of the tube until it exits at the bottom.

(b) The speed of the fluid in the tube is everywhere the same. Then the pressure difference at various points are only functions of height. The fluid exits at C, and assuming that it remains in contact with the walls of the tube the pressure difference is given by $\Delta p = \rho(h_1 + d + h_2)$, so the pressure at B is

$$p = p_0 - \rho(h_1 + d + h_2).$$

(c) The lowest possible pressure at B is zero. Assume the flow rate is *so* slow that Pascal's principle applies. Then the maximum height is given by $0 = p_0 + \rho g h_1$, or

$$h_1 = (1.01 \times 10^5 \mathrm{Pa})/[(9.81\,\mathrm{m/s^2})(1000\,\mathrm{kg/m^3})] = 10.3\,\mathrm{m}.$$

P16-5 (a) The momentum per kilogram of the fluid in the smaller pipe is v_1. The momentum per kilogram of the fluid in the larger pipe is v_2. The change in momentum per kilogram is $v_2 - v_1$. There are $\rho a_2 v_2$ kilograms per second flowing past any point, so the change in momentum per second is $\rho a_2 v_2(v_2 - v_1)$. The change in momentum per second is related to the net force according to $F = \Delta p/\Delta t$, so $F = \rho a_2 v_2(v_2 - v_1)$. But $F \approx \Delta p/a_2$, so $p_1 - p_2 \approx \rho v_2(v_2 - v_1)$.

(b) Applying the streamline equation,

$$\begin{aligned} p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2, \\ \frac{1}{2}\rho(v_1^2 - v_2^2) &= p_2 - p_1 \end{aligned}$$

(c) This question asks for the loss of pressure *beyond* that which would occur from a gradually widened pipe. Then we want

$$\begin{aligned} \Delta p &= \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho v_2(v_1 - v_2), \\ &= \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho v_2 v_1 + \rho v_2^2, \\ &= \frac{1}{2}\rho(v_1^2 - 2v_1 v_2 + v_2^2) = \frac{1}{2}\rho(v_1 - v_2)^2. \end{aligned}$$

P16-6 The juice leaves the jug with a speed $v = \sqrt{2gy}$, where y is the height of the juice in the jug. If A is the cross sectional area of the base of the jug and a the cross sectional area of the hole, then the juice flows out the hole with a rate $dV/dt = va = a\sqrt{2gy}$, which means the level of jug varies as $dy/dt = -(a/A)\sqrt{2gy}$. Rearrange and integrate,

$$\int_y^h dy/\sqrt{y} = \int_0^t \sqrt{2g}(a/A)dt,$$

$$
\begin{aligned}
2(\sqrt{h}-\sqrt{y}) &= \sqrt{2g}at/A. \\
\left(\frac{A}{a}\sqrt{\frac{2h}{g}}\right)(\sqrt{h}-\sqrt{y}) &= t
\end{aligned}
$$

When $y = 14h/15$ we have $t = 12.0\,\mathrm{s}$. Then the part in the parenthesis on the left is $3.539\times10^2\mathrm{s}$. The time to empty completely is then 354 seconds, or 5 minutes and 54 seconds. But we want the remaining time, which is 12 seconds less than this.

$\boxed{\textbf{P16-7}}$ The greatest possible value for v will be the value over the wing which results in an air pressure of zero. If the air at the leading edge is stagnant (not moving) and has a pressure of p_0, then Bernoulli's equation gives

$$
p_0 = \frac{1}{2}\rho v^2,
$$

or $v = \sqrt{2p_0/\rho} = \sqrt{2(1.01\times10^5\mathrm{Pa})/(1.2\,\mathrm{kg/m^3})} = 410\,\mathrm{m/s}$. This value is only slightly larger than the speed of sound; they are related because sound waves involve the movement of air particles which "shove" other air particles out of the way.

P16-8 Bernoulli's equation together with continuity gives

$$
\begin{aligned}
p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\
p_1 - p_2 &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right), \\
&= \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2}v_1^2 - v_1^2\right), \\
&= \frac{v_1^2}{2A_2^2}\rho\left(A_1^2 - A_2^2\right).
\end{aligned}
$$

But $p_1 - p_2 = (\rho' - \rho)gh$. Note that we are *not* assuming ρ is negligible compared to ρ'. Combining,

$$
v_1 = A_2\sqrt{\frac{2(\rho'-\rho)gh}{\rho(A_1^2 - A_2^2)}}.
$$

P16-9 (a) Bernoulli's equation together with continuity gives

$$
\begin{aligned}
p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\
p_1 &= \frac{1}{2}\rho\left(v_2^2 - v_1^2\right), \\
&= \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2}v_1^2 - v_1^2\right), \\
&= v_1^2\rho\left[(4.75)^2 - 1\right]/2.
\end{aligned}
$$

Then

$$
v_1 = \sqrt{\frac{2(2.12)(1.01\times10^5\mathrm{Pa})}{(1000\,\mathrm{kg/m^3})(21.6)}} = 4.45\,\mathrm{m/s},
$$

and then $v_2 = (4.75)(4.45\,\mathrm{m/s}) = 21.2\,\mathrm{m/s}$.

(b) $R = \pi(2.60\times10^{-2}\mathrm{m})^2(4.45\,\mathrm{m/s}) = 9.45\times10^{-3}\,\mathrm{m^3/s}$.

P16-10 (a) For Fig. 16-13 the velocity is constant, or $\vec{\mathbf{v}} = v\hat{\mathbf{i}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{i}}dx + \hat{\mathbf{j}}dy$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dx = 0,$$

because $\oint dx = 0$.

(b) For Fig. 16-16 the velocity is $\vec{\mathbf{v}} = (k/r)\hat{\mathbf{r}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{r}}dr + \hat{\theta}r\, d\phi$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dr = 0,$$

because $\oint dr = 0$.

P16-11 (a) For an element of the fluid of mass dm the net force as it moves around the circle is $dF = (v^2/r)dm$. $dm/dV = \rho$ and $dV = A\, dr$ and $dF/A = dp$. Then $dp/dr = \rho v^2/r$.

(b) From Bernoulli's equation $p + \rho v^2/2$ is a constant. Then

$$\frac{dp}{dr} + \rho v \frac{dv}{dr} = 0,$$

or $v/r + dv/dr = 0$, or $d(vr) = 0$. Consequently vr is a constant.

(c) The velocity is $\vec{\mathbf{v}} = (k/r)\hat{\mathbf{r}}$. $d\vec{\mathbf{s}} = \hat{\mathbf{r}}dr + \hat{\theta}r\, d\phi$. Then

$$\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = v \oint dr = 0,$$

because $\oint dr = 0$. This means the flow is irrotational.

P16-12 $F/A = \eta v/D$, so

$$F/A = (4.0\times10^{19}\mathrm{N\cdot s/m^2})(0.048\,\mathrm{m}/3.16\times10^7\mathrm{s})/(1.9\times10^5\mathrm{m}) = 3.2\times10^5\mathrm{Pa}.$$

$\boxed{\textbf{P16-13}}$ A flow will be irrotational if and only if $\oint \vec{\mathbf{v}} \cdot d\vec{\mathbf{s}} = 0$ for all possible paths. It is fairly easy to construct a rectangular path which is parallel to the flow on the top and bottom sides, but perpendicular on the left and right sides. Then only the top and bottom paths contribute to the integral. $\vec{\mathbf{v}}$ is constant for either path (but not the same), so the magnitude v will come out of the integral sign. Since the lengths of the two paths are the same but v is different the two terms *don't* cancel, so the flow is not irrotational.

P16-14 (a) The area of a cylinder is $A = 2\pi rL$. The velocity gradient if dv/dr. Then the retarding force on the cylinder is $F = -\eta(2\pi rL)dv/dr$.

(b) The force pushing a cylinder through is $F' = A\Delta p = \pi r^2\Delta p$.

(c) Equate, rearrange, and integrate:

$$\begin{aligned}
\pi r^2 \Delta p &= -\eta(2\pi rL)\frac{dv}{dr}, \\
\Delta p \int_r^R r\, dr &= 2\eta L \int_0^v dv, \\
\Delta p \frac{1}{2}(R^2 - r^2) &= 2\eta L v.
\end{aligned}$$

Then

$$v = \frac{\Delta p}{4\eta L}(R^2 - r^2).$$

$\boxed{\textbf{P16-15}}$ The volume flux (called R_f to distinguish it from the radius R) through an annular ring of radius r and width δr is

$$\delta R_f = \delta A\, v = 2\pi r\, \delta r\, v,$$

where v is a function of r given by Eq. 16-18. The mass flux is the volume flux times the density, so the total mass flux is

$$\begin{aligned}
\frac{dm}{dt} &= \rho \int_0^R \frac{\delta R_f}{\delta r} dr, \\
&= \rho \int_0^R 2\pi r \left(\frac{\Delta p}{4\eta L}(R^2 - r^2) \right) dr, \\
&= \frac{\pi \rho \Delta p}{2\eta L} \int_0^R (rR^2 - r^3) dr, \\
&= \frac{\pi \rho \Delta p}{2\eta L}(R^4/2 - R^4/4), \\
&= \frac{\pi \rho \Delta p R^4}{8\eta L}.
\end{aligned}$$

P16-16 The pressure difference in the tube is $\Delta p = 4\gamma/r$, where r is the (changing) radius of the bubble. The mass flux through the tube is

$$\frac{dm}{dt} = \frac{4\rho\pi R^4\gamma}{8\eta L r},$$

R is the radius of the tube. $dm = \rho dV$, and $dV = 4\pi r^2 dr$. Then

$$\begin{aligned}
\int_{r_1}^{r_2} r^3\, dr &= \int_0^t \frac{R^4\gamma}{8\eta L} dt, \\
r_1^4 - r_2^4 &= \frac{\rho R^4 \gamma}{2\eta L} t,
\end{aligned}$$

Then

$$t = \frac{2(1.80\times10^{-5}\mathrm{N}\cdot\mathrm{s/m^2})(0.112\,\mathrm{m})}{(0.54\times10^{-3}\mathrm{m})^4(2.50\times10^{-2}\mathrm{N/m})}[(38.2\times10^{-3}\mathrm{m})^4 - (21.6\times10^{-3}\mathrm{m})^4] = 3630\,\mathrm{s}.$$

$\boxed{\textbf{E17-1}}$ For a perfect spring $|F| = k|x|$. $x = 0.157\,\text{m}$ when $3.94\,\text{kg}$ is suspended from it. There would be two forces on the object— the force of gravity, $W = mg$, and the force of the spring, F. These two force must balance, so $mg = kx$ or

$$k = \frac{mg}{x} = \frac{(3.94\,\text{kg})(9.81\,\text{m/s}^2)}{(0.157\,\text{m})} = 0.246\,\text{N/m}.$$

Now that we know k, the spring constant, we can find the period of oscillations from Eq. 17-8,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(0.520\,\text{kg})}{(0.246\,\text{N/m})}} = 0.289\,\text{s}.$$

E17-2 (a) $T = 0.484\,\text{s}$.
(b) $f = 1/T = 1/(0.484\,\text{s}) = 2.07\,\text{s}^{-1}$.
(c) $\omega = 2\pi f = 13.0$ rad/s.
(d) $k = m\omega^2 = (0.512\,\text{kg})(13.0\text{ rad/s})^2 = 86.5\,\text{N/m}$.
(e) $v_\text{m} = \omega x_\text{m} = (13.0\text{ rad/s})(0.347\,\text{m}) = 4.51\,\text{m/s}$.
(f) $F_\text{m} = ma_\text{m} = (0.512\,\text{kg})(13.0\text{ rad/s})^2(0.347\,\text{m}) = 30.0\,\text{N}$.

E17-3 $a_\text{m} = (2\pi f)^2 x_\text{m}$. Then

$$f = \sqrt{(9.81\,\text{m/s}^2)/(1.20\times 10^{-6}\text{m})}/(2\pi) = 455\text{ Hz}.$$

E17-4 (a) $\omega = (2\pi)/(0.645\,\text{s}) = 9.74$ rad/s. $k = m\omega^2 = (5.22\,\text{kg})(9.74\text{ rad/s})^2 = 495\,\text{N/m}$.
(b) $x_\text{m} = v_\text{m}/\omega = (0.153\,\text{m/s})/(9.74\text{ rad/s}) = 1.57\times 10^{-2}\text{m}$.
(c) $f = 1/(0.645\,\text{s}) = 1.55$ Hz.

$\boxed{\textbf{E17-5}}$ (a) The amplitude is half of the distance between the extremes of the motion, so $A = (2.00\text{ mm})/2 = 1.00$ mm.
(b) The maximum blade speed is given by $v_\text{m} = \omega x_\text{m}$. The blade oscillates with a frequency of 120 Hz, so $\omega = 2\pi f = 2\pi(120\,\text{s}^{-1}) = 754$ rad/s, and then $v_\text{m} = (754\text{ rad/s})(0.001\,\text{m}) = 0.754\,\text{m/s}$.
(c) Similarly, $a_\text{m} = \omega^2 x_\text{m}$, $a_\text{m} = (754\text{ rad/s})^2(0.001\,\text{m}) = 568\,\text{m/s}^2$.

E17-6 (a) $k = m\omega^2 = (1460\,\text{kg}/4)(2\pi 2.95/\text{s})^2 = 1.25\times 10^5\text{N/m}$
(b) $f = \sqrt{k/m}/2\pi = \sqrt{(1.25\times 10^5\text{N/m})/(1830\,\text{kg}/4)}/2\pi = 2.63/\text{s}$.

E17-7 (a) $x = (6.12\,\text{m})\cos[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = 3.27\,\text{m}$.
(b) $v = -(6.12\,\text{m})(8.38/\text{s})\sin[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = 43.4\,\text{m/s}$.
(c) $a = -(6.12\,\text{m})(8.38/\text{s})^2\cos[(8.38\text{ rad/s})(1.90\,\text{s}) + 1.92\text{ rad}] = -229\,\text{m/s}^2$.
(d) $f = (8.38\text{ rad/s})/2\pi = 1.33/\text{s}$.
(e) $T = 1/f = 0.750\,\text{s}$.

E17-8 $k = (50.0\text{ lb})/(4.00\text{ in}) = 12.5$ lb/in.

$$mg = \frac{(32\text{ ft/s}^2)(12\text{ in/ft})(12.5\text{ lb/in})}{[2\pi(2.00/\text{s})]^2} = 30.4\text{ lb}.$$

$\boxed{\textbf{E17-9}}$ If the drive wheel rotates at 193 rev/min then

$$\omega = (193\text{ rev/min})(2\pi\text{ rad/rev})(1/60\text{ s/min}) = 20.2\text{ rad/s},$$

then $v_\text{m} = \omega x_\text{m} = (20.2\text{ rad/s})(0.3825\,\text{m}) = 7.73\,\text{m/s}$.

E17-10 $k = (0.325\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/(1.80\times10^{-2}\,\mathrm{m}) = 177\,\mathrm{N/m}$..

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(2.14\,\mathrm{kg})}{(177\,\mathrm{N/m})}} = 0.691\,\mathrm{s}.$$

E17-11 For the tides $\omega = 2\pi/(12.5\ \mathrm{h})$. Half the maximum occurs when $\cos\omega t = 1/2$, or $\omega t = \pi/3$. Then $t = (12.5\ \mathrm{h})/6 = 2.08\ \mathrm{h}$.

E17-12 The two will separate if the (maximum) acceleration exceeds g.

(a) Since $\omega = 2\pi/T = 2\pi/(1.18\,\mathrm{s}) = 5.32$ rad/s the maximum amplitude is

$$x_\mathrm{m} = (9.81\,\mathrm{m/s^2})/(5.32\ \mathrm{rad/s})^2 = 0.347\,\mathrm{m}.$$

(b) In this case $\omega = \sqrt{(9.81\,\mathrm{m/s^2})/(0.0512\,\mathrm{m})} = 13.8$ rad/s. Then $f = (13.8\ \mathrm{rad/s})/2\pi = 2.20/\mathrm{s}$.

E17-13 (a) $a_x/x = -\omega^2$. Then

$$\omega = \sqrt{-(-123\,\mathrm{m/s})/(0.112\,m)} = 33.1\ \mathrm{rad/s},$$

so $f = (33.1\ \mathrm{rad/s})/2\pi = 5.27/\mathrm{s}$.

(b) $m = k/\omega^2 = (456\,\mathrm{N/m})/(33.1\ \mathrm{rad/s})^2 = 0.416\,\mathrm{kg}$.

(c) $x = x_\mathrm{m}\cos\omega t$; $v = -x_\mathrm{m}\omega\sin\omega t$. Combining,

$$x^2 + (v/\omega)^2 = {x_\mathrm{m}}^2\cos^2\omega t + {x_\mathrm{m}}^2\sin^2\omega t = {x_\mathrm{m}}^2.$$

Consequently,

$$x_\mathrm{m} = \sqrt{(0.112\,\mathrm{m})^2 + (-13.6\,\mathrm{m/s})^2/(33.1\ \mathrm{rad/s})^2} = 0.426\,\mathrm{m}.$$

E17-14 $x_1 = x_\mathrm{m}\cos\omega t$, $x_2 = x_\mathrm{m}\cos(\omega t + \phi)$. The crossing happens when $x_1 = x_\mathrm{m}/2$, or when $\omega t = \pi/3$ (and other values!). The same constraint happens for x_2, except that it is moving in the other direction. The closest value is $\omega t + \phi = 2\pi/3$, or $\phi = \pi/3$.

$\boxed{\textbf{E17-15}}$ (a) The net force on the three cars is zero before the cable breaks. There are three forces on the cars: the weight, W, a normal force, N, and the upward force from the cable, F. Then

$$F = W\sin\theta = 3mg\sin\theta.$$

This force is from the elastic properties of the cable, so

$$k = \frac{F}{x} = \frac{3mg\sin\theta}{x}$$

The frequency of oscillation of the remaining two cars after the bottom car is released is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} = \frac{1}{2\pi}\sqrt{\frac{3mg\sin\theta}{2mx}} = \frac{1}{2\pi}\sqrt{\frac{3g\sin\theta}{2x}}.$$

Numerically, the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{3g\sin\theta}{2x}} = \frac{1}{2\pi}\sqrt{\frac{3(9.81\,\mathrm{m/s^2})\sin(26^\circ)}{2(0.142\,\mathrm{m})}} = 1.07\ \mathrm{Hz}.$$

(b) Each car contributes equally to the stretching of the cable, so one car causes the cable to stretch $14.2/3 = 4.73$ cm. The amplitude is then 4.73 cm.

E17-16 Let the height of one side over the equilibrium position be x. The net restoring force on the liquid is $2\rho Axg$, where A is the cross sectional area of the tube and g is the acceleration of free-fall. This corresponds to a spring constant of $k = 2\rho Ag$. The mass of the fluid is $m = \rho AL$. The period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{2L}{g}}.$$

$\boxed{\textbf{E17-17}}$ (a) There are two forces on the log. The weight, $W = mg$, and the buoyant force B. We'll assume the log is cylindrical. If x is the length of the log beneath the surface and A the cross sectional area of the log, then $V = Ax$ is the volume of the displaced water. Furthermore, $m_{\rm w} = \rho_{\rm w}V$ is the mass of the displaced water and $B = m_{\rm w}g$ is then the buoyant force on the log. Combining,

$$B = \rho_{\rm w}Agx,$$

where $\rho_{\rm w}$ is the density of water. This certainly looks similar to an elastic spring force law, with $k = \rho_{\rm w}Ag$. We would then expect the motion to be simple harmonic.

(b) The period of the oscillation would be

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\rho_{\rm w}Ag}},$$

where m is the total mass of the log and lead. We are told the log is in equilibrium when $x = L = 2.56$ m. This would give us the *weight* of the log, since $W = B$ is the condition for the log to float. Then

$$m = \frac{B}{g} = \frac{\rho_{\rm w}AgL}{g} = \rho AL.$$

From this we can write the period of the motion as

$$T = 2\pi\sqrt{\frac{\rho AL}{\rho_{\rm w}Ag}} = 2\pi\sqrt{L/g} = 2\pi\sqrt{\frac{(2.56\,\text{m})}{(9.81\,\text{m/s}^2)}} = 3.21\,\text{s}.$$

E17-18 (a) $k = 2(1.18\,\text{J})/(0.0984\,\text{m})^2 = 244\,\text{N/m}$.

(b) $m = 2(1.18\,\text{J})/(1.22\,\text{m/s})^2 = 1.59\,\text{kg}$.

(c) $f = [(1.22\,\text{m/s})/(0.0984\,\text{m})]/(2\pi) = 1.97/\text{s}$.

E17-19 (a) Equate the kinetic energy of the object just after it leaves the slingshot with the potential energy of the stretched slingshot.

$$k = \frac{mv^2}{x^2} = \frac{(0.130\,\text{kg})(11.2\times10^3\text{m/s})^2}{(1.53\,\text{m})^2} = 6.97\times10^6\text{N/m}.$$

(b) $N = (6.97\times10^6\,\text{N/m})(1.53\,\text{m})/(220\,\text{N}) = 4.85\times10^4$ people.

E17-20 (a) $E = k{x_{\rm m}}^2/2$, $U = kx^2/2 = k(x_{\rm m}/2)^2/2 = E/4$. $K = E - U = 3E/4$. The the energy is 25% potential and 75% kinetic.

(b) If $U = E/2$ then $kx^2/2 = k{x_{\rm m}}^2/4$, or $x = x_{\rm m}/\sqrt{2}$.

$\boxed{\textbf{E17-21}}$ (a) $a_\mathrm{m} = \omega^2 x_\mathrm{m}$ so

$$\omega = \sqrt{\frac{a_\mathrm{m}}{x_\mathrm{m}}} = \sqrt{\frac{(7.93\times10^3\,\mathrm{m/s^2})}{(1.86\times10^{-3}\,\mathrm{m})}} = 2.06\times10^3\ \mathrm{rad/s}$$

The period of the motion is then

$$T = \frac{2\pi}{\omega} = 3.05\times10^{-3}\mathrm{s}.$$

(b) The maximum speed of the particle is found by

$$v_\mathrm{m} = \omega x_\mathrm{m} = (2.06\times10^3\ \mathrm{rad/s})(1.86\times10^{-3}\,\mathrm{m}) = 3.83\,\mathrm{m/s}.$$

(c) The mechanical energy is given by Eq. 17-15, except that we will focus on when $v_x = v_\mathrm{m}$, because then $x = 0$ and

$$E = \frac{1}{2}m{v_\mathrm{m}}^2 = \frac{1}{2}(12.3\,\mathrm{kg})(3.83\,\mathrm{m/s})^2 = 90.2\,\mathrm{J}.$$

E17-22 (a) $f = \sqrt{k/m}/2\pi = \sqrt{(988\,\mathrm{N/m})/(5.13\,\mathrm{kg})}/2\pi = 2.21/\mathrm{s}.$
(b) $U_\mathrm{i} = (988\,\mathrm{N/m})(0.535\,\mathrm{m})^2/2 = 141\,\mathrm{J}.$
(c) $K_\mathrm{i} = (5.13\,\mathrm{kg})(11.2\,\mathrm{m/s})^2/2 = 322\,\mathrm{J}.$
(d) $x_\mathrm{m} = \sqrt{2E/k} = \sqrt{2(322\,\mathrm{J} + 141\,\mathrm{J})/(988\,\mathrm{N/m})} = 0.968\,\mathrm{m}.$

E17-23 (a) $\omega = \sqrt{(538\,\mathrm{N/m})/(1.26\,\mathrm{kg})} = 20.7\ \mathrm{rad/s}.$

$$x_\mathrm{m} = \sqrt{(0.263\,\mathrm{m})^2 + (3.72\,\mathrm{m/s})^2/(20.7\ \mathrm{rad/s})^2} = 0.319\,\mathrm{m}.$$

(b) $\phi = \arctan\{-(-3.72\,\mathrm{m/s})/[(20.7\ \mathrm{rad/s})(0.263\,\mathrm{m})]\} = 34.3°.$

E17-24 Before doing anything else apply conservation of momentum. If v_0 is the speed of the bullet just before hitting the block and v_1 is the speed of the bullet/block system just after the two begin moving as one, then $v_1 = mv_0/(m+M)$, where m is the mass of the bullet and M is the mass of the block.

For this system $\omega = \sqrt{k/(m+M)}$.

(a) The total energy of the oscillation is $\frac{1}{2}(m+M)v_1^2$, so the amplitude is

$$x_\mathrm{m} = \sqrt{\frac{m+M}{k}}v_1 = \sqrt{\frac{m+M}{k}}\frac{mv_0}{m+M} = mv_0\sqrt{\frac{1}{k(m+M)}}.$$

The numerical value is

$$x_\mathrm{m} = (0.050\,\mathrm{kg})(150\,\mathrm{m/s})\sqrt{\frac{1}{(500\,\mathrm{N/m})(0.050\,\mathrm{kg} + 4.00\,\mathrm{kg})}} = 0.167\,\mathrm{m}.$$

(b) The fraction of the energy is

$$\frac{(m+M)v_1^2}{mv_0^2} = \frac{m+M}{m}\left(\frac{m}{m+M}\right)^2 = \frac{m}{m+M} = \frac{(0.050\,\mathrm{kg})}{(0.050\,\mathrm{kg} + 4.00\,\mathrm{kg})} = 1.23\times10^{-2}.$$

E17-25 $L = (9.82\,\mathrm{m/s^2})(1.00\,\mathrm{s}/2\pi)^2 = 0.249\,\mathrm{m}.$

E17-26 $T = (180\,\mathrm{s})/(72.0)$. Then

$$g = \left(\frac{2\pi(72.0)}{(180\,\mathrm{s})}\right)^2 (1.53\,\mathrm{m}) = 9.66\,\mathrm{m/s}.$$

$\boxed{\textbf{E17-27}}$ We are interested in the value of θ_m which will make the second term 2% of the first term. We want to solve

$$0.02 = \frac{1}{2^2}\sin^2\frac{\theta_\mathrm{m}}{2},$$

which has solution

$$\sin\frac{\theta_\mathrm{m}}{2} = \sqrt{0.08}$$

or $\theta_m = 33°$.

(b) How large is the third term at this angle?

$$\frac{3^2}{2^2 4^2}\sin^4\frac{\theta_m}{2} = \frac{3^2}{2^2}\left(\frac{1}{2^2}\sin^2\frac{\theta_m}{2}\right)^2 = \frac{9}{4}(0.02)^2$$

or 0.0009, which is very small.

E17-28 Since $T \propto \sqrt{1/g}$ we have

$$T_\mathrm{p} = T_\mathrm{e}\sqrt{g_\mathrm{e}/g_\mathrm{p}} = (1.00\,\mathrm{s})\sqrt{\frac{(9.78\,\mathrm{m/s^2})}{(9.834\,\mathrm{m/s^2})}} = 0.997\,\mathrm{s}.$$

$\boxed{\textbf{E17-29}}$ Let the period of the clock in Paris be T_1. In a day of length $D_1 = 24$ hours it will undergo $n = D/T_1$ oscillations. In Cayenne the period is T_2. n oscillations should occur in 24 hours, but since the clock runs slow, D_2 is 24 hours + 2.5 minutes elapse. So

$$T_2 = D_2/n = (D_2/D_1)T_1 = [(1442.5\text{ min})/(1440.0\text{ min})]T_1 = 1.0017T_1.$$

Since the ratio of the periods is $(T_2/T_1) = \sqrt{(g_1/g_2)}$, the g_2 in Cayenne is

$$g_2 = g_1(T_1/T_2)^2 = (9.81\,\mathrm{m/s^2})/(1.0017)^2 = 9.78\,\mathrm{m/s^2}.$$

E17-30 (a) Take the differential of

$$g = \left(\frac{2\pi(100)}{T}\right)^2 (10\,\mathrm{m}) = \frac{4\pi^2\times 10^5\,\mathrm{m}}{T^2},$$

so $\delta g = (-8\pi^2\times 10^5\,\mathrm{m}/T^3)\delta T$. Note that T is not the period here, it is the time for 100 oscillations! The relative error is then

$$\frac{\delta g}{g} = -2\frac{\delta T}{T}.$$

If $\delta g/g = 0.1\%$ then $\delta T/T = 0.05\%$.

(b) For $g \approx 10\,\mathrm{m/s^2}$ we have

$$T \approx 2\pi(100)\sqrt{(10\,\mathrm{m})/(10\,\mathrm{m/s^2})} = 628\,\mathrm{s}.$$

Then $\delta T \approx (0.0005)(987\,\mathrm{s}) \approx 300$ ms.

E17-31 $T = 2\pi\sqrt{(17.3\,\mathrm{m})/(9.81\,\mathrm{m/s^2})} = 8.34\,\mathrm{s}$.

E17-32 The spring will extend until the force from the spring balances the weight, or when $Mg = kh$. The frequency of this system is then

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{Mg/h}{M}} = \frac{1}{2\pi}\sqrt{\frac{g}{h}},$$

which is the frequency of a pendulum of length h. The mass of the bob is irrelevant.

$\boxed{\textbf{E17-33}}$ The frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}},$$

where d is the distance from the pivot about which the hoop oscillates and the center of mass of the hoop.

The rotational inertia I is about an axis through the pivot, so we apply the parallel axis theorem. Then

$$I = Md^2 + I_{\mathrm{cm}} = Md^2 + Mr^2.$$

But d is r, since the pivot point is on the rim of the hoop. So $I = 2Md^2$, and the frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgd}{2Md^2}} = \frac{1}{2\pi}\sqrt{\frac{g}{2d}} = \frac{1}{2\pi}\sqrt{\frac{(9.81\,\mathrm{m/s^2})}{2(0.653\,\mathrm{m})}} = 0.436\ \mathrm{Hz}.$$

(b) Note the above expression looks like the simple pendulum equation if we replace $2d$ with l. Then the equivalent length of the simple pendulum is $2(0.653\,\mathrm{m}) = 1.31\,\mathrm{m}$.

E17-34 Apply Eq. 17-21:

$$I = \frac{T^2\kappa}{4\pi^2} = \frac{(48.7\,\mathrm{s}/20.0)^2(0.513\,\mathrm{N\cdot m})}{4\pi^2} = 7.70\times10^{-2}\mathrm{kg\cdot m^2}.$$

E17-35 $\kappa = (0.192\,\mathrm{N\cdot m})/(0.850\ \mathrm{rad}) = 0.226\,\mathrm{N\cdot m}$. $I = \frac{2}{5}(95.2\,\mathrm{kg})(0.148\,\mathrm{m})^2 = 0.834\,\mathrm{kg\cdot m^2}$. Then

$$T = 2\pi\sqrt{I/\kappa} = 2\pi\sqrt{(0.834\,\mathrm{kg\cdot m^2})/(0.226\,\mathrm{N\cdot m})} = 12.1\,\mathrm{s}.$$

E17-36 x is d in Eq. 17-29. Since the hole is drilled off center we apply the principle axis theorem to find the rotational inertia:

$$I = \frac{1}{12}ML^2 + Mx^2.$$

Then

$$\begin{aligned}
\frac{1}{12}ML^2 + Mx^2 &= \frac{T^2Mgx}{4\pi^2},\\
\frac{1}{12}(1.00\,\mathrm{m})^2 + x^2 &= \frac{(2.50\,\mathrm{s})^2(9.81\,\mathrm{m/s^2})}{4\pi^2}x,\\
(8.33\times10^{-2}\mathrm{m^2}) - (1.55\,\mathrm{m})x + x^2 &= 0.
\end{aligned}$$

This has solutions $x = 1.49\,\mathrm{m}$ and $x = 0.0557\,\mathrm{m}$. Use the latter.

$\boxed{\textbf{E17-37}}$ For a stick of length L which can pivot about the end, $I = \frac{1}{3}ML^2$. The center of mass of such a stick is located $d = L/2$ away from the end.

The frequency of oscillation of such a stick is

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{Mgd}{I}}, \\ f &= \frac{1}{2\pi}\sqrt{\frac{Mg\,(L/2)}{\frac{1}{3}ML^2}}, \\ f &= \frac{1}{2\pi}\sqrt{\frac{3g}{2L}}. \end{aligned}$$

This means that f is proportional to $\sqrt{1/L}$, regardless of the mass or density of the stick. The ratio of the frequency of two such sticks is then $f_2/f_1 = \sqrt{L_1/L_2}$, which in our case gives

$$f_2 = f_1\sqrt{L_2/L_1} = f_1\sqrt{(L_1)/(2L_1/3)} = 1.22 f_1.$$

E17-38 The rotational inertia of the pipe section about the cylindrical axis is

$$I_{\text{cm}} = \frac{M}{2}\left[r_1^2 + r_2^2\right] = \frac{M}{2}\left[(0.102\,\text{m})^2 + (0.1084\,\text{m})^2\right] = (1.11\times 10^{-2}\text{m}^2)M$$

(a) The total rotational inertia about the pivot axis is

$$I = 2I_{\text{cm}} + M(0.102\,\text{m})^2 + M(0.3188\,\text{m})^2 = (0.134\,\text{m}^2)M.$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{(0.134\,\text{m}^2)M}{M(9.81\,\text{m/s}^2)(0.2104\,\text{m})}} = 1.60\,\text{s}$$

(b) The rotational inertia of the pipe section about a diameter is

$$I_{\text{cm}} = \frac{M}{4}\left[r_1^2 + r_2^2\right] = \frac{M}{4}\left[(0.102\,\text{m})^2 + (0.1084\,\text{m})^2\right] = (5.54\times 10^{-3}\text{m}^2)M$$

The total rotational inertia about the pivot axis is now

$$I = M(1.11\times 10^{-2}\text{m}^2) + M(0.102\,\text{m})^2 + M(5.54\times 10^{-3}\text{m}^2) + M(0.3188\,\text{m})^2 = (0.129\,\text{m}^2)M$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{(0.129\,\text{m}^2)M}{M(9.81\,\text{m/s}^2)(0.2104\,\text{m})}} = 1.57\,\text{s}.$$

The percentage difference with part (a) is $(0.03\,\text{s})/(1.60\,\text{s}) = 1.9\%$.

E17-39

E17-40

E17-41 (a) Since effectively $x = y$, the path is a diagonal line.

(b) The path will be an ellipse which is symmetric about the line $x = y$.

(c) Since $\cos(\omega t + 90^\circ) = -\sin(\omega t)$, the path is a circle.

E17-42 (a)

(b) Take two time derivatives and multiply by m,

$$\vec{\mathbf{F}} = -mA\omega^2 \left(\hat{\mathbf{i}} \cos\omega t + 9\hat{\mathbf{j}} \cos 3\omega t\right).$$

(c) $U = -\int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, so

$$U = \frac{1}{2} mA^2\omega^2 \left(\cos^2 \omega t + 9\cos^2 3\omega t\right).$$

(d) $K = \frac{1}{2}mv^2$, so

$$K = \frac{1}{2} mA^2\omega^2 \left(\sin^2 \omega t + 9\sin^2 3\omega t\right);$$

And then $E = K + U = 5mA^2\omega^2$.

(e) Yes; the period is $2\pi/\omega$.

$\boxed{\textbf{E17-43}}$ The ω which describes the angular velocity in uniform circular motion is effectively the same ω which describes the angular frequency of the corresponding simple harmonic motion. Since $\omega = \sqrt{k/m}$, we can find the effective force constant k from knowledge of the Moon's mass and the period of revolution.

The moon orbits with a period of T, so

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(27.3 \times 24 \times 3600\,\text{s})} = 2.66\times 10^{-6}\text{rad/s}.$$

This can be used to find the value of the effective force constant k from

$$k = m\omega^2 = (7.36\times 10^{22}\text{kg})(2.66\times 10^{-6}\text{rad/s})^2 = 5.21\times 10^{11}\text{N/m}.$$

E17-44 (a) We want to know when $e^{-bt/2m} = 1/3$, or

$$t = \frac{2m}{b} \ln 3 = \frac{2(1.52\,\text{kg})}{(0.227\,\text{kg/s})} \ln 3 = 14.7\,\text{s}$$

(b) The (angular) frequency is

$$\omega' = \sqrt{\left(\frac{(8.13\,\text{N/m})}{(1.52\,\text{kg})}\right) - \left(\frac{(0.227\,\text{kg/s})}{2(1.52\,\text{kg})}\right)^2} = 2.31\text{ rad/s}.$$

The number of oscillations is then

$$(14.7\,\text{s})(2.31\text{ rad/s})/2\pi = 5.40$$

$\boxed{\textbf{E17-45}}$ The first derivative of Eq. 17-39 is

$$\begin{aligned}
\frac{dx}{dt} &= x_\text{m}(-b/2m)e^{-bt/2m}\cos(\omega' t + \phi) + x_\text{m} e^{-bt/2m}(-\omega')\sin(\omega' t + \phi), \\
&= -x_\text{m} e^{-bt/2m}\left((b/2m)\cos(\omega' t + \phi) + \omega' \sin(\omega' t + \phi)\right)
\end{aligned}$$

The second derivative is quite a bit messier;

$$\begin{aligned}
\frac{d^2}{dx^2} &= -x_\text{m}(-b/2m)e^{-bt/2m}\left((b/2m)\cos(\omega' t + \phi) + \omega' \sin(\omega' t + \phi)\right) \\
&\quad -x_\text{m} e^{-bt/2m}\left((b/2m)(-\omega')\sin(\omega' t + \phi) + (\omega')^2\cos(\omega' t + \phi)\right), \\
&= x_\text{m} e^{-bt/2m}\left((\omega' b/m)\sin(\omega' t + \phi) + (b^2/4m^2 - \omega'^2)\cos(\omega' t + \phi)\right).
\end{aligned}$$

Substitute these three expressions into Eq. 17-38. There are, however, some fairly obvious simplifications. Every one of the terms above has a factor of $x_{\rm m}$, and every term above has a factor of $e^{-bt/2m}$, so simultaneously with the substitution we will cancel out those factors. Then Eq. 17-38 becomes

$$\begin{aligned} &m\left[(\omega' b/m)\sin(\omega' t+\phi)+(b^2/4m^2-\omega'^2)\cos(\omega' t+\phi)\right]\\ &\quad -b\left[(b/2m)\cos(\omega' t+\phi)+\omega'\sin(\omega' t+\phi)\right]+k\cos(\omega' t+\phi)=0\end{aligned}$$

Now we collect terms with cosine and terms with sine,

$$(\omega' b-\omega' b)\sin(\omega' t+\phi)+\left(mb^2/4m^2-\omega'^2-b^2/2m+k\right)\cos(\omega' t+\phi)=0.$$

The coefficient for the sine term is identically zero; furthermore, because the cosine term must then vanish regardless of the value of t, the coefficient for the sine term must also vanish. Then

$$\left(mb^2/4m^2-m\omega'^2-b^2/2m+k\right)=0,$$

or

$$\omega'^2=\frac{k}{m}-\frac{b^2}{4m^2}.$$

If this condition is met, then Eq. 17-39 is indeed a solution of Eq. 17-38.

E17-46 (a) Four complete cycles requires a time $t_4=8\pi/\omega'$. The amplitude decays to 3/4 the original value in this time, so $0.75=e^{-bt_4/2m}$, or

$$\ln(4/3)=\frac{8\pi b}{2m\omega'}.$$

It is probably reasonable at this time to assume that $b/2m$ is small compared to ω so that $\omega'\approx\omega$. We'll do it the hard way anyway. Then

$$\begin{aligned}\omega'^2 &= \left(\frac{8\pi}{\ln(4/3)}\right)^2\left(\frac{b}{2m}\right)^2,\\ \frac{k}{m}-\left(\frac{b}{2m}\right)^2 &= \left(\frac{8\pi}{\ln(4/3)}\right)^2\left(\frac{b}{2m}\right)^2,\\ \frac{k}{m} &= (7630)\left(\frac{b}{2m}\right)^2\end{aligned}$$

Numerically, then,

$$b=\sqrt{\frac{4(1.91\,\text{kg})(12.6\,\text{N/m})}{(7630)}}=0.112\,\text{kg/s}.$$

(b)

E17-47 (a) Use Eqs. 17-43 and 17-44. At resonance $\omega''=\omega$, so

$$G=\sqrt{b^2\omega^2}=b\omega,$$

and then $x_{\rm m}=F_{\rm m}/b\omega$.

(b) $v_{\rm m}=\omega x_{\rm m}=F_{\rm m}/b$.

E17-48 We need the first two derivatives of

$$x = \frac{F_\mathrm{m}}{G}\cos(\omega'' t - \beta)$$

The derivatives are easy enough to find,

$$\frac{dx}{dt} = \frac{F_\mathrm{m}}{G}(-\omega'')\sin(\omega'' t - \beta),$$

and

$$\frac{d^2x}{dt^2} = -\frac{F_\mathrm{m}}{G}(\omega'')^2\cos(\omega'' t - \beta),$$

We'll substitute this into Eq. 17-42,

$$m\left(-\frac{F_\mathrm{m}}{G}(\omega'')^2\cos(\omega'' t - \beta)\right) \\ +b\left(\frac{F_\mathrm{m}}{G}(-\omega'')\sin(\omega'' t - \beta)\right) + k\frac{F_\mathrm{m}}{G}\cos(\omega'' t - \beta) = F_\mathrm{m}\cos\omega'' t.$$

Then we'll cancel out as much as we can and collect the sine and cosine terms,

$$\left(k - m(\omega'')^2\right)\cos(\omega'' t - \beta) - (b\omega'')\sin(\omega'' t - \beta) = G\cos\omega'' t.$$

We can write the left hand side of this equation in the form

$$A\cos\alpha_1\cos\alpha_2 - A\sin\alpha_1\sin\alpha_2,$$

if we let $\alpha_2 = \omega'' t - \beta$ and choose A and α_1 correctly. The best choice is

$$\begin{aligned} A\cos\alpha_1 &= k - m(\omega'')^2, \\ A\sin\alpha_1 &= b\omega'', \end{aligned}$$

and then taking advantage of the fact that $\sin^2 + \cos^2 = 1$,

$$A^2 = \left(k - m(\omega'')^2\right)^2 + (b\omega'')^2,$$

which looks like Eq. 17-44! But then we can apply the cosine angle addition formula, and

$$A\cos(\alpha_1 + \omega'' t - \beta) = G\cos\omega'' t.$$

This expression needs to be true for all time. This means that $A = G$ and $\alpha_1 = \beta$.

$\boxed{\textbf{E17-49}}$ The derivatives are easy enough to find,

$$\frac{dx}{dt} = \frac{F_\mathrm{m}}{G}(-\omega''')\sin(\omega''' t - \beta),$$

and

$$\frac{d^2x}{dt^2} = -\frac{F_\mathrm{m}}{G}(\omega''')^2\cos(\omega''' t - \beta),$$

We'll substitute this into Eq. 17-42,

$$m\left(-\frac{F_\mathrm{m}}{G}(\omega''')^2\cos(\omega''' t - \beta)\right) \\ +b\left(\frac{F_\mathrm{m}}{G}(-\omega''')\sin(\omega''' t - \beta)\right) + k\frac{F_\mathrm{m}}{G}\cos(\omega''' t - \beta) = F_\mathrm{m}\cos\omega'' t.$$

Then we'll cancel out as much as we can and collect the sine and cosine terms,

$$\left(k - m(\omega''')^2\right)\cos(\omega'''t - \beta) - (b\omega''')\sin(\omega'''t - \beta) = G\cos\omega''t.$$

We can write the left hand side of this equation in the form

$$A\cos\alpha_1\cos\alpha_2 - A\sin\alpha_1\sin\alpha_2,$$

if we let $\alpha_2 = \omega'''t - \beta$ and choose A and α_1 correctly. The best choice is

$$\begin{aligned} A\cos\alpha_1 &= k - m(\omega''')^2, \\ A\sin\alpha_1 &= b\omega''', \end{aligned}$$

and then taking advantage of the fact that $\sin^2 + \cos^2 = 1$,

$$A^2 = \left(k - m(\omega''')^2\right)^2 + (b\omega''')^2,$$

which looks like Eq. 17-44! But then we can apply the cosine angle addition formula, and

$$A\cos(\alpha_1 + \omega'''t - \beta) = G\cos\omega''t.$$

This expression needs to be true for all time. This means that $A = G$ and $\alpha_1 + \omega'''t - \beta = \omega''t$ and $\alpha_1 = \beta$ and $\omega''' = \omega''$.

E17-50 Actually, Eq. 17-39 is *not* a solution to Eq. 17-42 by itself, this is a wording mistake in the exercise. Instead, Eq. 17-39 can be added to *any* solution of Eq. 17-42 and the result will still be a solution.

Let x_n be *any* solution to Eq. 17-42 (such as Eq. 17-43.) Let x_h be given by Eq. 17-39. Then

$$x = x_\mathrm{n} + x_\mathrm{h}.$$

Take the first two time derivatives of this expression.

$$\begin{aligned} \frac{dx}{dt} &= \frac{dx_\mathrm{n}}{dt} + \frac{dx_\mathrm{h}}{dt}, \\ \frac{d^2x}{dt^2} &= \frac{d^2x_\mathrm{n}}{dt^2} + \frac{d^2x_\mathrm{h}}{dt^2} \end{aligned}$$

Substitute these three expressions into Eq. 17-42.

$$m\left(\frac{d^2x_\mathrm{n}}{dt^2} + \frac{d^2x_\mathrm{h}}{dt^2}\right) + b\left(\frac{dx_\mathrm{n}}{dt} + \frac{dx_\mathrm{h}}{dt}\right) + k\left(x_\mathrm{n} + x_\mathrm{h}\right) = F_\mathrm{m}\cos\omega''t.$$

Rearrange and regroup.

$$\left(m\frac{d^2x_\mathrm{n}}{dt^2} + b\frac{dx_\mathrm{n}}{dt} + kx_\mathrm{n}\right) + \left(m\frac{d^2x_\mathrm{h}}{dt^2} + b\frac{dx_\mathrm{h}}{dt} + kx_\mathrm{h}\right) = F_\mathrm{m}\cos\omega''t.$$

Consider the second term on the left. The parenthetical expression is just Eq. 17-38, the damped harmonic oscillator equation. It is given in the text (and proved in Ex. 17-45) the x_h is a solution, so this term is identically zero. What remains is Eq. 17-42; and we took as a given that x_n was a solution.

(b) The "add-on" solution of x_h represents the transient motion that will die away with time.

$\boxed{\textbf{E17-51}}$ The time between "bumps" is the solution to

$$\begin{aligned} vt &= x, \\ t &= \frac{(13\text{ ft})}{(10\text{ mi/hr})}\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{3600\text{ s}}{1\text{ hr}}\right) = 0.886\,\text{s} \end{aligned}$$

The angular frequency is

$$\omega = \frac{2\pi}{T} = 7.09\text{ rad/s}$$

This is the driving frequency, and the problem states that at this frequency the up-down bounce oscillation is at a maximum. This occurs when the driving frequency is approximately equal to the natural frequency of oscillation. The force constant for the car is k, and this is related to the natural angular frequency by

$$k = m\omega^2 = \frac{W}{g}\omega^2,$$

where $W = (2200 + 4 \times 180)$ lb$=$ 2920 lb is the weight of the car and occupants. Then

$$k = \frac{(2920\text{ lb})}{(32\text{ ft/s}^2)}(7.09\text{ rad/s})^2 = 4590\text{ lb/ft}$$

When the four people get out of the car there is less downward force on the car springs. The important relationship is

$$\Delta F = k\Delta x.$$

In this case $\Delta F = 720$ lb, the weight of the four people who got out of the car. Δx is the distance the car will rise when the people get out. So

$$\Delta x = \frac{\Delta F}{k} = \frac{(720\text{ lb})}{4590\text{ lb/ft}} = 0.157\text{ ft} \approx 2\text{ in.}$$

E17-52 The derivative is easy enough to find,

$$\frac{dx}{dt} = \frac{F_\text{m}}{G}(-\omega'')\sin(\omega'' t - \beta),$$

The velocity amplitude is

$$\begin{aligned} v_\text{m} &= \frac{F_\text{m}}{G}\omega'', \\ &= \frac{F_\text{m}}{\frac{1}{\omega''}\sqrt{m^2(\omega''^2 - \omega^2)^2 + b^2\omega''^2}}, \\ &= \frac{F_\text{m}}{\sqrt{(m\omega'' - k/\omega'')^2 + b^2}}. \end{aligned}$$

Note that this *is exactly* a maximum when $\omega'' = \omega$.

E17-53 The reduced mass is

$$m = (1.13\,\text{kg})(3.24\,\text{kg})/(1.12\,\text{kg} + 3.24\,\text{kg}) = 0.840\,\text{kg}.$$

The period of oscillation is

$$T = 2\pi\sqrt{(0.840\,\text{kg})/(252\,\text{N/m})} = 0.363\,\text{s}$$

E17-54

E17-55 Start by multiplying the kinetic energy expression by $(m_1+m_2)/(m_1+m_2)$.

$$\begin{aligned} K &= \frac{(m_1+m_2)}{2(m_1+m_2)}\left(m_1v_1^2+m_2v_2^2\right), \\ &= \frac{1}{2(m_1+m_2)}\left(m_1^2v_1^2+m_1m_2(v_1^2+v_2^2)+m_2^2v_2^2\right), \end{aligned}$$

and then add $2m_1m_2v_1v_2 - 2m_1m_2v_1v_2$,

$$\begin{aligned} K &= \frac{1}{2(m_1+m_2)}\left(m_1^2v_1^2+2m_1m_2v_1v_2+m_2^2v_2^2+m_1m_2(v_1^2+v_2^2-2v_1v_2)\right), \\ &= \frac{1}{2(m_1+m_2)}\left((m_1v_1+m_2v_2)^2+m_1m_2(v_1-v_2)^2\right). \end{aligned}$$

But $m_1v_2+m_2v_2=0$ by conservation of momentum, so

$$\begin{aligned} K &= \frac{(m_1m_2)}{2(m_1+m_2)}(v_1-v_2)^2, \\ &= \frac{m}{2}(v_1-v_2)^2. \end{aligned}$$

P17-1 The mass of one silver atom is $(0.108\,\mathrm{kg})/(6.02\times10^{23}) = 1.79\times10^{-25}\mathrm{kg}$. The effective spring constant is

$$k = (1.79\times10^{-25}\mathrm{kg})4\pi^2(10.0\times10^{12}/\mathrm{s})^2 = 7.07\times10^2\mathrm{N/m}.$$

P17-2 (a) Rearrange Eq. 17-8 except replace m with the total mass, or $m+M$. Then $(M+m)/k = T^2/(4\pi^2)$, or

$$M = (k/4\pi^2)T^2 - m.$$

(b) When $M=0$ we have

$$m = [(605.6\,\mathrm{N/m})/(4\pi^2)](0.90149\,\mathrm{s})^2 = 12.467\,\mathrm{kg}.$$

(c) $M = [(605.6\,\mathrm{N/m})/(4\pi^2)](2.08832\,\mathrm{s})^2 - (12.467\,\mathrm{kg}) = 54.432\,\mathrm{kg}.$

$\boxed{\textbf{P17-3}}$ The maximum static friction is $F_f \le \mu_s N$. Then

$$F_f = \mu_s N = \mu_s W = \mu_s mg$$

is the maximum available force to accelerate the upper block. So the maximum acceleration is

$$a_\mathrm{m} = \frac{F_f}{m} = \mu_s g$$

The maximum possible amplitude of the oscillation is then given by

$$x_\mathrm{m} = \frac{a_\mathrm{m}}{\omega^2} = \frac{\mu_s g}{k/(m+M)},$$

where in the last part we substituted the total mass of the two blocks because both blocks are oscillating. Now we put in numbers, and find

$$x_\mathrm{m} = \frac{(0.42)(1.22\,\mathrm{kg}+8.73\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(344\,\mathrm{N/m})} = 0.119\,\mathrm{m}.$$

P17-4 (a) Equilibrium occurs when $F = 0$, or $b/r^3 = a/r^2$. This happens when $r = b/a$.

(b) $dF/dr = 2a/r^3 - 3b/r^4$. At $r = b/a$ this becomes

$$dF/dr = 2a^4/b^3 - 3a^4/b^3 = -a^4/b^3,$$

which corresponds to a force constant of a^4/b^3.

(c) $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{mb^3}/a^2$, where m is the reduced mass.

P17-5 Each spring helps to restore the block. The net force on the block is then of magnitude $F_1 + F_2 = k_1 x + k_2 x = (k_1 + k_2)x = kx$. We can then write the frequency as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}.$$

With a little algebra,

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}, \\ &= \sqrt{\frac{1}{4\pi^2}\frac{k_1}{m} + \frac{1}{4\pi^2}\frac{k_2}{m}}, \\ &= \sqrt{f_1^2 + f_2^2}. \end{aligned}$$

P17-6 The tension in the two spring is the same, so $k_1 x_1 = k_2 x_2$, where x_i is the extension of the ith spring. The total extension is $x_1 + x_2$, so the effective spring constant of the combination is

$$\frac{F}{x} = \frac{F}{x_1 + x_2} = \frac{1}{x_1/F + x_2/F} = \frac{1}{1/k_1 + 1/k_2} = \frac{k_1 k_2}{k_1 + k_2}.$$

The period is then

$$T = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}$$

With a little algebra,

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1 + k_2)m}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{1}{m/k_1 + m/k_2}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{1}{1/\omega_1^2 + 1/\omega_2^2}}, \\ &= \frac{1}{2\pi}\sqrt{\frac{\omega_1^2\omega_2^2}{\omega_1^2 + \omega_2^2}}, \\ &= \frac{f_1 f_2}{\sqrt{f_1^2 + f_2^2}}. \end{aligned}$$

$\boxed{\textbf{P17-7}}$ (a) When a spring is stretched the tension is the same everywhere in the spring. The stretching, however, is distributed over the entire length of the spring, so that the relative amount of stretch is proportional to the length of the spring under consideration. Half a spring, half the extension. But $k = -F/x$, so half the extension means twice the spring constant.

In short, cutting the spring in half will create two stiffer springs with twice the spring constant, so $k = 7.20$ N/cm for each spring.

(b) The two spring halves now support a mass M. We can view this as each spring is holding one-half of the total mass, so in effect

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M/2}}$$

or, solving for M,

$$M = \frac{2k}{4\pi^2 f^2} = \frac{2(720\,\text{N/m})}{4\pi^2(2.87\,\text{s}^{-1})^2} = 4.43\,\text{kg}.$$

P17-8 Treat the spring a being composed of N massless springlets each with a point mass m_s/N at the end. The spring constant for each springlet will be kN. An expression for the conservation of energy is then

$$\frac{m}{2}v^2 + \frac{m_\text{s}}{2N}\sum_1^N {v_n}^2 + \frac{Nk}{2}\sum_1^N {x_n}^2 = E.$$

Since the spring stretches proportionally along the length then we conclude that each springlet compresses the same amount, and then $x_n = A/N \sin\omega t$ could describe the *change* in length of each springlet. The energy conservation expression becomes

$$\frac{m}{2}v^2 + \frac{m_\text{s}}{2N}\sum_1^N {v_n}^2 + \frac{k}{2}A^2\sin^2\omega t = E.$$

$v = A\omega\cos\omega t$. The hard part to sort out is the v_n, since the displacements for all springlets to one side of the nth must be added to get the net displacement. Then

$$v_n = n\frac{A}{N}\omega\cos\omega t,$$

and the energy expression becomes

$$\left(\frac{m}{2} + \frac{m_\text{s}}{2N^3}\sum_1^N n^2\right)A^2\omega^2\cos^2\omega t + \frac{k}{2}A^2\sin^2\omega t = E.$$

Replace the sum with an integral, then

$$\frac{1}{N^3}\int_0^N n^2\,dn = \frac{1}{3},$$

and the energy expression becomes

$$\frac{1}{2}\left(m + \frac{m_\text{s}}{3}\right)A^2\omega^2\cos^2\omega t + \frac{k}{2}A^2\sin^2\omega t = E.$$

This will only be constant if

$$\omega^2 = \left(m + \frac{m_\text{s}}{3}\right)/k,$$

or $T = 2\pi\sqrt{(m + m_\text{s}/3)/k}$.

P17-9 (a) Apply conservation of energy. When $x = x_m$ $v = 0$, so

$$\begin{aligned}\frac{1}{2}k{x_m}^2 &= \frac{1}{2}m[v(0)]^2 + \frac{1}{2}k[x(0)]^2,\\ {x_m}^2 &= \frac{m}{k}[v(0)]^2 + [x(0)]^2,\\ x_m &= \sqrt{[v(0)/\omega]^2 + [x(0)]^2}.\end{aligned}$$

(b) When $t = 0$ $x(0) = x_m \cos\phi$ and $v(0) = -\omega x_m \sin\phi$, so

$$\frac{v(0)}{\omega x(0)} = -\frac{\sin\phi}{\cos\phi} = \tan\phi.$$

P17-10

$\boxed{\textbf{P17-11}}$ Conservation of momentum for the bullet block collision gives $mv = (m + M)v_f$ or

$$v_f = \frac{m}{m+M}v.$$

This v_f will be equal to the maximum oscillation speed v_m. The angular frequency for the oscillation is given by

$$\omega = \sqrt{\frac{k}{m+M}}.$$

Then the amplitude for the oscillation is

$$x_m = \frac{v_m}{\omega} = v\frac{m}{m+M}\sqrt{\frac{m+M}{k}} = \frac{mv}{\sqrt{k(m+M)}}.$$

P17-12 (a) $W = F_s$, or $mg = kx$, so $x = mg/k$.

(b) $F = ma$, but $F = W - F_s = mg - kx$, and since $ma = m\,d^2x/dt^2$,

$$m\frac{d^2x}{dt^2} + kx = mg.$$

The solution can be verified by direct substitution.

(c) Just look at the answer!

(d) dE/dt is

$$\begin{aligned}mv\frac{dv}{dt} + kx\frac{dx}{dt} - mg\frac{dx}{dt} &= 0,\\ m\frac{dv}{dt} + kx &= mg.\end{aligned}$$

P17-13 The initial energy stored in the spring is $k{x_m}^2/2$. When the cylinder passes through the equilibrium point it has a translational velocity v_m and a rotational velocity $\omega_r = v_m/R$, where R is the radius of the cylinder. The total kinetic energy at the equilibrium point is

$$\frac{1}{2}m{v_m}^2 + \frac{1}{2}I{\omega_r}^2 = \frac{1}{2}\left(m + \frac{1}{2}m\right){v_m}^2.$$

Then the kinetic energy is 2/3 translational and 1/3 rotational. The total energy of the system is

$$E = \frac{1}{2}(294\,\mathrm{N/m})(0.239\,\mathrm{m})^2 = 8.40\,\mathrm{J}.$$

(a) $K_\mathrm{t} = (2/3)(8.40\,\mathrm{J}) = 5.60\,\mathrm{J}$.
(b) $K_\mathrm{r} = (1/3)(8.40\,\mathrm{J}) = 2.80\,\mathrm{J}$.
(c) The energy expression is

$$\frac{1}{2}\left(\frac{3m}{2}\right)v^2 + \frac{1}{2}kx^2 = E,$$

which leads to a standard expression for the period with $3M/2$ replacing m. Then $T = 2\pi\sqrt{3M/2k}$.

P17-14 (a) Integrate the potential energy expression over one complete period and then divide by the time for one period:

$$\begin{aligned}
\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{1}{2}kx^2\,dt &= \frac{k\omega}{4\pi}\int_0^{2\pi/\omega}{x_\mathrm{m}}^2\cos^2\omega t\,dt, \\
&= \frac{k\omega}{4\pi}{x_\mathrm{m}}^2\frac{\pi}{\omega}, \\
&= \frac{1}{4}k{x_\mathrm{m}}^2.
\end{aligned}$$

This is half the total energy; since the average total energy is E, then the average kinetic energy must be the other half of the average total energy, or $(1/4)k{x_\mathrm{m}}^2$.

(b) Integrate over half a cycle and divide by twice the amplitude.

$$\begin{aligned}
\frac{1}{2x_\mathrm{m}}\int_{-x_\mathrm{m}}^{x_\mathrm{m}}\frac{1}{2}kx^2\,dx &= \frac{1}{2x_\mathrm{m}}\frac{1}{3}k{x_\mathrm{m}}^3, \\
&= \frac{1}{6}k{x_\mathrm{m}}^2.
\end{aligned}$$

This is one-third the total energy. The average kinetic energy must be two-thirds the total energy, or $(1/3)k{x_\mathrm{m}}^2$.

P17-15 The rotational inertia is

$$I = \frac{1}{2}MR^2 + Md^2 = M\left(\frac{1}{2}(0.144\,\mathrm{m})^2 + (0.102\,\mathrm{m})^2\right) = (2.08\times10^{-2}\mathrm{m}^2)M.$$

The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{(2.08\times10^{-2}\mathrm{m}^2)M}{M(9.81\,\mathrm{m/s}^2)(0.102\,\mathrm{m})}} = 0.906\,\mathrm{s}.$$

P17-16 (a) The rotational inertia of the pendulum about the pivot is

$$(0.488\,\mathrm{kg})\left(\frac{1}{2}(0.103\,\mathrm{m})^2 + (0.103\,\mathrm{m} + 0.524\,\mathrm{m})^2\right) + \frac{1}{3}(0.272\,\mathrm{kg})(0.524\,\mathrm{m})^2 = 0.219\,\mathrm{kg}\cdot\mathrm{m}^2.$$

(b) The center of mass location is

$$d = \frac{(0.524\,\mathrm{m})(0.272\,\mathrm{kg})/2 + (0.103\,\mathrm{m} + 0.524\,\mathrm{m})(0.488\,\mathrm{kg})}{(0.272\,\mathrm{kg}) + (0.488\,\mathrm{kg})} = 0.496\,\mathrm{m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{(0.291\,\mathrm{kg}\cdot\mathrm{m}^2)/(0.272\,\mathrm{kg} + 0.488\,\mathrm{kg})(9.81\,\mathrm{m/s}^2)(0.496\,\mathrm{m})} = 1.76\,\mathrm{s}.$$

$\boxed{\textbf{P17-17}}$ (a) The rotational inertia of a stick about an axis through a point which is a distance d from the center of mass is given by the parallel axis theorem,

$$I = I_{\rm cm} + md^2 = \frac{1}{12}mL^2 + md^2.$$

The period of oscillation is given by Eq. 17-28,

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}}$$

(b) We want to find the minimum period, so we need to take the derivative of T with respect to d. It'll look weird, but

$$\frac{dT}{dd} = \pi\frac{12d^2 - L^2}{\sqrt{12gd^3(L^2 + 12d^2)}}.$$

This will vanish when $12d^2 = L^2$, or when $d = L/\sqrt{12}$.

P17-18 The energy stored in the spring is given by $kx^2/2$, the kinetic energy of the rotating wheel is

$$\frac{1}{2}(MR^2)\left(\frac{v}{r}\right)^2,$$

where v is the tangential velocity of the point of attachment of the spring to the wheel. If $x = x_{\rm m}\sin\omega t$, then $v = x_{\rm m}\omega\cos\omega t$, and the energy will only be constant if

$$\omega^2 = \frac{k}{M}\frac{r^2}{R^2}.$$

P17-19 The method of solution is identical to the approach for the simple pendulum on page 381 *except* replace the tension with the normal force of the bowl on the particle. The effective pendulum with have a length R.

P17-20 Let x be the distance from the center of mass to the first pivot point. Then the period is given by

$$T = 2\pi\sqrt{\frac{I + Mx^2}{Mgx}}.$$

Solve this for x by expressing the above equation as a quadratic:

$$\left(\frac{MgT^2}{4\pi^2}\right)x = I + Mx^2,$$

or

$$Mx^2 - \left(\frac{MgT^2}{4\pi^2}\right)x + I = 0$$

There are *two* solutions. One corresponds to the first location, the other the second location. Adding the two solutions together will yield L; in this case the discriminant of the quadratic will drop out, leaving

$$L = x_1 + x_2 = \frac{MgT^2}{M4\pi^2} = \frac{gT^2}{4\pi^2}.$$

Then $g = 4\pi^2 L/T^2$.

P17-21 In this problem

$$I = (2.50\,\text{kg})\left(\frac{(0.210\,\text{m})^2}{2} + (0.760\,\text{m} + 0.210\,\text{m})^2\right) = 2.41\,\text{kg}/\cdot\text{m}^2.$$

The center of mass is at the center of the disk.

(a) $T = 2\pi\sqrt{(2.41\,\text{kg}/\cdot\text{m}^2)/(2.50\,\text{kg})(9.81\,\text{m/s}^2)(0.760\,\text{m} + 0.210\,\text{m})} = 2.00\,\text{s}$.

(b) Replace Mgd with $Mgd + \kappa$ and $2.00\,\text{s}$ with $1.50\,\text{s}$. Then

$$\kappa = \frac{4\pi^2(2.41\,\text{kg}/\cdot\text{m}^2)}{(1.50\,\text{s})^2} - (2.50\,\text{kg})(9.81\,\text{m/s}^2)(0.760\,\text{m} + 0.210\,\text{m}) = 18.5\,\text{N}\cdot\text{m/rad}.$$

P17-22 The net force on the bob is toward the center of the circle, and has magnitude $F_{\text{net}} = mv^2/R$. This net force comes from the horizontal component of the tension. There is also a vertical component of the tension of magnitude mg. The tension then has magnitude

$$T = \sqrt{(mg)^2 + (mv^2/R)^2} = m\sqrt{g^2 + v^4/R^2}.$$

It is this tension which is important in finding the restoring force in Eq. 17-22; in effect we want to replace g with $\sqrt{g^2 + v^4/R^2}$ in Eq. 17-24. The frequency will then be

$$f = \frac{1}{2\pi}\sqrt{\frac{L}{\sqrt{g^2 + v^4/R^2}}}.$$

$\boxed{\textbf{P17-23}}$ (a) Consider an object of mass m at a point P on the axis of the ring. It experiences a gravitational force of attraction to all points on the ring; by symmetry, however, the net force is not directed toward the circumference of the ring, but instead along the axis of the ring. There is then a factor of $\cos\theta$ which will be thrown in to the mix.

The distance from P to *any* point on the ring is $r = \sqrt{R^2 + z^2}$, and θ is the angle between the axis on the line which connects P and *any* point on the circumference. Consequently,

$$\cos\theta = z/r,$$

and then the net force on the star of mass m at P is

$$F = \frac{GMm}{r^2}\cos\theta = \frac{GMmz}{r^3} = \frac{GMmz}{(R^2 + z^2)^{3/2}}.$$

(b) If $z \ll R$ we can apply the binomial expansion to the denominator, and

$$(R^2 + z^2)^{-3/2} = R^{-3}\left(1 + \left(\frac{z}{R}\right)^2\right)^{-3/2} \approx R^{-3}\left(1 - \frac{3}{2}\left(\frac{z}{R}\right)^2\right).$$

Keeping terms only linear in z we have

$$F = \frac{GMm}{R^3}z,$$

which corresponds to a spring constant $k = GMm/R^3$. The frequency of oscillation is then

$$f = \sqrt{k/m}/(2\pi) = \sqrt{GM/R^3}/(2\pi).$$

(c) Using some numbers from the Milky Way galaxy,

$$f = \sqrt{(7\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(2\times10^{43}\text{kg})/(6\times10^{19}\text{m})^3}/(2\pi) = 1\times10^{-14}\ \text{Hz}.$$

P17-24 (a) The acceleration of the center of mass (point C) is $a = F/M$. The torque about an axis through the center of mass is $tau = FR/2$, since O is $R/2$ away from the center of mass. The angular acceleration of the disk is then

$$\alpha = \tau/I = (FR/2)/(MR^2/2) = F/(MR).$$

Note that the angular acceleration will tend to rotate the disk anti-clockwise. The tangential component to the angular acceleration at P is $a_\text{T} = -\alpha R = -F/M$; this is exactly the opposite of the linear acceleration, so P will not (initially) accelerate.

(b) There is no net force at P.

P17-25 The value for k is closest to

$$k \approx (2000\,\text{kg}/4)(9.81\,\text{m/s}^2)/(0.10\,\text{m}) = 4.9\times10^4\text{N/m}.$$

One complete oscillation requires a time $t_1 = 2\pi/\omega'$. The amplitude decays to 1/2 the original value in this time, so $0.5 = e^{-bt_1/2m}$, or

$$\ln(2) = \frac{2\pi b}{2m\omega'}.$$

It is *not* reasonable at this time to assume that $b/2m$ is small compared to ω so that $\omega' \approx \omega$. Then

$$\begin{aligned}
\omega'^2 &= \left(\frac{2\pi}{\ln(2)}\right)^2\left(\frac{b}{2m}\right)^2, \\
\frac{k}{m} - \left(\frac{b}{2m}\right)^2 &= \left(\frac{2\pi}{\ln(2)}\right)^2\left(\frac{b}{2m}\right)^2, \\
\frac{k}{m} &= (81.2)\left(\frac{b}{2m}\right)^2
\end{aligned}$$

Then the value for b is

$$b = \sqrt{\frac{4(2000\,\text{kg}/4)(4.9\times10^4\text{N/m})}{(81.2)}} = 1100\,\text{kg/s}.$$

P17-26 $a = d^2x/dt^2 = -A\omega^2\cos\omega t$. Substituting into the non-linear equation,

$$-mA\omega^2\cos\omega t + kA^3\cos^3\omega t = F\cos\omega_\text{d} t.$$

Now let $\omega_\text{d} = 3\omega$. Then

$$kA^3\cos^3\omega t - mA\omega^2\cos\omega t = F\cos 3\omega t$$

Expand the right hand side as $\cos 3\omega t = 4\cos^3\omega t - 3\cos\omega t$, then

$$kA^3\cos^3\omega t - mA\omega^2\cos\omega t = F(4\cos^3\omega t - 3\cos\omega t)$$

This will only work if $4F = kA^3$ and $3F = mA\omega^2$. Dividing one condition by the other means $4mA\omega^2 = kA^3$, so $A \propto \omega$ and then $F \propto \omega^3 \propto {\omega_\text{d}}^3$.

P17-27 (a) Divide the top and the bottom by m_2. Then

$$\frac{m_1 m_2}{m_1 + m_2} = \frac{m_1}{(m_1/m_2) + 1},$$

and in the limit as $m_2 \to \infty$ the value of $(m_1/m_2) \to 0$, so

$$\lim_{m_2 \to \infty} \frac{m_1 m_2}{m_1 + m_2} = \lim_{m_2 \to \infty} \frac{m_1}{(m_1/m_2) + 1} = m_1.$$

(b) m is called the reduced mass because it is always less than either m_1 or m_2. Think about it in terms of

$$m = \frac{m_1}{(m_1/m_2) + 1} = \frac{m_2}{(m_2/m_1) + 1}.$$

Since mass is always positive, the denominator is always greater than or equal to 1. Equality only occurs if one of the masses is infinite. Now $\omega = \sqrt{k/m}$, and since m is always less than m_1, so the existence of a finite wall will cause ω to be larger, and the period to be smaller.

(c) If the bodies have equal mass then $m = m_1/2$. This corresponds to a value of $\omega = \sqrt{2k/m_1}$. In effect, the spring constant is doubled, which is what happens if a spring is cut in half.

E18-1 (a) $f = v/\lambda = (243\,\mathrm{m/s})/(0.0327\,\mathrm{m}) = 7.43\times10^3$ Hz.
(b) $T = 1/f = 1.35\times10^{-4}$s.

E18-2 (a) $f = (12)/(30\,\mathrm{s}) = 0.40$ Hz.
(b) $v = (15\,\mathrm{m})/(5.0\,\mathrm{s}) = 3.0\,\mathrm{m/s}$.
(c) $\lambda = v/f = (3.0\,\mathrm{m/s})/(0.40\text{ Hz}) = 7.5\,\mathrm{m}$.

$\boxed{\textbf{E18-3}}$ (a) The time for a particular point to move from maximum displacement to zero displacement is one-quarter of a period; the point must then go to maximum negative displacement, zero displacement, and finally maximum positive displacement to complete a cycle. So the period is 4(178 ms) = 712 ms.
(b) The frequency is $f = 1/T = 1/(712\times10^{-3}\mathrm{s}) = 1.40$ Hz.
(c) The wave-speed is $v = f\lambda = (1.40\text{ Hz})(1.38\,\mathrm{m}) = 1.93\,\mathrm{m/s}$.

E18-4 Use Eq. 18-9, except let $f = 1/T$:

$$y = (0.0213\,\mathrm{m})\sin 2\pi\left(\frac{x}{(0.114\,\mathrm{m})} - (385\text{ Hz})t\right) = (0.0213\,\mathrm{m})\sin\left[(55.1\text{ rad/m})x - (2420\text{ rad/s})t\right].$$

$\boxed{\textbf{E18-5}}$ The dimensions for tension are $[\mathrm{F}] = [\mathrm{M}][\mathrm{L}]/[\mathrm{T}]^2$ where M stands for mass, L for length, T for time, and F stands for force. The dimensions for linear mass density are [M]/[L]. The dimensions for velocity are [L]/[T].

Inserting this into the expression $v = F^a/\mu^b$,

$$\frac{[\mathrm{L}]}{[\mathrm{T}]} = \left(\frac{[\mathrm{M}][\mathrm{L}]}{[\mathrm{T}]^2}\right)^{\mathrm{a}} / \left(\frac{[\mathrm{M}]}{[\mathrm{L}]}\right)^{\mathrm{b}},$$
$$\frac{[\mathrm{L}]}{[\mathrm{T}]} = \frac{[\mathrm{M}]^{\mathrm{a}}[\mathrm{L}]^{\mathrm{a}}}{[\mathrm{T}]^{2\mathrm{a}}}\frac{[\mathrm{L}]^{\mathrm{b}}}{[\mathrm{M}]^{\mathrm{b}}},$$
$$\frac{[\mathrm{L}]}{[\mathrm{T}]} = \frac{[\mathrm{M}]^{\mathrm{a-b}}[\mathrm{L}]^{\mathrm{a+b}}}{[\mathrm{T}]^{2\mathrm{a}}}$$

There are three equations here. One for time, $-1 = -2a$; one for length, $1 = a + b$; and one for mass, $0 = a - b$. We need to satisfy all three equations. The first is fairly quick; $a = 1/2$. Either of the other equations can be used to show that $b = 1/2$.

E18-6 (a) $y_{\mathrm{m}} = 2.30\,\mathrm{mm}$.
(b) $f = (588\text{ rad/s})/(2\pi\text{ rad}) = 93.6$ Hz.
(c) $v = (588\text{ rad/s})/(1822\text{ rad/m}) = 0.323\,\mathrm{m/s}$.
(d) $\lambda = (2\pi\text{ rad})/(1822\text{ rad/m}) = 3.45\,\mathrm{mm}$.
(e) $u_y = y_{\mathrm{m}}\omega = (2.30\,\mathrm{mm})(588\text{ rad/s}) = 1.35\,\mathrm{m/s}$.

E18-7 (a) $y_{\mathrm{m}} = 0.060\,\mathrm{m}$.
(b) $\lambda = (2\pi\text{ rad})/(2.0\pi\text{ rad/m}) = 1.0\,\mathrm{m}$.
(c) $f = (4.0\pi\text{ rad/s})/(2\pi\text{ rad}) = 2.0$ Hz.
(d) $v = (4.0\pi\text{ rad/s})/(2.0\pi\text{ rad/m}) = 2.0\,\mathrm{m/s}$.
(e) Since the second term is positive the wave is moving in the $-x$ direction.
(f) $u_y = y_{\mathrm{m}}\omega = (0.060\,\mathrm{m})(4.0\pi\text{ rad/s}) = 0.75\,\mathrm{m/s}$.

E18-8 $v = \sqrt{F/\mu} = \sqrt{(487\,\mathrm{N})/[(0.0625\,\mathrm{kg})/(2.15\,\mathrm{m})]} = 129\,\mathrm{m/s}$.

$\boxed{\textbf{E18-9}}$ We'll first find the linear mass density by rearranging Eq. 18-19,

$$\mu = \frac{F}{v^2}$$

Since this is the same string, we expect that changing the tension will not significantly change the linear mass density. Then for the two different instances,

$$\frac{F_1}{v_1^2} = \frac{F_2}{v_2^2}$$

We want to know the new tension, so

$$F_2 = F_1 \frac{v_2^2}{v_1^2} = (123\,\mathrm{N}) \frac{(180\,\mathrm{m/s})^2}{(172\,\mathrm{m/s})^2} = 135\,\mathrm{N}$$

E18-10 First $v = (317\ \mathrm{rad/s})/(23.8\ \mathrm{rad/m}) = 13.32\,\mathrm{m/s}$. Then

$$\mu = F/v^2 = (16.3\,\mathrm{N})/(13.32\,\mathrm{m/s})^2 = 0.0919\,\mathrm{kg/m}.$$

E18-11 (a) $y_m = 0.05\,\mathrm{m}$.

(b) $\lambda = (0.55\,\mathrm{m}) - (0.15\,\mathrm{m}) = 0.40\,\mathrm{m}$.

(c) $v = \sqrt{F/\mu} = \sqrt{(3.6\,\mathrm{N})/(0.025\,\mathrm{kg/m})} = 12\,\mathrm{m/s}$.

(d) $T = 1/f = \lambda/v = (0.40\,\mathrm{m})/(12\,\mathrm{m/s}) = 3.33\times10^{-2}\mathrm{s}$.

(e) $u_y = y_\mathrm{m}\omega = 2\pi y_\mathrm{m}/T = 2\pi(0.05\,\mathrm{m})/(3.33\times10^{-2}\mathrm{s}) = 9.4\,\mathrm{m/s}$.

(f) $k = (2\pi\ \mathrm{rad})/(0.40\,\mathrm{m}) = 5.0\pi\ \mathrm{rad/m}$; $\omega = kv = (5.0\pi\ \mathrm{rad/m})(12\,\mathrm{m/s}) = 60\pi\ \mathrm{rad/s}$. The phase angle can be found from

$$(0.04\,\mathrm{m}) = (0.05\,\mathrm{m})\sin(\phi),$$

or $\phi = 0.93$ rad. Then

$$y = (0.05\,\mathrm{m})\sin[(5.0\pi\ \mathrm{rad/m})x + (60\pi\ \mathrm{rad/s})t + (0.93\ \mathrm{rad})].$$

E18-12 (a) The tensions in the two strings are equal, so $F = (0.511\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/2 = 2.506\,\mathrm{N}$. The wave speed in string 1 is

$$v = \sqrt{F/\mu} = \sqrt{(2.506\,\mathrm{N})/(3.31\times10^{-3}\mathrm{kg/m})} = 27.5\,\mathrm{m/s},$$

while the wave speed in string 2 is

$$v = \sqrt{F/\mu} = \sqrt{(2.506\,\mathrm{N})/(4.87\times10^{-3}\mathrm{kg/m})} = 22.7\,\mathrm{m/s}.$$

(b) We have $\sqrt{F_1/\mu_1} = \sqrt{F_2/\mu_2}$, or $F_1/\mu_1 = F_2/\mu_2$. But $F_i = M_i g$, so $M_1/\mu_1 = M_2/\mu_2$. Using $M = M_1 + M_2$,

$$\begin{aligned}
\frac{M_1}{\mu_1} &= \frac{M - M_1}{\mu_2}, \\
\frac{M_1}{\mu_1} + \frac{M_1}{\mu_2} &= \frac{M}{\mu_2}, \\
M_1 &= \frac{M}{\mu_2} \bigg/ \left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right), \\
&= \frac{(0.511\,\mathrm{kg})}{(4.87\times10^{-3}\mathrm{kg/m})} \bigg/ \left(\frac{1}{(3.31\times10^{-3}\mathrm{kg/m})} + \frac{1}{(4.87\times10^{-3}\mathrm{kg/m})}\right), \\
&= 0.207\,\mathrm{kg}
\end{aligned}$$

and $M_2 = (0.511\,\mathrm{kg}) - (0.207\,\mathrm{kg}) = 0.304\,\mathrm{kg}$.

$\boxed{\textbf{E18-13}}$ We need to know the wave speed before we do anything else. This is found from Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{(248\,\mathrm{N})}{(0.0978\,\mathrm{kg})/(10.3\,\mathrm{m})}} = 162\,\mathrm{m/s}.$$

The two pulses travel in opposite directions on the wire; one travels as distance x_1 in a time t, the other travels a distance x_2 in a time $t + 29.6$ ms, and since the pulses meet, we have $x_1 + x_2 = 10.3$ m.

Our equations are then $x_1 = vt = (162\,\mathrm{m/s})t$, and $x_2 = v(t{+}29.6\text{ ms}) = (162\,\mathrm{m/s})(t{+}29.6\text{ ms}) = (162\,\mathrm{m/s})t + 4.80\,\mathrm{m}$. We can add these two expressions together to solve for the time t at which the pulses meet,

$$10.3\,\mathrm{m} = x_1 + x_2 = (162\,\mathrm{m/s})t + (162\,\mathrm{m/s})t + 4.80\,\mathrm{m} = (324\,\mathrm{m/s})t + 4.80\,\mathrm{m}.$$

which has solution $t = 0.0170\,\mathrm{s}$. The two pulses meet at $x_1 = (162\,\mathrm{m/s})(0.0170\,\mathrm{s}) = 2.75\,\mathrm{m}$, or $x_2 = 7.55\,\mathrm{m}$.

E18-14 (a) $\partial y/\partial r = (A/r)k\cos(kr - \omega t) - (A/r^2)\sin(kr - \omega t)$. Multiply this by r^2, and then find

$$\frac{\partial}{\partial r} r^2 \frac{\partial y}{\partial r} = Ak\cos(kr - \omega t) - Ak^2 r\sin(kr - \omega t) - Ak\cos(kr - \omega t).$$

Simplify, and then divide by r^2 to get

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial y}{\partial r} = -(Ak^2/r)\sin(kr - \omega t).$$

Now find $\partial^2 y/\partial t^2 = -A\omega^2 \sin(kr - \omega t)$. But since $1/v^2 = k^2/\omega^2$, the two sides are equal.

(b) [length]^2.

E18-15 The liner mass density is $\mu = (0.263\,\mathrm{kg})/(2.72\,\mathrm{m}) = 9.669{\times}10^{-2}\mathrm{kg/m}$. The wave speed is $v = \sqrt{(36.1\,\mathrm{N})/(9.669{\times}10^{-2}\mathrm{kg/m})} = 19.32\,\mathrm{m/s}$.

$P_\mathrm{av} = \frac{1}{2}\mu\omega^2 {y_\mathrm{m}}^2 v$, so

$$\omega = \sqrt{\frac{2(85.5\,\mathrm{W})}{(9.669{\times}10^{-2}\mathrm{kg/m})(7.70{\times}10^{-3}\mathrm{m})^2(19.32\,\mathrm{m/s})}} = 1243\text{ rad/s}.$$

Then $f = (1243\text{ rad/s})/2\pi = 199$ Hz.

E18-16 (a) If the medium absorbs no energy then the power flow through any closed surface which contains the source must be constant. Since for a cylindrical surface the area grows as r, then intensity must fall of as $1/r$.

(b) Intensity is proportional to the amplitude squared, so the amplitude must fall off as $1/\sqrt{r}$.

$\boxed{\textbf{E18-17}}$ The intensity is the average power per unit area (Eq. 18-33); as you get farther from the source the intensity falls off because the perpendicular area increases. At some distance r from the source the total possible area is the area of a spherical shell of radius r, so intensity as a function of distance from the source would be

$$I = \frac{P_\mathrm{av}}{4\pi r^2}$$

We are given two intensities: $I_1 = 1.13\,\mathrm{W/m^2}$ at a distance r_1; $I_2 = 2.41\,\mathrm{W/m^2}$ at a distance $r_2 = r_1 - 5.30\,\mathrm{m}$. Since the average power of the source is the same in both cases we can equate these two values as

$$\begin{aligned}4\pi r_1^2 I_1 &= 4\pi r_2^2 I_2,\\ 4\pi r_1^2 I_1 &= 4\pi (r_1 - d)^2 I_2,\end{aligned}$$

where $d = 5.30\,\mathrm{m}$, and then solve for r_1. Doing this we find a quadratic expression which is

$$\begin{aligned}r_1^2 I_1 &= (r_1^2 - 2dr_1 + d^2) I_2,\\ 0 &= \left(1 - \frac{I_1}{I_2}\right) r_1^2 - 2dr_1 + d^2,\\ 0 &= \left(1 - \frac{(1.13\,\mathrm{W/m^2})}{(2.41\,\mathrm{W/m^2})}\right) r_1^2 - 2(5.30\,\mathrm{m}) r_1 + (5.30\,\mathrm{m})^2,\\ 0 &= (0.531) r_1^2 - (10.6\,\mathrm{m}) r_1 + (28.1\,\mathrm{m^2}).\end{aligned}$$

The solutions to this are $r_1 = 16.8\,\mathrm{m}$ and $r_1 = 3.15\,\mathrm{m}$; but since the person walked 5.3 m toward the lamp we will assume they started at least that far away. Then the power output from the light is

$$P = 4\pi r_1^2 I_1 = 4\pi (16.8\,\mathrm{m})^2 (1.13\,\mathrm{W/m^2}) = 4.01 \times 10^3 \mathrm{W}.$$

E18-18 Energy density is energy per volume, or $u = U/V$. A wave front of cross sectional area A sweeps out a volume of $V = Al$ when it travels a distance l. The wave front travels that distance l in a time $t = l/v$. The energy flow per time is the power, or $P = U/t$. Combine this with the definition of intensity, $I = P/A$, and

$$I = \frac{P}{A} = \frac{U}{At} = \frac{uV}{At} = \frac{uAl}{At} = uv.$$

$\boxed{\textbf{E18-19}}$ Refer to Eq. 18-40, where the amplitude of the combined wave is

$$2y_\mathrm{m} \cos(\Delta\phi/2),$$

where y_m is the amplitude of the combining waves. Then

$$\cos(\Delta\phi/2) = (1.65 y_\mathrm{m})/(2y_\mathrm{m}) = 0.825,$$

which has solution $\Delta\phi = 68.8^\circ$.

E18-20 Consider only the point $x = 0$. The displacement y at that point is given by

$$y = y_\mathrm{m1} \sin(\omega t) + y_\mathrm{m2} \sin(\omega t + \pi/2) = y_\mathrm{m1} \sin(\omega t) + y_\mathrm{m2} \cos(\omega t).$$

This can be written as

$$y = y_\mathrm{m}(A_1 \sin \omega t + A_2 \cos \omega t),$$

where $A_i = y_{\mathrm{m}i}/y_\mathrm{m}$. But if y_m is judiciously chosen, $A_1 = \cos\beta$ and $A_2 = \sin\beta$, so that

$$y = y_\mathrm{m} \sin(\omega t + \beta).$$

Since we then require $A_1^2 + A_2^2 = 1$, we must have

$$y_\mathrm{m} = \sqrt{(3.20\ \mathrm{cm})^2 + (4.19\ \mathrm{cm})^2} = 5.27\ \mathrm{cm}.$$

E18-21 The easiest approach is to use a phasor representation of the waves.

Write the phasor components as

$$\begin{aligned} x_1 &= y\mathrm{m}1 \cos\phi_1, \\ y_1 &= y\mathrm{m}1 \sin\phi_1, \\ x_2 &= y\mathrm{m}2 \cos\phi_2, \\ y_2 &= y\mathrm{m}2 \sin\phi_2, \end{aligned}$$

and then use the cosine law to find the magnitude of the resultant.

The phase angle can be found from the arcsine of the opposite over the hypotenuse.

E18-22 (a) The diagrams for all times except $t = 15\,\mathrm{ms}$ should show two distinct pulses, first moving closer together, then moving farther apart. The pulses don not flip over when passing each other. The $t = 15\,\mathrm{ms}$ diagram, however, should simply be a flat line.

E18-23 Use a program such as Maple or Mathematica to plot this.

E18-24 Use a program such as Maple or Mathematica to plot this.

$\boxed{\textbf{E18-25}}$ (a) The wave speed can be found from Eq. 18-19; we need to know the linear mass density, which is $\mu = m/L = (0.122\,\mathrm{kg})/(8.36\,\mathrm{m}) = 0.0146\,\mathrm{kg/m}$. The wave speed is then given by

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{(96.7\,\mathrm{N})}{(0.0146\,\mathrm{kg/m})}} = 81.4\,\mathrm{m/s}.$$

(b) The longest possible standing wave will be twice the length of the string; so $\lambda = 2L = 16.7\,\mathrm{m}$.

(c) The frequency of the wave is found from Eq. 18-13, $v = f\lambda$.

$$f = \frac{v}{\lambda} = \frac{(81.4\,\mathrm{m/s})}{(16.7\,\mathrm{m})} = 4.87\text{ Hz}$$

E18-26 (a) $v = \sqrt{(152\,\mathrm{N})/(7.16\times10^{-3}\mathrm{kg/m})} = 146\,\mathrm{m/s}$.

(b) $\lambda = (2/3)(0.894\,\mathrm{m}) = 0.596\,\mathrm{m}$.

(c) $f = v/\lambda = (146\,\mathrm{m/s})/(0.596\,\mathrm{m}) = 245$ Hz.

E18-27 (a) $y = -3.9$ cm.

(b) $y = (0.15\,\mathrm{m})\sin[(0.79\text{ rad/m})x + (13\text{ rad/s})t]$.

(c) $y = 2(0.15\,\mathrm{m})\sin[(0.79\text{ rad/m})(2.3\,\mathrm{m})]\cos[(13\text{ rad/s})(0.16\,\mathrm{s})] = -0.14\,\mathrm{m}$.

E18-28 (a) The amplitude is half of 0.520 cm, or 2.60 mm. The speed is

$$v = (137\text{ rad/s})/(1.14\text{ rad/cm}) = 1.20\,\mathrm{m/s}.$$

(b) The nodes are $(\pi\mathrm{rad})/(1.14\text{ rad/cm}) = 2.76$ cm apart.

(c) The velocity of a particle on the string at position x and time t is the derivative of the wave equation with respect to time, or

$$u_y = -(0.520\text{ cm})(137\text{ rad/s})\sin[(1.14\text{ rad/cm})(1.47\text{ cm})]\sin[(137\text{ rad/s})(1.36\,\mathrm{s})] = -0.582\,\mathrm{m/s}.$$

$\boxed{\textbf{E18-29}}$ (a) We are given the wave frequency and the wave-speed, the wavelength is found from Eq. 18-13,

$$\lambda = \frac{v}{f} = \frac{(388\,\text{m/s})}{(622\text{ Hz})} = 0.624\,\text{m}$$

The standing wave has four loops, so from Eq. 18-45

$$L = n\frac{\lambda}{2} = (4)\frac{(0.624\,\text{m})}{2} = 1.25\,\text{m}$$

is the length of the string.

(b) We can just write it down,

$$y = (1.90\text{ mm})\sin[(2\pi/0.624\,\text{m})x]\cos[(2\pi 622\,\text{s}^{-1})t].$$

E18-30 (a) $f_n = nv/2L = (1)(250\,\text{m/s})/2(0.150\,\text{m}) = 833$ Hz.

(b) $\lambda = v/f = (348\,\text{m/s})/(833\text{ Hz}) = 0.418\,\text{m}$.

E18-31 $v = \sqrt{F/\mu} = \sqrt{FL/m}$. Then $f_n = nv/2L = n\sqrt{F/4mL}$, so

$$f_1 = (1)\sqrt{(236\,\text{N})/4(0.107\,\text{kg})(9.88\,\text{m})}7.47\text{ Hz},$$

and $f_2 = 2f_1 = 14.9$ Hz while $f_3 = 3f_1 = 22.4$ Hz.

E18-32 (a) $v = \sqrt{F/\mu} = \sqrt{FL/m} = \sqrt{(122\,\text{N})(1.48\,\text{m})/(8.62\times10^{-3}\text{kg})} = 145\,\text{m/s}$.

(b) $\lambda_1 = 2(1.48\,\text{m}) = 2.96\,\text{m}$; $\lambda_2 = 1.48\,\text{m}$.

(c) $f_1 = (145\,\text{m/s})/(2.96\,\text{m}) = 49.0$ Hz; $f_2 = (145\,\text{m/s})/(1.48\,\text{m}) = 98.0$ Hz.

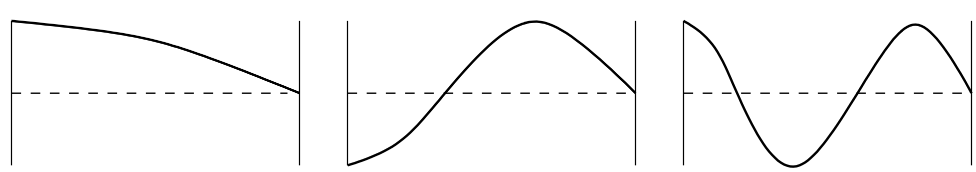
$\boxed{\textbf{E18-33}}$ Although the tied end of the string forces it to be a node, the fact that the other end is loose means that it should be an anti-node. The discussion of Section 18-10 indicated that the spacing between nodes is always $\lambda/2$. Since anti-nodes occur between nodes, we can expect that the distance between a node and the nearest anti-node is $\lambda/4$.

The longest possible wavelength will have one node at the tied end, an anti-node at the loose end, and no other nodes or anti-nodes. In this case $\lambda/4 = 120$ cm, or $\lambda = 480$ cm.

The next longest wavelength will have a node somewhere in the middle region of the string. But this means that there must be an anti-node between this new node and the node at the tied end of the string. Moving from left to right, we then have an anti-node at the loose end, a node, and anti-node, and finally a node at the tied end. There are four points, each separated by $\lambda/4$, so the wavelength would be given by $3\lambda/4 = 120$ cm, or $\lambda = 160$ cm.

To progress to the next wavelength we will add another node, and another anti-node. This will add another two lengths of $\lambda/4$ that need to be fit onto the string; hence $5\lambda/4 = 120$ cm, or $\lambda = 100$ cm.

In the figure below we have sketched the first three standing waves.



E18-34 (a) Note that $f_n = nf_1$. Then $f_{n+1} - f_n = f_1$. Since there is no resonant frequency between these two then they must differ by 1, and consequently $f_1 = (420\text{ Hz}) - (315\text{ Hz}) = 105\text{ Hz}$.

(b) $v = f\lambda = (105\text{ Hz})[2(0.756\,\text{m})] = 159\,\text{m/s}$.

P18-1 (a) $\lambda = v/f$ and $k = 360°/\lambda$. Then

$$x = (55°)\lambda/(360°) = 55(353\,\text{m/s})/360(493\text{ Hz}) = 0.109\,\text{m}.$$

(b) $\omega = 360° f$, so

$$\phi = \omega t = (360°)(493\text{ Hz})(1.12\times10^{-3}\text{s}) = 199°.$$

P18-2 $\omega = (2\pi\text{ rad})(548\text{ Hz}) = 3440\text{ rad/s}$; $\lambda = v/f$ and then

$$k = (2\pi\text{ rad})/[(326\,\text{m/s})/(548\text{ Hz})] = 10.6\text{ rad/m}.$$

Finally, $y = (1.12\times10^{-2}\text{m})\sin[(10.6\text{ rad/m})x + (3440\text{ rad/s})t]$.

$\boxed{\textbf{P18-3}}$ (a) This problem really isn't as bad as it might look. The tensile stress S is tension per unit cross sectional area, so

$$S = \frac{F}{A}\text{ or }F = SA.$$

We already know that linear mass density is $\mu = m/L$, where L is the length of the wire. Substituting into Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{SA}{m/L}}\sqrt{\frac{S}{m/(AL)}}.$$

But AL is the volume of the wire, so the denominator is just the mass density ρ.

(b) The maximum speed of the transverse wave will be

$$v = \sqrt{\frac{S}{\rho}} = \sqrt{\frac{(720\times10^6\text{ Pa})}{(7800\text{ kg/m}^3)}} = 300\,\text{m/s}.$$

P18-4 (a) $f = \omega/2\pi = (4.08\text{ rad/s})/(2\pi\text{ rad}) = 0.649\text{ Hz}$.

(b) $\lambda = v/f = (0.826\,\text{m/s})/(0.649\text{ Hz}) = 1.27\,\text{m}$.

(c) $k = (2\pi\text{ rad})/(1.27\,\text{m}) = 4.95\text{ rad/m}$, so

$$y = (5.12\text{ cm})\sin[(4.95\text{ rad/m}x - (4.08\text{ rad/s})t + \phi],$$

where ϕ is determined by $(4.95\text{ rad/m})(9.60\times10^{-2}\text{m}) + \phi = (1.16\text{ rad})$, or $\phi = 0.685\text{ rad}$.

(d) $F = \mu v^2 = (0.386\,\text{kg/m})(0.826\,\text{m/s})^2 = 0.263\,\text{m/s}$.

P18-5 We want to show that $dy/dx = u_y/v$. The easy way, although not mathematically rigorous:

$$\frac{dy}{dx} = \frac{dy}{dx}\frac{dt}{dt} = \frac{dy}{dt}\frac{dt}{dx} = u_y\frac{1}{v} = \frac{u_y}{x}.$$

P18-6 The maximum value for u_y occurs when the cosine function in Eq. 18-14 returns unity. Consequently, $u_{\text{m}}/y_{\text{m}} = \omega$.

$\boxed{\textbf{P18-7}}$ (a) The linear mass density changes as the rubber band is stretched! In this case,

$$\mu = \frac{m}{L+\Delta L}.$$

The tension in the rubber band is given by $F = k\Delta L$. Substituting this into Eq. 18-19,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{k\Delta L(L+\Delta L)}{m}}.$$

(b) We want to know the time it will take to travel the length of the rubber band, so

$$v = \frac{L+\Delta L}{t} \text{ or } t = \frac{L+\Delta L}{v}.$$

Into this we will substitute our expression for wave speed

$$t = (L+\Delta L)\sqrt{\frac{m}{k\Delta L(L+\Delta L)}} = \sqrt{\frac{m(L+\Delta L)}{k\Delta L}}$$

We have to possibilities to consider: either $\Delta L \ll L$ or $\Delta L \gg L$. In either case we are only interested in the part of the expression with $L + \Delta L$; whichever term is much larger than the other will be the only significant part.

Then if $\Delta L \ll L$ we get $L + \Delta L \approx L$ and

$$t = \sqrt{\frac{m(L+\Delta L)}{k\Delta L}} \approx \sqrt{\frac{mL}{k\Delta L}},$$

so that t is proportional to $1/\sqrt{\Delta L}$.

But if $\Delta L \gg L$ we get $L + \Delta L \approx \Delta L$ and

$$t = \sqrt{\frac{m(L+\Delta L)}{k\Delta L}} \approx \sqrt{\frac{m\Delta L}{k\Delta L}} = \sqrt{\frac{m}{k}},$$

so that t is constant.

P18-8 (a) The tension in the rope at some point is a function of the weight of the cable beneath it. If the bottom of the rope is $y = 0$, then the weight beneath some point y is $W = y(m/L)g$. The speed of the wave at that point is $v = \sqrt{T/(m/L)} = \sqrt{y(M/L)g/(m/L)} = \sqrt{gy}$.

(b) $dy/dt = \sqrt{gy}$, so

$$\begin{aligned} dt &= \frac{dy}{\sqrt{gy}}, \\ t &= \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{L/g}. \end{aligned}$$

(c) No.

P18-9 (a) $M = \int \mu\, dx$, so

$$M = \int_0^L kx\, dx = \frac{1}{2}kL^2.$$

Then $k = 2M/L^2$.

(b) $v = \sqrt{F/\mu} = \sqrt{F/kx}$, then

$$\begin{aligned} dt &= \sqrt{kx/F}\, dx, \\ t &= \int_0^L \sqrt{2M/FL^2}\sqrt{x}\, dx = \frac{2}{3}\sqrt{2M/FL^2}L^{3/2} = \sqrt{8ML/9F}. \end{aligned}$$

P18-10 Take a cue from pressure and surface tension. In the rotating *non-inertial* reference frame for which the hoop appears to be at rest there is an effective force per unit length acting to push on each part of the loop directly away from the center. This force per unit length has magnitude

$$\frac{\Delta F}{\Delta L} = (\Delta m/\Delta L)\frac{v^2}{r} = \mu\frac{v^2}{r}.$$

There must be a tension T in the string to hold the loop together. Imagine the loop to be replaced with two semicircular loops. Each semicircular loop has a diameter part; the force tending to pull off the diameter section is $(\Delta F/\Delta L)2r = 2\mu v^2$. There are two connections to the diameter section, so the tension in the string must be half the force on the diameter section, or $T = \mu v^2$.

The wave speed is $v_{\rm w} = \sqrt{T/\mu} = v$.

Note that the wave on the string can travel in either direction relative to an inertial observer. One wave will appear to be fixed in space; the other will move around the string with twice the speed of the string.

$\boxed{\textbf{P18-11}}$ If we assume that Handel wanted his violins to play in tune with the other instruments then all we need to do is find an instrument from Handel's time that will accurately keep pitch over a period of several hundred years. Most instruments won't keep pitch for even a few days because of temperature and humidity changes; some (like the piccolo?) can't even play in tune for more than a few notes! But if someone found a tuning fork...

Since the length of the string doesn't change, and we are using a string with the same mass density, the only choice is to change the tension. But $f \propto v \propto \sqrt{T}$, so the percentage change in the tension of the string is

$$\frac{T_{\rm f} - T_{\rm i}}{T_{\rm i}} = \frac{{f_{\rm f}}^2 - {f_{\rm i}}^2}{{f_{\rm i}}^2} = \frac{(440\ {\rm Hz})^2 - (422.5\ {\rm Hz})^2}{(422.5\ {\rm Hz})^2} = 8.46\,\%.$$

P18-12

P18-13 (a) The point sources emit spherical waves; the solution to the appropriate wave equation is found in Ex. 18-14:

$$y_i = \frac{A}{r_i}\sin(kr_i - \omega t).$$

If r_i is sufficiently large compared to A, and $r_1 \approx r_2$, then let $r_1 = r - \delta r$ and $r_2 = r - \delta r$;

$$\frac{A}{r_1} + \frac{A}{r_2} \approx \frac{2A}{r},$$

with an error of order $(\delta r/r)^2$. So ignore it.

Then

$$\begin{aligned} y_1 + y_2 &\approx \frac{A}{r}\left[\sin(kr_1 - \omega t) + \sin(kr_2 - \omega t)\right], \\ &= \frac{2A}{r}\sin(kr - \omega t)\cos\frac{k}{2}(r_1 - r_2), \\ y_m &= \frac{2A}{r}\cos\frac{k}{2}(r_1 - r_2). \end{aligned}$$

(b) A maximum (minimum) occurs when the operand of the cosine, $k(r_1 - r_2)/2$ is an integer multiple of π (a half odd-integer multiple of π)

P18-14 The direct wave travels a distance d from S to D. The wave which reflects off the original layer travels a distance $\sqrt{d^2 + 4H^2}$ between S and D. The wave which reflects off the layer after it has risen a distance h travels a distance $\sqrt{d^2 + 4(H+h)^2}$. Waves will interfere constructively if there is a difference of an integer number of wavelengths between the two path lengths. In other words originally we have

$$\sqrt{d^2 + 4H^2} - d = n\lambda,$$

and later we have destructive interference so

$$\sqrt{d^2 + 4(H+h)^2} - d = (n + 1/2)\lambda.$$

We don't know n, but we can subtract the top equation from the bottom and get

$$\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} = \lambda/2$$

P18-15 The wavelength is

$$\lambda = v/f = (3.00\times 10^8\mathrm{m/s})/(13.0\times 10^6\mathrm{Hz}) = 23.1\,\mathrm{m}.$$

The direct wave travels a distance d from S to D. The wave which reflects off the original layer travels a distance $\sqrt{d^2 + 4H^2}$ between S and D. The wave which reflects off the layer one minute later travels a distance $\sqrt{d^2 + 4(H+h)^2}$. Waves will interfere constructively if there is a difference of an integer number of wavelengths between the two path lengths. In other words originally we have

$$\sqrt{d^2 + 4H^2} - d = n_1\lambda,$$

and then one minute later we have

$$\sqrt{d^2 + 4(H+h)^2} - d = n_2\lambda.$$

We don't know either n_1 or n_2, but we do know the difference is 6, so we can subtract the top equation from the bottom and get

$$\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} = 6\lambda$$

We could use that expression as written, do some really obnoxious algebra, and then get the answer. But we don't want to; we want to take advantage of the fact that h is small compared to d and H. Then the first term can be written as

$$\begin{aligned}
\sqrt{d^2 + 4(H+h)^2} &= \sqrt{d^2 + 4H^2 + 8Hh + 4h^2}, \\
&\approx \sqrt{d^2 + 4H^2 + 8Hh}, \\
&\approx \sqrt{d^2 + 4H^2}\sqrt{1 + \frac{8H}{d^2 + 4H^2}h}, \\
&\approx \sqrt{d^2 + 4H^2}\left(1 + \frac{1}{2}\frac{8H}{d^2 + 4H^2}h\right).
\end{aligned}$$

Between the second and the third lines we factored out $d^2 + 4H^2$; that last line is from the binomial expansion theorem. We put this into the previous expression, and

$$\begin{aligned}
\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} &= 6\lambda, \\
\sqrt{d^2 + 4H^2}\left(1 + \frac{4H}{d^2 + 4H^2}h\right) - \sqrt{d^2 + 4H^2} &= 6\lambda, \\
\frac{4H}{\sqrt{d^2 + 4H^2}}h &= 6\lambda.
\end{aligned}$$

Now what were we doing? We were trying to find the speed at which the layer is moving. We know H, d, and λ; we can then find h,

$$h = \frac{6(23.1\,\mathrm{m})}{4(510\times10^3\mathrm{m})}\sqrt{(230\times10^3\mathrm{m})^2 + 4(510\times10^3\mathrm{m})^2} = 71.0\,\mathrm{m}.$$

The layer is then moving at $v = (71.0\,\mathrm{m})/(60\,\mathrm{s}) = 1.18\,\mathrm{m/s}$.

P18-16 The equation of the standing wave is

$$y = 2y_\mathrm{m}\sin kx\cos\omega t.$$

The transverse speed of a point on the string is the derivative of this, or

$$u_y = -2y_\mathrm{m}\omega\sin kx\sin\omega t,$$

this has a maximum value when $\omega t - \pi/2$ is a integer multiple of π. The maximum value is

$$u_\mathrm{m} = 2y_\mathrm{m}\omega\sin kx.$$

Each mass element on the string dm then has a maximum kinetic energy

$$dK_\mathrm{m} = (dm/2){u_\mathrm{m}}^2 = {y_\mathrm{m}}^2\omega^2\sin^2 kx\ dm.$$

Using $dm = \mu dx$, and integrating over one loop from $kx = 0$ to $kx = \pi$, we get

$$K_\mathrm{m} = {y_\mathrm{m}}^2\omega^2\mu/2k = 2\pi^2{y_\mathrm{m}}^2 f\mu v.$$

P18-17 (a) For 100% reflection the amplitudes of the incident and reflected wave are equal, or $A_i = A_r$, which puts a zero in the denominator of the equation for SWR. If there is no reflection, $A_r = 0$ leaving the expression for SWR to reduce to $A_i/A_i = 1$.

(b) $P_r/P_i = A_r^2/A_i^2$. Do the algebra:

$$\begin{aligned} \frac{A_i + A_r}{A_i - A_r} &= \mathrm{SWR}, \\ A_i + A_r &= \mathrm{SWR}(A_i - A_r), \\ A_r(\mathrm{SWR}+1) &= A_i(\mathrm{SWR}-1), \\ A_r/A_i &= (\mathrm{SWR}-1)/(\mathrm{SWR}+1). \end{aligned}$$

Square this, and multiply by 100.

P18-18 Measure with a ruler; I get $2A_\mathrm{max} = 1.1$ cm and $2A_\mathrm{min} = 0.5$ cm.

(a) $\mathrm{SWR} = (1.1/0.5) = 2.2$

(b) $(2.2-1)^2/(2.2+1)^2 = 0.14\,\%$.

$\boxed{\textbf{P18-19}}$ (a) Call the three waves

$$\begin{aligned} y_\mathrm{i} &= A\sin k_1(x - v_1 t), \\ y_\mathrm{t} &= B\sin k_2(x - v_2 t), \\ y_\mathrm{r} &= C\sin k_1(x + v_1 t), \end{aligned}$$

where the subscripts i, t, and r refer to the incident, transmitted, and reflected waves respectively.

Apply the principle of superposition. Just to the left of the knot the wave has amplitude $y_\mathrm{i} + y_\mathrm{r}$ while just to the right of the knot the wave has amplitude y_t. These two amplitudes must line up at the knot for all times t, or the knot will come undone. Remember the knot is at $x = 0$, so

$$\begin{aligned} y_\mathrm{i} + y_\mathrm{r} &= y_\mathrm{t}, \\ A \sin k_1(-v_1 t) + C \sin k_1(+v_1 t) &= B \sin k_2(-v_2 t), \\ -A \sin k_1 v_1 t + C \sin k_1 v_1 t &= -B \sin k_2 v_2 t \end{aligned}$$

We know that $k_1 v_1 = k_2 v_2 = \omega$, so the three sin functions are all equivalent, and can be canceled. This leaves $A = B + C$.

(b) We need to match more than the displacement, we need to match the slope just on either side of the knot. In that case we need to take the derivative of

$$y_\mathrm{i} + y_\mathrm{r} = y_\mathrm{t}$$

with respect to x, and then set $x = 0$. First we take the derivative,

$$\begin{aligned} \frac{d}{dx}\left(y_\mathrm{i} + y_\mathrm{r}\right) &= \frac{d}{dx}\left(y_\mathrm{t}\right), \\ k_1 A \cos k_1(x - v_1 t) + k_1 C \cos k_1(x + v_1 t) &= k_2 B \cos k_2(x - v_2 t), \end{aligned}$$

and then we set $x = 0$ and simplify,

$$\begin{aligned} k_1 A \cos k_1(-v_1 t) + k_1 C \cos k_1(+v_1 t) &= k_2 B \cos k_2(-v_2 t), \\ k_1 A \cos k_1 v_1 t + k_1 C \cos k_1 v_1 t &= k_2 B \cos k_2 v_2 t. \end{aligned}$$

This last expression simplifies like the one in part (a) to give

$$k_1(A + C) = k_2 B$$

We can combine this with $A = B + C$ to solve for C,

$$\begin{aligned} k_1(A + C) &= k_2(A - C), \\ C(k_1 + k_2) &= A(k_2 - k_1), \\ C &= A\frac{k_2 - k_1}{k_1 + k_2}. \end{aligned}$$

If $k_2 < k_1$ C will be negative; this means the reflected wave will be inverted.

P18-20

P18-21 Find the wavelength from

$$\lambda = 2(0.924\,\mathrm{m})/4 = 0.462\,\mathrm{m}.$$

Find the wavespeed from

$$v = f\lambda = (60.0\text{ Hz})(0.462\,\mathrm{m}) = 27.7\,\mathrm{m/s}.$$

Find the tension from

$$F = \mu v^2 = (0.0442\,\mathrm{kg})(27.7\,\mathrm{m/s})^2/(0.924\,\mathrm{m}) = 36.7\,\mathrm{N}.$$

P18-22 (a) The frequency of vibration f is the same for both the aluminum and steel wires; they don not, however, need to vibrate in the same mode. The speed of waves in the aluminum is v_1, that in the steel is v_2. The aluminum vibrates in a mode given by $n_1 = 2L_1 f/v_1$, the steel vibrates in a mode given by $n_2 = 2L_2 f/v_2$. Both n_1 and n_2 need be integers, so the ratio must be a rational fraction. Note that the ratio is independent of f, so that L_1 and L_2 must be chosen correctly for this problem to work at all!

This ratio is

$$\frac{n_2}{n_1} = \frac{L_2}{L_1}\sqrt{\frac{\mu_2}{\mu_1}} = \frac{(0.866\,\mathrm{m})}{(0.600\,\mathrm{m})}\sqrt{\frac{(7800\,\mathrm{kg/m^3})}{(2600\,\mathrm{kg/m^3})}} = 2.50 \approx \frac{5}{2}$$

Note that since the wires have the same tension and the same cross sectional area it is acceptable to use the volume density instead of the linear density in the problem.

The smallest integer solution is then $n_1 = 2$ and $n_2 = 5$. The frequency of vibration is then

$$f = \frac{n_1 v}{2L_1} = \frac{n_1}{2L_1}\sqrt{\frac{T}{\rho_1 A}} = \frac{(2)}{2(0.600\,\mathrm{m})}\sqrt{\frac{(10.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{(2600\,\mathrm{kg/m^3})(1.00\times 10^{-6}\mathrm{m^2})}} = 323\ \mathrm{Hz}.$$

(b) There are three nodes in the aluminum and six in the steel. But one of those nodes is shared, and two are on the ends of the wire. The answer is then six.

E19-1 (a) $v = f\lambda = (25\text{ Hz})(0.24\,\text{m}) = 6.0\,\text{m/s}$.

(b) $k = (2\pi\text{ rad})/(0.24\,\text{m}) = 26\text{ rad/m}$; $\omega = (2\pi\text{ rad})(25\text{ Hz}) = 160\text{ rad/s}$. The wave equation is

$$s = (3.0\times10^{-3}\text{m})\sin[(26\text{ rad/m})x + (160\text{ rad/s})t]$$

E19-2 (a) $[\Delta P]_\text{m} = 1.48\,\text{Pa}$.

(b) $f = (334\pi\text{ rad/s})/(2\pi\text{ rad}) = 167\text{ Hz}$.

(c) $\lambda = (2\pi\text{ rad})/(1.07\pi\text{ rad/m}) = 1.87\,\text{m}$.

(d) $v = (167\text{ Hz})(1.87\,\text{m}) = 312\,\text{m/s}$.

$\boxed{\textbf{E19-3}}$ (a) The wavelength is given by $\lambda = v/f = (343\,\text{m/s})/(4.50\times10^6\text{Hz} = 7.62\times10^{-5}\text{m}$.

(b) The wavelength is given by $\lambda = v/f = (1500\,\text{m/s})/(4.50\times10^6\text{Hz} = 3.33\times10^{-4}\text{m}$.

E19-4 Note: There is a typo; the mean free path should have been measured in "μm" instead of "pm".

$\lambda_\text{min} = 1.0\times10^{-6}\text{m}$; $f_\text{max} = (343\,\text{m/s})/(1.0\times10^{-6}\text{m}) = 3.4\times10^8\text{Hz}$.

E19-5 (a) $\lambda = (240\,\text{m/s})/(4.2\times10^9\text{Hz}) = 5.7\times10^{-8}\text{m}$.

E19-6 (a) The speed of sound is

$$v = (331\,\text{m/s})(6.21\times10^{-4}\text{ mi/m}) = 0.206\text{ mi/s}.$$

In five seconds the sound travels $(0.206\text{ mi/s})(5.0\,\text{s}) = 1.03$ mi, which is 3% too large.

(b) Count seconds and divide by 3.

$\boxed{\textbf{E19-7}}$ Marching at 120 paces per minute means that you move a foot every half a second. The soldiers in the back are moving the wrong foot, which means they are moving the correct foot half a second later than they should. If the speed of sound is 343 m/s, then the column of soldiers must be $(343\,\text{m/s})(0.5\,\text{s}) = 172\,\text{m}$ long.

E19-8 It takes $(300\,\text{m})/(343\,\text{m/s}) = 0.87\,\text{s}$ for the concert goer to hear the music after it has passed the microphone. It takes $(5.0\times10^6\text{m})/(3.0\times10^8\text{m/s}) = 0.017\,\text{s}$ for the radio listener to hear the music after it has passed the microphone. The radio listener hears the music first, 0.85 s before the concert goer.

E19-9 $x/v_\text{P} = t_\text{P}$ and $x/v_\text{S} = t_\text{S}$; subtracting and rearranging,

$$x = \Delta t/[1/v_\text{S} - 1/v_\text{P}] = (180\,\text{s})/[1/(4.5\text{ km/s}) - 1/(8.2\text{ km/s})] = 1800\text{ km}.$$

E19-10 Use Eq. 19-8, $s_\text{m} = [\Delta p]_\text{m}/kB$, and Eq. 19-14, $v = \sqrt{B/\rho_0}$. Then

$$[\Delta p]_\text{m} = kBs_\text{m} = kv^2\rho_0 s_\text{m} = 2\pi f v\rho_0 s_\text{m}.$$

Insert into Eq. 19-18, and

$$I = 2\pi^2\rho v f^2 {s_\text{m}}^2.$$

$\boxed{\textbf{E19-11}}$ If the source emits equally in all directions the intensity at a distance r is given by the average power divided by the surface area of a sphere of radius r centered on the source.

The power output of the source can then be found from

$$P = IA = I(4\pi r^2) = (197\times10^{-6}\text{W/m}^2)4\pi(42.5\,\text{m})^2 = 4.47\,\text{W}.$$

E19-12 Use the results of Exercise 19-10.

$$s_{\rm m} = \sqrt{\frac{(1.13\times 10^{-6}{\rm W/m^2})}{2\pi^2(1.21\,{\rm kg/m^3})(343\,{\rm m/s})(313\ {\rm Hz})^2}} = 3.75\times 10^{-8}{\rm m}.$$

E19-13 $U = IAt = (1.60\times 10^{-2}{\rm W/m^2})(4.70\times 10^{-4}{\rm m^2})(3600\,{\rm s}) = 2.71\times 10^{-2}{\rm W}.$

E19-14 Invert Eq. 19-21:

$$I_1/I_2 = 10^{(1.00{\rm dB})/10} = 1.26.$$

E19-15 (a) Relative sound level is given by Eq. 19-21,

$$SL_1 - SL_2 = 10\log\frac{I_1}{I_2} \text{ or } \frac{I_1}{I_2} = 10^{(SL_1 - SL_2)/10},$$

so if $\Delta SL = 30$ then $I_1/I_2 = 10^{30/10} = 1000$.

(b) Intensity is proportional to pressure amplitude squared according to Eq. 19-19; so

$$\Delta p_{\rm m,1}/\Delta p_{\rm m,2} = \sqrt{I_1/I_2} = \sqrt{1000} = 32.$$

E19-16 We know where her ears hurt, so we know the intensity at that point. The power output is then

$$P = 4\pi(1.3\,{\rm m})^2(1.0\,{\rm W/m^2}) = 21\,{\rm W}.$$

This is less than the advertised power.

E19-17 Use the results of Exercise 18-18, $I = uv$. The intensity is

$$I = (5200\,{\rm W})/4\pi(4820\,{\rm m})^2 = 1.78\times 10^{-5}{\rm W/m^2},$$

so the energy density is

$$u = I/v = (1.78\times 10^{-5}{\rm W/m^2})/(343\,{\rm m/s}) = 5.19\times 10^{-8}{\rm J/m^3}.$$

E19-18 $I_2 = 2I_1$, since $I \propto 1/r^2$ then $r_1^2 = 2r_2^2$. Then

$$\begin{aligned} D &= \sqrt{2}(D - 51.4\,{\rm m}), \\ D(\sqrt{2} - 1) &= \sqrt{2}(51.4\,{\rm m}), \\ D &= 176\,{\rm m}. \end{aligned}$$

E19-19 The sound level is given by Eq. 19-20,

$$SL = 10\log\frac{I}{I_0}$$

where I_0 is the threshold intensity of 10^{-12} W/m^2. Intensity is given by Eq. 19-19,

$$I = \frac{(\Delta p_{\rm m})^2}{2\rho v}$$

If we assume the maximum possible pressure amplitude is equal to one atmosphere, then

$$I = \frac{(\Delta p_{\rm m})^2}{2\rho v} = \frac{(1.01\times 10^5{\rm Pa})^2}{2(1.21\,{\rm kg/m^3})(343\,{\rm m/s})} = 1.22\times 10^7{\rm W/m^2}.$$

The sound level would then be

$$SL = 10 \log \frac{I}{I_0} = 10 \log \frac{1.22\times10^7 \mathrm{W/m^2}}{(10^{-12}\ \mathrm{W/m^2})} = 191\ \mathrm{dB}$$

E19-20 Let one person speak with an intensity I_1. N people would have an intensity NI_1. The ratio is N, so by inverting Eq. 19-21,

$$N = 10^{(15\mathrm{dB})/10} = 31.6,$$

so 32 people would be required.

E19-21 Let one leaf rustle with an intensity I_1. N leaves would have an intensity NI_1. The ratio is N, so by Eq. 19-21,

$$SL_N = (8.4\ \mathrm{dB}) + 10 \log(2.71\times10^5) = 63\ \mathrm{dB}.$$

E19-22 Ignoring the finite time means that we can assume the sound waves travels vertically, which considerably simplifies the algebra.

The intensity ratio can be found by inverting Eq. 19-21,

$$I_1/I_2 = 10^{(30\mathrm{dB})/10} = 1000.$$

But intensity is proportional to the inverse distance squared, so $I_1/I_2 = (r_2/r_1)^2$, or

$$r_2 = (115\,\mathrm{m})\sqrt{(1000)} = 3640\,\mathrm{m}.$$

$\boxed{\textbf{E19-23}}$ A minimum will be heard at the detector if the path length difference between the straight path and the path through the curved tube is half of a wavelength. Both paths involve a straight section from the source to the start of the curved tube, and then from the end of the curved tube to the detector. Since it is the path difference that matters, we'll only focus on the part of the path between the start of the curved tube and the end of the curved tube. The length of the straight path is one diameter, or $2r$. The length of the curved tube is half a circumference, or πr. The difference is $(\pi - 2)r$. This difference is equal to half a wavelength, so

$$\begin{aligned} (\pi - 2)r &= \lambda/2, \\ r &= \frac{\lambda}{2\pi - 4} = \frac{(42.0\ \mathrm{cm})}{2\pi - 4} = 18.4\ \mathrm{cm}. \end{aligned}$$

E19-24 The path length difference here is

$$\sqrt{(3.75\,\mathrm{m})^2 + (2.12\,\mathrm{m})^2} - (3.75\,\mathrm{m}) = 0.5578\,\mathrm{m}.$$

(a) A minimum will occur if this is equal to a half integer number of wavelengths, or $(n-1/2)\lambda = 0.5578\,\mathrm{m}$. This will occur when

$$f = (n - 1/2)\frac{(343\,\mathrm{m/s})}{(0.5578\,\mathrm{m})} = (n - 1/2)(615\ \mathrm{Hz}).$$

(b) A maximum will occur if this is equal to an integer number of wavelengths, or $n\lambda = 0.5578\,\mathrm{m}$. This will occur when

$$f = n\frac{(343\,\mathrm{m/s})}{(0.5578\,\mathrm{m})} = n(615\ \mathrm{Hz}).$$

E19-25 The path length difference here is

$$\sqrt{(24.4\,\text{m} + 6.10\,\text{m})^2 + (15.2\,\text{m})^2} - \sqrt{(24.4\,\text{m})^2 + (15.2\,\text{m})^2} = 5.33\,\text{m}.$$

A maximum will occur if this is equal to an integer number of wavelengths, or $n\lambda = 5.33\,\text{m}$. This will occur when

$$f = n(343\,\text{m/s})/(5.33\,\text{m}) = n(64.4\text{ Hz})$$

The two lowest frequencies are then 64.4 Hz and 129 Hz.

E19-26 The wavelength is $\lambda = (343\,\text{m/s})/(300\text{ Hz}) = 1.143\,\text{m}$. This means that the sound maxima will be half of this, or $0.572\,\text{m}$ apart. Directly in the center the path length difference is zero, but since the waves are out of phase, this will be a minimum. The maxima should be located on either side of this, a distance $(0.572\,\text{m})/2 = 0.286\,\text{m}$ from the center. There will then be maxima located each $0.572\,\text{m}$ farther along.

E19-27 (a) $f_1 = v/2L$ and $f_2 = v/2(L - \Delta L)$. Then

$$\frac{1}{r} = \frac{f_1}{f_2} = \frac{L - \Delta L}{L} = 1 - \frac{\Delta L}{L},$$

or $\Delta L = L(1 - 1/r)$.

(b) The answers are $\Delta L = (0.80\,\text{m})(1 - 5/6) = 0.133\,\text{m}$; $\Delta L = (0.80\,\text{m})(1 - 4/5) = 0.160\,\text{m}$; $\Delta L = (0.80\,\text{m})(1 - 3/4) = 0.200\,\text{m}$; and $\Delta L = (0.80\,\text{m})(1 - 2/3) = 0.267\,\text{m}$.

E19-28 The wavelength is twice the distance between the nodes in this case, so $\lambda = 7.68$ cm. The frequency is

$$f = (1520\,\text{m/s})/(7.68\times10^{-2}\text{m}) = 1.98\times10^{4}\text{Hz}.$$

$\boxed{\textbf{E19-29}}$ The well is a tube open at one end and closed at the other; Eq. 19-28 describes the allowed frequencies of the resonant modes. The lowest frequency is when $n = 1$, so $f_1 = v/4L$. We know f_1; to find the depth of the well, L, we need to know the speed of sound.

We should use the information provided, instead of looking up the speed of sound, because maybe the well is filled with some kind of strange gas.

Then, from Eq. 19-14,

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{(1.41\times10^5\text{ Pa})}{(1.21\,\text{kg/m}^3)}} = 341\,\text{m/s}.$$

The depth of the well is then

$$L = v/(4f_1) = (341\,\text{m/s})/[4(7.20\text{ Hz})] = 11.8\,\text{m}.$$

E19-30 (a) The resonant frequencies of the pipe are given by $f_n = nv/2L$, or

$$f_n = n(343\,\text{m/s})/2(0.457\,\text{m}) = n(375\text{ Hz}).$$

The lowest frequency in the specified range is $f_3 = 1130$ Hz; the other allowed frequencies in the specified range are $f_4 = 1500$ Hz, and $f_5 = 1880$ Hz.

$\boxed{\textbf{E19-31}}$ The maximum reflected frequencies will be the ones that undergo constructive interference, which means the path length difference will be an integer multiple of a wavelength. A wavefront will strike a terrace wall and part will reflect, the other part will travel on to the next terrace, and then reflect. Since part of the wave had to travel to the next terrace and back, the path length difference will be $2 \times 0.914\,\mathrm{m} = 1.83\,\mathrm{m}$.

If the speed of sound is $v = 343\,\mathrm{m/s}$, the lowest frequency wave which undergoes constructive interference will be

$$f = \frac{v}{\lambda} = \frac{(343\,\mathrm{m/s})}{(1.83\,\mathrm{m})} = 187 \text{ Hz}$$

Any integer multiple of this frequency will also undergo constructive interference, and will also be heard. The ear and brain, however, will most likely interpret the complex mix of frequencies as a single tone of frequency 187 Hz.

E19-32 Assume there is no frequency between these two that is amplified. Then one of these frequencies is $f_n = nv/2L$, and the other is $f_{n+1} = (n+1)v/2L$. Subtracting the larger from the smaller, $\Delta f = v/2L$, or

$$L = v/2\Delta f = (343\,\mathrm{m/s})/2(138 \text{ Hz} - 135 \text{ Hz}) = 57.2\,\mathrm{m}.$$

E19-33 (a) $v = 2Lf = 2(0.22\,\mathrm{m})(920 \text{ Hz}) = 405\,\mathrm{m/s}$.
(b) $F = v^2\mu = (405\,\mathrm{m/s})^2(820\times 10^{-6}\mathrm{kg})/(0.220\,\mathrm{m}) = 611\,\mathrm{N}$.

E19-34 $f \propto v$, and $v \propto \sqrt{F}$, so $f \propto \sqrt{F}$. Doubling f requires F increase by a factor of 4.

$\boxed{\textbf{E19-35}}$ The speed of a wave on the string is the same, regardless of where you put your finger, so $f\lambda$ is a constant. The string will vibrate (mostly) in the lowest harmonic, so that $\lambda = 2L$, where L is the length of the part of the string that is allowed to vibrate. Then

$$\begin{aligned} f_2\lambda_2 &= f_1\lambda_1, \\ 2f_2L_2 &= 2f_1L_1, \\ L_2 &= L_1\frac{f_1}{f_2} = (30 \text{ cm})\frac{(440 \text{ Hz})}{(528 \text{ Hz})} = 25 \text{ cm}. \end{aligned}$$

So you need to place your finger 5 cm from the end.

E19-36 The open organ pipe has a length

$$L_\mathrm{o} = v/2f_1 = (343\,\mathrm{m/s})/2(291 \text{ Hz}) = 0.589\,\mathrm{m}.$$

The second harmonic of the open pipe has frequency $2f_1$; this is the first overtone of the closed pipe, so the closed pipe has a length

$$L_\mathrm{c} = (3)v/4(2f_1) = (3)(343\,\mathrm{m/s})/4(2)(291 \text{ Hz}) = 0.442\,\mathrm{m}.$$

$\boxed{\textbf{E19-37}}$ The unknown frequency is either 3 Hz higher or lower than the standard fork. A small piece of wax placed on the fork of this unknown frequency tuning fork will result in a lower frequency because $f \propto \sqrt{k/m}$. If the beat frequency decreases then the two tuning forks are getting *closer* in frequency, so the frequency of the first tuning fork must be above the frequency of the standard fork. Hence, 387 Hz.

E19-38 If the string is too taut then the frequency is too high, or $f = (440 + 4)$Hz. Then $T = 1/f = 1/(444\text{ Hz}) = 2.25\times 10^{-3}$s.

E19-39 One of the tuning forks need to have a frequency 1 Hz different from another. Assume then one is at 501 Hz. The next fork can be played against the first or the second, so it could have a frequency of 503 Hz to pick up the 2 and 3 Hz beats. The next one needs to pick up the 5, 7, and 8 Hz beats, and 508 Hz will do the trick. There are other choices.

E19-40 $f = v/\lambda = (5.5\,\text{m/s})/(2.3\,\text{m}) = 2.39$ Hz. Then

$$f' = f(v + v_O)/v = (2.39\text{ Hz})(5.5\,\text{m/s} + 3.3\,\text{m/s})/(5.5\,\text{m/s}) = 3.8\text{ Hz}.$$

$\boxed{\textbf{E19-41}}$ We'll use Eq. 19-44, since both the observer and the source are in motion. Then

$$f' = f\frac{v \pm v_O}{v \mp v_S} = (15.8\text{ kHz})\frac{(343\,\text{m/s}) + (246\,\text{m/s})}{(343\,\text{m/s}) + (193\,\text{m/s})} = 17.4\text{ kHz}r$$

E19-42 Solve Eq. 19-44 for v_S;

$$v_S = (v + v_O)f/f' - v = (343\,\text{m/s} + 2.63\,\text{m/s})(1602\text{ Hz})/(1590\text{ Hz}) - (343\,\text{m/s}) = 5.24\,\text{m/s}.$$

E19-43 $v_S = (14.7\text{ Rad/s})(0.712\,\text{m}) = 10.5\,\text{m/s}$.

(a) The low frequency heard is

$$f' = (538\text{ Hz})(343\,\text{m/s})/(343\,\text{m/s} + 10.5\,\text{m/s}) = 522\text{ Hz}.$$

(a) The high frequency heard is

$$f' = (538\text{ Hz})(343\,\text{m/s})/(343\,\text{m/s} - 10.5\,\text{m/s}) = 555\text{ Hz}.$$

E19-44 Solve Eq. 19-44 for v_S;

$$v_S = v - vf/f' = (343\,\text{m/s}) - (343\,\text{m/s})(440\text{ Hz})/(444\text{ Hz}) = 3.1\,\text{m/s}.$$

E19-45

E19-46 The approaching car "hears"

$$f' = f\frac{v + v_O}{v - v_S} = (148\text{ Hz})\frac{(343\,\text{m/s}) + (44.7\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 167\text{ Hz}$$

This sound is reflected back at the same frequency, so the police car "hears"

$$f' = f\frac{v + v_O}{v - v_S} = (167\text{ Hz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) - (44.7\,\text{m/s})} = 192\text{ Hz}$$

E19-47 The departing intruder "hears"

$$f' = f\frac{v - v_O}{v + v_S} = (28.3\text{ kHz})\frac{(343\,\text{m/s}) - (0.95\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 28.22\text{ kHz}$$

This sound is reflected back at the same frequency, so the alarm "hears"

$$f' = f\frac{v - v_O}{v + v_S} = (28.22\text{ kHz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) + (0.95\,\text{m/s})} = 28.14\text{ kHz}$$

The beat frequency is 28.3 kHz − 28.14 kHz = 160 Hz.

E19-48 (a) $f' = (1000 \text{ Hz})(330\,\text{m/s})(330\,\text{m/s} + 10.0\,\text{m/s}) = 971 \text{ Hz}$.
(b) $f' = (1000 \text{ Hz})(330\,\text{m/s})(330\,\text{m/s} - 10.0\,\text{m/s}) = 1030 \text{ Hz}$.
(c) $1030 \text{ Hz} - 971 \text{ Hz} = 59 \text{ Hz}$.

$\boxed{\textbf{E19-49}}$ (a) The frequency "heard" by the wall is

$$f' = f\frac{v + v_O}{v - v_S} = (438 \text{ Hz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) - (19.3\,\text{m/s})} = 464 \text{ Hz}$$

(b) The wall then reflects a frequency of 464 Hz back to the trumpet player. Sticking with Eq. 19-44, the source is now at rest while the observer moving,

$$f' = f\frac{v + v_O}{v - v_S} = (464 \text{ Hz})\frac{(343\,\text{m/s}) + (19.3\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 490 \text{ Hz}$$

E19-50 The body part "hears"

$$f' = f\frac{v + v_b}{v}.$$

This sound is reflected back to the detector which then "hears"

$$f'' = f'\frac{v}{v - v_b} = f\frac{v + v_b}{v - v_b}.$$

Rearranging,

$$v_b/v = \frac{f'' - f}{f'' + f} \approx \frac{1}{2}\frac{\Delta f}{f},$$

so $v \approx 2(1\times10^{-3}\text{m/s})/(1.3\times10^{-6}) \approx 1500\,\text{m/s}$.

E19-51 The wall "hears"

$$f' = f\frac{v + v_O}{v - v_S} = (39.2 \text{ kHz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) - (8.58\,\text{m/s})} = 40.21 \text{ kHz}$$

This sound is reflected back at the same frequency, so the bat "hears"

$$f' = f\frac{v + v_O}{v - v_S} = (40.21 \text{ kHz})\frac{(343\,\text{m/s}) + (8.58\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 41.2 \text{ kHz}.$$

P19-1 (a) $t_{\text{air}} = L/v_{\text{air}}$ and $t_{\text{m}} = L/v$, so the difference is

$$\Delta t = L(1/v_{\text{air}} - 1/v)$$

(b) Rearrange the above result, and

$$L = (0.120\,\text{s})/[1/(343\,\text{m/s}) - 1/(6420\,\text{m/s})] = 43.5\,\text{m}.$$

P19-2 The stone falls for a time t_1 where $y = gt_1^2/2$ is the depth of the well. Note y is positive in this equation. The sound travels back in a time t_2 where $v = y/t_2$ is the speed of sound in the well. $t_1 + t_2 = 3.00\,\text{s}$, so

$$2y = g(3.00\,\text{s} - t_2)^2 = g[(9.00\,\text{s}^2) - (6.00\,\text{s})y/v + y^2/v^2],$$

or, using $g = 9.81\,\text{m/s}^2$ and $v = 343\,\text{m/s}$,

$$y^2 - (2.555\times10^5\,\text{m})y + (1.039\times10^7\,\text{m}^2) = 0,$$

which has a positive solution $y = 40.7\,\text{m}$.

$\boxed{\textbf{P19-3}}$ (a) The intensity at 28.5 m is found from the $1/r^2$ dependence;

$$I_2 = I_1(r_1/r_2)^2 = (962\,\mu\text{W/m}^2)(6.11\,\text{m}/28.5\,\text{m})^2 = 44.2\,\mu\text{W/m}^2.$$

(c) We'll do this part first. The pressure amplitude is found from Eq. 19-19,

$$\Delta p_\text{m} = \sqrt{2\rho v I} = \sqrt{2(1.21\,\text{kg/m}^3)(343\,\text{m/s})(962\times10^{-6}\text{W/m}^2)} = 0.894\,\text{Pa}.$$

(b) The displacement amplitude is found from Eq. 19-8,

$$s_\text{m} = \Delta p_\text{m}/(kB),$$

where $k = 2\pi f/v$ is the wave number. From Eq. 19-14 w know that $B = \rho v^2$, so

$$s_\text{m} = \frac{\Delta p_\text{m}}{2\pi f \rho v} = \frac{(0.894\,\text{Pa})}{2\pi(2090\text{ Hz})(1.21\,\text{kg/m}^3)(343\,\text{m/s})} = 1.64\times10^{-7}\text{m}.$$

P19-4 (a) If the intensities are equal, then $\Delta p_\text{m} \propto \sqrt{\rho v}$, so

$$\frac{[\Delta p_\text{m}]_\text{water}}{[\Delta p_\text{m}]_\text{air}} = \sqrt{\frac{(998\,\text{kg/m}^3)(1482\,\text{m/s})}{(1.2\,\text{kg/m}^3)(343\,\text{m/s})}} = 59.9.$$

(b) If the pressure amplitudes are equal, then $I \propto\propto 1/\rho v$, so

$$\frac{I_\text{water}}{I_\text{air}} = \frac{(1.2\,\text{kg/m}^3)(343\,\text{m/s})}{(998\,\text{kg/m}^3)(1482\,\text{m/s})} = 2.78\times10^{-4}.$$

P19-5 The energy is dissipated on a cylindrical surface which grows in area as r, so the intensity is proportional to $1/r$. The amplitude is proportional to the square root of the intensity, so $s_\text{m} \propto 1/\sqrt{r}$.

P19-6 (a) The first position corresponds to maximum destructive interference, so the waves are half a wavelength out of phase; the second position corresponds to maximum constructive interference, so the waves are in phase. Shifting the tube has in effect added half a wavelength to the path through B. But each segment is added, so

$$\lambda = (2)(2)(1.65\text{ cm}) = 6.60\text{ cm},$$

and $f = (343\,\text{m/s})/(6.60\text{ cm}) = 5200$ Hz.

(b) $I_\text{min} \propto (s_1 - s_2)^2$, $I_\text{max} \propto (s_1 + s_2)^2$, then dividing one expression by the other and rearranging we find

$$\frac{s_1}{s_2} = \frac{\sqrt{I_\text{max}} + \sqrt{I_\text{min}}}{\sqrt{I_\text{max}} - \sqrt{I_\text{min}}} = \frac{\sqrt{90} + \sqrt{10}}{\sqrt{90} - \sqrt{10}} = 2$$

P19-7 (a) $I = P/4\pi r^2 = (31.6\,\text{W})/4\pi(194\,\text{m})^2 = 6.68\times10^{-5}\text{W/m}^2$.

(b) $P = IA = (6.68\times10^{-5}\text{W/m}^2)(75.2\times10^{-6}\text{m}^2) = 5.02\times10^{-9}\text{W}$.

(c) $U = Pt = (5.02\times10^{-9}\text{W})(25.0\text{ min})(60.0\text{ s/min}) = 7.53\,\mu\text{J}$.

P19-8 Note that the reverberation time is logarithmically related to the intensity, but linearly related to the sound level. As such, the reverberation time is the amount of time for the sound level to decrease by

$$\Delta SL = 10\log(10^{-6}) = 60\text{ dB}.$$

Then

$$t = (87\text{ dB})(2.6\,s)/(60\text{ dB}) = 3.8\,\text{s}$$

$\boxed{\textbf{P19-9}}$ What the device is doing is taking all of the energy which strikes a large surface area and concentrating it into a small surface area. It doesn't succeed; only 12% of the energy is concentrated. We can think, however, in terms of power: 12% of the average power which strikes the parabolic reflector is transmitted into the tube.

If the sound intensity on the reflector is I_1, then the average power is $P_1 = I_1 A_1 = I_1 \pi r_1^2$, where r_1 is the radius of the reflector. The average power in the tube will be $P_2 = 0.12 P_1$, so the intensity in the tube will be

$$I_2 = \frac{P_2}{A_2} = \frac{0.12 I_1 \pi r_1^2}{\pi r_2^2} = 0.12 I_1 \frac{r_1^2}{r_2^2}$$

Since the lowest audible sound has an intensity of $I_0 = 10^{-12}\ \mathrm{W/m^2}$, we can set $I_2 = I_0$ as the condition for "hearing" the whisperer through the apparatus. The minimum sound intensity at the parabolic reflector is

$$I_1 = \frac{I_0}{0.12} \frac{r_2^2}{r_1^2}.$$

Now for the whisperers. Intensity falls off as $1/d^2$, where d is the distance from the source. We are told that when $d = 1.0$ m the sound level is 20 dB; this sound level has an intensity of

$$I = I_0 10^{20/10} = 100 I_0$$

Then at a distance d from the source the intensity must be

$$I_1 = 100 I_0 \frac{(1\,\mathrm{m})^2}{d^2}.$$

This would be the intensity "picked-up" by the parabolic reflector. Combining this with the condition for being able to hear the whisperers through the apparatus, we have

$$\frac{I_0}{0.12} \frac{r_2^2}{r_1^2} = 100 I_0 \frac{(1\,\mathrm{m})^2}{d^2}$$

or, upon some rearranging,

$$d = (\sqrt{12}\,\mathrm{m}) \frac{r_1}{r_2} = (\sqrt{12}\,\mathrm{m}) \frac{(0.50\,\mathrm{m})}{(0.005\,\mathrm{m})} = 346\,\mathrm{m}.$$

P19-10 (a) A displacement node; at the center the particles have nowhere to go.

(b) This systems acts like a pipe which is closed at one end.

(c) $v\sqrt{B/\rho}$, so

$$T = 4(0.009)(6.96\times10^{8}\mathrm{m})\sqrt{(1.0\times10^{10}\mathrm{kg/m^3})/(1.33\times10^{22}\mathrm{Pa})} = 22\,\mathrm{s}.$$

P19-11 The cork filing collect at pressure antinodes when standing waves are present, and the antinodes are each half a wavelength apart. Then $v = f\lambda = f(2d)$.

P19-12 (a) $f = v/4L = (343\,\mathrm{m/s})/4(1.18\,\mathrm{m}) = 72.7$ Hz.

(b) $F = \mu v^2 = \mu f^2 \lambda^2$, or

$$F = (9.57\times10^{-3}\mathrm{kg}/0.332\,\mathrm{m})(72.7\ \mathrm{Hz})^2[2(0.332\,\mathrm{m})]^2 = 67.1\,N.$$

$\boxed{\textbf{P19-13}}$ In this problem the string is observed to resonate at 880 Hz and then again at 1320 Hz, so the two corresponding values of n must differ by 1. We can then write two equations

$$(880\text{ Hz}) = \frac{nv}{2L} \text{ and } (1320\text{ Hz}) = \frac{(n+1)v}{2L}$$

and solve these for v. It is somewhat easier to first solve for n. Rearranging both equations, we get

$$\frac{(880\text{ Hz})}{n} = \frac{v}{2L} \text{ and } \frac{(1320\text{ Hz})}{n+1} = \frac{v}{2L}.$$

Combining these two equations we get

$$\begin{aligned} \frac{(880\text{ Hz})}{n} &= \frac{(1320\text{ Hz})}{n+1}, \\ (n+1)(880\text{ Hz}) &= n(1320\text{ Hz}), \\ n &= \frac{(880\text{ Hz})}{(1320\text{ Hz}) - (880\text{ Hz})} = 2. \end{aligned}$$

Now that we know n we can find v,

$$v = 2(0.300\,\text{m})\frac{(880\text{ Hz})}{2} = 264\,\text{m/s}$$

And, finally, we are in a position to find the tension, since

$$F = \mu v^2 = (0.652\times 10^{-3}\text{kg/m})(264\,\text{m/s})^2 = 45.4\,\text{N}.$$

P19-14 (a) There are five choices for the first fork, and four for the second. That gives 20 pairs. But order doesn't matter, so we need divide that by two to get a maximum of 10 possible beat frequencies.

(b) If the forks are ordered to have equal differences (say, 400 Hz, 410 Hz, 420 Hz, 430 Hz, and 440 Hz) then there will actually be only 4 beat frequencies.

P19-15 $v = (2.25\times 10^8\text{m/s})/\sin(58.0°) = 2.65\times 10^8\text{m/s}$.

P19-16 (a) $f_1 = (442\text{ Hz})(343\,\text{m/s})/(343\,\text{m/s} - 31.3\,\text{m/s}) = 486\text{ Hz}$, while

$$f_2 = (442\text{ Hz})(343\,\text{m/s})/(343\,\text{m/s} + 31.3\,\text{m/s}) = 405\text{ Hz},$$

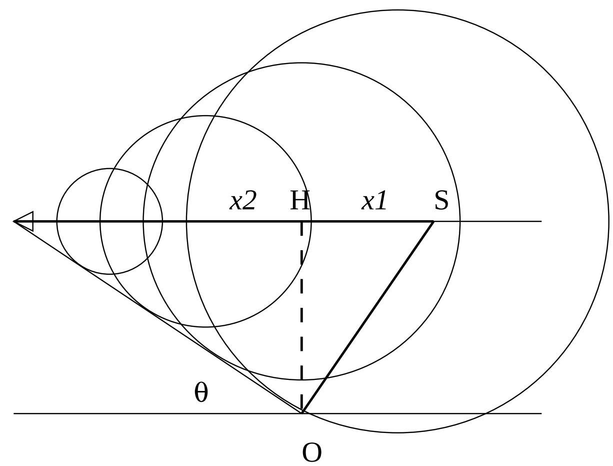
so $\Delta f = 81$ Hz.

(b) $f_1 = (442\text{ Hz})(343\,\text{m/s} - 31.3\,\text{m/s})/(343\,\text{m/s}) = 402\text{ Hz}$, while

$$f_2 = (442\text{ Hz})(343\,\text{m/s} + 31.3\,\text{m/s})/(343\,\text{m/s}) = 482\text{ Hz},$$

so $\Delta f = 80$ Hz.

$\boxed{\textbf{P19-17}}$ The sonic boom that you hear is not from the sound given off by the plane when it is overhead, it is from the sound given off *before* the plane was overhead. So this problem isn't as simple as distance equals velocity × time. It is *very* useful to sketch a picture.



We can find the angle θ from the figure, we'll get Eq. 19-45, so

$$\sin\theta = \frac{v}{v_s} = \frac{(330\,\text{m/s})}{(396\,\text{m/s})} = 0.833 \text{ or } \theta = 56.4^\circ$$

Note that v_s is the speed of the source, not the speed of sound!

Unfortunately $t = 12\,\text{s}$ is *not* the time between when the sonic boom leaves the plane and when it arrives at the observer. It is the time between when the plane is overhead and when the sonic boom arrives at the observer. That's why there are so many marks and variables on the figure. x_1 is the distance from where the sonic boom which is heard by the observer is emitted to the point directly overhead; x_2 is the distance from the point which is directly overhead to the point where the plane is when the sonic boom is heard by the observer. We do have $x_2 = v_s(12.0\,\text{s})$. This length forms one side of a right triangle HSO, the opposite side of this triangle is the side HO, which is the height of the plane above the ground, so

$$h = x_2 \tan\theta = (343\,\text{m/s})(12.0\,\text{s})\tan(56.4^\circ) = 7150\,\text{m}.$$

P19-18 (a) The target "hears"

$$f' = f_s \frac{v+V}{v}.$$

This sound is reflected back to the detector which then "hears"

$$f_r = f' \frac{v}{v-V} = f_s \frac{v+V}{v-V}.$$

(b) Rearranging,

$$V/v = \frac{f_r - f_s}{f_r + f_s} \approx \frac{1}{2}\frac{f_r - f_s}{f_s},$$

where we have assumed that the source frequency and the reflected frequency are almost identical, so that when added $f_r + f_s \approx 2f_s$.

P19-19 (a) We apply Eq. 19-44

$$f' = f\frac{v+v_O}{v-v_S} = (1030\text{ Hz})\frac{(5470\text{ km/h}) + (94.6\text{ km/h})}{(5470\text{ km/h}) - (20.2\text{ km/h})} = 1050\text{ Hz}$$

(b) The reflected signal has a frequency equal to that of the signal received by the second sub originally. Applying Eq. 19-44 again,

$$f' = f\frac{v + v_O}{v - v_S} = (1050 \text{ Hz})\frac{(5470 \text{ km/h}) + (20.2 \text{ km/h})}{(5470 \text{ km/h}) - (94.6 \text{ km/h})} = 1070 \text{ Hz}$$

P19-20 In this case $v_S = 75.2 \text{ km/h} - 30.5 \text{ km/h} = 12.4\,\text{m/s}$. Then

$$f' = (989 \text{ Hz})(1482\,\text{m/s})(1482\,\text{m/s} - 12.4\,\text{m/s}) = 997 \text{ Hz}.$$

P19-21 There is no relative motion between the source and observer, so there is no frequency shift regardless of the wind direction.

P19-22 (a) $v_S = 34.2\,\text{m/s}$ and $v_O = 34.2\,\text{m/s}$, so

$$f' = (525 \text{ Hz})(343\,\text{m/s} + 34.2\,\text{m/s})/(343\,\text{m/s} - 34.2\,\text{m/s}) = 641 \text{ Hz}.$$

(b) $v_S = 34.2\,\text{m/s} + 15.3\,\text{m/s} = 49.5\,\text{m/s}$ and $v_O = 34.2\,\text{m/s} - 15.3\,\text{m/s} = 18.9\,\text{m/s}$, so

$$f' = (525 \text{ Hz})(343\,\text{m/s} + 18.9\,\text{m/s})/(343\,\text{m/s} - 49.5\,\text{m/s}) = 647 \text{ Hz}.$$

(c) $v_S = 34.2\,\text{m/s} - 15.3\,\text{m/s} = 18.9\,\text{m/s}$ and $v_O = 34.2\,\text{m/s} + 15.3\,\text{m/s} = 49.5\,\text{m/s}$, so

$$f' = (525 \text{ Hz})(343\,\text{m/s} + 49.5\,\text{m/s})/(343\,\text{m/s} - 18.9\,\text{m/s}) = 636 \text{ Hz}.$$

E20-1 (a) $t = x/v = (0.20\,\text{m})/(0.941)(3.00\times10^8\text{m/s}) = 7.1\times10^{-10}\text{s}$.
(b) $y = -gt^2/2 = -(9.81\,\text{m/s}^2)(7.1\times10^{-10}\text{s})^2/2 = 2.5\times10^{-18}\text{m}$.

E20-2 $L = L_0\sqrt{1-u^2/c^2} = (2.86\,\text{m})\sqrt{1-(0.999987)^2} = 1.46$ cm.

E20-3 $L = L_0\sqrt{1-u^2/c^2} = (1.68\,\text{m})\sqrt{1-(0.632)^2} = 1.30\,\text{m}$.

E20-4 Solve $\Delta t = \Delta t_0/\sqrt{1-u^2/c^2}$ for u:

$$u = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = (3.00\times10^8\text{m/s})\sqrt{1-\left(\frac{(2.20\,\mu\text{s})}{(16.0\,\mu\text{s})}\right)^2} = 2.97\times10^8\text{m/s}.$$

E20-5 We can apply $\Delta x = v\Delta t$ to find the time the particle existed before it decayed. Then

$$\Delta t = \frac{x}{v}\frac{(1.05\times10^{-3}\,\text{m})}{(0.992)(3.00\times10^8\,\text{m/s})} = 3.53\times10^{-12}\,\text{s}.$$

The *proper lifetime* of the particle is

$$\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (3.53\times10^{-12}\,\text{s})\sqrt{1-(0.992)^2} = 4.46\times10^{-13}\,\text{s}.$$

E20-6 Apply Eq. 20-12:

$$v = \frac{(0.43c)+(0.587c)}{1+(0.43c)(0.587c)/c^2} = 0.812c.$$

E20-7 (a) $L = L_0\sqrt{1-u^2/c^2} = (130\,\text{m})\sqrt{1-(0.740)^2} = 87.4\,\text{m}$
(b) $\Delta t = L/v = (87.4\,\text{m})/(0.740)(3.00\times10^8\text{m/s}) = 3.94\times10^{-7}\text{s}$.

E20-8 $\Delta t = \Delta t_0/\sqrt{1-u^2/c^2} = (26\text{ ns})\sqrt{1-(0.99)^2} = 184$ ns. Then

$$L = v\Delta t = (0.99)(3.00\times10^8\text{m/s})(184\times10^{-9}\text{s}) = 55\,\text{m}.$$

E20-9 (a) $v_\text{g} = 2v = (7.91+7.91)$ km/s $= 15.82$ km/s.
(b) A relativistic treatment yields $v_\text{r} = 2v/(1+v^2/c^2)$. The fractional error is

$$\frac{v_\text{g}}{v_\text{r}} - 1 = \left(1+\frac{v^2}{c^2}\right) - 1 = \frac{v^2}{c^2} = \frac{(7.91\times10^3\text{m/s})^2}{(3.00\times10^8\text{m/s})^2} = 6.95\times10^{-10}.$$

E20-10 Invert Eq. 20-15 to get $\beta = \sqrt{1-1/\gamma^2}$.
(a) $\beta = \sqrt{1-1/(1.01)^2} = 0.140$.
(b) $\beta = \sqrt{1-1/(10.0)^2} = 0.995$.
(c) $\beta = \sqrt{1-1/(100)^2} = 0.99995$.
(d) $\beta = \sqrt{1-1/(1000)^2} = 0.9999995$.

$\boxed{\textbf{E20-11}}$ The distance traveled by the particle is (6.0 y)c; the time required for the particle to travel this distance is 8.0 years. Then the speed of the particle is

$$v = \frac{\Delta x}{\Delta t} = \frac{(6.0\text{ y})c}{(8.0\text{ y})} = \frac{3}{4}c.$$

The speed parameter β is given by

$$\beta = \frac{v}{c} = \frac{\frac{3}{4}c}{c} = \frac{3}{4}.$$

E20-12 $\gamma = 1/\sqrt{1-(0.950)^2} = 3.20$. Then

$$\begin{aligned} x' &= (3.20)[(1.00\times10^5\text{m}) - (0.950)(3.00\times10^8\text{m/s})(2.00\times10^{-4}\text{s})] = 1.38\times10^5\text{m}, \\ t' &= (3.20)[(2.00\times10^{-4}\text{s}) - (1.00\times10^5\text{m})(0.950)/(3.00\times10^8\text{m/s})] = -3.73\times10^{-4}\text{s}. \end{aligned}$$

E20-13 (a) $\gamma = 1/\sqrt{1-(0.380)^2} = 1.081$. Then

$$\begin{aligned} x' &= (1.081)[(3.20\times10^8\text{m}) - (0.380)(3.00\times10^8\text{m/s})(2.50\,\text{s})] = 3.78\times10^7\text{m}, \\ t' &= (1.081)[(2.50\,\text{s}) - (3.20\times10^8\text{m})(0.380)/(3.00\times10^8\text{m/s})] = 2.26\,\text{s}. \end{aligned}$$

(b) $\gamma = 1/\sqrt{1-(0.380)^2} = 1.081$. Then

$$\begin{aligned} x' &= (1.081)[(3.20\times10^8\text{m}) - (-0.380)(3.00\times10^8\text{m/s})(2.50\,\text{s})] = 6.54\times10^8\text{m}, \\ t' &= (1.081)[(2.50\,\text{s}) - (3.20\times10^8\text{m})(-0.380)/(3.00\times10^8\text{m/s})] = 3.14\,\text{s}. \end{aligned}$$

E20-14

E20-15 (a) $v'_x = (-u)/(1-0)$ and $v'_y = c\sqrt{1-u^2/c^2}$.
(b) $(v'_x)^2 + (v'_y)^2 = u^2 + c^2 - u^2 = c^2$.

E20-16 $v' = (0.787c + 0.612c)/[1 + (0.787)(0.612)] = 0.944c$.

$\boxed{\textbf{E20-17}}$ (a) The first part is easy; we appear to be moving away from A at the same speed as A appears to be moving away from us: $0.347c$.

(b) Using the velocity transformation formula, Eq. 20-18,

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} = \frac{(0.347c) - (-0.347c)}{1 - (-0.347c)(0.347c)/c^2} = 0.619c.$$

The negative sign reflects the fact that these two velocities are in *opposite* directions.

E20-18 $v' = (0.788c - 0.413c)/[1 + (0.788)(-0.413)] = 0.556c$.

E20-19 (a) $\gamma = 1/\sqrt{1-(0.8)^2} = 5/3$.

$$\begin{aligned} v'_x &= \frac{v_x}{\gamma(1 - uv_y/c^2)} = \frac{3(0.8c)}{5[1-(0)]} = \frac{12}{25}c, \\ v'_y &= \frac{v_y - u}{1 - uv_y/c^2} = \frac{(0) - (0.8c)}{1 - (0)} = -\frac{4}{5}c. \end{aligned}$$

Then $v' = c\sqrt{(-4/5)^2 + (12/25)^2} = 0.933c$ directed $\theta = \arctan(-12/20) = 31°$ East of South.

(b) $\gamma = 1/\sqrt{1-(0.8)^2} = 5/3$.

$$\begin{aligned} v'_x &= \frac{v_x - u}{1 - uv_x/c^2} = \frac{(0) - (-0.8c)}{1 - (0)} = +\frac{4}{5}c, \\ v'_y &= \frac{v_y}{\gamma(1 - uv_x/c^2)} = \frac{3(0.8c)}{5[1 - (0)]} = \frac{12}{25}c. \end{aligned}$$

Then $v' = c\sqrt{(4/5)^2 + (12/25)^2} = 0.933c$ directed $\theta = \arctan(20/12) = 59^\circ$ West of North.

E20-20 This exercise should occur in Section 20-9.
(a) $v = 2\pi(6.37\times10^6\text{m})c/(1\,\text{s})(3.00\times10^8\text{m/s}) = 0.133c$.
(b) $K = (\gamma - 1)mc^2 = (1/\sqrt{1-(0.133)^2} - 1)(511\text{ keV}) = 4.58\text{ keV}$.
(c) $K_c = mv^2/2 = mc^2(v^2/c^2)/2 = (511\text{ keV})(0.133)^2/2 = 4.52\text{ keV}$. The percent error is

$$(4.52 - 4.58)/(4.58) = -1.31\%.$$

E20-21 $\Delta L = L' - L_0$ so

$$\Delta L = 2(6.370\times10^6\text{m})(1 - \sqrt{1 - (29.8\times10^3\text{m/s})^2/(3.00\times10^8\text{m/s})^2}) = 6.29\times10^{-2}\text{m}.$$

E20-22 (a) $\Delta L/L_0 = 1 - L'/L_0$ so

$$\Delta L = (1 - \sqrt{1 - (522\,\text{m/s})^2/(3.00\times10^8\text{m/s})^2}) = 1.51\times10^{-12}.$$

(b) We want to solve $\Delta t - \Delta t' = 1\,\mu\text{s}$, or

$$1\,\mu\text{s} = \Delta t(1 - 1/\sqrt{1 - (522\,\text{m/s})^2/(3.00\times10^8\text{m/s})^2}),$$

which has solution $\Delta t = 6.61\times10^5$s. That's 7.64 days.

$\boxed{\textbf{E20-23}}$ The length of the ship as measured in the "certain" reference frame is

$$L = L_0\sqrt{1 - v^2/c^2} = (358\,\text{m})\sqrt{1 - (0.728)^2} = 245\,\text{m}.$$

In a time Δt the ship will move a distance $x_1 = v_1\Delta t$ while the micrometeorite will move a distance $x_2 = v_2\Delta t$; since they are moving toward each other then the micrometeorite will pass then ship when $x_1 + x_2 = L$. Then

$$\Delta t = L/(v_1 + v_2) = (245\,\text{m})/[(0.728 + 0.817)(3.00\times10^8\text{m/s})] = 5.29\times10^{-7}\,\text{s}.$$

This answer is the time measured in the "certain" reference frame. We can use Eq. 20-21 to find the time as measured on the ship,

$$\Delta t = \frac{\Delta t' + u\Delta x'/c^2}{\sqrt{1 - u^2/c^2}} = \frac{(5.29\times10^{-7}\,\text{s}) + (0.728c)(116\,\text{m})/c^2}{\sqrt{1 - (0.728)^2}} = 1.23\times10^{-6}\text{s}.$$

E20-24 (a) $\gamma = 1/\sqrt{1 - (0.622)^2} = 1.28$.
(b) $\Delta t = (183\,\text{m})/(0.622)(3.00\times10^8\text{m/s}) = 9.81\times10^{-7}$s. On the clock, however,

$$\Delta t' = \Delta t/\gamma = (9.81\times10^{-7}\text{s})/(1.28) = 7.66\times10^{-7}\text{s}.$$

E20-25 (a) $\Delta t = (26.0 \text{ ly})/(0.988)(1.00 \text{ ly/y}) = 26.3$ y.

(b) The signal takes 26 years to return, so $26 + 26.3 = 52.3$ years.

(c) $\Delta t' = (26.3 \text{ y})\sqrt{1 - (0.988)^2} = 4.06$ y.

E20-26 (a) $\gamma = (1000 \text{ y})(1 \text{ y}) = 1000$;

$$v = c\sqrt{1 - 1/\gamma^2} \approx c(1 - 1/2\gamma^2) = 0.9999995c$$

(b) No.

E20-27 $(5.61\times10^{29} \text{ MeV}/c^2)c/(3.00\times10^8\text{m/s}) = 1.87\times10^{21}$ MeV/c.

E20-28 $p^2 = m^2c^2 = m^2v^2/(1 - v^2/c^2)$, so $2v^2/c^2 = 1$, or $v = \sqrt{2}c$.

E20-29 The magnitude of the momentum of a relativistic particle in terms of the magnitude of the velocity is given by Eq. 20-23,

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}}.$$

The speed parameter, β, is what we are looking for, so we need to rearrange the above expression for the quantity v/c.

$$\begin{aligned}
p/c &= \frac{mv/c}{\sqrt{1 - v^2/c^2}}, \\
\frac{p}{c} &= \frac{m\beta}{\sqrt{1 - \beta^2}}, \\
\frac{mc}{p} &= \frac{\sqrt{1 - \beta^2}}{\beta}, \\
\frac{mc}{p} &= \sqrt{1/\beta^2 - 1}.
\end{aligned}$$

Rearranging,

$$\begin{aligned}
\frac{mc^2}{pc} &= \sqrt{1/\beta^2 - 1}, \\
\left(\frac{mc^2}{pc}\right)^2 &= \frac{1}{\beta^2} - 1, \\
\sqrt{\left(\frac{mc^2}{pc}\right)^2 + 1} &= \frac{1}{\beta}, \\
\frac{pc}{\sqrt{m^2c^4 + p^2c^2}} &= \beta
\end{aligned}$$

(a) For the electron,

$$\beta = \frac{(12.5 \text{ MeV}/c)c}{\sqrt{(0.511 \text{ MeV}/c^2)^2c^4 + (12.5 \text{ MeV}/c)^2c^2}} = 0.999.$$

(b) For the proton,

$$\beta = \frac{(12.5 \text{ MeV}/c)c}{\sqrt{(938 \text{ MeV}/c^2)^2c^4 + (12.5 \text{ MeV}/c)^2c^2}} = 0.0133.$$

E20-30 $K = mc^2(\gamma - 1)$, so $\gamma = 1 + K/mc^2$. $\beta = \sqrt{1 - 1/\gamma^2}$.

(a) $\gamma = 1 + (1.0\text{ keV})/(511\text{ keV}) = 1.00196$. $\beta = 0.0625c$.

(b) $\gamma = 1 + (1.0\text{ MeV})/(0.511\text{ MeV}) = 2.96$. $\beta = 0.941c$.

(c) $\gamma = 1 + (1.0\text{ GeV})/(0.511\text{ MeV}) = 1960$. $\beta = 0.99999987c$.

$\boxed{\textbf{E20-31}}$ The kinetic energy is given by Eq. 20-27,

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2.$$

We rearrange this to solve for $\beta = v/c$,

$$\beta = \sqrt{1 - \left(\frac{mc^2}{K + mc^2}\right)^2}.$$

It is actually *much* easier to find γ, since

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}},$$

so $K = \gamma mc^2 - mc^2$ implies

$$\gamma = \frac{K + mc^2}{mc^2}$$

(a) For the electron,

$$\beta = \sqrt{1 - \left(\frac{(0.511\text{ MeV}/c^2)c^2}{(10\text{ MeV}) + (0.511\text{ MeV}/c^2)c^2}\right)^2} = 0.9988,$$

and

$$\gamma = \frac{(10\text{ MeV}) + (0.511\text{ MeV}/c^2)c^2}{(0.511\text{ MeV}/c^2)c^2} = 20.6.$$

(b) For the proton,

$$\beta = \sqrt{1 - \left(\frac{(938\text{ MeV}/c^2)c^2}{(10\text{ MeV}) + (938\text{ MeV}/c^2)c^2}\right)^2} = 0.0145,$$

and

$$\gamma = \frac{(10\text{ MeV}) + (938\text{ MeV}/c^2)c^2}{(938\text{ MeV}/c^2)c^2} = 1.01.$$

(b) For the alpha particle,

$$\beta = \sqrt{1 - \left(\frac{4(938\text{ MeV}/c^2)c^2}{(10\text{ MeV}) + 4(938\text{ MeV}/c^2)c^2}\right)^2} = 0.73,$$

and

$$\gamma = \frac{(10\text{ MeV}) + 4(938\text{ MeV}/c^2)c^2}{4(938\text{ MeV}/c^2)c^2} = 1.0027.$$

E20-32 $\gamma = 1/\sqrt{1 - (0.99)^2} = 7.089$.

(a) $E = \gamma mc^2 = (7.089)(938.3\text{ MeV}) = 6650\text{ MeV}$. $K = E - mc^2 = 5710\text{ MeV}$. $p = mv\gamma = (938.3\text{ MeV}/c^2)(0.99c)(7.089) = 6580\text{ MeV}/c$.

(b) $E = \gamma mc^2 = (7.089)(0.511\text{ MeV}) = 3.62\text{ MeV}$. $K = E - mc^2 = 3.11\text{ MeV}$. $p = mv\gamma = (0.511\text{ MeV}/c^2)(0.99c)(7.089) = 3.59\text{ MeV}/c$.

E20-33 $\Delta m/\Delta t = (1.2\times 10^{41}\text{W})/(3.0\times 10^{8}\text{m/s})^2 = 1.33\times 10^{24}\text{kg/s}$, which is

$$\frac{\Delta m}{\Delta t} = \frac{(1.33\times 10^{24}\text{kg/s})(3.16\times 10^{7}\text{s/y})}{(1.99\times 10^{30}\text{kg/sun})} = 21.1$$

E20-34 (a) If $K = E - mc^2 = 2mc^2$, then $E = 3mc^2$, so $\gamma = 3$, and

$$v = c\sqrt{1-1/\gamma^2} = c\sqrt{1-1/(3)^2} = 0.943c.$$

(b) If $E = 2mc^2$, then $\gamma = 2$, and

$$v = c\sqrt{1-1/\gamma^2} = c\sqrt{1-1/(2)^2} = 0.866c.$$

$\boxed{\textbf{E20-35}}$ (a) The kinetic energy is given by Eq. 20-27,

$$K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2\left(\left(1-\beta^2\right)^{-1/2} - 1\right).$$

We want to expand the $1-\beta^2$ part for small β,

$$\left(1-\beta^2\right)^{-1/2} = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \cdots$$

Inserting this into the kinetic energy expression,

$$K = \frac{1}{2}mc^2\beta^2 + \frac{3}{8}mc^2\beta^4 + \cdots$$

But $\beta = v/c$, so

$$K = \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots$$

(b) We want to know when the error because of neglecting the second (and higher) terms is 1%; or

$$0.01 = (\frac{3}{8}m\frac{v^4}{c^2})/(\frac{1}{2}mv^2) = \frac{3}{4}\left(\frac{v}{c}\right)^2.$$

This will happen when $v/c = \sqrt{(0.01)4/3)} = 0.115$.

E20-36 $K_c = (1000\,\text{kg})(20\,\text{m/s})^2/2 = 2.0\times 10^5\text{J}$. The relativistic calculation is slightly harder:

$$\begin{aligned} K_r &= (1000\,\text{kg})(3\times 10^8\text{m/s})^2(1/\sqrt{1-(20\,\text{m/s})^2/(3\times 10^8\text{m/s})^2} - 1), \\ &\approx (1000\,\text{kg})\left[\frac{1}{2}(20\,\text{m/s})^2 + \frac{3}{8}(20\,\text{m/s})^4/(3\times 10^8\text{m/s})^2 + ...\right], \\ &= 2.0\times 10^5\text{J} + 6.7\times 10^{-10}\text{J}. \end{aligned}$$

$\boxed{\textbf{E20-37}}$ Start with Eq. 20-34 in the form

$$E^2 = (pc)^2 + (mc^2)^2$$

The rest energy is mc^2, and if the total energy is three times this then $E = 3mc^2$, so

$$\begin{aligned} (3mc^2)^2 &= (pc)^2 + (mc^2)^2, \\ 8(mc^2)^2 &= (pc)^2, \\ \sqrt{8}\,mc &= p. \end{aligned}$$

E20-38 The initial kinetic energy is

$$K_{\rm i} = \frac{1}{2}m\left(\frac{2v}{1+v^2/c^2}\right) = \frac{2mv^2}{(1+v^2/c^2)^2}.$$

The final kinetic energy is

$$K_{\rm f} = 2\frac{1}{2}m\left(v\sqrt{2-v^2/c^2}\right)^2 = mv^2(2-v^2/c^2).$$

E20-39 This exercise is much more involved than the previous one!

The initial kinetic energy is

$$\begin{aligned}
K_{\rm i} &= \frac{mc^2}{\sqrt{1-\left(\frac{2v}{1+v^2/c^2}\right)^2/c^2}} - mc^2, \\
&= \frac{mc^2(1+v^2/c^2)}{\sqrt{(1+v^2/c^2)^2-4v^2/c^2}} - mc^2, \\
&= \frac{m(c^2+v^2)}{1-v^2/c^2} - \frac{m(c^2-v^2)}{1-v^2/c^2}, \\
&= \frac{2mv^2}{1-v^2/c^2}.
\end{aligned}$$

The final kinetic energy is

$$\begin{aligned}
K_{\rm f} &= 2\frac{mc^2}{\sqrt{1-\left(v\sqrt{2-v^2/c^2}\right)^2/c^2}} - 2mc^2, \\
&= 2\frac{mc^2}{\sqrt{1-(v^2/c^2)(2-v^2/c^2)}} - 2mc^2, \\
&= 2\frac{mc^2}{1-v^2/c^2} - 2mc^2, \\
&= 2\frac{mc^2}{1-v^2/c^2} - 2\frac{m(c^2-v^2)}{1-v^2/c^2}, \\
&= \frac{2mv^2}{1-v^2/c^2}.
\end{aligned}$$

E20-40 For a particle with mass, $\gamma = K/mc^2 + 1$. For the electron, $\gamma = (0.40)/(0.511) + 1 = 1.78$. For the proton, $\gamma = (10)/(938) + 1 = 1.066$.

For the photon, $pc = E$. For a particle with mass, $pc = \sqrt{(K+mc^2)^2 - m^2c^4}$. For the electron,

$$pc = \sqrt{[(0.40\text{ MeV}) + (0.511\text{ MeV})]^2 - (0.511\text{ MeV})^2} = 0.754\text{ MeV}.$$

For the proton,

$$pc = \sqrt{[(10\text{ MeV}) + (938\text{ MeV})]^2 - (938\text{ MeV})^2} = 137\text{ MeV}.$$

(a) Only photons move at the speed of light, so it is moving the fastest.
(b) The proton, since it has smallest value for γ.
(c) The proton has the greatest momentum.
(d) The photon has the least.

$\boxed{\textbf{E20-41}}$ Work is change in energy, so

$$W = mc^2/\sqrt{1-(v_\mathrm{f}/c)^2} - mc^2/\sqrt{1-(v_\mathrm{i}/c)^2}.$$

(a) Plug in the numbers,

$$W = (0.511\ \mathrm{MeV})(1/\sqrt{1-(0.19)^2} - 1/\sqrt{1-(0.18)^2}) = 0.996\ \mathrm{keV}.$$

(b) Plug in the numbers,

$$W = (0.511\ \mathrm{MeV})(1/\sqrt{1-(0.99)^2} - 1/\sqrt{1-(0.98)^2}) = 1.05\ \mathrm{MeV}.$$

E20-42 $E = 2\gamma m_0 c^2 = mc^2$, so

$$m = 2\gamma m_0 = 2(1.30\ \mathrm{mg})/\sqrt{1-(0.580)^2} = 3.19\ \mathrm{mg}.$$

E20-43 (a) Energy conservation requires $E_\mathrm{k} = 2E_\pi$, or $m_\mathrm{k}c^2 = 2\gamma m_\pi c^2$. Then

$$\gamma = (498\ \mathrm{MeV})/2(140\ \mathrm{MeV}) = 1.78$$

This corresponds to a speed of $v = c\sqrt{1-1/(1.78)^2} = 0.827c$.

(b) $\gamma = (498\ \mathrm{MeV} + 325\ \mathrm{MeV})/(498\ \mathrm{MeV}) = 1.65$, so $v = c\sqrt{1-1/(1.65)^2} = 0.795c$.

(c) The lab frame velocities are then

$$v_1' = \frac{(0.795)+(-0.827)}{1+(0.795)(-0.827)}c = -0.0934c,$$

and

$$v_2' = \frac{(0.795)+(0.827)}{1+(0.795)(0.827)}c = 0.979c,$$

The corresponding kinetic energies are

$$K_1 = (140\ \mathrm{MeV})(1/\sqrt{1-(-0.0934)^2} - 1) = 0.614\ \mathrm{MeV}$$

and

$$K_1 = (140\ \mathrm{MeV})(1/\sqrt{1-(0.979)^2} - 1) = 548\ \mathrm{MeV}$$

E20-44

P20-1 (a) $\gamma = 2$, so $v = \sqrt{1-1/(2)^2} = 0.866c$.

(b) $\gamma = 2$.

P20-2 (a) Classically, $v' = (0.620c) + (0.470c) = 1.09c$. Relativistically,

$$v' = \frac{(0.620c)+(0.470c)}{1+(0.620)(0.470)} = 0.844c.$$

(b) Classically, $v' = (0.620c) + (-0.470c) = 0.150c$. Relativistically,

$$v' = \frac{(0.620c)+(-0.470c)}{1+(0.620)(-0.470)} = 0.211c.$$

P20-3 (a) $\gamma = 1/\sqrt{1-(0.247)^2} = 1.032$. Use the equations from Table 20-2.

$$\Delta t = (1.032)[(0) - (0.247)(30.4\times10^3\text{m})/(3.00\times10^8\text{m/s})] = -2.58\times10^{-5}\text{s}.$$

(b) The red flash appears to go first.

P20-4 Once again, the "pico" should have been a μ.
$\gamma = 1/\sqrt{1-(0.60)^2} = 1.25$. Use the equations from Table 20-2.

$$\Delta t = (1.25)[(4.0\times10^{-6}\text{s}) - (0.60)(3.0\times10^3\text{m})/(3.00\times10^8\text{m/s})] = -2.5\times10^{-6}\text{s}.$$

$\boxed{\textbf{P20-5}}$ We can choose our coordinate system so that u is directed along the x axis without any loss of generality. Then, according to Table 20-2,

$$\begin{aligned}
\Delta x' &= \gamma(\Delta x - u\Delta t), \\
\Delta y' &= \Delta y, \\
\Delta z' &= \Delta z, \\
c\Delta t' &= \gamma(c\Delta t - u\Delta x/c).
\end{aligned}$$

Square these expressions,

$$\begin{aligned}
(\Delta x')^2 &= \gamma^2(\Delta x - u\Delta t)^2 = \gamma^2\left((\Delta x)^2 - 2u(\Delta x)(\Delta t) + (\Delta t)^2\right), \\
(\Delta y')^2 &= (\Delta y)^2, \\
(\Delta z')^2 &= (\Delta z)^2, \\
c^2(\Delta t')^2 &= \gamma^2(c\Delta t - u\Delta x/c)^2 = \gamma^2\left(c^2(\Delta t)^2 - 2u(\Delta t)(\Delta x) + u^2(\Delta x)^2/c^2\right).
\end{aligned}$$

We'll add the first three equations and then subtract the fourth. The left hand side is the equal to

$$(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - c^2(\Delta t')^2,$$

while the right hand side will equal

$$\gamma^2\left((\Delta x)^2 + u^2(\Delta t)^2 - c^2(\Delta t)^2 - u^2/c^2(\Delta x)^2\right) + (\Delta y)^2 + (\Delta z)^2,$$

which can be rearranged as

$$\gamma^2\left(1-u^2/c^2\right)(\Delta x)^2 + \gamma^2\left(u^2-c^2\right)(\Delta t)^2 + (\Delta y)^2 + (\Delta z)^2,$$
$$\gamma^2\left(1-u^2/c^2\right)(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2\gamma^2\left(1-u^2/c^2\right)(\Delta t)^2.$$

But

$$\gamma^2 = \frac{1}{1-u^2/c^2},$$

so the previous expression will simplify to

$$(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2.$$

P20-6 (a) $v_x = [(0.780c) + (0.240c)]/[1 + (0.240)(0.780)] = 0.859c$.
(b) $v_x = [(0) + (0.240c)]/[1 + (0)] = 0.240c$, while

$$v_y = (0.780c)\sqrt{1-(0.240)^2}/[1 + (0)] = 0.757c.$$

Then $v = \sqrt{(0.240c)^2 + (0.757c)^2} = 0.794c$.
(b) $v'_x = [(0) - (0.240c)]/[1 + (0)] = -0.240c$, while

$$v'_y = (0.780c)\sqrt{1-(0.240)^2}/[1 + (0)] = 0.757c.$$

Then $v' = \sqrt{(-0.240c)^2 + (0.757c)^2} = 0.794c$.

$\boxed{\textbf{P20-7}}$ If we look back at the boost equation we might notice that it looks *very* similar to the rule for the tangent of the sum of two angles. It is exactly the same as the rule for the *hyperbolic* tangent,

$$\tanh(\alpha_1 + \alpha_2) = \frac{\tanh\alpha_1 + \tanh\alpha_2}{1 + \tanh\alpha_1\,\tanh\alpha_2}.$$

This means that each boost of $\beta = 0.5$ is the same as a "hyperbolic" rotation of α_r where $\tanh\alpha_r = 0.5$. We need only add these rotations together until we get to α_f, where $\tanh\alpha_\mathrm{f} = 0.999$.

$\alpha_\mathrm{f} = 3.800$, and $\alpha_R = 0.5493$. We can fit $(3.800)/(0.5493) = 6.92$ boosts, but we need an integral number, so there are seven boosts required. The final speed after these seven boosts will be $0.9991c$.

P20-8 (a) If $\Delta x' = 0$, then $\Delta x = u\Delta t$, or

$$u = (730\,\mathrm{m})/(4.96\times10^{-6}\mathrm{s}) = 1.472\times10^{8}\mathrm{m/s} = 0.491c.$$

(b) $\gamma = 1/\sqrt{1-(0.491)^2} = 1.148$,

$$\Delta t' = (1.148)[(4.96\times10^{-6}\mathrm{s}) - (0.491)(730\,\mathrm{m})/(3\times10^{8})] = 4.32\times10^{-6}\mathrm{s}.$$

P20-9 Since the maximum value for u is c, then the minimum Δt is

$$\Delta t \geq (730\,\mathrm{m})/(3.00\times10^{8}\mathrm{m/s}) = 2.43\times10^{-6}\mathrm{s}.$$

P20-10 (a) Yes.

(b) The speed will be very close to the speed of light, consequently $\gamma \approx (23,000)/(30) = 766.7$. Then

$$v = \sqrt{1 - 1/\gamma^2} \approx 1 - 1/2\gamma^2 = 1 - 1/2(766.7)^2 = 0.99999915c.$$

P20-11 (a) $\Delta t' = (5.00\,\mu\mathrm{s})\sqrt{1-(0.6)^2} = 4.00\,\mu\mathrm{s}$.

(b) Note: it takes time for the reading on the S' clock to be seen by the S clock. In this case, $\Delta t_1 + \Delta t_2 = 5.00\,\mu\mathrm{s}$, where $\Delta t_1 = x/u$ and $\Delta t_2 = x/c$. Solving for Δt_1,

$$\Delta t_1 = \frac{(5.00\,\mu\mathrm{s})/(0.6c)}{1/(0.6c) + 1/c} = 3.125\,\mathrm{s},$$

and

$$\Delta t_1' = (3.125\,\mu\mathrm{s})\sqrt{1-(0.6)^2} = 2.50\,\mu\mathrm{s}.$$

P20-12 The only change in the components of Δr occur parallel to the boost. Then we can choose the boost to be parallel to Δr and then

$$\Delta r' = \gamma[\Delta r - u(0)] = \gamma\Delta r \geq \Delta r,$$

since $\gamma \geq 1$.

$\boxed{\textbf{P20-13}}$ (a) Start with Eq. 20-34,

$$E^2 = (pc)^2 + (mc^2)^2,$$

and substitute into this $E = K + mc^2$,

$$K^2 + 2Kmc^2 + (mc^2)^2 = (pc)^2 + (mc^2)^2.$$

We can rearrange this, and then

$$\begin{aligned} K^2 + 2Kmc^2 &= (pc)^2, \\ m &= \frac{(pc)^2 - K^2}{2Kc^2} \end{aligned}$$

(b) As $v/c \to 0$ we have $K \to \frac{1}{2}mv^2$ and $p \to mv$, the classical limits. Then the above expression becomes

$$\begin{aligned} m &= \frac{m^2v^2c^2 - \frac{1}{4}m^2v^4}{mv^2c^2}, \\ &= m\frac{v^2c^2 - \frac{1}{4}v^4}{v^2c^2}, \\ &= m\left(1 - \frac{1}{4}\frac{v^2}{c^2}\right) \end{aligned}$$

But $v/c \to 0$, so this expression reduces to $m = m$ in the classical limit, which is a *good thing.*

(c) We get

$$m = \frac{(121\text{ MeV})^2 - (55.0\text{ MeV})^2}{2(55.0\text{ MeV})c^2} = 1.06\text{ MeV}/c^2,$$

which is $(1.06\text{ MeV}/c^2)/(0.511\text{ MeV}/c^2) = 207m_e$. A muon.

P20-14 Since $E \gg mc^2$ the particle is ultra-relativistic and $v \approx c$. $\gamma = (135)/(0.1396) = 967$. Then the particle has a lab-life of $\Delta t' = (967)(35.0\times10^{-9}\text{s}) = 3.385\times10^{-5}\text{s}$. The distance traveled is

$$x = (3.00\times10^8\text{m/s})(3.385\times10^{-5}\text{s}) = 1.016\times10^4\text{m},$$

so the pion decays 110 km above the Earth.

$\boxed{\textbf{P20-15}}$ (a) A completely inelastic collision means the two particles, each of mass m_1, stick together after the collision, in effect becoming a new particle of mass m_2. We'll use the subscript 1 for moving particle of mass m_1, the subscript 0 for the particle which is originally at rest, and the subscript 2 for the new particle after the collision. We need to conserve momentum,

$$\begin{aligned} p_1 + p_0 &= p_2, \\ \gamma_1 m_1 u_1 + (0) &= \gamma_2 m_2 u_2, \end{aligned}$$

and we need to conserve total energy,

$$\begin{aligned} E_1 + E_0 &= E_2, \\ \gamma_1 m_1 c^2 + m_1 c^2 &= \gamma_2 m_2 c^2, \end{aligned}$$

Divide the momentum equation by the energy equation and then

$$\frac{\gamma_1 u_1}{\gamma_1 + 1} = u_2.$$

But $u_1 = c\sqrt{1 - 1/\gamma_1^2}$, so

$$\begin{aligned} u_2 &= c\frac{\gamma_1\sqrt{1 - 1/\gamma_1^2}}{\gamma_1 + 1}, \\ &= c\frac{\sqrt{\gamma_1^2 - 1}}{\gamma_1 + 1}, \end{aligned}$$

$$
\begin{aligned}
&= c\frac{\sqrt{(\gamma_1+1)(\gamma_1-1)}}{\gamma_1+1},\\
&= c\sqrt{\frac{\gamma_1-1}{\gamma_1+1}}.
\end{aligned}
$$

(b) Using the momentum equation,

$$
\begin{aligned}
m_2 &= m_1\frac{\gamma_1 u_1}{\gamma_2 u_2},\\
&= m_1\frac{c\gamma_1\sqrt{1-1/\gamma_1^2}}{u_2/\sqrt{1-(u_2/c)^2}},\\
&= m_1\frac{\sqrt{\gamma_1^2-1}}{1/\sqrt{(c/u_2)^2-1}},\\
&= m_1\frac{\sqrt{\gamma_1^2-1}}{1/\sqrt{(\gamma_1+1)/(\gamma_1-1)-1}},\\
&= m_1\frac{\sqrt{(\gamma_1+1)(\gamma_1-1)}}{\sqrt{(\gamma_1-1)/2}},\\
&= m_1\sqrt{2(\gamma_1+1)}.
\end{aligned}
$$

P20-16 (a) $K = W = \int F\,dx = \int (dp/dt)dx = \int (dx/dt)dp = \int v\,dp$.
(b) $dp = m\gamma\,dv + mv(d\gamma/dv)dv$. Now use Maple or Mathematica to save time, and get

$$
dp = \frac{m\,dv}{(1-v^2/c^2)^{1/2}} + \frac{mv^2\,dv}{c^2(1-v^2/c^2)^{3/2}}.
$$

Now integrate:

$$
\begin{aligned}
K &= \int v\left(\frac{m}{(1-v^2/c^2)^{1/2}} + \frac{mv^2}{c^2(1-v^2/c^2)^{3/2}}\right)dv,\\
&= \frac{mv^2}{\sqrt{1-v^2/c^2}}.
\end{aligned}
$$

P20-17 (a) Since $E = K + mc^2$, then

$$
E_{\text{new}} = 2E = 2mc^2 + 2K = 2mc^2(1 + K/mc^2).
$$

(b) $E_{\text{new}} = 2(0.938\text{ GeV}) + 2(100\text{ GeV}) = 202\text{ GeV}$.
(c) $K = (100\text{ GeV})/2 - (0.938\text{ GeV}) = 49.1\text{ GeV}$.

P20-18 (a) Assume only one particle is formed. That particle can later decay, but it sets the standard on energy and momentum conservation. The momentum of this one particle must equal that of the incident proton, or

$$
p^2c^2 = [(mc^2+K)^2 - m^2c^4].
$$

The initial energy was $K + 2mc^2$, so the mass of the "one" particle is given by

$$
M^2c^4 = [(K+2mc^2)^2 - p^2c^2] = 2Kmc^2 + 4m^2c^4.
$$

This is a measure of the available energy; the remaining energy is required to conserve momentum. Then

$$
E_{\text{new}} = \sqrt{M^2c^4} = 2mc^2\sqrt{1 + K/2mc^2}.
$$

P20-19 The initial momentum is $m\gamma_\mathrm{i}v_\mathrm{i}$. The final momentum is $(M-m)\gamma_\mathrm{f}v_\mathrm{f}$. Manipulating the momentum conservation equation,

$$\begin{aligned}
m\gamma_\mathrm{i}v_\mathrm{i} &= (M-m)\gamma_\mathrm{f}v_\mathrm{f}, \\
\frac{1}{m\gamma_\mathrm{i}\beta_\mathrm{i}} &= \frac{\sqrt{1-{\beta_\mathrm{f}}^2}}{(M-m)\beta_\mathrm{f}}, \\
\frac{M-m}{m\gamma_\mathrm{i}\beta_\mathrm{i}} &= \left(\frac{1}{{\beta_\mathrm{f}}^2}-1\right), \\
\frac{M-m}{m\gamma_\mathrm{i}\beta_\mathrm{i}}+1 &= \frac{1}{{\beta_\mathrm{f}}^2},
\end{aligned}$$

$\boxed{\textbf{E21-1}}$ (a) We'll assume that the new temperature scale is related to the Celsius scale by a linear transformation; then $T_S = mT_C + b$, where m and b are constants to be determined, T_S is the temperature measurement in the "new" scale, and T_C is the temperature measurement in Celsius degrees.

One of our known points is absolute zero;

$$\begin{aligned} T_S &= mT_C + b, \\ (0) &= m(-273.15^\circ\mathrm{C}) + b. \end{aligned}$$

We have two other points, the melting and boiling points for water,

$$\begin{aligned} (T_S)_{\mathrm{bp}} &= m(100^\circ\mathrm{C}) + b, \\ (T_S)_{\mathrm{mp}} &= m(0^\circ\mathrm{C}) + b; \end{aligned}$$

we can subtract the top equation from the bottom equation to get

$$(T_S)_{\mathrm{bp}} - (T_s)S_{\mathrm{mp}} = 100\ \mathrm{C}^\circ m.$$

We are told this is 180 S°, so $m = 1.8\ \mathrm{S}^\circ/\mathrm{C}^\circ$. Put this into the first equation and then find b, $b = 273.15^\circ\mathrm{C}m = 491.67^\circ\mathrm{S}$. The conversion is then

$$T_S = (1.8\ \mathrm{S}^\circ/\mathrm{C}^\circ)T_C + (491.67^\circ\mathrm{S}).$$

(b) The melting point for water is 491.67°S; the boiling point for water is 180 S° above this, or 671.67°S.

E21-2 $T_{\mathrm{F}} = 9(-273.15\,\mathrm{deg\,C})/5 + 32^\circ\mathrm{F} = -459.67^\circ\mathrm{F}$.

E21-3 (a) We'll assume that the new temperature scale is related to the Celsius scale by a linear transformation; then $T_S = mT_C + b$, where m and b are constants to be determined, T_S is the temperature measurement in the "new" scale, and T_C is the temperature measurement in Celsius degrees.

One of our known points is absolute zero;

$$\begin{aligned} T_S &= mT_C + b, \\ (0) &= m(-273.15^\circ\mathrm{C}) + b. \end{aligned}$$

We have two other points, the melting and boiling points for water,

$$\begin{aligned} (T_S)_{\mathrm{bp}} &= m(100^\circ\mathrm{C}) + b, \\ (T_S)_{\mathrm{mp}} &= m(0^\circ\mathrm{C}) + b; \end{aligned}$$

we can subtract the top equation from the bottom equation to get

$$(T_S)_{\mathrm{bp}} - (T_s)S_{\mathrm{mp}} = 100\ \mathrm{C}^\circ m.$$

We are told this is 100 Q°, so $m = 1.0\ \mathrm{Q}^\circ/\mathrm{C}^\circ$. Put this into the first equation and then find b, $b = 273.15^\circ\mathrm{C} = 273.15^\circ\mathrm{Q}$. The conversion is then

$$T_S = T_C + (273.15^\circ\mathrm{S}).$$

(b) The melting point for water is 273.15°Q; the boiling point for water is 100 Q° above this, or 373.15°Q.

(c) Kelvin Scale.

E21-4 (a) $T = (9/5)(6000\,\mathrm{K} - 273.15) + 32 = 10000°\mathrm{F}$.

(b) $T = (5/9)(98.6°\mathrm{F} - 32) = 37.0°\mathrm{C}$.

(c) $T = (5/9)(-70°\mathrm{F} - 32) = -57°\mathrm{C}$.

(d) $T = (9/5)(-183°\mathrm{C}) + 32 = -297°\mathrm{F}$.

(e) It depends on what you think is hot. My mom thinks 79°F is too warm; that's $T = (5/9)(79°\mathrm{F} - 32) = 26°\mathrm{C}$.

E21-5 $T = (9/5)(310\,\mathrm{K} - 273.15) + 32 = 98.3°\mathrm{F}$, which is fine.

E21-6 (a) $T = 2(5/9)(T - 32)$, so $-T/10 = -32$, or $T = 320°\mathrm{F}$.

(b) $2T = (5/9)(T - 32)$, so $13T/5 = -32$, or $T = -12.3°\mathrm{F}$.

$\boxed{\textbf{E21-7}}$ If the temperature (in Kelvin) is directly proportional to the resistance then $T = kR$, where k is a constant of proportionality. We are given one point, $T = 273.16$ K when $R = 90.35\ \Omega$, but that is okay; we only have one unknown, k. Then $(273.16\text{ K}) = k(90.35\ \Omega)$ or $k = 3.023\text{ K}/\Omega$.

If the resistance is measured to be $R = 96.28\ \Omega$, we have a temperature of

$$T = kR = (3.023\text{ K}/\Omega)(96.28\ \Omega) = 291.1\text{ K}.$$

E21-8 $T = (510°\mathrm{C})/(0.028\,\mathrm{V})V$, so $T = (1.82\times10^{4}\,°\mathrm{C/V})(0.0102\,\mathrm{V}) = 186°\mathrm{C}$.

$\boxed{\textbf{E21-9}}$ We must first find the equation which relates gain to temperature, and then find the gain at the specified temperature. If we let G be the gain we can write this linear relationship as

$$G = mT + b,$$

where m and b are constants to be determined. We have two known points:

$$\begin{aligned}(30.0) &= m(20.0°\,\mathrm{C}) + b,\\ (35.2) &= m(55.0°\,\mathrm{C}) + b.\end{aligned}$$

If we subtract the top equation from the bottom we get $5.2 = m(35.0°\,\mathrm{C})$, or $m = 1.49\,\mathrm{C}^{-1}$. Put this into either of the first two equations and

$$(30.0) = (0.149\,\mathrm{C}^{-1})(20.0°\,\mathrm{C}) + b,$$

which has a solution $b = 27.0$

Now to find the gain when $T = 28.0\,°\mathrm{C}$:

$$G = mT + b = (0.149\,\mathrm{C}^{-1})(28.0°\,\mathrm{C}) + (27.0) = 31.2$$

E21-10 $p/p_{\mathrm{tr}} = (373.15\,\mathrm{K})/(273.16\,\mathrm{K}) = 1.366$.

E21-11 100 cm Hg is 1000 torr. $P_{\mathrm{He}} = (100\text{ cm Hg})(373\,\mathrm{K})/(273.16\,\mathrm{K}) = 136.550$ cm Hg. Nitrogen records a temperature which is 0.2 K higher, so $P_{\mathrm{N}} = (100\text{ cm Hg})(373.2\,\mathrm{K})/(273.16\,\mathrm{K}) = 136.623$ cm Hg. The difference is 0.073 cm Hg.

E21-12 $\Delta L = (23\times10^{-6}/\mathrm{C}°)(33\,\mathrm{m})(15\mathrm{C}°) = 1.1\times10^{-2}\mathrm{m}$.

E21-13 $\Delta L = (3.2\times10^{-6}/\mathrm{C}°)(200\text{ in})(60\mathrm{C}°) = 3.8\times10^{-2}\text{in}$.

E21-14 $L' = (2.725\text{cm})[1 + (23\times10^{-6}/\text{C}^\circ)(128\text{C}^\circ)] = 2.733$ cm.

$\boxed{\textbf{E21-15}}$ We want to focus on the temperature change, not the absolute temperature. In this case, $\Delta T = T_\text{f} - T_\text{i} = (42^\circ\text{C}) - (-5.0^\circ\text{C}) = 47\,\text{C}^\circ$.

Then

$$\Delta L = (11 \times 10^{-6}\,\text{C}^{-1})(12.0\,\text{m})(47\,\text{C}^\circ) = 6.2 \times 10^{-3}\,\text{m}.$$

E21-16 $\Delta A = 2\alpha A\Delta T$, so

$$\Delta A = 2(9\times10^{-6}/\text{C}^\circ)(2.0\,\text{m})(3.0\,\text{m})(30\text{C}^\circ) = 3.2\times10^{-3}\text{m}^2.$$

$\boxed{\textbf{E21-17}}$ (a) We'll apply Eq. 21-10. The surface area of a cube is six times the area of one face, which is the edge length squared. So $A = 6(0.332\,\text{m})^2 = 0.661\,\text{m}^2$. The temperature change is $\Delta T = (75.0^\circ\text{C}) - (20.0^\circ\text{C}) = 55.0\,\text{C}^\circ$. Then the increase in surface area is

$$\Delta A = 2\alpha A\Delta T = 2(19 \times 10^{-6}\,\text{C}^{-1})(0.661\,\text{m}^2)(55.0\,\text{C}^\circ) = 1.38 \times 10^{-3}\,\text{m}^2$$

(b) We'll now apply Eq. 21-11. The volume of the cube is the edge length cubed, so

$$V = (0.332\,\text{m})^3 = 0.0366\,\text{m}^3.$$

and then from Eq. 21-11,

$$\Delta V = 2\alpha V\Delta T = 3(19 \times 10^{-6}\,\text{C}^{-1})(0.0366\,\text{m}^3)(55.0\,\text{C}^\circ) = 1.15 \times 10^{-4}\,\text{m}^3,$$

is the change in volume of the cube.

E21-18 $V' = V(1 + 3\alpha\Delta T)$, so

$$V' = (530\ \text{cm}^3)[1 + 3(29\times10^{-6}/\text{C}^\circ)(-172\,\text{C}^\circ)] = 522\ \text{cm}^3.$$

E21-19 (a) The slope is approximately $1.6\times10^{-4}/\text{C}^\circ$.

(b) The slope is zero.

E21-20 $\Delta r = (\beta/3)r\Delta T$, so

$$\Delta r = [(3.2\times10^{-5}/\text{K})/3](6.37\times10^6\text{m})(2700\,\text{K}) = 1.8\times10^5\text{m}.$$

$\boxed{\textbf{E21-21}}$ We'll assume that the steel ruler measures length correctly at room temperature. Then the 20.05 cm measurement of the rod is correct. But both the rod and the ruler will expand in the oven, so the 20.11 cm measurement of the rod is *not* the actual length of the rod in the oven. What is the actual length of the rod in the oven? We can only answer that after figuring out how the 20.11 cm mark on the ruler moves when the ruler expands.

Let $L = 20.11$ cm correspond to the ruler mark at room temperature. Then

$$\Delta L = \alpha_\text{steel} L\Delta T = (11 \times 10^{-6}\,\text{C}^{-1})(20.11\ \text{cm})(250\,\text{C}^\circ) = 5.5 \times 10^{-2}\ \text{cm}$$

is the shift in position of the mark as the ruler is raised to the higher temperature. Then the change in length of the rod is *not* (20.11 cm) – (20.05 cm) = 0.06 cm, because the 20.11 cm mark is shifted out. We need to add 0.055 cm to this; the rod changed length by 0.115 cm.

The coefficient of thermal expansion for the rod is

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{(0.115\ \text{cm})}{(20.05\ \text{cm})(250\,\text{C}^\circ)} = 23 \times 10^{-6}\,\text{C}^{-1}.$$

E21-22 $A = ab$, $A' = (a + \Delta a)(b + \Delta b) = ab + a\Delta b + b\Delta a + \Delta a\Delta b$, so

$$\begin{aligned}
\Delta A &= a\Delta b + b\Delta a + \Delta a\Delta b, \\
&= A(\Delta b/b + \Delta a/a + \Delta a\Delta b/ab), \\
&\approx A(\alpha\Delta T + \alpha\Delta T), \\
&= 2\alpha A\Delta T.
\end{aligned}$$

$\boxed{\textbf{E21-23}}$ Solve this problem by assuming the solid is in the form of a cube.

If the length of one side of a cube is originally L_0, then the volume is originally $V_0 = L_0^3$. After heating, the volume of the cube will be $V = L^3$, where $L = L_0 + \Delta L$.

Then

$$\begin{aligned}
V &= L^3, \\
&= (L_0 + \Delta L)^3, \\
&= (L_0 + \alpha L_0 \Delta T)^3, \\
&= L_0^3(1 + \alpha\Delta T)^3.
\end{aligned}$$

As long as the quantity $\alpha\Delta T$ is much less than one we can expand the last line in a binomial expansion as

$$V \approx V_0(1 + 3\alpha\Delta T + \cdots),$$

so the change in volume is $\Delta V \approx 3\alpha V_0 \Delta T$.

E21-24 (a) $\Delta A/A = 2(0.18\%) = (0.36\%)$.

(b) $\Delta L/L = 0.18\%$.

(c) $\Delta V/V = 3(0.18\%) = (0.54\%)$.

(d) Zero.

(e) $\alpha = (0.0018)/(100\,\mathrm{C}^\circ) = 1.8\times10^{-5}/\mathrm{C}^\circ$.

E21-25 $\rho' - \rho = m/V' - m/V = m/(V + \Delta V) - m/V \approx -m\Delta V/V^2$. Then

$$\Delta\rho = -(m/V)(\Delta V/V) = -\rho\beta\Delta T.$$

E21-26 Use the results of Exercise 21-25.

(a) $\Delta V/V = 3\Delta L/L = 3(0.092\%) = 0.276\%$. The change in density is

$$\Delta\rho/\rho = -\Delta V/V = -(0.276\%) = -0.28$$

(b) $\alpha = \beta/3 = (0.28\%)/3(40\,\mathrm{C}^\circ) = 2.3\times10^{-5}/\mathrm{C}^\circ$. Must be aluminum.

$\boxed{\textbf{E21-27}}$ The diameter of the rod as a function of temperature is

$$d_\mathrm{s} = d_{\mathrm{s},0}(1 + \alpha_\mathrm{s}\Delta T),$$

The diameter of the ring as a function of temperature is

$$d_\mathrm{b} = d_{\mathrm{b},0}(1 + \alpha_\mathrm{b}\Delta T).$$

We are interested in the temperature when the diameters are equal,

$$
\begin{aligned}
d_{s,0}(1 + \alpha_s \Delta T) &= d_{b,0}(1 + \alpha_b \Delta T), \\
\alpha_s d_{s,0} \Delta T - \alpha_b d_{b,0} \Delta T &= d_{b,0} - d_{s,0}, \\
\Delta T &= \frac{d_{b,0} - d_{s,0}}{\alpha_s d_{s,0} - \alpha_b d_{b,0}}, \\
\Delta T &= \frac{(2.992 \text{ cm}) - (3.000 \text{ cm})}{(11\times10^{-6}/\mathrm{C}^\circ)(3.000 \text{ cm}) - (19\times10^{-6}/\mathrm{C}^\circ)(2.992 \text{ cm})}, \\
&= 335\,\mathrm{C}^\circ.
\end{aligned}
$$

The final temperature is then $T_f = (25^\circ) + 335\,\mathrm{C}^\circ = 360^\circ$.

E21-28 (a) $\Delta L = \Delta L_1 + \Delta L_2 = (L_1\alpha_1 + L_2\alpha_2)\Delta T$. The effective value for α is then

$$\alpha = \frac{\Delta L}{L \Delta T} = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L}.$$

(b) Since $L_2 = L - L_1$ we can write

$$
\begin{aligned}
\alpha_1 L_1 + \alpha_2 (L - L_1) &= \alpha L, \\
L_1 &= L \frac{\alpha - \alpha_2}{\alpha_1 - \alpha_2}, \\
&= (0.524\,\mathrm{m}) \frac{(13\times10^{-6}) - (11\times10^{-6})}{(19\times10^{-6}) - (11\times10^{-6})} = 0.131\,\mathrm{m}.
\end{aligned}
$$

The brass length is then 13.1 cm and the steel is 39.3 cm.

E21-29 At 100°C the glass and mercury each have a volume V_0. After cooling, the *difference* in volume changes is given by

$$\Delta V = V_0 (3\alpha_g - \beta_m)\Delta T.$$

Since $m = \rho V$, the mass of mercury that needs to be added can be found by multiplying though by the density of mercury. Then

$$\Delta m = (0.891\,\mathrm{kg})[3(9.0\times10^{-6}/\mathrm{C}^\circ) - (1.8\times10^{-4}/\mathrm{C}^\circ)](-135\mathrm{C}^\circ) = 0.0184\,\mathrm{kg}.$$

This is the additional amount required, so the total is now 909 g.

E21-30 (a) The rotational inertia is given by $I = \int r^2 dm$; changing the temperature requires $r \to r' = r + \Delta r = r(1 + \alpha \Delta T)$. Then

$$I' = \int (1 + \alpha \Delta T)^2 r^2 dm \approx (1 + 2\alpha \Delta T) \int r^2 dm,$$

so $\Delta I = 2\alpha I \Delta T$.

(b) Since $L = I\omega$, then $0 = \omega \Delta I + I \Delta \omega$. Rearranging, $\Delta\omega/\omega = -\Delta I/I = -2\alpha\Delta T$. Then

$$\Delta\omega = -2(19\times10^{-6}/\mathrm{C}^\circ)(230 \text{ rev/s})(170\,\mathrm{C}^\circ) = -1.5 \text{ rev/s}.$$

$\boxed{\textbf{E21-31}}$ This problem is related to objects which expand when heated, but we never actually need to calculate any temperature changes. We will, however, be interested in the change in rotational inertia. Rotational inertia is directly proportional to the square of the (appropriate) linear dimension, so

$$I_\mathrm{f}/I_\mathrm{i} = (r_\mathrm{f}/r_\mathrm{i})^2.$$

(a) If the bearings are frictionless then there are no external torques, so the angular momentum is constant.

(b) If the angular momentum is constant, then

$$\begin{aligned} L_\mathrm{i} &= L_\mathrm{f}, \\ I_\mathrm{i}\omega_\mathrm{i} &= I_\mathrm{f}\omega_\mathrm{f}. \end{aligned}$$

We are interested in the percent change in the angular velocity, which is

$$\frac{\omega_\mathrm{f} - \omega_\mathrm{i}}{\omega_\mathrm{i}} = \frac{\omega_\mathrm{f}}{\omega_\mathrm{i}} - 1 = \frac{I_\mathrm{i}}{I_\mathrm{f}} - 1 = \left(\frac{r_\mathrm{i}}{r_\mathrm{f}}\right)^2 - 1 = \left(\frac{1}{1.0018}\right)^2 - 1 = -0.36\%.$$

(c) The rotational kinetic energy is proportional to $I\omega^2 = (I\omega)\omega = L\omega$, but L is constant, so

$$\frac{K_\mathrm{f} - K_\mathrm{i}}{K_\mathrm{i}} = \frac{\omega_\mathrm{f} - \omega_\mathrm{i}}{\omega_\mathrm{i}} = -0.36\%.$$

E21-32 (a) The period of a physical pendulum is given by Eq. 17-28. There are two variables in the equation that depend on length. I, which is proportional to a length squared, and d, which is proportional to a length. This means that the period have an overall dependence on length proportional to $\sqrt{r}$. Taking the derivative,

$$\Delta P \approx dP = \frac{1}{2}\frac{P}{r}dr \approx \frac{1}{2}P\alpha\Delta T.$$

(b) $\Delta P/P = (0.7\times 10^{-6}\mathrm{C}^\circ)(10\mathrm{C}^\circ)/2 = 3.5\times 10^{-6}$. After 30 days the clock will be slow by

$$\Delta t = (30 \times 24 \times 60 \times 60\,\mathrm{s})(3.5\times 10^{-6}) = 9.07\,\mathrm{s}.$$

E21-33 Refer to the Exercise 21-32.

$$\Delta P = (3600\,\mathrm{s})(19\times 10^{-6}\mathrm{C}^\circ)(-20\mathrm{C}^\circ)/2 = 0.68\,\mathrm{s}.$$

E21-34 At 22°C the aluminum cup and glycerin each have a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(3\alpha_\mathrm{a} - \beta_\mathrm{g})\Delta T.$$

The amount that spills out is then

$$\Delta V = (110\ \mathrm{cm}^3)[3(23\times 10^{-6}/\mathrm{C}^\circ) - (5.1\times 10^{-4}/\mathrm{C}^\circ)](6\mathrm{C}^\circ) = -0.29\ \mathrm{cm}^3.$$

E21-35 At 20.0°C the glass tube is filled with liquid to a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(3\alpha_\mathrm{g} - \beta_\mathrm{l})\Delta T.$$

The cross sectional area of the tube changes according to

$$\Delta A = A_0 2\alpha_\mathrm{g}\Delta T.$$

Consequently, the height of the liquid changes according to

$$\begin{aligned}\Delta V &= (h_0+\Delta h)(A_0+\Delta A)-h_0 A,\\ &\approx h_0\Delta A + A_0\Delta h,\\ \Delta V/V_0 &= \Delta A/A_0 + \Delta h/h_0.\end{aligned}$$

Then

$$\Delta h = (1.28\,\mathrm{m}/2)[(1.1\times10^{-5}/\mathrm{C}^\circ)-(4.2\times10^{-5}/\mathrm{C}^\circ)](13\,\mathrm{C}^\circ) = 2.6\times10^{-4}\mathrm{m}.$$

E21-36 (a) $\beta = (dV/dT)/V$. If $pV = nRT$, then $p\,dV = nR\,dT$, so

$$\beta = (nR/p)/V = nR/pV = 1/T.$$

(b) Kelvins.
(c) $\beta \approx 1/(300/\mathrm{K}) = 3.3\times10^{-3}/\mathrm{K}$.

E21-37 (a) $V = (1\text{ mol})(8.31\,\mathrm{J/mol\cdot K})(273\,\mathrm{K})/(1.01\times10^5\mathrm{Pa}) = 2.25\times10^{-2}\mathrm{m}^3$.
(b) $(6.02\times10^{23}\mathrm{mol}^{-1})/(2.25\times10^4/\mathrm{cm}^3) = 2.68\times10^{19}$.

E21-38 $n/V = p/kT$, so

$$n/V = (1.01\times10^{-13}\mathrm{Pa})/(1.38\times10^{-23}\mathrm{J/K})(295\,\mathrm{K}) = 25\text{ part/cm}^3.$$

$\boxed{\textbf{E21-39}}$ (a) Using Eq. 21-17,

$$n = \frac{pV}{RT} = \frac{(108\times10^3\mathrm{Pa})(2.47\,\mathrm{m}^3)}{(8.31\,\mathrm{J/mol{\cdot}K})([12+273]\,\mathrm{K})} = 113\text{ mol}.$$

(b) Use the same expression again,

$$V = \frac{nRT}{p} = \frac{(113\text{ mol})(8.31\,\mathrm{J/mol{\cdot}K})([31+273]\,\mathrm{K})}{(316\times10^3\mathrm{Pa})} = 0.903\,\mathrm{m}^3.$$

E21-40 (a) $n = pV/RT = (1.01\times10^5\mathrm{Pa})(1.13\times10^{-3}\mathrm{m}^3)/(8.31\,\mathrm{J/mol{\cdot}K})(315\,\mathrm{K}) = 4.36\times10^{-2}\mathrm{mol}$.
(b) $T_\mathrm{f} = T_\mathrm{i}p_\mathrm{f}V_\mathrm{f}/p_\mathrm{i}V_\mathrm{i}$, so

$$T_\mathrm{f} = \frac{(315\,\mathrm{K})(1.06\times10^5\mathrm{Pa})(1.530\times10^{-3}\mathrm{m}^3)}{(1.01\times10^5\mathrm{Pa})(1.130\times10^{-3}\mathrm{m}^3)} = 448\,\mathrm{K}.$$

E21-41 $p_\mathrm{i} = (14.7+24.2)\text{ lb/in}^2 = 38.9\text{ lb/in}^2$. $p_\mathrm{f} = p_\mathrm{i}T_\mathrm{f}V_\mathrm{i}/T_\mathrm{i}V_\mathrm{f}$, so

$$p_\mathrm{f} = \frac{(38.9\text{ lb/in}^2)(299\mathrm{K})(988\text{ in}^3)}{(270\,\mathrm{K})(1020\text{ in}^3)} = 41.7\text{ lb/in}^2.$$

The gauge pressure is then $(41.7-14.7)\text{ lb/in}^2 = 27.0\text{ lb/in}^2$.

E21-42 Since $p = F/A$ and $F = mg$, a reasonable estimate for the mass of the atmosphere is

$$m = pA/g = (1.01\times10^5\mathrm{Pa})4\pi(6.37\times10^6\mathrm{m})^2/(9.81\,\mathrm{m/s^2}) = 5.25\times10^{18}\mathrm{kg}.$$

E21-43 $p = p_0 + \rho g h$, where h is the depth. Then $P_\mathrm{f} = 1.01\times10^5\mathrm{Pa}$ and

$$p_\mathrm{i} = (1.01\times10^5\mathrm{Pa}) + (998\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})(41.5\,\mathrm{m}) = 5.07\times10^5\mathrm{Pa}.$$

$V_\mathrm{f} = V_\mathrm{i} p_\mathrm{i} T_\mathrm{f}/p_\mathrm{f} T_\mathrm{i}$, so

$$V_\mathrm{f} = \frac{(19.4\ \mathrm{cm^3})(5.07\times10^5\mathrm{Pa.})(296\,\mathrm{K})}{(1.01\times10^5\mathrm{Pa})(277\,\mathrm{K})} = 104\ \mathrm{cm^3}.$$

E21-44 The new pressure in the pipe is

$$p_\mathrm{f} = p_\mathrm{i} V_\mathrm{i}/V_\mathrm{f} = (1.01\times10^5\mathrm{Pa})(2) = 2.02\times10^5\mathrm{Pa}.$$

The water pressure at some depth y is given by $p = p_0 + \rho g y$, so

$$y = \frac{(2.02\times10^5\mathrm{Pa}) - (1.01\times10^5\mathrm{Pa})}{(998\,\mathrm{kg/m^3})(9.81\,\mathrm{m/s^2})} = 10.3\,\mathrm{m}.$$

Then the water/air interface inside the tube is at a depth of 10.3 m; so $h = (10.3\,\mathrm{m}) + (25.0\,\mathrm{m})/2 = 22.8\,\mathrm{m}$.

$\boxed{\textbf{P21-1}}$ (a) The dimensions of A must be $[\mathrm{time}]^{-1}$, as can be seen with a quick inspection of the equation. We would expect that A would depend on the surface area at the very least; however, that means that it must also depend on some other factor to fix the dimensionality of A.

(b) Rearrange and integrate,

$$\begin{aligned} \int_{\Delta T_0}^{T} \frac{d\Delta T}{\Delta T} &= -\int_0^t A\,dt, \\ \ln(\Delta T/\Delta T_0) &= -At, \\ \Delta T &= \Delta T_0 e^{-At}. \end{aligned}$$

P21-2 First find A.

$$A = \frac{\ln(\Delta T_0/\Delta T)}{t} = \frac{\ln[(29\,\mathrm{C^\circ})/(25\,\mathrm{C^\circ})]}{(45\ \mathrm{min})} = 3.30\times10^{-3}/\mathrm{min}.$$

Then find time to new temperature difference.

$$t = \frac{\ln(\Delta T_0/\Delta T)}{t} = \frac{\ln[(29\,\mathrm{C^\circ})/(21\,\mathrm{C^\circ})]}{(3.30\times10^{-3}/\mathrm{min})} = 97.8\mathrm{min}$$

This happens $97.8 - 45 = 53$ minutes later.

P21-3 If we neglect the expansion of the tube then we can assume the cross sectional area of the tube is constant. Since $V = Ah$, we can assume that $\Delta V = A\Delta h$. Then since $\Delta V = \beta V_0 \Delta T$, we can write $\Delta h = \beta h_0 \Delta T$.

P21-4 For either container we can write $p_i V_i = n_i R T_i$. We are told that V_i and n_i are constants. Then $\Delta p = AT_1 - BT_2$, where A and B are constants. When $T_1 = T_2$ $\Delta p = 0$, so $A = B$. When $T_1 = T_\mathrm{tr}$ and $T_2 = T_\mathrm{b}$ we have

$$(120\ \mathrm{mm\ Hg}) = A(373\,\mathrm{K} - 273.16\,\mathrm{K}),$$

so $A = 1.202$ mm Hg/K. Then

$$T = \frac{(90\ \mathrm{mm\ Hg}) + (1.202\ \mathrm{mm\ Hg/K})(273.16\,\mathrm{K})}{(1.202\ \mathrm{mm\ Hg/K})} = 348\,\mathrm{K}.$$

Actually, we could have assumed A was negative, and then the answer would be 198 K.

$\boxed{\textbf{P21-5}}$ Start with a differential form for Eq. 21-8, $dL/dT = \alpha L_0$, rearrange, and integrate:

$$\begin{aligned}
\int_{L_0}^{L} dL &= \int_{T_0}^{T} \alpha L_0 \, dT, \\
L - L_0 &= L_0 \int_{T_0}^{T} \alpha \, dT, \\
L &= L_0 \left(1 + \int_{T_0}^{T} \alpha \, dT \right).
\end{aligned}$$

P21-6 $\Delta L = \alpha L \Delta T$, so

$$\frac{\Delta T}{\Delta t} = \frac{1}{\alpha L} \frac{\Delta L}{\Delta t} = \frac{(96 \times 10^{-9}\text{m/s})}{(23 \times 10^{-6}/\text{C}^\circ)(1.8 \times 10^{-2}\text{m})} = 0.23^\circ\text{C/s}.$$

$\boxed{\textbf{P21-7}}$ (a) Consider the work that was done for Ex. 21-27. The length of rod a is

$$L_a = L_{a,0}(1 + \alpha_a \Delta T),$$

while the length of rod b is

$$L_b = L_{b,0}(1 + \alpha_b \Delta T).$$

The difference is

$$\begin{aligned}
L_a - L_b &= L_{a,0}(1 + \alpha_a \Delta T) - L_{b,0}(1 + \alpha_b \Delta T), \\
&= L_{a,0} - L_{b,0} + (L_{a,0}\alpha_a - L_{b,0}\alpha_b)\Delta T,
\end{aligned}$$

which will be a constant is $L_{a,0}\alpha_a = L_{b,0}\alpha_b$ or

$$L_{i,0} \propto 1/\alpha_i.$$

(b) We want $L_{a,0} - L_{b,0} = 0.30\,\text{m}$ so

$$k/\alpha_a - k/\alpha_b = 0.30\,\text{m},$$

where k is a constant of proportionality;

$$k = (0.30\,\text{m}) / \left(1/(11 \times 10^{-6}/\text{C}^\circ) - 1/(19 \times 10^{-6}/\text{C}^\circ) \right) = 7.84 \times 10^{-6}\text{m/C}^\circ.$$

The two lengths are

$$L_a = (7.84 \times 10^{-6}\text{m/C}^\circ)/(11 \times 10^{-6}/\text{C}^\circ) = 0.713\,\text{m}$$

for steel and

$$L_b = (7.84 \times 10^{-6}\text{m/C}^\circ)/(19 \times 10^{-6}/\text{C}^\circ) = 0.413\,\text{m}$$

for brass.

P21-8 The fractional increase in length of the bar is $\Delta L/L_0 = \alpha \Delta T$. The right triangle on the left has base $L_0/2$, height x, and hypotenuse $(L_0 + \Delta L)/2$. Then

$$x = \frac{1}{2}\sqrt{(L_0 + \Delta L)^2 - L_0^2} = \frac{L_0}{2}\sqrt{2\frac{\Delta L}{L_0}}.$$

With numbers,

$$x = \frac{(3.77\,\text{m})}{2}\sqrt{2(25 \times 10^{-6}/\text{C}^\circ)(32\,\text{C}^\circ)} = 7.54 \times 10^{-2}\text{m}.$$

P21-9 We want to evaluate $V = V_0(1 + \int \beta\, dT)$; the integral is the area under the graph; the graph looks like a triangle, so the result is

$$V = V_0[1 + (16\,\mathrm{C}^\circ)(0.0002/\mathrm{C}^\circ)/2] = (1.0016)V_0.$$

The density is then

$$\rho = \rho_0(V_0/V) = (1000\,\mathrm{kg/m^3})/(1.0016) = 0.9984\,\mathrm{kg/m^3}.$$

P21-10 At 0.00°C the glass bulb is filled with mercury to a volume V_0. After heating, the *difference* in volume changes is given by

$$\Delta V = V_0(\beta - 3\alpha)\Delta T.$$

Since $T_0 = 0.0$°C, then $\Delta T = T$, if it is measured in °C. The amount of mercury in the capillary is ΔV, and since the cross sectional area is fixed at A, then the length is $L = \Delta V/A$, or

$$L = \frac{V}{A}(\beta - 3\alpha)\Delta T.$$

P21-11 Let a, b, and c correspond to aluminum, steel, and invar, respectively. Then

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

We can replace a with $a_0(1 + \alpha_a \Delta T)$, and write similar expressions for b and c. Since $a_0 = b_0 = c_0$, this can be simplified to

$$\cos C = \frac{(1 + \alpha_a\Delta T)^2 + (1 + \alpha_b\Delta T)^2 - (1 + \alpha_c\Delta T)^2}{2(1 + \alpha_a\Delta T)(1 + \alpha_b\Delta T)}.$$

Expand this as a Taylor series in terms of ΔT, and we find

$$\cos C \approx \frac{1}{2} + \frac{1}{2}\left(\alpha_a + \alpha_b - 2\alpha_c\right)\Delta T.$$

Now solve:

$$\Delta T = \frac{2\cos(59.95^\circ) - 1}{(23\times10^{-6}/\mathrm{C}^\circ) + (11\times10^{-6}/\mathrm{C}^\circ) - 2(0.7\times10^{-6}/\mathrm{C}^\circ)} = 46.4^\circ\mathrm{C}.$$

The final temperature is then 66.4°C.

P21-12 The bottom of the iron bar moves downward according to $\Delta L = \alpha L \Delta T$. The center of mass of the iron bar is located in the center; it moves downward half the distance. The mercury expands in the glass upwards; subtracting off the distance the iron moves we get

$$\Delta h = \beta h \Delta T - \Delta L = (\beta h - \alpha L)\Delta T.$$

The center of mass in the mercury is located in the center. If the center of mass of the system is to remain constant we require

$$m_\mathrm{i}\Delta L/2 = m_\mathrm{m}(\Delta h - \Delta L)/2;$$

or, since $\rho = mV = mAy$,

$$\rho_\mathrm{i}\alpha L = \rho_\mathrm{m}(\beta h - 2\alpha L).$$

Solving for h,

$$h = \frac{(12\times10^{-6}/\mathrm{C}^\circ)(1.00\,\mathrm{m})[(7.87\times10^3\mathrm{kg/m^3}) + 2(13.6\times10^3\mathrm{kg/m^3})]}{(13.6\times10^3\mathrm{kg/m^3})(18\times10^{-5}/\mathrm{C}^\circ)} = 0.17\,\mathrm{m}.$$

$\boxed{\textbf{P21-13}}$ The volume of the block which is beneath the surface of the mercury displaces a mass of mercury equal to the mass of the block. The mass of the block is independent of the temperature but the volume of the displaced mercury changes according to

$$V_{\rm m} = V_{\rm m,0}(1 + \beta_{\rm m}\Delta T).$$

This volume is equal to the depth which the block sinks times the cross sectional area of the block (which *does* change with temperature). Then

$$h_{\rm s}{h_{\rm b}}^2 = h_{\rm s,0}{h_{\rm b,0}}^2(1 + \beta_{\rm m}\Delta T),$$

where $h_{\rm s}$ is the depth to which the block sinks and $h_{\rm b,0} = 20$ cm is the length of the side of the block. But

$$h_{\rm b} = h_{\rm b,0}(1 + \alpha_{\rm b}\Delta T),$$

so

$$h_{\rm s} = h_{\rm s,0}\frac{1 + \beta_{\rm m}\Delta T}{(1 + \alpha_{\rm b}\Delta T)^2}.$$

Since the changes are small we can expand the right hand side using the binomial expansion; keeping terms only in ΔT we get

$$h_{\rm s} \approx h_{\rm s,0}(1 + (\beta_{\rm m} - 2\alpha_{\rm b})\Delta T),$$

which means the block will sink a distance $h_{\rm s} - h_{\rm s,0}$ given by

$$h_{\rm s,0}(\beta_{\rm m} - 2\alpha_{\rm b})\Delta T = h_{\rm s,0}\left[(1.8\times10^{-4}/{\rm C}^\circ) - 2(23\times10^{-6}/{\rm C}^\circ)\right](50\,{\rm C}^\circ) = (6.7\times10^{-3})h_{\rm s,0}.$$

In order to finish we need to know how much of the block was submerged in the first place. Since the fraction submerged is equal to the ratio of the densities, we have

$$h_{\rm s,0}/h_{\rm b,0} = \rho_{\rm b}/\rho_{\rm m} = (2.7\times10^3{\rm kg/m^3})/(1.36\times10^4{\rm kg/m^3}),$$

so $h_{\rm s,0} = 3.97$ cm, and the change in depth is 0.27 mm.

P21-14 The area of glass expands according to $\Delta A_{\rm g} = 2\alpha_{\rm g}A_{\rm g}\Delta T$. The are of Dumet wire expands according to

$$\Delta A_{\rm c} + \Delta A{\rm i} = 2(\alpha_{\rm c}A_{\rm c} + \alpha_{\rm i}A_{\rm i})\Delta T.$$

We need these to be equal, so

$$\begin{aligned}
\alpha_{\rm g}A_{\rm g} &= \alpha_{\rm c}A_{\rm c} + \alpha_{\rm i}A_{\rm i},\\
\alpha_{\rm g}{r_{\rm g}}^2 &= \alpha_{\rm c}({r_{\rm c}}^2 - {r_{\rm i}}^2) + \alpha_{\rm i}{r_{\rm i}}^2,\\
\alpha_{\rm g}({r_{\rm c}}^2 + {r_{\rm i}}^2) &= \alpha_{\rm c}({r_{\rm c}}^2 - {r_{\rm i}}^2) + \alpha_{\rm i}{r_{\rm i}}^2,\\
\frac{{r_{\rm i}}^2}{{r_{\rm c}}^2} &= \frac{\alpha_{\rm c} - \alpha_{\rm g}}{\alpha_{\rm c} - \alpha_{\rm i}}.
\end{aligned}$$

P21-15

P21-16 $V_2 = V_1(p_1/p_2)(T_1/T_2)$, so

$$V_2 = (3.47\,{\rm m}^3)[(76\text{ cm Hg})/(36\text{ cm Hg})][(225\,{\rm K})/(295\,{\rm K})] = 5.59\,{\rm m}^3.$$

$\boxed{\textbf{P21-17}}$ Call the containers one and two so that $V_1 = 1.22$ L and $V_2 = 3.18$ L. Then the initial number of moles in the two containers are

$$n_{1,\mathrm{i}} = \frac{p_\mathrm{i} V_1}{RT_\mathrm{i}} \text{ and } n_{2,\mathrm{i}} = \frac{p_\mathrm{i} V_2}{RT_\mathrm{i}}.$$

The total is

$$n = p_\mathrm{i}(V_1 + V_2)/(RT_\mathrm{i}).$$

Later the temperatures are changed and then the number of moles of gas in each container is

$$n_{1,\mathrm{f}} = \frac{p_\mathrm{f} V_1}{RT_{1,\mathrm{f}}} \text{ and } n_{2,\mathrm{f}} = \frac{p_\mathrm{f} V_2}{RT_{2,\mathrm{f}}}.$$

The total is still n, so

$$\frac{p_\mathrm{f}}{R}\left(\frac{V_1}{T_{1,\mathrm{f}}} + \frac{V_2}{T_{2,\mathrm{f}}}\right) = \frac{p_\mathrm{i}(V_1 + V_2)}{RT_\mathrm{i}}.$$

We can solve this for the final pressure, so long as we remember to convert all temperatures to Kelvins,

$$p_\mathrm{f} = \frac{p_\mathrm{i}(V_1 + V_2)}{T_\mathrm{i}}\left(\frac{V_1}{T_{1,\mathrm{f}}} + \frac{V_2}{T_{2,\mathrm{f}}}\right)^{-1},$$

or

$$p_\mathrm{f} = \frac{(1.44\text{ atm})(1.22\text{L} + 3.18\text{ L})}{(289\,\text{K})}\left(\frac{(1.22\text{ L})}{(289\,\text{K})} + \frac{(3.18\text{ L})}{(381\,\text{K})}\right)^{-1} = 1.74\text{ atm}.$$

P21-18 Originally $n_A = p_A V_A/RT_A$ and $n_B = p_B V_B/RT_B$; $V_B = 4V_A$. Label the final state of A as C and the final state of B as D. After mixing, $n_C = p_C V_A/RT_A$ and $n_D = p_D V_B/RT_B$, but $P_C = P_D$ and $n_A + n_B = n_C + n_D$. Then

$$p_A/T_A + 4p_B/T_B = p_C(1/T_A + 4/T_B),$$

or

$$p_C = \frac{(5\times10^5\text{Pa})/(300\,\text{K}) + 4(1\times10^5\text{Pa})/(400\,\text{K})}{1/(300\,\text{K}) + 4/(400\,K)} = 2.00\times10^5\text{Pa}.$$

P21-19 If the temperature is uniform then all that is necessary is to substitute $p_0 = nRT/V$ and $p = nRT/V$; cancel RT from both sides, and then equate n/V with n_V.

P21-20 Use the results of Problem 15-19. The initial pressure inside the bubble is $p_\mathrm{i} = p_0 + 4\gamma/r_\mathrm{i}$. The final pressure inside the bell jar is zero, so$p_\mathrm{f} = 4\gamma/r_\mathrm{f}$. The initial and final pressure inside the bubble are related by $p_\mathrm{i} {r_\mathrm{i}}^3 = p_\mathrm{f} {r_\mathrm{f}}^3 = 4\gamma {r_\mathrm{f}}^2$. Now for numbers:

$$p_\mathrm{i} = (1.01\times10^5\text{Pa}) + 4(2.5\times10^{-2}\text{N/m})/(2.0\times10^{-3}\text{m}) = 1.0105\times10^5\text{Pa}.$$

and

$$r_\mathrm{f} = \sqrt{\frac{(1.0105\times10^5\text{Pa})(2.0\times10^{-3}\text{m})^3}{4(2.5\times10^{-2}\text{N/m})}} = 8.99\times10^{-2}\text{m}.$$

P21-21

P21-22

E22-1 (a) $n = (2.56 \text{ g})/(197 \text{ g/mol}) = 1.30\times10^{-2}\text{mol}$.
(b) $N = (6.02\times10^{23}\text{mol}^{-1})(1.30\times10^{-2}\text{mol}) = 7.83\times10^{21}$.

E22-2 (a) $N = pV/kT = (1.01\times10^{5}\text{Pa})(1.00\,\text{m}^3)/(1.38\times10^{-23}\text{J/K})(293\,\text{K}) = 2.50\times10^{25}$.
(b) $n = (2.50\times10^{25})/(6.02\times10^{23}\text{mol}^{-1}) = 41.5$ mol. Then

$$m = (41.5 \text{ mol})[75\%(28 \text{ g/mol}) + 25\%(32 \text{ g/mol})] = 1.20\,\text{kg}.$$

$\boxed{\textbf{E22-3}}$ (a) We first need to calculate the molar mass of ammonia. This is

$$M = M(\text{N}) + 3M(\text{H}) = (14.0 \text{ g/mol}) + 3(1.01 \text{ g/mol}) = 17.0 \text{ g/mol}$$

The number of moles of nitrogen present is

$$n = m/M_\text{r} = (315 \text{ g})/(17.0 \text{ g/mol}) = 18.5 \text{ mol}.$$

The volume of the tank is

$$V = nRT/p = (18.5 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(350\,\text{K})/(1.35\times10^{6}\text{Pa}) = 3.99\times10^{-2}\text{m}^3.$$

(b) After the tank is checked the number of moles of gas in the tank is

$$n = pV/(RT) = (8.68\times10^{5}\text{Pa})(3.99\times10^{-2}\text{m}^3)/[(8.31\,\text{J/mol}\cdot\text{K})(295\,\text{K})] = 14.1 \text{ mol}.$$

In that case, 4.4 mol must have escaped; that corresponds to a mass of

$$m = nM_\text{r} = (4.4 \text{ mol})(17.0 \text{ g/mol}) = 74.8 \text{ g}.$$

E22-4 (a) The volume per particle is $V/N = kT/P$, so

$$V/N = (1.38\times10^{-23}\text{J/K})(285\,\text{K})/(1.01\times10^{5}\text{Pa}) = 3.89\times10^{-26}\text{m}^3.$$

The edge length is the cube root of this, or 3.39×10^{-9}m. The ratio is 11.3.
(b) The volume per particle is V/N_A, so

$$V/N_A = (18\times10^{-6}\text{m}^3)/(6.02\times10^{23}) = 2.99\times10^{-29}\text{m}^3.$$

The edge length is the cube root of this, or 3.10×10^{-10}m. The ratio is 1.03.

E22-5 The volume per particle is $V/N = kT/P$, so

$$V/N = (1.38\times10^{-23}\text{J/K})(308\,\text{K})/(1.22)(1.01\times10^{5}\text{Pa}) = 3.45\times10^{-26}\text{m}^3.$$

The fraction actually occupied by the particle is

$$\frac{4\pi(0.710\times10^{-10}\text{m})^3/3)}{(3.45\times10^{-26}\text{m}^3)} = 4.34\times10^{-5}.$$

E22-6 The component of the momentum normal to the wall is

$$p_y = (3.3\times10^{-27}\text{kg})(1.0\times10^{3}\text{m/s})\cos(55^\circ) = 1.89\times10^{-24}\text{kg}\cdot\text{m/s}.$$

The pressure exerted on the wall is

$$p = \frac{F}{A} = \frac{(1.6\times10^{23}/\text{s})2(1.89\times10^{-24}\text{kg}\cdot\text{m/s})}{(2.0\times10^{-4}\text{m}^2)} = 3.0\times10^{3}\text{Pa}.$$

$\boxed{\textbf{E22-7}}$ (a) From Eq. 22-9,

$$v_{\rm rms} = \sqrt{\frac{3p}{\rho}}.$$

Then

$$p = 1.23 \times 10^{-3} \text{ atm} \left(\frac{1.01 \times 10^5 \,\text{Pa}}{1 \text{ atm}}\right) = 124\,\text{Pa}$$

and

$$\rho = 1.32 \times 10^{-5} \text{ g/cm}^3 \left(\frac{1\text{kg}}{1000 \text{ g}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 1.32 \times 10^{-2} \text{ kg/m}^3.$$

Finally,

$$v_{\rm rms} = \sqrt{\frac{3(1240\,\text{Pa})}{(1.32 \times 10^{-2} \text{ kg/m}^3)}} = 531\,\text{m/s}.$$

(b) The molar density of the gas is just n/V; but this can be found quickly from the ideal gas law as

$$\frac{n}{V} = \frac{p}{RT} = \frac{(1240\,\text{Pa})}{(8.31\,\text{J/mol}\cdot\text{K})(317\,\text{K})} = 4.71 \times 10^{-1} \text{ mol/m}^3.$$

(c) We were given the density, which is mass per volume, so we could find the molar mass from

$$\frac{\rho}{n/V} = \frac{(1.32 \times 10^{-2} \text{ kg/m}^3)}{(4.71 \times 10^{-1} \text{ mol/m}^3)} = 28.0 \text{ g/mol}.$$

But what gas is it? It could contain any atom lighter than silicon; trial and error is the way to go. Some of my guesses include C_2H_4 (ethene), CO (carbon monoxide), and N_2. There's no way to tell which is correct at this point, in fact, the gas could be a mixture of all three.

E22-8 The density is $\rho = m/V = nM_r/V$, or

$$\rho = (0.350 \text{ mol})(0.0280 \text{ kg/mol})/\pi(0.125\,\text{m}/2)^2(0.560\,\text{m}) = 1.43\,\text{kg/m}^3.$$

The rms speed is

$$v_{\rm rms} = \sqrt{\frac{3(2.05)(1.01\times10^5\text{Pa})}{(1.43\,\text{kg/m}^3)}} = 659\,\text{m/s}.$$

E22-9 (a) $N/V = p/kT = (1.01\times10^5\text{Pa})/(1.38\times10^{-23}\text{J/K})(273\,\text{K}) = 2.68\times10^{25}/\text{m}^3$.

(b) Note that Eq. 22-11 is *wrong*; for the explanation read the last two paragraphs in the first column on page 502. We need an extra factor of $\sqrt{2}$, so $\pi d^2 = V/\sqrt{2}N\lambda$, so

$$d = \sqrt{1/\sqrt{2}\pi(2.68\times10^{25}/\text{m}^3)(285\times10^{-9}\text{m})} = 1.72\times10^{-10}\text{m}.$$

E22-10 (a) $\lambda = V/\sqrt{2}N\pi d^2$, so

$$\lambda = \frac{1}{\sqrt{2}(1.0\times10^6/\text{m}^3)\pi(2.0\times10^{-10}\text{m})^2} = 5.6\times10^{12}\text{m}.$$

(b) Particles effectively follow ballistic trajectories.

$\boxed{\textbf{E22-11}}$ We have $v = f\lambda$, where λ is the wavelength (which we will set equal to the mean free path), and v is the speed of sound. The mean free path is, from Eq. 22-13,

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 p}$$

so

$$f = \frac{\sqrt{2}\pi d^2 pv}{kT} = \frac{\sqrt{2}\pi(315\times10^{-12}\text{m})^2(1.02\times1.01\times10^5\text{Pa})(343\,\text{m/s})}{(1.38\times10^{-23}\text{J/K})(291\,\text{K})} = 3.88\times10^9\text{Hz}.$$

E22-12 (a) $p = (1.10\times10^{-6}$ mm Hg$)(133\,\text{Pa/mm Hg}) = 1.46\times10^{-4}\text{Pa}$. The particle density is

$$N/V = (1.46\times10^{-4}\text{Pa})/(1.38\times10^{-23}\text{J/K})(295\,\text{K}) = 3.59\times10^{16}/\text{m}^3.$$

(b) The mean free path is

$$\lambda = 1/\sqrt{(2)}(3.59\times10^{16}/\text{m}^3)\pi(2.20\times10^{-10}\text{m})^2 = 130\,\text{m}.$$

E22-13 Note that $v_{\text{av}} \propto \sqrt{T}$, while $\lambda \propto T$. Then the collision rate is proportional to $1/\sqrt{T}$. Then

$$T = (300\,\text{K})\frac{(5.1\times10^9/\text{s})^2}{(6.0\times10^9/\text{s})^2} = 216\,\text{K}.$$

E22-14 (a) $v_{\text{av}} = (65\text{ km/s})/(10) = 6.5\text{ km/s}$.
(b) $v_{\text{rms}} = \sqrt{(505\text{ km/s})/(10)} = 7.1\text{ km/s}$.

$\boxed{\textbf{E22-15}}$ (a) The average is

$$\frac{4(200\,\text{s}) + 2(500\,\text{m/s}) + 4(600\,\text{m/s})}{4+2+4} = 420\,\text{m/s}.$$

The mean-square value is

$$\frac{4(200\,\text{s})^2 + 2(500\,\text{m/s})^2 + 4(600\,\text{m/s})^2}{4+2+4} = 2.1\times10^5\text{ m}^2/\text{s}^2.$$

The root-mean-square value is the square root of this, or 458 m/s.

(b) I'll be lazy. Nine particles are not moving, and the tenth has a speed of 10 m/s. Then the average speed is 1 m/s, and the root-mean-square speed is 3.16 m/s. Look, v_{rms} is larger than v_{av}!

(c) Can $v_{\text{rms}}{=}v_{\text{av}}$? Assume that the speeds are *not* all the same. Transform to a frame of reference where $v_{\text{av}} = 0$, then some of the individual speeds must be greater than zero, and some will be less than zero. Squaring these speeds will result in positive, non-zero, numbers; the mean square will necessarily be greater than zero, so $v_{\text{rms}} > 0$.

Only if *all* of the particles have the same speed will $v_{\text{rms}}{=}v_{\text{av}}$.

E22-16 Use Eq. 22-20:

$$v_{\text{rms}} = \sqrt{\frac{3(1.38\times10^{-23}\text{J/K})(329\,\text{K})}{(2.33\times10^{-26}\text{kg} + 3\times1.67\times10^{-27}\text{kg})}} = 694\,\text{m/s}.$$

E22-17 Use Eq. 22-20:

$$v_{\text{rms}} = \sqrt{\frac{3(1.38\times10^{-23}\text{J/K})(2.7\,\text{K})}{(2\times1.67\times10^{-27}\text{kg})}} = 180\,\text{m/s}.$$

E22-18 Eq. 22-14 is in the form $N = Av^2 e^{-Bv^2}$. Taking the derivative,

$$\frac{dN}{dv} = 2Ave^{-Bv^2} - 2ABv^3 e^{-Bv^2},$$

and setting this equal to zero,

$$v^2 = 1/B = 2kT/m.$$

$\boxed{\textbf{E22-19}}$ We want to integrate

$$\begin{aligned}
v_{\text{av}} &= \frac{1}{N}\int_0^\infty N(v)v\,dv, \\
&= \frac{1}{N}\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} v\,dv, \\
&= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^2 e^{-mv^2/2kT} v\,dv.
\end{aligned}$$

The easiest way to attack this is first with a change of variables— let $x = mv^2/2kT$, then $kT\,dx = mv\,dv$. The limits of integration don't change, since $\sqrt{\infty} = \infty$. Then

$$\begin{aligned}
v_{\text{av}} &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty \frac{2kT}{m} xe^{-x}\frac{kT}{m}dx, \\
&= 2\left(\frac{2kT}{\pi m}\right)^{1/2}\int_0^\infty xe^{-\alpha x}dx
\end{aligned}$$

The factor of α that was introduced in the last line is a Feynman trick; we'll set it equal to one when we are finished, so it won't change the result.

Feynman's trick looks like

$$\frac{d}{d\alpha}\int e^{-\alpha x}dx = \int \frac{\partial}{\partial\alpha} e^{-\alpha x}dx = \int (-x)e^{-\alpha x}dx.$$

Applying this to our original problem,

$$\begin{aligned}
v_{\text{av}} &= 2\left(\frac{2kT}{\pi m}\right)^{1/2}\int_0^\infty xe^{-\alpha x}dx, \\
&= -\frac{d}{d\alpha}2\left(\frac{2kT}{\pi m}\right)^{1/2}\int_0^\infty e^{-\alpha x}dx, \\
&= -2\left(\frac{2kT}{\pi m}\right)^{1/2}\frac{d}{d\alpha}\left(\left.\frac{-1}{\alpha}e^{-\alpha x}\right|_0^\infty\right), \\
&= -2\left(\frac{2kT}{\pi m}\right)^{1/2}\frac{d}{d\alpha}\left(\frac{1}{\alpha}\right), \\
&= -2\left(\frac{2kT}{\pi m}\right)^{1/2}\frac{-1}{\alpha^2}.
\end{aligned}$$

We promised, however, that we would set $\alpha = 1$ in the end, so this last line is

$$\begin{aligned}
v_{\text{av}} &= 2\left(\frac{2kT}{\pi m}\right)^{1/2}, \\
&= \sqrt{\frac{8kT}{\pi m}}.
\end{aligned}$$

E22-20 We want to integrate

$$\begin{aligned}(v^2)_{\rm av} &= \frac{1}{N}\int_0^\infty N(v)v^2\,dv,\\ &= \frac{1}{N}\int_0^\infty 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}v^2\,dv,\\ &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^2 e^{-mv^2/2kT}v^2\,dv.\end{aligned}$$

The easiest way to attack this is first with a change of variables— let $x^2 = mv^2/2kT$, then $\sqrt{2kT/m}dx = dv$. The limits of integration don't change. Then

$$\begin{aligned}(v^2)_{\rm av} &= 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty \left(\frac{2kT}{m}\right)^{5/2} x^4 e^{-x^2}\,dx,\\ &= \frac{8kT}{\sqrt{\pi}\,m}\int_0^\infty x^4 e^{-x^2}\,dx\end{aligned}$$

Look up the integral; although you can solve it by first applying a Feynman trick (see solution to Exercise 22-21) and then squaring the integral and changing to polar coordinates. I looked it up. I found $3\sqrt{\pi}/8$, so

$$(v^2)_{\rm av} = \frac{8kT}{\sqrt{\pi}\,m}3\sqrt{\pi}/8 = 3kT/m.$$

E22-21 Apply Eq. 22-20:

$$v_{\rm rms} = \sqrt{3(1.38\times10^{-23}{\rm J/K})(287\,{\rm K})/(5.2\times10^{-17}{\rm kg})} = 1.5\times10^{-2}{\rm m/s}.$$

E22-22 Since $v_{\rm rms} \propto \sqrt{T/m}$, we have

$$T = (299\,{\rm K})(4/2) = 598\,{\rm K},$$

or 325°C.

E22-23 (a) The escape speed is found on page 310; $v = 11.2\times10^3$m/s. For hydrogen,

$$T = (2)(1.67\times10^{-27}{\rm kg})(11.2\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 1.0\times10^4{\rm K}.$$

For oxygen,

$$T = (32)(1.67\times10^{-27}{\rm kg})(11.2\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 1.6\times10^5{\rm K}.$$

(b) The escape speed is found on page 310; $v = 2.38\times10^3$m/s. For hydrogen,

$$T = (2)(1.67\times10^{-27}{\rm kg})(2.38\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 460{\rm K}.$$

For oxygen,

$$T = (32)(1.67\times10^{-27}{\rm kg})(2.38\times10^3{\rm m/s})^2/3(1.38\times10^{-23}{\rm J/K}) = 7300{\rm K}.$$

(c) There should be more oxygen than hydrogen.

E22-24 (a) $v_{\rm av} = (70\text{ km/s})/(22) = 3.18$ km/s.
(b) $v_{\rm rms} = \sqrt{(250\text{ km}^2/\text{s}^2)/(22)} = 3.37$ km/s.
(c) 3.0 km/s.

$\boxed{\textbf{E22-25}}$ According to the equation directly beneath Fig. 22-8,

$$\omega = v\phi/L = (212\,\text{m/s})(0.0841\text{ rad})/(0.204\,\text{m}) = 87.3\text{ rad/s}.$$

E22-26 If $v_\text{p} = v_\text{rms}$ then $2T_2 = 3T_1$, or $T_2/T_1 = 3/2$.

E22-27 Read the last paragraph on the first column of page 505. The distribution of speeds is proportional to

$$v^3 e^{-mv^2/2kT} = v^3 e^{-Bv^2},$$

taking the derivative dN/dv and setting equal to zero yields

$$\frac{dN}{dv} = 3v^2 e^{-Bv^2} - 2Bv^4 e^{-Bv^2},$$

and setting this equal to zero,

$$v^2 = 3/2B = 3kT/m.$$

E22-28 (a) $v = \sqrt{3(8.31\,\text{J/mol}\cdot\text{K})(4220\,\text{K})/(0.07261\,\text{kg/mol})} = 1200\,\text{m/s}.$

(b) Half of the sum of the diameters, or 273 pm.

(c) The mean free path of the germanium in the argon is

$$\lambda = 1/\sqrt{2}(4.13\times10^{25}/\text{m}^3)\pi(273\times10^{-12}\text{m})^2 = 7.31\times10^{-8}\text{m}.$$

The collision rate is

$$(1200\,\text{m/s})/(7.31\times10^{-8}\text{m}) = 1.64\times10^{10}/\text{s}.$$

$\boxed{\textbf{E22-29}}$ The fraction of particles that interests us is

$$\frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\int_{0.01kT}^{0.03kT} E^{1/2}e^{-E/kT}\,dE.$$

Change variables according to $E/kT = x$, so that $dE = kT\,dx$. The integral is then

$$\frac{2}{\sqrt{\pi}}\int_{0.01}^{0.03} x^{1/2}e^{-x}\,dx.$$

Since the value of x is so small compared to 1 throughout the range of integration, we can expand according to

$$e^{-x} \approx 1 - x \text{ for } x \ll 1.$$

The integral then simplifies to

$$\frac{2}{\sqrt{\pi}}\int_{0.01}^{0.03} x^{1/2}(1-x)\,dx = \frac{2}{\sqrt{\pi}}\left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2}\right]_{0.01}^{0.03} = 3.09\times10^{-3}.$$

E22-30 Write $N(E) = N(E_\text{p} + \epsilon)$. Then

$$N(E_\text{p} + \epsilon) \approx N(E_\text{p}) + \epsilon\left.\frac{dN(E)}{dE}\right|_{E_\text{p}} + ...$$

But the very definition of E_p implies that the first derivative is zero. Then the fraction of [particles with energies in the range $E_\text{p} \pm 0.02kT$ is

$$\frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}(kT/2)^{1/2}e^{-1/2}(0.02kT),$$

or $0.04\sqrt{1/2e\pi} = 9.68\times10^{-3}$.

$\boxed{\textbf{E22-31}}$ The volume correction is on page 508; we need first to find d. If we assume that the particles in water are arranged in a cubic lattice (a bad guess, but we'll use it anyway), then 18 grams of water has a volume of $18\times10^{-6}\text{m}^3$, and

$$d^3 = \frac{(18\times10^{-6}\,\text{m}^3)}{(6.02\times10^{23})} = 3.0\times10^{-29}\text{m}^3$$

is the volume allocated to each water molecule. In this case $d = 3.1\times10^{-10}\text{m}$. Then

$$b = \frac{1}{2}(6.02\times10^{23})(\frac{4}{3}\pi(3.1\times10^{-10}\text{m})^3) = 3.8\,\text{m}^3/\text{mol}.$$

E22-32 $d^3 = 3b/2\pi N_A$, or

$$d = \sqrt[3]{\frac{3(32\times10^{-6}\text{m}^3/\text{mol})}{2\pi(6.02\times10^{23}/\text{mol})}} = 2.9\times10^{-10}\text{m}.$$

E22-33 a has units of energy volume per square mole, which is the same as energy per mole times volume per mole.

P22-1 Solve $(1-x)(1.429) + x(1.250) = 1.293$ for x. The result is $x = 0.7598$.

P22-2

$\boxed{\textbf{P22-3}}$ The only thing that matters is the total number of moles of gas (2.5) and the number of moles of the second gas (0.5). Since 1/5 of the total number of moles of gas is associated with the second gas, then 1/5 of the total pressure is associated with the second gas.

P22-4 Use Eq. 22-11 with the appropriate $\sqrt{2}$ inserted.

$$\lambda = \frac{(1.0\times10^{-3}\text{m}^3)}{\sqrt{2}(35)\pi(1.0\times10^{-2}\text{m})^2} = 6.4\times10^{-2}\text{m}.$$

P22-5 (a) Since $\lambda \propto 1/d^2$, we have

$$\frac{d_\text{a}}{d_\text{n}} = \sqrt{\frac{\lambda_\text{n}}{\lambda_\text{a}}} = \sqrt{\frac{(27.5\times10^{-8}\text{m})}{(9.90\times10^{-8}\text{m})}} = 1.67.$$

(b) Since $\lambda \propto 1/p$, we have

$$\lambda_2 = \lambda_1\frac{p_1}{p_2} = (9.90\times10^{-8}\text{m})\frac{(75.0\text{ cm Hg})}{(15.0\text{ cm Hg})} = 49.5\times10^{-8}\text{m}.$$

(c) Since $\lambda \propto T$, we have

$$\lambda_2 = \lambda_1\frac{T_2}{T_1} = (9.90\times10^{-8}\text{m})\frac{(233\,\text{K})}{(293\,\text{K})} = 7.87\times10^{-8}\,\text{m}.$$

P22-6 We can assume the molecule will collide with something. Then

$$1 = \int_0^\infty Ae^{-cr}dr = A/c,$$

so $A = c$. If the molecule has a mean free path of λ, then

$$\lambda = \int_0^\infty rce^{-cr}dr = 1/c,$$

so $A = c = 1/\lambda$.

$\boxed{\textbf{P22-7}}$ What is important here is the temperature; since the temperatures are the same then the average kinetic energies per particle are the same. Then

$$\frac{1}{2}m_1(v_{\mathrm{rms},1})^2 = \frac{1}{2}m_2(v_{\mathrm{rms},2})^2.$$

We are given in the problem that $v_{\mathrm{av},2} = 2v_{\mathrm{rms},1}$. According to Eqs. 22-18 and 22-20 we have

$$v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\pi}{8}}\sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{3\pi}{8}}v_{\mathrm{av}}.$$

Combining this with the kinetic energy expression above,

$$\frac{m_1}{m_2} = \left(\frac{v_{\mathrm{rms},2}}{v_{\mathrm{rms},1}}\right)^2 = \left(2\sqrt{\frac{3\pi}{8}}\right)^2 = 4.71.$$

P22-8 (a) Assume that the speeds are *not* all the same. Transform to a frame of reference where $v_{\mathrm{av}} = 0$, then some of the individual speeds must be greater than zero, and some will be less than zero. Squaring these speeds will result in positive, non-zero, numbers; the mean square will necessarily be greater than zero, so $v_{\mathrm{rms}} > 0$.

(b) Only if *all* of the particles have the same speed will v_{rms}=v_{av}.

$\boxed{\textbf{P22-9}}$ (a) We need to first find the number of particles by integrating

$$\begin{aligned} N &= \int_0^\infty N(v)\,dv, \\ &= \int_0^{v_0} Cv^2\,dv + \int_{v_0}^\infty (0)\,dv = C\int_0^{v_0} v^2\,dv = \frac{C}{3}v_0^3. \end{aligned}$$

Invert, then $C = 3N/v_0^3$.

(b) The average velocity is found from

$$v_{\mathrm{av}} = \frac{1}{N}\int_0^\infty N(v)v\,dv.$$

Using our result from above,

$$\begin{aligned} v_{\mathrm{av}} &= \frac{1}{N}\int_0^{v_0}\left(\frac{3N}{v_0^3}v^2\right)v\,dv, \\ &= \frac{3}{v_0^3}\int_0^{v_0} v^3\,dv = \frac{3}{v_0^3}\frac{v_0^4}{4} = \frac{3}{4}v_0. \end{aligned}$$

As expected, the average speed is less than the maximum speed. We can make a prediction about the root mean square speed; it will be larger than the average speed (see Exercise 22-15 above) but smaller than the maximum speed.

(c) The root-mean-square velocity is found from

$$v_{\text{rms}}^2 = \frac{1}{N}\int_0^\infty N(v)v^2\,dv.$$

Using our results from above,

$$\begin{aligned} v_{\text{rms}}^2 &= \frac{1}{N}\int_0^{v_0}\left(\frac{3N}{v_0^3}v^2\right)v^2\,dv, \\ &= \frac{3}{v_0^3}\int_0^{v_0} v^4\,dv = \frac{3}{v_0^3}\frac{v_0^5}{5} = \frac{3}{5}v_0^2. \end{aligned}$$

Then, taking the square root,

$$v_{\text{rms}}^2 = \sqrt{\frac{3}{5}}\,v_0$$

Is $\sqrt{3/5} > 3/4$? It had better be.

P22-10

P22-11

P22-12

P22-13

P22-14

$\boxed{\textbf{P22-15}}$ The mass of air displaced by 2180 m^3 is $m = (1.22\,\text{kg/m}^3)(2180\,\text{m}^3) = 2660\,\text{kg}$. The mass of the balloon and basket is 249 kg and we want to lift 272 kg; this leaves a remainder of 2140 kg for the mass of the air inside the balloon. This corresponds to $(2140\,\text{kg})/(0.0289\,\text{kg/mol}) = 7.4{\times}10^4\text{mol}$.

The temperature of the gas inside the balloon is then

$$T = (pV)/(nR) = [(1.01{\times}10^5\text{Pa})(2180\,\text{m}^3)]/[(7.4{\times}10^4\text{mol})(8.31\,\text{J/mol}\cdot\text{K}) = 358\,\text{K}.$$

That's 85°C.

P22-16

P22-17

$\boxed{\textbf{E23-1}}$ We apply Eq. 23-1,

$$H = kA\frac{\Delta T}{\Delta x}$$

The rate at which heat flows out is given as a power per area (mW/m^2), so the quantity given is really H/A. Then the temperature difference is

$$\Delta T = \frac{H}{A}\frac{\Delta x}{k} = (0.054\,\mathrm{W/m^2})\frac{(33,000\,\mathrm{m})}{(2.5\,\mathrm{W/m\cdot K})} = 710\,\mathrm{K}$$

The heat flow is out, so that the temperature is higher at the base of the crust. The temperature there is then

$$710 + 10 = 720\,^\circ\mathrm{C}.$$

E23-2 We apply Eq. 23-1,

$$H = kA\frac{\Delta T}{\Delta x} = (0.74\,\mathrm{W/m\cdot K})(6.2\,\mathrm{m})(3.8\,\mathrm{m})\frac{(44\,\mathrm{C^\circ})}{(0.32\,\mathrm{m})} = 2400\,\mathrm{W}.$$

E23-3 (a) $\Delta T/\Delta x = (136\,\mathrm{C^\circ})/(0.249\,\mathrm{m}) = 546\,\mathrm{C^\circ/m}$.

(b) $H = kA\Delta T/\Delta x = (401\,\mathrm{W/m\cdot K})(1.80\,\mathrm{m^2})(546\,\mathrm{C^\circ/m}) = 3.94\times10^5\mathrm{W}$.

(c) $T_\mathrm{H} = (-12^\circ\mathrm{C} + 136\,\mathrm{C^\circ}) = 124^\circ\mathrm{C}$. Then

$$T = (124^\circ\mathrm{C}) - (546\,\mathrm{C^\circ/m})(0.11\,\mathrm{m}) = 63.9^\circ\mathrm{C}.$$

E23-4 (a) $H = (0.040\,\mathrm{W/m\cdot K})(1.8\,\mathrm{m^2})(32\,\mathrm{C^\circ})/(0.012\,\mathrm{m}) = 190\,\mathrm{W}$.

(b) Since k has increased by a factor of $(0.60)/(0.04) = 15$ then H should also increase by a factor of 15.

$\boxed{\textbf{E23-5}}$ There are three possible arrangements: a sheet of type 1 with a sheet of type 1; a sheet of type 2 with a sheet of type 2; and a sheet of type 1 with a sheet of type 2. We can look back on Sample Problem 23-1 to see how to start the problem; the heat flow will be

$$H_{12} = \frac{A\Delta T}{(L/k_1) + (L/k_2)}$$

for substances of different types; and

$$H_{11} = \frac{A\Delta T/L}{(L/k_1) + (L/k_1)} = \frac{1}{2}\frac{A\Delta T k_1}{L}$$

for a double layer if substance 1. There is a similar expression for a double layer of substance 2.

For configuration (a) we then have

$$H_{11} + H_{22} = \frac{1}{2}\frac{A\Delta T k_1}{L} + \frac{1}{2}\frac{A\Delta T k_2}{L} = \frac{A\Delta T}{2L}(k_1 + k_2),$$

while for configuration (b) we have

$$H_{12} + H_{21} = 2\frac{A\Delta T}{(L/k_1) + (L/k_2)} = \frac{2A\Delta T}{L}\left((1/k_1) + (1/k_2)\right)^{-1}.$$

We want to compare these, so expanding the relevant part of the second configuration

$$\left((1/k_1) + (1/k_2)\right)^{-1} = \left((k_1 + k_2)/(k_2 k_2)\right)^{-1} = \frac{k_1 k_2}{k_1 + k_2}.$$

Then which is larger

$$(k_1 + k_2)/2 \text{ or } \frac{2k_1k_2}{k_1 + k_2}\,?$$

If $k_1 \gg k_2$ then the expression become

$$k_1/2 \text{ and } 2k_2,$$

so the first expression is larger, and therefore configuration (b) has the lower heat flow. Notice that we get the same result if $k_1 \ll k_2$!

E23-6 There's a typo in the exercise.

$H = A\Delta T/R$; since the heat flows through one slab and then through the other, we can write $(T_1 - T_x)/R_1 = (T_x - T_2)/R_2$. Rearranging,

$$T_x = (T_1R_2 + T_2R_1)/(R_1 + R_2).$$

E23-7 Use the results of Exercise 23-6. At the interface between ice and water $T_x = 0°\text{C}$. Then $R_1T_2 + R_2T_1 = 0$, or $k_1T_1/L_1 + k_2T_2/L_2 = 0$. Not only that, $L_1 + L_2 = L$, so

$$k_1T_1L_2 + (L - L_2)k_2T_2 = 0,$$

so

$$L_2 = \frac{(1.42\,\text{m})(1.67\,\text{W/m}\cdot\text{K})(-5.20°\text{C})}{(1.67\,\text{W/m}\cdot\text{K})(-5.20°\text{C}) - (0.502\,\text{W/m}\cdot\text{K})(3.98°\text{C})} = 1.15\,\text{m}.$$

E23-8 ΔT is the same in both cases. So is k. The top configuration has $H_\text{t} = kA\Delta T/(2L)$. The bottom configuration has $H_\text{b} = k(2A)\Delta T/L$. The ratio of $H_\text{b}/H_\text{t} = 4$, so heat flows through the bottom configuration at 4 times the rate of the top. For the top configuration $H_\text{t} = (10\,\text{J})/(2\text{ min}) = 5\,\text{J/min}$. Then $H_\text{b} = 20\,\text{J/min}$. It will take

$$t = (30\,\text{J})/(20\,\text{J/min}) = 1.5\text{ min}.$$

$\boxed{\textbf{E23-9}}$ (a) This exercise has a distraction: it asks about the heat flow through the window, but what you need to find first is the heat flow through the air near the window. We are given the temperature gradient both inside and outside the window. Inside,

$$\frac{\Delta T}{\Delta x} = \frac{(20°\text{C}) - (5°\text{C})}{(0.08\,\text{m})} = 190\,\text{C}°/\text{m};$$

a similar expression exists for outside.

From Eq. 23-1 we find the heat flow *through the air*;

$$H = kA\frac{\Delta T}{\Delta x} = (0.026\,\text{W/m}\cdot\text{K})(0.6\,\text{m})^2(190\,\text{C}°/\text{m}) = 1.8\,\text{W}.$$

The value that we arrived at is the rate that heat flows through the air across an area the size of the window on either side of the window. This heat flow had to occur through the window as well, so

$$H = 1.8\,\text{W}$$

answers the window question.

(b) Now that we know the rate that heat flows through the window, we are in a position to find the temperature difference across the window. Rearranging Eq. 32-1,

$$\Delta T = \frac{H\Delta x}{kA} = \frac{(1.8\,\text{W})(0.005\,\text{m})}{(1.0\,\text{W/m}\cdot\text{K})(0.6\,\text{m})^2} = 0.025\,\text{C}°,$$

so we were well justified in our approximation that the temperature drop across the glass is very small.

E23-10 (a) $W = +214\,\text{J}$, done on means positive.
(b) $Q = -293\,\text{J}$, extracted from means negative.
(c) $\Delta E_{\text{int}} = Q + W = (-293\,\text{J}) + (+214\,\text{J}) = -79.0\,\text{J}$.

E23-11 (a) ΔE_{int} along *any* path between these two points is

$$\Delta E_{\text{int}} = Q + W = (50\,\text{J}) + (-20\,\text{J}) = 30\,\text{J}.$$

Then along ibf $W = (30\,\text{J}) - (36\,\text{J}) = -6\,\text{J}$.
(b) $Q = (-30\,\text{J}) - (+13\,\text{J}) = -43\,\text{J}$.
(c) $E_{\text{int},f} = E_{\text{int},i} + \Delta E_{\text{int}} = (10\,\text{J}) + (30\,\text{J}) = 40\,\text{J}$.
(d) $\Delta E_{\text{int}ib} = (22\,\text{J}) - (10\,\text{J}) = 12\,\text{J}$; while $\Delta E_{\text{int}bf} = (40\,\text{J}) - (22\,\text{J}) = 18\,\text{J}$. There is no work done on the path bf, so

$$Q_{bf} = \Delta E_{\text{int}bf} - W_{bf} = (18\,\text{J}) - (0) = 18\,\text{J},$$

and $Q_{ib} = Q_{ibf} - Q_{bf} = (36\,\text{J}) - (18\,\text{J}) = 18\,\text{J}$.

E23-12 $Q = mL = (0.10)(2.1\times10^8\,\text{kg})(333\times10^3\,\text{J/kg}) = 7.0\times10^{12}\,\text{J}$.

$\boxed{\textbf{E23-13}}$ We don't need to know the outside temperature because the amount of heat energy required is explicitly stated: 5.22 GJ. We just need to know how much water is required to transfer this amount of heat energy. Use Eq. 23-11, and then

$$m = \frac{Q}{c\Delta T} = \frac{(5.22\times10^9\,\text{J})}{(4190\,\text{J/kg}\cdot\text{K})(50.0^\circ\text{C} - 22.0^\circ\text{C})} = 4.45\times10^4\,\text{kg}.$$

This is the mass of the water, we want to know the volume, so we'll use the density, and then

$$V = \frac{m}{\rho} = \frac{(4.45\times10^4\,\text{kg})}{(998\,\text{kg/m}^3)} = 44.5\,\text{m}^3.$$

E23-14 The heat energy required is $Q = mc\Delta T$. The time required is $t = Q/P$. Then

$$t = \frac{(0.136\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(100^\circ\text{C} - 23.5^\circ\text{C})}{(220\,\text{W})} = 198\,\text{s}.$$

E23-15 $Q = mL$, so $m = (50.4\times10^3\text{J})/(333\times10^3\text{J/kg}) = 0.151\,\text{kg}$ is the mount which freezes. Then $(0.258\,\text{kg}) - (0.151\,\text{kg}) = 0.107\,\text{kg}$ is the amount which does not freeze.

E23-16 (a) $W = mg\Delta y$; if $|Q| = |W|$, then

$$\Delta T = \frac{mg\Delta y}{mc} = \frac{(9.81\,\text{m/s}^2)(49.4\,\text{m})}{(4190\,\text{J/kg}\cdot\text{K})} = 0.112\,\text{C}^\circ.$$

$\boxed{\textbf{E23-17}}$ There are three "things" in this problem: the copper bowl (b), the water (w), and the copper cylinder (c). The total internal energy changes must add up to zero, so

$$\Delta E_{\text{int,b}} + \Delta E_{\text{int,w}} + \Delta E_{\text{int,c}} = 0.$$

As in Sample Problem 23-3, no work is done on any object, so

$$Q_{\text{b}} + Q_{\text{w}} + Q_{\text{c}} = 0.$$

The heat transfers for these three objects are

$$\begin{aligned} Q_{\mathrm{b}} &= m_{\mathrm{b}}c_{\mathrm{b}}(T_{\mathrm{f,b}} - T_{\mathrm{i,b}}), \\ Q_{\mathrm{w}} &= m_{\mathrm{w}}c_{\mathrm{w}}(T_{\mathrm{f,w}} - T_{\mathrm{i,w}}) + L_{\mathrm{v}}m_2, \\ Q_{\mathrm{c}} &= m_{\mathrm{c}}c_{\mathrm{c}}(T_{\mathrm{f,c}} - T_{\mathrm{i,c}}). \end{aligned}$$

For the most part, this looks exactly like the presentation in Sample Problem 23-3; but there is an extra term in the second line. This term reflects the extra heat required to vaporize $m_2 = 4.70$ g of water at 100°C into steam 100°C.

Some of the initial temperatures are specified in the exercise: $T_{\mathrm{i,b}} = T_{\mathrm{i,w}} = 21.0°\mathrm{C}$ and $T_{\mathrm{f,b}} = T_{\mathrm{f,w}} = T_{\mathrm{f,c}} = 100°\mathrm{C}$.

(a) The heat transferred to the water, then, is

$$\begin{aligned} Q_{\mathrm{w}} &= (0.223\,\mathrm{kg})(4190\,\mathrm{J/kg{\cdot}K})\left((100°\mathrm{C}) - (21.0°\mathrm{C})\right), \\ &\quad +(2.26\times 10^6\mathrm{J/kg})(4.70\times 10^{-3}\mathrm{kg}), \\ &= 8.44 \times 10^4\,\mathrm{J}. \end{aligned}$$

This answer differs from the back of the book. I think that they (or was it me) used the latent heat of fusion when they should have used the latent heat of vaporization!

(b) The heat transfered to the bowl, then, is

$$Q_{\mathrm{w}} = (0.146\,\mathrm{kg})(387\,\mathrm{J/kg \cdot K})\left((100°\mathrm{C}) - (21.0°\mathrm{C})\right) = 4.46 \times 10^3\,\mathrm{J}.$$

(c) The heat transfered from the cylinder was transfered into the water and bowl, so

$$Q_{\mathrm{c}} = -Q_{\mathrm{b}} - Q_{\mathrm{w}} = -(4.46 \times 10^3\,\mathrm{J}) - (8.44 \times 10^4\,\mathrm{J}) = -8.89 \times 10^4\,\mathrm{J}.$$

The initial temperature of the cylinder is then given by

$$T_{\mathrm{i,c}} = T_{\mathrm{f,c}} - \frac{Q_{\mathrm{c}}}{m_{\mathrm{c}}c_{\mathrm{c}}} = (100°\mathrm{C}) - \frac{(-8.89 \times 10^4\,\mathrm{J})}{(0.314\,\mathrm{kg})(387\,\mathrm{J/kg \cdot K})} = 832°\mathrm{C}.$$

E23-18 The temperature of the silver must be raised to the melting point and then the heated silver needs to be melted. The heat required is

$$Q = mL + mc\Delta T = (0.130\,\mathrm{kg})[(105\times 10^3\mathrm{J/kg}) + (236\,\mathrm{J/kg \cdot K})(1235\,\mathrm{K} - 289\,\mathrm{K})] = 4.27\times 10^4\mathrm{J}.$$

E23-19 (a) Use $Q = mc\Delta T$, $m = \rho V$, and $t = Q/P$. Then

$$\begin{aligned} t &= \frac{[m_{\mathrm{a}}c_{\mathrm{a}} + \rho_{\mathrm{w}}V_{\mathrm{w}}c_{\mathrm{w}}]\Delta T}{P}, \\ &= \frac{[(0.56\,\mathrm{kg})(900\,\mathrm{J/kg{\cdot}K}) + (998\,\mathrm{kg/m^3})(0.64\times 10^{-3}\mathrm{m^3})(4190\,\mathrm{J/kg{\cdot}K})](100°\mathrm{C} - 12°\mathrm{C})}{(2400\,\mathrm{W})} = 117\,\mathrm{s}. \end{aligned}$$

(b) Use $Q = mL$, $m = \rho V$, and $t = Q/P$. Then

$$t = \frac{\rho_{\mathrm{w}}V_{\mathrm{w}}L_{\mathrm{w}}}{P} = \frac{(998\,\mathrm{kg/m^3})(0.640\times 10^{-3}\mathrm{m^3})(2256\times 10^3\,\mathrm{J/kg})}{(2400\,\mathrm{W})} = 600\,\mathrm{s}$$

is the *additional* time required.

E23-20 The heat given off by the steam will be

$$Q_{\rm s} = m_{\rm s}L_{\rm v} + m_{\rm s}c_{\rm w}(50\,{\rm C}^\circ).$$

The hear taken in by the ice will be

$$Q_{\rm i} = m_{\rm i}L_{\rm f} + m_{\rm i}c_{\rm w}(50\,{\rm C}^\circ).$$

Equating,

$$\begin{aligned} m_{\rm s} &= mrm_i\frac{L_{\rm f} + c_{\rm w}(50\,{\rm C}^\circ)}{L_{\rm v} + c_{\rm w}(50\,{\rm C}^\circ)}, \\ &= (0.150\,{\rm kg})\frac{(333\times 10^3\,{\rm J/kg}) + (4190\,{\rm J/kg\cdot K})(50\,{\rm C}^\circ)}{(2256\times 10^3\,{\rm J/kg}) + (4190\,{\rm J/kg\cdot K})(50\,{\rm C}^\circ)} = 0.033\,{\rm kg}. \end{aligned}$$

$\boxed{\textbf{E23-21}}$ The linear dimensions of the ring and sphere change with the temperature change according to

$$\begin{aligned} \Delta d_{\rm r} &= \alpha_{\rm r}d_{\rm r}(T_{\rm f,r} - T_{\rm i,r}), \\ \Delta d_{\rm s} &= \alpha_{\rm s}d_{\rm s}(T_{\rm f,s} - T_{\rm i,s}). \end{aligned}$$

When the ring and sphere are at the same (final) temperature the ring and the sphere have the same diameter. This means that

$$d_{\rm r} + \Delta d_{\rm r} = d_{\rm s} + \Delta d_{\rm s}$$

when $T_{\rm f,s} = T_{\rm f,r}$. We'll solve these expansion equations first, and then go back to the heat equations.

$$\begin{aligned} d_{\rm r} + \Delta d_{\rm r} &= d_{\rm s} + \Delta d_{\rm s}, \\ d_{\rm r}\left(1 + \alpha_{\rm r}(T_{\rm f,r} - T_{\rm i,r})\right) &= d_{\rm s}\left(1 + \alpha_{\rm s}(T_{\rm f,s} - T_{\rm i,s})\right), \end{aligned}$$

which can be rearranged to give

$$\alpha_{\rm r}d_{\rm r}T_{\rm f,r} - \alpha_{\rm s}d_{\rm s}T_{\rm f,s} = d_{\rm s}\left(1 - \alpha_{\rm s}T_{\rm i,s}\right) - d_{\rm r}\left(1 - \alpha_{\rm r}T_{\rm i,r}\right),$$

but since the final temperatures are the same,

$$T_{\rm f} = \frac{d_{\rm s}\left(1 - \alpha_{\rm s}T_{\rm i,s}\right) - d_{\rm r}\left(1 - \alpha_{\rm r}T_{\rm i,r}\right)}{\alpha_{\rm r}d_{\rm r} - \alpha_{\rm s}d_{\rm s}}$$

Putting in the numbers,

$$\begin{aligned} T_{\rm f} &= \\ &\frac{(2.54533{\rm cm})[1-(23\times 10^{-6}/{\rm C}^\circ)(100^\circ{\rm C})]-(2.54000{\rm cm})[1-(17\times 10^{-6}/{\rm C}^\circ)(0^\circ{\rm C})]}{(2.54000{\rm cm})(17\times 10^{-6}/{\rm C}^\circ)-(2.54533{\rm cm})(23\times 10^{-6}/{\rm C}^\circ)}, \\ &= 34.1^\circ{\rm C}. \end{aligned}$$

No work is done, so we only have the issue of heat flow, then

$$Q_{\rm r} + Q_{\rm s} = 0.$$

Where "r" refers to the copper ring and "s" refers to the aluminum sphere. The heat equations are

$$\begin{aligned} Q_{\rm r} &= m_{\rm r}c_{\rm r}(T_{\rm f} - T_{\rm i,r}), \\ Q_{\rm s} &= m_{\rm s}c_{\rm s}(T_{\rm f} - T_{\rm i,s}). \end{aligned}$$

Equating and rearranging,

$$m_{\rm s} = \frac{m_{\rm r}c_{\rm r}(T_{\rm i,r} - T_{\rm f})}{c_{\rm s}(T_{\rm f} - T_{\rm i,s})}$$

or

$$m_{\rm s} = \frac{(21.6\ {\rm g})(387\,{\rm J/kg\cdot K})(0^\circ{\rm C} - 34.1^\circ{\rm C})}{(900\,{\rm J/kg\cdot K})(34.1^\circ{\rm C} - 100^\circ{\rm C})} = 4.81\ {\rm g}.$$

E23-22 The problem is compounded because we don't know if the final state is only water, only ice, or a mixture of the two.

Consider first the water. Cooling it to 0°C would require the removal of

$$Q_\text{w} = (0.200\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(0^\circ\text{C} - 25^\circ\text{C}) = -2.095\times10^4\,\text{J}.$$

Consider now the ice. Warming the ice to would require the addition of

$$Q_\text{i} = (0.100\,\text{kg})(2220\,\text{J/kg}\cdot\text{K})(0^\circ\text{C} + 15^\circ\text{C}) = 3.33\times10^3\,\text{J}.$$

The heat absorbed by the warming ice isn't enough to cool the water to freezing. However, the ice can melt; and if it does it will require the addition of

$$Q_\text{im} = (0.100\,\text{kg})(333\times10^3\,\text{J/kg}) = 3.33\times10^4\,\text{J}.$$

This is far more than will be liberated by the cooling water, so the final temperature is 0°C, and consists of a mixture of ice and water.

(b) Consider now the ice. Warming the ice to would require the addition of

$$Q_\text{i} = (0.050\,\text{kg})(2220\,\text{J/kg}\cdot\text{K})(0^\circ\text{C} + 15^\circ\text{C}) = 1.665\times10^3\,\text{J}.$$

The heat absorbed by the warming ice isn't enough to cool the water to freezing. However, the ice can melt; and if it does it will require the addition of

$$Q_\text{im} = (0.050\,\text{kg})(333\times10^3\,\text{J/kg}) = 1.665\times10^4\,\text{J}.$$

This is still not enough to cool the water to freezing. Hence, we need to solve

$$Q_\text{i} + Q_\text{im} + m_\text{i}c_\text{w}(T - 0^\circ\text{C}) + m_\text{w}c_\text{w}(T - 25^\circ\text{C}) = 0,$$

which has solution

$$T = \frac{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg})(25^\circ\text{C}) - (1.665\times10^3\,\text{J}) + (1.665\times10^4\,\text{J})}{(4190\,\text{J/kg}\cdot\text{K})(0.250\,\text{kg})} = 2.5^\circ\text{C}.$$

E23-23 (a) $c = (320\,\text{J})/(0.0371\,\text{kg})(42.0^\circ\text{C} - 26.1^\circ\text{C}) = 542\,\text{J/kg}\cdot\text{K}$.

(b) $n = m/M = (37.1\text{ g})/(51.4\text{ g/mol}) = 0.722$ mol.

(c) $c = (542\,\text{J/kg}\cdot\text{K})(51.4\times10^{-3}\text{ kg/mol}) = 27.9\,\text{J/mol}\cdot\text{K}$.

E23-24 (1) $W = -p\Delta V = (15\,\text{Pa})(4\,\text{m}^3) = -60\,\text{J}$ for the horizontal path; no work is done during the vertical path; the net work done on the gas is $-60\,\text{J}$.

(2) It is easiest to consider work as the (negative of) the area under the curve; then $W = -(15\,\text{Pa} + 5\,\text{Pa})(4\,\text{m}^3)/2 = -40\,\text{J}$.

(3) No work is done during the vertical path; $W = -p\Delta V = (5\,\text{Pa})(4\,\text{m}^3) = -20\,\text{J}$ for the horizontal path; the net work done on the gas is $-20\,\text{J}$.

E23-25 Net work done on the gas is given by Eq. 23-15,

$$W = -\int p\,dV.$$

But integrals are just the area under the curve; and that's the easy way to solve this problem. In the case of closed paths, it becomes the area inside the curve, with a clockwise sense giving a positive value for the integral.

The magnitude of the area is the same for either path, since it is a rectangle divided in half by a square. The area of the rectangle is

$$(15\times10^3\text{Pa})(6\,\text{m}^3) = 90\times10^3\,\text{J},$$

so the area of path 1 (counterclockwise) is -45 kJ; this means the work done on the gas is -(-45 kJ) or 45 kJ. The work done on the gas for path 2 is the negative of this because the path is clockwise.

E23-26 During the isothermal expansion,

$$W_1 = -nRT \ln \frac{V_2}{V_1} = -p_1 V_1 \ln \frac{p_1}{p_2}.$$

During cooling at constant pressure,

$$W_2 = -p_2 \Delta V = -p_2(V_1 - V_2) = -p_2 V_1(1 - p_1/p_2) = V_1(p_1 - p_2).$$

The work done is the sum, or

$$-(204\times10^3\text{Pa})(0.142\,\text{m}^3)\ln\frac{(204\times10^3\text{Pa})}{(101\times10^3\text{Pa})} + (0.142\,\text{m}^3)(103\,\text{Pa}) = -5.74\times10^3\text{J}.$$

E23-27 During the isothermal expansion,

$$W = -nRT \ln \frac{V_2}{V_1} = -p_1 V_1 \ln \frac{V_2}{V_1},$$

so

$$W = -(1.32)(1.01\times10^5\text{Pa})(0.0224\,\text{m}^3)\ln\frac{(0.0153\,\text{m}^3)}{(0.0224\,\text{m}^3)} = 1.14\times10^3\text{J}.$$

E23-28 (a) pV^γ is a constant, so

$$p_2 = p_1(V_1/V_2)^\gamma = (1.00\text{ atm})[(1\text{ l})/(0.5\text{ l})]^{1.32} = 2.50\text{ atm};$$

$T_2 = T_1(p_2/p_1)(V_2/V_1)$, so

$$T_2 = (273\,\text{K})\frac{(2.50\text{ atm})}{(1.00\text{ atm})}\frac{(0.5\text{ l})}{(1\text{ l})} = 341\,\text{K}.$$

(b) $V_3 = V_2(p_2/p_1)(T_3/T_2)$, so

$$V_3 = (0.5\text{ l})\frac{(273\,\text{K})}{(341\,\text{K})} = 0.40\text{ l}.$$

(c) During the adiabatic process,

$$W_{12} = \frac{(1.01\times10^5\text{Pa/atm})(1\times10^{-3}\text{m}^3/\text{l})}{(1.32)-1}[(2.5\text{ atm})(0.5\text{ l}) - (1.0\text{ atm})(1\text{ l})] = 78.9\,\text{J}.$$

During the cooling process,

$$W_{23} = -p\Delta V = -(1.01\times10^5\text{Pa/atm})(2.50\text{ atm})(1\times10^{-3}\text{m}^3/\text{l})[(0.4\text{ l}) - (0.5\text{ l})] = 25.2\,\text{J}.$$

The net work done is $W_{123} = 78.9\,\text{J} + 25.2\,\text{J} = 104.1\,\text{J}$.

$\boxed{\textbf{E23-29}}$ (a) According to Eq. 23-20,

$$p_\text{f} = \frac{p_\text{i}V_\text{i}{}^\gamma}{V_\text{f}{}^\gamma} = \frac{(1.17\text{ atm})(4.33\text{ L})^(1.40)}{(1.06\text{ L})^(1.40)} = 8.39\text{ atm}.$$

(b) The final temperature can be found from the ideal gas law,

$$T_\text{f} = T_\text{i}\frac{p_\text{f}V_\text{f}}{p_\text{i}V_\text{i}} = (310\,\text{K})\frac{(8.39\text{ atm})(1.06\text{ L})}{(1.17\text{ atm})(4.33\text{ L})} = 544\,\text{K}.$$

(c) The work done (for an adiabatic process) is given by Eq. 23-22,

$$\begin{aligned} W &= \frac{1}{(1.40)-1}\left[(8.39\times1.01\times10^5\text{Pa})(1.06\times10^{-3}\text{m}^3)\right.\\ &\qquad\left. -(1.17\times1.01\times10^5\text{Pa})(4.33\times10^{-3}\text{m}^3)\right],\\ &= 966\,\text{J}. \end{aligned}$$

E23-30 Air is mostly diatomic (N_2 and O_2), so use $\gamma = 1.4$.

(a) pV^γ is a constant, so

$$V_2 = V_1 \sqrt[\gamma]{p_1/p_2} = V_1 \sqrt[1.4]{(1.0 \text{ atm})/(2.3 \text{ atm})} = 0.552 V_1.$$

$T_2 = T_1(p_2/p_1)(V_2/V_1)$, so

$$T_2 = (291\,\text{K})\frac{(2.3 \text{ atm})}{(1.0 \text{ atm})}\frac{(0.552V_1)}{V_1} = 369\,\text{K},$$

or 96°C.

(b) The work required for delivering 1 liter of compressed air is

$$W_{12} = \frac{(1.01\times10^5\text{Pa/atm})(1\times10^{-3}\text{m}^3/\text{l})}{(1.40)-1}[(2.3 \text{ atm})(1.0 \text{ l}) - (1.0 \text{ atm})(1.0 \text{ l}/0.552)] = 123\,\text{J}.$$

The number of liters per second that can be delivered is then

$$\Delta V/\Delta t = (230\,\text{W})/(123\,\text{J/l}) = 1.87 \text{ l}.$$

E23-31 $E_{\text{int,rot}} = nRT = (1 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(298\,\text{K}) = 2480\,\text{J}.$

E23-32 $E_{\text{int,rot}} = \frac{3}{2}nRT = (1.5)(1 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(523\,\text{K}) = 6520\,\text{J}.$

E23-33 (a) Invert Eq. 32-20,

$$\gamma = \frac{\ln(p_1/p_2)}{\ln(V_2/V_1)} = \frac{\ln(122 \text{ kPa}/1450 \text{ kPa})}{\ln(1.36\,\text{m}^3/10.7\,\text{m}^3)} = 1.20.$$

(b) The final temperature is found from the ideal gas law,

$$T_\text{f} = T_\text{i}\frac{p_\text{f}V_\text{f}}{p_\text{i}V_\text{i}} = (250\,\text{K})\frac{(1450\times10^3\text{Pa})(1.36\,\text{m}^3)}{(122\times10^3\text{Pa})(10.7\,\text{m}^3)} = 378\,\text{K},$$

which is the same as 105°C.

(c) Ideal gas law, again:

$$n = [pV]/[RT] = [(1450\times10^3\text{Pa})(1.36\,\text{m}^3)]/[(8.31\,\text{J/mol}\cdot\text{K})(378\,\text{K})] = 628 \text{ mol}.$$

(d) From Eq. 23-24,

$$E_\text{int} = \frac{3}{2}nRT = \frac{3}{2}(628 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(250\,\text{K}) = 1.96\times10^6\text{J}$$

before the compression and

$$E_\text{int} = \frac{3}{2}nRT = \frac{3}{2}(628 \text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(378\,\text{K}) = 2.96\times10^6\text{J}$$

after the compression.

(e) The ratio of the rms speeds will be proportional to the square root of the ratio of the internal energies,

$$\sqrt{(1.96\times10^6\text{J})/(2.96\times10^6\text{J})} = 0.813;$$

we can do this because the number of particles is the same before and after, hence the ratio of the energies per particle is the same as the ratio of the total energies.

E23-34 We can assume neon is an ideal gas. Then $\Delta T = 2\Delta E_{\rm int}/3nR$, or

$$\Delta T = \frac{2(1.34\times 10^{12}\mathrm{eV})(1.6\times 10^{-19}\,\mathrm{J/eV})}{3(0.120 \text{ mol})(8.31\,\mathrm{J/mol\cdot K})} = 1.43\times 10^{-7}\mathrm{J}.$$

E23-35 At constant pressure, doubling the volume is the same as doubling the temperature. Then

$$Q = nC_p\Delta T = (1.35 \text{ mol})\frac{7}{2}(8.31\,\mathrm{J/mol\cdot K})(568\,\mathrm{K} - 284\,\mathrm{K}) = 1.12\times 10^{4}\mathrm{J}.$$

E23-36 (a) $n = m/M = (12 \text{ g})/(28 \text{ g/mol}) = 0.429$ mol.
(b) This is a constant volume process, so

$$Q = nC_V\Delta T = (0.429 \text{ mol})\frac{5}{2}(8.31\,\mathrm{J/mol\cdot K})(125^\circ\mathrm{C} - 25^\circ\mathrm{C}) = 891\mathrm{J}.$$

E23-37 (a) From Eq. 23-37,

$$Q = nc_{\rm p}\Delta T = (4.34 \text{ mol})(29.1\,\mathrm{J/mol\cdot K})(62.4\,\mathrm{K}) = 7880\,\mathrm{J}.$$

(b) From Eq. 23-28,

$$E_{\rm int} = \frac{5}{2}nR\Delta T = \frac{5}{2}(4.34 \text{ mol})(8.31\,\mathrm{J/mol\cdot K})(62.4\,\mathrm{K}) = 5630\,\mathrm{J}.$$

(c) From Eq. 23-23,

$$K_{\rm trans} = \frac{3}{2}nR\Delta T = \frac{5}{2}(4.34 \text{ mol})(8.31\,\mathrm{J/mol\cdot K})(62.4\,\mathrm{K}) = 3380\,\mathrm{J}.$$

E23-38 $c_V = \frac{3}{2}(8.31\,\mathrm{J/mol\cdot K})/(4.00 \text{ g/mol}) = 3120\,\mathrm{J/kg\cdot K}$.

E23-39 Each species will experience the same temperature change, so

$$\begin{aligned} Q &= Q_1 + Q_2 + Q_3, \\ &= n_1C_1\Delta T + n_2C_2\Delta T + n_3C_3\Delta T, \end{aligned}$$

Dividing this by $n = n_1 + n_2 + n_3$ and ΔT will return the specific heat capacity of the mixture, so

$$C = \frac{n_1C_1 + n_2C_2 + n_3C_3}{n_1 + n_2 + n_3}.$$

E23-40 $W_{AB} = 0$, since it is a constant volume process, consequently, $W = W_{AB}+W_{ABC} = -15\,\mathrm{J}$. But around a closed path $Q = -W$, so $Q = 15\,\mathrm{J}$. Then

$$Q_{CA} = Q - Q_{AB} - Q_{BC} = (15\,\mathrm{J}) - (20\,\mathrm{J}) - (0\,\mathrm{J}) = -5\,\mathrm{J}.$$

Note that this heat is *removed* from the system!

$\boxed{\textbf{E23-41}}$ According to Eq. 23-25 (which is specific to ideal gases),

$$\Delta E_{\text{int}} = \frac{3}{2} nR\Delta T,$$

and for an isothermal process $\Delta T = 0$, so for an ideal gas $\Delta E_{\text{int}} = 0$. Consequently, $Q + W = 0$ for an ideal gas which undergoes an isothermal process.

But we know W for an isotherm, Eq. 23-18 shows

$$W = -nRT \ln \frac{V_\text{f}}{V_\text{i}}$$

Then finally

$$Q = -W = nRT \ln \frac{V_\text{f}}{V_\text{i}}$$

E23-42 Q is greatest for constant pressure processes and least for adiabatic. W is greatest (in magnitude, it is negative for increasing volume processes) for constant pressure processes and least for adiabatic. ΔE_{int} is greatest for constant pressure (for which it is positive), and least for adiabatic (for which is is negative).

E23-43 (a) For a monatomic gas, $\gamma = 1.667$. Fast process are often adiabatic, so

$$T_2 = T_1(V_1/V_2)^{\gamma-1} = (292\,\text{K})[(1)(1/10)]^{1.667-1} = 1360\,\text{K}.$$

(b) For a diatomic gas, $\gamma = 1.4$. Fast process are often adiabatic, so

$$T_2 = T_1(V_1/V_2)^{\gamma-1} = (292\,\text{K})[(1)(1/10)]^{1.4-1} = 733\,\text{K}.$$

E23-44 This problem cannot be solved without making some assumptions about the type of process occurring on the two curved portions.

$\boxed{\textbf{E23-45}}$ If the pressure and volume are both doubled along a straight line then the process can be described by

$$p = \frac{p_1}{V_1} V$$

The final point involves the doubling of both the pressure and the volume, so according to the ideal gas law, $pV = nRT$, the final temperature T_2 will be *four* times the initial temperature T_1.

Now for the exercises.

(a) The work done on the gas is

$$W = -\int_1^2 p\,dV = -\int_1^2 \frac{p_1}{V_1} V\,dV = -\frac{p_1}{V_1}\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)$$

We want to express our answer in terms of T_1. First we take advantage of the fact that $V_2 = 2V_1$, then

$$W = -\frac{p_1}{V_1}\left(\frac{4V_1^2}{2} - \frac{V_1^2}{2}\right) = -\frac{3}{2} p_1 V_1 = -\frac{3}{2} nRT_1$$

(b) The nice thing about ΔE_{int} is that it is path independent, we care only of the initial and final points. From Eq. 23-25,

$$\Delta E_{\text{int}} = \frac{3}{2} nR\Delta T = \frac{3}{2} nR\,(T_2 - T_1) = \frac{9}{2} nRT_1$$

(c) Finally we are in a position to find Q by applying the first law,

$$Q = \Delta E_{\text{int}} - W = \frac{9}{2}nRT_1 + \frac{3}{2}nRT_1 = 6nRT_1.$$

(d) If we define specific heat as heat divided by temperature change, then

$$c = \frac{Q}{n\Delta T} = \frac{6RT_1}{4T_1 - T_1} = 2R.$$

E23-46 The work done is the area enclosed by the path. If the pressure is measured in units of 10MPa, then the shape is a semi-circle, and the area is

$$W = (\pi/2)(1.5)^2(10\text{MPa})(1\times10^{-3}\text{m}^3) = 3.53\times10^4\text{J}.$$

The heat is given by $Q = -W = -3.53\times10^4$ J.

E23-47 (a) Internal energy changes according to $\Delta E_{\text{int}} = Q + W$, so

$$\Delta E_{\text{int}} = (20.9\,\text{J}) - (1.01\times10^5\text{Pa})(113\times10^{-6}\text{m}^3 - 63\times10^{-6}\text{m}^3) = 15.9\,\text{J}.$$

(b) $T_1 = p_1V_1/nR$ and $T_2 = p_2V_2/nR$, but p is constant, so $\Delta T = p\Delta V/nR$. Then

$$C_P = \frac{Q}{n\Delta T} = \frac{QR}{p\Delta V} = \frac{(20.9\,\text{J})(8.31\,\text{J/mol}\cdot\text{K})}{(1.01\times10^5\text{Pa})(113\times10^{-6}\text{m}^3 - 63\times10^{-6}\text{m}^3)} = 34.4\,\text{J/mol}\cdot\text{K}.$$

(c) $C_V = C_P - R = (34.4\,\text{J/mol}\cdot\text{K}) - (8.31\,\text{J/mol}\cdot\text{K}) = 26.1\,\text{J/mol}\cdot\text{K}$.

E23-48 Constant Volume
(a) $Q = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.
(b) $W = 0$.
(c) $\Delta Erm_{int} = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.
Constant Pressure
(a) $Q = 4(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 5450\,\text{J}$.
(b) $W = -p\Delta V = -nR\Delta T = -(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = -1360\,\text{J}$.
(c) $\Delta Erm_{int} = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.
Adiabatic
(a) $Q = 0$.
(b) $W = (p_\text{f}V_\text{f} - p_\text{i}V_\text{i})/(\gamma - 1) = nR\Delta T/(\gamma - 1) = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.
(c) $\Delta Erm_{int} = 3(3.15\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(52.0\,\text{K}) = 4080\,\text{J}$.

P23-1 (a) The temperature difference is

$$(5\,\text{C}^\circ/9\text{ F}^\circ)(72^\circ\text{F} - -20^\circ\text{F}) = 51.1\,\text{C}^\circ.$$

The rate of heat loss is

$$H = (1.0\,\text{W/m}\cdot\text{K})(1.4\,\text{m}^2)(51.1\,\text{C}^\circ)/(3.0\times10^{-3}\text{m}) = 2.4\times10^4\text{W}.$$

(b) Start by finding the R values.

$$R_\text{g} = (3.0\times10^{-3}\text{m})/(1.0\,\text{W/m}\cdot\text{K}) = 3.0\times10^{-3}\text{m}^2\cdot\text{K/W},$$

$$R_\text{a} = (7.5\times10^{-2}\text{m})/(0.026\text{W/m}\cdot\text{K}) = 2.88\text{m}^2\cdot\text{K/W}.$$

Then use Eq. 23-5,

$$H = \frac{(1.4\,\text{m}^2)(51.1\,\text{C}^\circ)}{2(3.0\times10^{-3}\text{m}^2\cdot\text{K/W}) + (2.88\text{m}^2\cdot\text{K/W})} = 25\,\text{W}.$$

Get double pane windows!

P23-2 (a) $H = (428\,\mathrm{W/m\cdot K})(4.76\times10^{-4}\mathrm{m}^2)(100\,\mathrm{C}^\circ)/(1.17\,\mathrm{m}) = 17.4\,\mathrm{W}$.
(b) $\Delta m/\Delta t = H/L = (17.4\,\mathrm{W})/(333\times10^3\mathrm{J/kg}) = 5.23\times10^{-5}\mathrm{kg/s}$, which is the same as 188 g/h.

$\boxed{\textbf{P23-3}}$ Follow the example in Sample Problem 23-2. We start with Eq. 23-1:

$$\begin{aligned}
H &= kA\frac{dT}{dr},\\
H &= k(4\pi r^2)\frac{dT}{dr},\\
\int_{r_1}^{r_2} H\frac{dr}{4\pi r^2} &= \int_{T_1}^{T_2} k\,dT,\\
\frac{H}{4pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) &= k(T_1-T_2),\\
H\left(\frac{r_2-r_1}{r_1r_2}\right) &= 4\pi k(T_1-T_2),\\
H &= \frac{4\pi k(T_1-T_2)r_1r_2}{r_2-r_1}.
\end{aligned}$$

P23-4 (a) $H = (54\times10^{-3}\mathrm{W/m}^2)4\pi(6.37\times10^6\mathrm{m})^2 = 2.8\times10^{13}\mathrm{W}$.
(b) Using the results of Problem 23-3,

$$\Delta T = \frac{(2.8\times10^{13}\mathrm{W})(6.37\times10^6\mathrm{m} - 3.47\times10^6\mathrm{m})}{4\pi(4.2\,\mathrm{W/m\cdot K})(6.37\times10^6\mathrm{m})(3.47\times10^6\mathrm{m})} = 7.0\times10^4\mathrm{C}^\circ.$$

Since $T_2 = 0^\circ\mathrm{C}$, we expect $T_1 = 7.0\times10^4\mathrm{C}^\circ$.

P23-5 Since $H = -kA\,dT/dx$, then $H\,dx = -aT\,dT$. H is a constant, so integrate both side according to

$$\begin{aligned}
\int H\,dx &= -\int aT\,dT,\\
HL &= -a\frac{1}{2}(T_2^2 - T_1^2),\\
H &= \frac{aA}{2L}(T_1^2 - T_2^2).
\end{aligned}$$

P23-6 Assume the water is all at 0°C. The heat flow through the ice is then $H = kA\Delta T/x$; the rate of ice formation is $\Delta m/\Delta t = H/L$. But $\Delta m = \rho A\Delta x$, so

$$\frac{\Delta x}{\Delta t} = \frac{H}{\rho AL} = \frac{k\Delta T}{\rho Lx},$$

$$\frac{(1.7\,\mathrm{W/m\cdot K})(10\,\mathrm{C}^\circ)}{(920\,\mathrm{kg/m}^3)(333\times10^3\mathrm{J/kg})(0.05\,\mathrm{m})} = 1.11\times10^{-6}\mathrm{m/s}.$$

That's the same as 0.40 cm/h.

$\boxed{\textbf{P23-7}}$ (a) Start with the heat equation:

$$Q_\mathrm{t} + Q_\mathrm{i} + Q_\mathrm{w} = 0,$$

where $Q_{\rm t}$ is the heat from the tea, $Q_{\rm i}$ is the heat from the ice when it melts, and $Q_{\rm w}$ is the heat from the water (which used to be ice). Then

$$m_{\rm t}c_{\rm t}(T_{\rm f} - T_{\rm t,i}) + m_{\rm i}L_{\rm f} + m_{\rm w}c_{\rm w}(T_{\rm f} - T_{\rm w,i}) = 0,$$

which, since we have assumed all of the ice melts and the masses are all equal, can be solved for T_f as

$$\begin{aligned} T_f &= \frac{c_{\rm t}T_{\rm t,i} + c_{\rm w}T_{\rm w,i} - L_{\rm f}}{c_{\rm t} + c_{\rm w}}, \\ &= \frac{(4190\mathrm{J/kg\cdot K})(90^\circ\mathrm{C}) + (4190\mathrm{J/kg\cdot K})(0^\circ\mathrm{C}) - (333\times10^3\mathrm{J/kg})}{(4190\mathrm{J/kg\cdot K}) + (4190\mathrm{J/kg\cdot K})}, \\ &= 5.3^\circ\mathrm{C}. \end{aligned}$$

(b) Once again, assume all of the ice melted. Then we can do the same steps, and we get

$$\begin{aligned} T_f &= \frac{c_{\rm t}T_{\rm t,i} + c_{\rm w}T_{\rm w,i} - L_{\rm f}}{c_{\rm t} + c_{\rm w}}, \\ &= \frac{(4190\mathrm{J/kg\cdot K})(70^\circ\mathrm{C}) + (4190\mathrm{J/kg\cdot K})(0^\circ\mathrm{C}) - (333\times10^3\mathrm{J/kg})}{(4190\mathrm{J/kg\cdot K}) + (4190\mathrm{J/kg\cdot K})}, \\ &= -4.7^\circ\mathrm{C}. \end{aligned}$$

So we must have guessed wrong when we assumed that all of the ice melted. The heat equation then simplifies to

$$m_{\rm t}c_{\rm t}(T_{\rm f} - T_{\rm t,i}) + m_{\rm i}L_{\rm f} = 0,$$

and then

$$\begin{aligned} m_{\rm i} &= \frac{m_{\rm t}c_{\rm t}(T_{\rm t,i} - T_{\rm f})}{L_{\rm f}}, \\ &= \frac{(0.520\,\mathrm{kg})(4190\mathrm{J/kg\cdot K})(90^\circ\mathrm{C} - 0^\circ)}{(333\times10^3\mathrm{J/kg})}, \\ &= 0.458\,\mathrm{kg}. \end{aligned}$$

P23-8 $c = Q/m\Delta T = H/(\Delta m/\Delta t)\Delta T$. But $\Delta m/\Delta t = \rho\Delta V/\Delta t$. Combining,

$$c = \frac{H}{(\Delta V/\Delta t)\rho\Delta T} = \frac{(250\,\mathrm{W})}{(8.2\times10^{-6}\mathrm{m^3/s})(0.85\times10^3\mathrm{kg/m^3})(15\,\mathrm{C^\circ})} = 2.4\times10^3\mathrm{J/kg\cdot K}.$$

P23-9 (a) $n = N_A/M$, so

$$\epsilon = \frac{(2256\times10^3\mathrm{J/kg})}{(6.02\times10^{23}/\mathrm{mol})/(0.018\ \mathrm{kg/mol})} = 6.75\times10^{-20}\mathrm{J}.$$

(b) $E_{\rm av} = \frac{3}{2}kT$, so

$$\frac{\epsilon}{E_{\rm av}} = \frac{2(6.75\times10^{-20}\mathrm{J})}{3(1.38\times10^{-23}\mathrm{J/K})(305\,\mathrm{K})} = 10.7.$$

P23-10 $Q_{\rm w} + Q_{\rm t} = 0$, so

$$C_{\rm t}\Delta T_{\rm t} + m_{\rm w}c_{\rm w}(T_{\rm f} - T_{\rm i}) = 0,$$

or

$$T_{\rm i} = \frac{(0.3\,\mathrm{kg})(4190\,\mathrm{J/kg\cdot m})(44.4^\circ\mathrm{C}) + (46.1\,\mathrm{J/K})(44.4^\circ\mathrm{C} - 15.0^\circ\mathrm{C})}{(0.3\,\mathrm{kg})(4190\,\mathrm{J/kg\cdot m})} = 45.5^\circ\mathrm{C}.$$

$\boxed{\textbf{P23-11}}$ We can use Eq. 23-10, but we will need to approximate c first. If we assume that the line is straight then we use $c = mT + b$. I approximate m from

$$m = \frac{(14\,\mathrm{J/mol \cdot K}) - (3\,\mathrm{J/mol \cdot K})}{(500\,\mathrm{K}) - (200\,\mathrm{K})} = 3.67\times10^{-2}\mathrm{J/mol}.$$

Then I find b from those same data points,

$$b = (3\,\mathrm{J/mol \cdot K}) - (3.67\times10^{-2}\mathrm{J/mol})(200\,\mathrm{K}) = -4.34\,\mathrm{J/mol \cdot K}.$$

Then from Eq. 23-10,

$$\begin{aligned}
Q &= n\int_{T_\mathrm{i}}^{T_\mathrm{f}} c\,dT,\\
&= n\int_{T_\mathrm{i}}^{T_\mathrm{f}} (mT + b)\,dT,\\
&= n\left[\frac{m}{2}T^2 + bT\right]_{T_\mathrm{i}}^{T_\mathrm{f}},\\
&= n\left(\frac{m}{2}({T_\mathrm{f}}^2 - {T_\mathrm{i}}^2) + b(T_\mathrm{f} - T_\mathrm{i})\right),\\
&= (0.45\mathrm{mol})\left(\frac{(3.67\times10^{-2}\mathrm{J/mol})}{2}((500\,\mathrm{K})^2 - (200\,\mathrm{K})^2)\right.\\
&\quad \left. + (-4.34\,\mathrm{J/mol \cdot K})(500\,\mathrm{K} - 200\,\mathrm{K})\right),\\
&= 1.15\times10^3\mathrm{J}.
\end{aligned}$$

P23-12 $\delta Q = nC\delta T$, so

$$\begin{aligned}
Q &= n\int C\,dT,\\
&= n\left[(0.318\ \mathrm{J/mol \cdot K^2})T^2/2 - (0.00109\ \mathrm{J/mol \cdot K^3})T^3/3 - (0.628\ \mathrm{J/mol \cdot K})T\right]_{50\,\mathrm{K}}^{90\,\mathrm{K}},\\
&= n(645.8\ \mathrm{J/mol}).
\end{aligned}$$

Finally,

$$Q = (645.8\ \mathrm{J/mol})(316\ \mathrm{g})/(107.87\ \mathrm{g/mol}) = 189\,\mathrm{J}.$$

P23-13 $TV^{\gamma-1}$ is a constant, so

$$T_2 = (292\,\mathrm{K})(1/1.28)^{(1.40)-1} = 265\,\mathrm{K}$$

P23-14 $W = -\int p\,dV$, so

$$\begin{aligned}
W &= -\int\left[\frac{nRT}{V - nb} - \frac{an^2}{V^2}\right]dV,\\
&= -nRT\ln(V - nb) - \left.\frac{an^2}{V}\right|_i^f,\\
&= -nRT\ln\frac{V_\mathrm{f} - nb}{V_\mathrm{i} - nb} - an^2\left(\frac{1}{V_\mathrm{f}} - \frac{1}{V_\mathrm{i}}\right).
\end{aligned}$$

$\boxed{\textbf{P23-15}}$ When the tube is horizontal there are two regions filled with gas, one at $p_{1,\mathrm{i}}$, $V_{1,\mathrm{i}}$; the other at $p_{2,\mathrm{i}}$, $V_{2,\mathrm{i}}$. Originally $p_{1,\mathrm{i}} = p_{2,\mathrm{i}} = 1.01\times10^5\mathrm{Pa}$ and $V_{1,\mathrm{i}} = V_{2,\mathrm{i}} = (0.45\,\mathrm{m})A$, where A is the cross sectional area of the tube.

When the tube is held so that region 1 is on top then the mercury has three forces on it: the force of gravity, mg; the force from the gas above pushing down $p_{2,\mathrm{f}}A$; and the force from the gas below pushing up $p_{1,\mathrm{f}}A$. The balanced force expression is

$$p_{1,\mathrm{f}}A = p_{2,\mathrm{f}}A + mg.$$

If we write $m = \rho l_\mathrm{m} A$ where $l_\mathrm{m} = 0.10\,\mathrm{m}$, then

$$p_{1,\mathrm{f}} = p_{2,\mathrm{f}} + \rho g l_\mathrm{m}.$$

Finally, since the tube has uniform cross section, we can write $V = Al$ everywhere.

(a) For an isothermal process $p_\mathrm{i} l_\mathrm{i} = p_\mathrm{f} l_\mathrm{f}$, where we have used $V = Al$, and then

$$p_{1,\mathrm{i}}\frac{l_{1,\mathrm{i}}}{l_{1,\mathrm{f}}} - p_{2,\mathrm{i}}\frac{l_{2,\mathrm{i}}}{l_{2,\mathrm{f}}} = \rho g l_\mathrm{m}.$$

But we can factor out $p_{1,\mathrm{i}} = p_{2,\mathrm{i}}$ and $l_{1,\mathrm{i}} = l_{2,\mathrm{i}}$, and we can apply $l_{1,\mathrm{f}} + l_{2,\mathrm{f}} = 0.90\,\mathrm{m}$. Then

$$\frac{1}{l_{1,\mathrm{f}}} - \frac{1}{0.90\,\mathrm{m} - l_{1,\mathrm{f}}} = \frac{\rho g l_\mathrm{m}}{p_\mathrm{i} l_\mathrm{i}}.$$

Put in some numbers and rearrange,

$$0.90\,\mathrm{m} - 2l_{1,\mathrm{f}} = (0.294\,\mathrm{m}^{-1})l_{1,\mathrm{f}}(0.90\,\mathrm{m} - l_{1,\mathrm{f}}),$$

which can be written as an ordinary quadratic,

$$(0.294\,\mathrm{m}^{-1}){l_{1,\mathrm{f}}}^2 - (2.265)l_{1,\mathrm{f}} + (0.90\,\mathrm{m}) = 0$$

The solutions are $l_{1,\mathrm{f}} = 7.284\,\mathrm{m}$ and $0.421\,\mathrm{m}$. Only one of these solutions is reasonable, so the mercury shifted down $0.450 - 0.421 = 0.029\,\mathrm{m}$.

(b) The math is a wee bit uglier here, but we can start with $p_\mathrm{i}{l_\mathrm{i}}^\gamma = p_\mathrm{f}{l_\mathrm{f}}^\gamma$, and this means that everywhere we had a $l_{1,\mathrm{f}}$ in the previous derivation we need to replace it with ${l_{1,\mathrm{f}}}^\gamma$. Then we have

$$\frac{1}{{l_{1,\mathrm{f}}}^\gamma} - \frac{1}{(0.90\,\mathrm{m} - l_{1,\mathrm{f}})^\gamma} = \frac{\rho g l_\mathrm{m}}{p_\mathrm{i}{l_\mathrm{i}}^\gamma}.$$

This can be written as

$$(0.90\,\mathrm{m} - l_{1,\mathrm{f}})^\gamma - {l_{1,\mathrm{f}}}^\gamma - = (0.404\,\mathrm{m}^{-\gamma}){l_{1,\mathrm{f}}}^\gamma(0.90\,\mathrm{m} - l(0._{1,\mathrm{f}})^\gamma,$$

which looks nasty to me! I'll use Maple to get the answer, and find $l_{1,\mathrm{f}} = 0.429$, so the mercury shifted down $0.450 - 0.429 = 0.021\,\mathrm{m}$.

Which is more likely? Turn the tube fast, and the adiabatic approximation works. Eventually the system will return to room temperature, and then the isothermal approximation is valid.

P23-16 Internal energy for an ideal diatomic gas can be written as

$$E_\mathrm{int} = \frac{5}{2}nRT = \frac{5}{2}pV,$$

simply by applying the ideal gas law. The room, however, has a fixed pressure and volume, so the internal energy is independent of the temperature. As such, any energy supplied by the furnace leaves the room, either as heat or as expanding gas doing work on the outside.

P23-17 The speed of sound in the iodine gas is

$$v = f\lambda = (1000\text{ Hz})(2 \times 0.0677\,\text{m}) = 135\,\text{m/s}.$$

Then

$$\gamma = \frac{Mv^2}{RT} = \frac{n(0.127\,\text{kg/mol})(135\,\text{m/s})^2}{(8.31\,\text{J/mol}\cdot\text{K})(400\,\text{K})} = n(0.696).$$

Since γ is greater than one, $n \geq 2$. If $n = 2$ then $\gamma = 1.39$, which is consistent; if $n = 3$ then $\gamma = 2.08$, which is not consistent.

Consequently, iodine gas is diatomic.

P23-18 $W = -Q = mL = (333\times 10^3\text{J/kg})(0.122\,\text{kg}) = 4.06\times 10^4\text{J}.$

P23-19 (a)

Process AB

$Q = \frac{3}{2}(1.0\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K}) = 3740\,\text{J}.$

$W = 0.$

$\Delta E_{\text{int}} = Q + W = 3740\,\text{J}.$

Process BC

$Q = 0.$

$W = (p_\text{f}V_\text{f} - p_\text{i}V_\text{i})/(\gamma - 1) = nR\Delta T/(\gamma - 1) = (1.0\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(-145\,\text{K})/(1.67 - 1) = -1800\,\text{J}$

$\Delta E_{\text{int}} = Q + W = -1800\,\text{J}.$

Process AB

$Q = \frac{5}{2}(1.0\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(-155\,\text{K}) = -3220\,\text{J}.$

$W = -p\Delta V = -nR\Delta T = -(1.0\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(-155\,\text{K}) = 1290\,\text{J}.$

$\Delta E_{\text{int}} = Q + W = 1930\,\text{J}.$

Cycle

$Q = 520\,\text{J}$; $W = -510\,\text{J}$ (rounding error!); $\Delta E_{\text{int}} = 10\,\text{J}$ (rounding error!)

P23-20

P23-21 $p_\text{f} = (16.0\text{ atm})(50/250)^{1.40} = 1.68\text{ atm}$. The work done by the gas during the expansion is

$$\begin{aligned} W &= [p_\text{i}V_\text{i} - p_\text{f}V_\text{f}]/(\gamma - 1), \\ &= \frac{(16.0\text{ atm})(50\times 10^{-6}\text{m}^3) - (1.68\text{ atm})(250\times 10^{-6}\text{m}^3)}{(1.40) - 1}(1.01\times 10^5\text{Pa/atm}), \\ &= 96.0\,\text{J}. \end{aligned}$$

This process happens 4000 times per minute, but the actual time to complete the process is half of the cycle, or 1/8000 of a minute. Then $P = (96\,\text{J})(8000)/(60\,\text{s}) = 12.8\times 10^3\text{W}.$

$\boxed{\textbf{E24-1}}$ For isothermal processes the entropy expression is almost trivial, $\Delta S = Q/T$, where if Q is positive (heat flow into system) the entropy increases.

Then $Q = T\Delta S = (405\,\text{K})(46.2\,\text{J/K}) = 1.87\times10^4\,\text{J}$.

E24-2 Entropy is a state variable and is path independent, so

(a) $\Delta S_{ab,2} = \Delta S_{ab,1} = +0.60\,\text{J/K}$,

(b) $\Delta S_{ba,2} = -\Delta S_{ab,2} = -0.60\,\text{J/K}$,

E24-3 (a) Heat only enters along the top path, so

$$Q_{\text{in}} = T\Delta S = (400\,\text{K})(0.6\,\text{J/K} - 0.1\,\text{J/K}) = 200\,\text{J}.$$

(b) Heat leaves only bottom path, so

$$Q_{\text{out}} = T\Delta S = (250\,\text{K})(0.1\,\text{J/K} - 0.6\,\text{J/K}) = -125\,\text{J}.$$

Since $Q + W = 0$ for a cyclic path,

$$W = -Q = -[(200\,\text{J}) + (-125\,\text{J})] = -75\,\text{J}.$$

E24-4 (a) The work done for isothermal expansion is given by Eq. 23-18,

$$W = -(4.00\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(410\,\text{K})\ln\frac{3.45V_1}{V_1} = -1.69\times10^4\,\text{J}.$$

(b) For isothermal process, $Q = -W$, then

$$\Delta S = Q/T = (1.69\times10^4\,\text{J})/(410\,\text{K}) = 41.2\,\text{J/K}.$$

(c) Entropy change is zero for reversible adiabatic processes.

$\boxed{\textbf{E24-5}}$ (a) We want to find the heat absorbed, so

$$Q = mc\Delta T = (1.22\,\text{kg})(387\,\text{J/mol}\cdot\text{K})\left((105^\circ\text{C}) - (25.0^\circ\text{C})\right) = 3.77\times10^4\,\text{J}.$$

(b) We want to find the entropy change, so, according to Eq. 24-1,

$$\begin{aligned}
\Delta S &= \int_{T_\text{i}}^{T_\text{f}} \frac{dQ}{T}, \\
&= \int_{T_\text{i}}^{T_\text{f}} \frac{mc\,dT}{T}, \\
&= mc\ln\frac{T_\text{f}}{T_\text{i}}.
\end{aligned}$$

The entropy change of the copper block is then

$$\Delta S = mc\ln\frac{T_\text{f}}{T_\text{i}} = (1.22\,\text{kg})(387\,\text{J/mol}\cdot\text{K})\ln\frac{(378\,\text{K})}{(298\,\text{K})} = 112\,\text{J/K}.$$

E24-6 $\Delta S = Q/T = mL/T$, so

$$\Delta S = (0.001\,\text{kg})(-333\times10^3\,\text{J/kg})/(263\,\text{K}) = -1.27\text{J/K}.$$

E24-7 Use the first equation on page 551.

$$n = \frac{\Delta S}{R \ln(V_\mathrm{f}/V_\mathrm{i})} = \frac{(24\,\mathrm{J/K})}{(8.31\,\mathrm{J/mol\cdot K})\ln(3.4/1.3)} = 3.00 \text{ mol.}$$

E24-8 $\Delta S = Q/T_\mathrm{c} - Q/T_\mathrm{h}$.
(a) $\Delta S = (260\,\mathrm{J})(1/100\,\mathrm{K} - 1/400\,\mathrm{K}) = 1.95\,\mathrm{J/K}$.
(b) $\Delta S = (260\,\mathrm{J})(1/200\,\mathrm{K} - 1/400\,\mathrm{K}) = 0.65\,\mathrm{J/K}$.
(c) $\Delta S = (260\,\mathrm{J})(1/300\,\mathrm{K} - 1/400\,\mathrm{K}) = 0.217\,\mathrm{J/K}$.
(d) $\Delta S = (260\,\mathrm{J})(1/360\,\mathrm{K} - 1/400\,\mathrm{K}) = 0.0722\,\mathrm{J/K}$.

$\boxed{\textbf{E24-9}}$ (a) If the rod is in a steady state we wouldn't expect the entropy of the rod to change. Heat energy is flowing out of the hot reservoir into the rod, but this process happens at a fixed temperature, so the entropy change in the hot reservoir is

$$\Delta S_\mathrm{H} = \frac{Q_\mathrm{H}}{T_\mathrm{H}} = \frac{(-1200\,\mathrm{J})}{(403\,\mathrm{K})} = -2.98\,\mathrm{J/K}.$$

The heat energy flows into the cold reservoir, so

$$\Delta S_\mathrm{C} = \frac{Q_\mathrm{H}}{T_\mathrm{H}} = \frac{(1200\,\mathrm{J})}{(297\,\mathrm{K})} = 4.04\,\mathrm{J/K}.$$

The total change in entropy of the system is the sum of these two terms

$$\Delta S = \Delta S_\mathrm{H} + \Delta S_\mathrm{C} = 1.06\,\mathrm{J/K}.$$

(b) Since the rod is in a steady state, nothing is changing, not even the entropy.

E24-10 (a) $Q_\mathrm{c} + Q_\mathrm{l} = 0$, so

$$m_\mathrm{c}c_\mathrm{c}(T - T_\mathrm{c}) + m_\mathrm{l}c_\mathrm{l}(T - T_\mathrm{l}) = 0,$$

which can be solved for T to give

$$T = \frac{(0.05\,\mathrm{kg})(387\,\mathrm{J/kg\cdot K})(400\,\mathrm{K}) + (0.10\,\mathrm{kg})(129\,\mathrm{J/kg\cdot K})(200\,\mathrm{K})}{(0.05\,\mathrm{kg})(387\,\mathrm{J/kg\cdot K}) + (0.10\,\mathrm{kg})(129\,\mathrm{J/kg\cdot K})} = 320\,\mathrm{K}.$$

(b) Zero.
(c) $\Delta S = mc \ln T_\mathrm{f}/T_\mathrm{i}$, so

$$\Delta S = (0.05\,\mathrm{kg})(387\,\mathrm{J/kg\cdot K})\ln\frac{(320\,\mathrm{K})}{(400\,\mathrm{K})} + (0.10\,\mathrm{kg})(129\,\mathrm{J/kg\cdot K})\ln\frac{(320\,\mathrm{K})}{(200\,\mathrm{K})} = 1.75\,\mathrm{J/K}.$$

$\boxed{\textbf{E24-11}}$ The total mass of ice and water is 2.04 kg. If eventually the ice and water have the same mass, then the final state will have 1.02 kg of each. This means that $1.78\,\mathrm{kg} - 1.02\,\mathrm{kg} = 0.76\,\mathrm{kg}$ of water changed into ice.
(a) The change of water at 0°C to ice at 0°C is isothermal, so the entropy change is

$$\Delta S = \frac{Q}{T} = \frac{-mL}{T} = \frac{(0.76\,\mathrm{kg})(333\times10^3\,\mathrm{J/kg})}{(273\,\mathrm{K})} = -927\,\mathrm{J/K}.$$

(b) The entropy change is now $+927\,\mathrm{J/K}$.

E24-12 (a) $Q_a + Q_w = 0$, so

$$m_a c_a (T - T_a) + m_w c_w (T - T_w) = 0,$$

which can be solved for T to give

$$T = \frac{(0.196\,\text{kg})(900\,\text{J/kg}\cdot\text{K})(380\,\text{K}) + (0.0523\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(292\,\text{K})}{(0.196\,\text{kg})(900\,\text{J/kg}\cdot\text{K}) + (0.0523\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})} = 331\,\text{K}.$$

That's the same as 58°C.

(b) $\Delta S = mc \ln T_f/T_i$, so

$$\Delta S_a = (0.196\,\text{kg})(900\,\text{J/kg}\cdot\text{K}) \ln\frac{(331\,\text{K})}{(380\,\text{K})} = -24.4\,\text{J/K}.$$

(c) For the water,

$$(0.0523\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}) \ln\frac{(331\,\text{K})}{(292\,\text{K})} = 27.5\,\text{J/K}.$$

(d) $\Delta S = (27.5\,\text{J/K}) + (-24.4\,\text{J/K}) = 3.1\,\text{J/K}$.

E24-13 (a) $e = 1 - (36.2\,\text{J}/52.4\,\text{J}) = 0.309$.

(b) $W = Q_h - Q_c = (52.4\,\text{J}) - (36.2\,\text{J}) = 16.2\,\text{J}$.

E24-14 (a) $Q_h = (8.18\text{ kJ})/(0.25) = 32.7\text{ kJ}$, $Q_c = Q_h - W = (32.7\text{ kJ}) - (8.18\text{ kJ}) = 24.5\text{ kJ}$.

(b) $Q_h = (8.18\text{ kJ})/(0.31) = 26.4\text{ kJ}$, $Q_c = Q_h - W = (26.4\text{ kJ}) - (8.18\text{ kJ}) = 18.2\text{ kJ}$.

$\boxed{\textbf{E24-15}}$ One hour's worth of coal, when burned, will provide energy equal to

$$(382\times10^3\text{kg})(28.0\times10^6\text{J/kg}) = 1.07\times10^{13}\text{J}.$$

In this hour, however, the plant only generates

$$(755\times10^6\text{ W})(3600\,s) = 2.72\times10^{12}\text{J}.$$

The efficiency is then

$$e = (2.72\times10^{12}\text{J})/(1.07\times10^{13}\text{J}) = 25.4\%.$$

E24-16 We use the convention that all quantities are positive, regardless of direction. $W_A = 5W_B$; $Q_{i,A} = 3Q_{i,B}$; and $Q_{o,A} = 2Q_{o,B}$. But $W_A = Q_{i,A} - Q_{o,A}$, so

$$5W_B = 3Q_{i,B} - 2Q_{o,B},$$

or, applying $W_B = Q_{i,B} - Q_{o,B}$,

$$\begin{aligned} 5W_B &= 3Q_{i,B} - 2(Q_{i,B} - W_B), \\ 3W_B &= Q_{i,B}, \\ W_B/Q_{i,B} &= 1/3 = e_B. \end{aligned}$$

Then

$$e_A = \frac{W_A}{Q_{i,A}} = \frac{5W_B}{3Q_{i,B}} = \frac{5}{3}\frac{1}{3} = \frac{5}{9}.$$

E24-17 (a) During an isothermal process $W = -Q = -2090\,\text{J}$. The negative indicates that the gas did work on the environment.

(b) The efficiency is $e = 1 - (297\,\text{K})/(412\,\text{K}) = 0.279$. Then

$$Q_\text{o} = Q_\text{i}(1-e) = (2090\,\text{J})[1-(0.279)] = 1510\,\text{J}.$$

Since this is rejected heat it should actually be negative.

(c) During an isothermal process $W = -Q = 1510\,\text{J}$. Positive indicates that the gas did work on the environment.

E24-18 $1 - e = T_\text{c}/T_\text{h}$, or $T_\text{c} = T_\text{h}(1-e)$. The difference is

$$\Delta T = T_\text{h} - T_\text{c} = T_\text{h} e,$$

so $T_\text{h} = (75\,\text{C}^\circ)/(0.22) = 341\,\text{K}$, and

$$T_\text{c} = (341\,\text{K})[(1-(0.22)] = 266\,\text{K}.$$

$\boxed{\textbf{E24-19}}$ The BC and DA processes are both adiabatic; so if we could find an expression for work done during an adiabatic process we might be almost done. But what is an adiabatic process? It is a process for which $Q = 0$, so according to the first law

$$\Delta E_\text{int} = W.$$

But for an ideal gas

$$\Delta E_\text{int} = nC_V\Delta T,$$

as was pointed out in Table 23-5. So we have

$$|W| = nC_V|\Delta T|$$

and since the adiabatic paths BC and DA operate between the same two isotherms, we can conclude that the magnitude of the work is the same for both paths.

E24-20 (a) To save typing, assume that all quantities are positive. Then

$$e_1 = 1 - T_2/T_1,$$

$W_1 = e_1 Q_1$, and $Q_2 = Q_1 - W_1$. Not only that, but

$$e_2 = 1 - T_3/T_2,$$

and $W_2 = e_2 Q_2$. Combining,

$$e = \frac{W_1 + W_2}{Q_1} = \frac{e_1 Q_1 + e_2(Q_1 - W_1)}{Q_1} = e_1 + e_2(1-e_1) = e_1 + e_2 - e_1 e_2,$$

or

$$e = 1 - \frac{T_2}{T_1} + 1 - \frac{T_3}{T_2} - 1 + \frac{T_2}{T_1} + \frac{T_3}{T_2} - \frac{T_3}{T_1} = 1 - -\frac{T_3}{T_1}.$$

(b) $e = 1 - (311\,\text{K})/(742\,\text{K}) = 0.581$.

E24-21 (a) $p_2 = (16.0\text{ atm})(1/5.6)^{(1.33)} = 1.62$ atm.

(b) $T_2 = T_1(1/5.6)^{(1.33)-1} = (0.567)T_1$, so

$$e = 1 - (0.567) = 0.433.$$

E24-22 (a) The area of the cycle is $\Delta V \Delta p = p_0 V_0$, so the work done by the gas is

$$W = (1.01 \times 10^5 \mathrm{Pa})(0.0225\,\mathrm{m}^3) = 2270\,\mathrm{J}.$$

(b) Let the temperature at a be T_a. Then

$$T_b = T_a (V_b/V_a)(p_b/p_a) = 2T_a.$$

Let the temperature at c be T_c. Then

$$T_c = T_a (V_c/V_a)(p_c/p_a) = 4T_a.$$

Consequently, $\Delta T_{ab} = T_a$ and $\Delta T_{bc} = 2T_a$. Putting this information into the constant volume and constant pressure heat expressions,

$$Q_{ab} = \frac{3}{2} nR\Delta T_{ab} = \frac{3}{2} nRT_a = \frac{3}{2} p_a V_a,$$

and

$$Q_{bc} = \frac{5}{2} nR\Delta T_{bc} = \frac{5}{2} nR2T_a = 5p_a V_a,$$

so that $Q_{ac} = \frac{13}{2} p_0 V_0$, or

$$Q_{ac} = \frac{13}{2} (1.01 \times 10^5 \mathrm{Pa})(0.0225\,\mathrm{m}^3) = 1.48 \times 10^4 \mathrm{J}.$$

(c) $e = (2270\,\mathrm{J})/(14800\,\mathrm{J}) = 0.153$.
(d) $e_\mathrm{c} = 1 - (T_a/4T_a) = 0.75$.

E24-23 According to Eq. 24-15,

$$K = \frac{T_L}{T_H - T_L} = \frac{(261\,\mathrm{K})}{(299\,\mathrm{K}) - (261\,\mathrm{K})} = 6.87$$

Now we solve the question out of order.
(b) The work required to run the freezer is

$$|W| = |Q_L|/K = (185\text{ kJ})/(5.70) = 32.5\text{ kJ}.$$

(a) The freezer will discharge heat into the room equal to

$$|Q_L| + |W| = (185\text{ kJ}) + (32.5\text{ kJ}) = 218\text{ kJ}.$$

E24-24 (a) $K = |Q_L|/|W| = (568\,\mathrm{J})/(153\,\mathrm{J}) = 3.71$.
(b) $|Q_H| = |Q_L| + |W| = (568\,\mathrm{J}) + (153\,\mathrm{J}) = 721\,\mathrm{J}$.

E24-25 $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$.
(a) $|W| = (10.0\,\mathrm{J})(300\,\mathrm{K}/280\mathrm{K} - 1) = 0.714\,\mathrm{J}$.
(b) $|W| = (10.0\,\mathrm{J})(300\,\mathrm{K}/200\mathrm{K} - 1) = 5.00\,\mathrm{J}$.
(c) $|W| = (10.0\,\mathrm{J})(300\,\mathrm{K}/100\mathrm{K} - 1) = 20.0\,\mathrm{J}$.
(d) $|W| = (10.0\,\mathrm{J})(300\,\mathrm{K}/50\mathrm{K} - 1) = 50.0\,\mathrm{J}$.

E24-26 $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$. Then

$$|Q_H| = |Q_L| + |W| = |Q_L|(T_H/T_L) = (0.150\,\mathrm{J})(296\,\mathrm{K}/4.0\,\mathrm{K}) = 11\,\mathrm{J}.$$

$\boxed{\textbf{E24-27}}$ We will start with the assumption that the air conditioner is a Carnot refrigerator. $K = T_L/(T_H - T_L)$; $|W| = |Q_L|/K = |Q_L|(T_H/T_L - 1)$. For fun, I'll convert temperature to the absolute Rankine scale! Then

$$|Q_L| = (1.0\,\mathrm{J})/(555°\mathrm{R}/530°\mathrm{R} - 1) = 21\,\mathrm{J}.$$

E24-28 The best coefficient of performance is

$$K_c = (276\,\mathrm{K})/(308\,\mathrm{K} - 276\,\mathrm{K}) = 8.62.$$

The inventor claims they have a machine with

$$K = (20\ \mathrm{kW} - 1.9\mathrm{kW})/(1.9\ \mathrm{kW}) = 9.53.$$

Can't be done.

E24-29 (a) $e = 1 - (258\,\mathrm{K}/322\,\mathrm{K}) = 0.199$. $|W| = (568\,\mathrm{J})(0.199) = 113\,\mathrm{J}$.
(b) $K = (258\,\mathrm{K})/(322\,\mathrm{K} - 258\,\mathrm{K}) = 4.03$. $|W| = (1230\,\mathrm{J})/(4.03) = 305\,\mathrm{J}$.

E24-30 The temperatures are distractors!

$$|W| = |Q_H| - |Q_L| = |Q_H| - K|W|,$$

so

$$|W| = |Q_H|/(1 + K) = (7.6\ \mathrm{MJ})/(1 + 3.8) = 1.58\ \mathrm{MJ}.$$

Then $P = (1.58\ \mathrm{MJ})/(3600\,\mathrm{s}) = 440\,\mathrm{W}$.

E24-31 $K = (260\,\mathrm{K})/(298\,\mathrm{K} - 260\,\mathrm{K}) = 6.8$.

E24-32 $K = (0.85)(270\mathrm{K})/(299\mathrm{K}–270\mathrm{K}) = 7.91$. In 15 minutes the motor can do $(210\,\mathrm{W})(900\,\mathrm{s}) = 1.89\times10^5\mathrm{J}$ of work. Then

$$|Q_L| = K|W| = (7.91)(1.89\times10^5\mathrm{J}) = 1.50\times10^6\mathrm{J}.$$

$\boxed{\textbf{E24-33}}$ The Carnot engine has an efficiency

$$\epsilon = 1 - \frac{T_2}{T_1} = \frac{|W|}{|Q_1|}.$$

The Carnot refrigerator has a coefficient of performance

$$K = \frac{T_4}{T_3 - T_4} = \frac{|Q_4|}{|W|}.$$

Lastly, $|Q_4| = |Q_3| - |W|$. We just need to combine these three expressions into one. Starting with the first, and solving for $|W|$,

$$|W| = |Q_1|\frac{T_1 - T_2}{T_1}.$$

Then we combine the last two expressions, and

$$\frac{T_4}{T_3 - T_4} = \frac{|Q_3| - |W|}{|W|} = \frac{|Q_3|}{|W|} - 1.$$

Finally, combine them all,

$$\frac{T_4}{T_3 - T_4} = \frac{|Q_3|}{|Q_1|}\frac{T_1}{T_1 - T_2} - 1.$$

Now, we rearrange,

$$\begin{aligned}
\frac{|Q_3|}{|Q_1|} &= \left(\frac{T_4}{T_3 - T_4} + 1\right)\frac{T_1 - T_2}{T_1}, \\
&= \left(\frac{T_3}{T_3 - T_4}\right)\frac{T_1 - T_2}{T_1}, \\
&= (1 - T_2/T_1)/(1 - T_4/T_3).
\end{aligned}$$

E24-34 (a) Integrate:

$$\ln N! \approx \int_1^N \ln x\, dx = N \ln N - N + 1 \approx N \ln N - N.$$

(b) 91, 752, and about 615,000. You will need to use the Stirling approximation extended to a double inequality to do the last two:

$$\sqrt{2\pi} n^{n+1/2} e^{-n+1/(12n+1)} < n! < \sqrt{2\pi} n^{n+1/2} e^{-n+1/(12n)}.$$

$\boxed{\textbf{E24-35}}$ (a) For this problem we don't care how the particles are arranged inside a section, we only care how they are divided up between the two sides.

Consequently, there is only one way to arrange the particles: you put them all on one side, and you have no other choices. So the multiplicity in this case is one, or $w_1 = 1$.

(b) Once the particles are allowed to mix we have more work in computing the multiplicity. Using Eq. 24-19, we have

$$w_2 = \frac{N!}{(N/2)!(N/2)!} = \frac{N!}{((N/2)!)^2}$$

(c) The entropy of a state of multiplicity w is given by Eq. 24-20,

$$S = k \ln w$$

For part (a), with a multiplicity of 1, $S_1 = 0$. Now for part (b),

$$S_2 = k \ln\left(\frac{N!}{((N/2)!)^2}\right) = k \ln N! - 2k \ln(N/2)!$$

and we need to expand each of those terms with Stirling's approximation.

Combining,

$$\begin{aligned}
S_2 &= = k\,(N \ln N - N) - 2k\,((N/2)\ln(N/2) - (N/2)), \\
&= kNlnN - kN - kN \ln N + kN \ln 2 + kN, \\
&= kN \ln 2
\end{aligned}$$

Finally, $\Delta S = S_2 - S_1 = kN \ln 2$.

(d) The answer should be the same; it is a free expansion problem in both cases!

$\boxed{\textbf{P24-1}}$ We want to evaluate

$$\begin{aligned}\Delta S &= \int_{T_\mathrm{i}}^{T_\mathrm{f}} \frac{nC_\mathrm{V}\,dT}{T},\\ &= \int_{T_\mathrm{i}}^{T_\mathrm{f}} \frac{nAT^3\,dT}{T},\\ &= \int_{T_\mathrm{i}}^{T_\mathrm{f}} nAT^2\,dT,\\ &= \frac{nA}{3}\left({T_\mathrm{f}}^3 - {T_\mathrm{i}}^3\right).\end{aligned}$$

Into this last expression, which is true for many substances at sufficiently low temperatures, we substitute the given numbers.

$$\Delta S = \frac{(4.8\text{ mol})(3.15\times10^{-5}\mathrm{J/mol}\cdot\mathrm{K}^4)}{3}\left((10\,\mathrm{K})^3 - (5.0\,\mathrm{K})^3\right) = 4.41\times10^{-2}\mathrm{J/K}.$$

P24-2

P24-3 (a) Work is only done along path ab, where $W_{ab} = -p\Delta V = -3p_0\Delta V_0$. So $W_{abc} = -3p_0V_0$.

(b) $\Delta E_{\int bc} = \frac{3}{2}nR\Delta T_{bc}$, with a little algebra,

$$\Delta E_{\mathrm{int}bc} = \frac{3}{2}(nRT_c - nRT_b) = \frac{3}{2}(p_cV_c - p_bV_b) = \frac{3}{2}(8-4)p_0V_0 = 6p_0V_0.$$

$\Delta S_{bc} = \frac{3}{2}nR\ln(T_c/T_b)$, with a little algebra,

$$\Delta S_{bc} = \frac{3}{2}nR\ln(p_c/p_b) = \frac{3}{2}nR\ln 2.$$

(c) Both are zero for a cyclic process.

P24-4 (a) For an isothermal process,

$$p_2 = p_1(V_1/V_2) = p_1/3.$$

For an adiabatic process,

$$p_3 = p_1(V_1/V_2)^\gamma = p_1(1/3)^{1.4} = 0.215p_1.$$

For a constant volume process,

$$T_3 = T_2(p_3/p_2) = T_1(0.215/0.333) = 0.646T_1.$$

(b) The easiest ones first: $\Delta E_\mathrm{i}\mathrm{nt}_{12} = 0$, $W_{23} = 0$, $Q_{31} = 0$, $\Delta S_{31} = 0$. The next easier ones:

$$\Delta E_\mathrm{i}\mathrm{nt}_{23} = \frac{5}{2}nR\Delta T_{23} = \frac{5}{2}nR(0.646T_1 - T_1) = -0.885p_1V_1,$$

$$Q_{23} = \Delta E_{\mathrm{int}23} - W_{23} = -0.885p_1V_1,$$

$$\Delta E_{\mathrm{int}31} = -\Delta E_{\mathrm{int}23} - \Delta E_\mathrm{i}\mathrm{nt}_{12} = 0.885p_1V_1,$$

$$W_{31} = \Delta E_{\mathrm{int}31} - Q_{31} = 0.885p_1V_1.$$

Finally, some harder ones:

$$W_{12} = -nRT_1 \ln(V_2/V_1) = -p_1V_1 \ln(3) = -1.10p_1V_1,$$

$$Q_{12} = \Delta E_{\mathrm{int}\,12} - W_{12} = 1.10p_1V_1.$$

And now, the hardest:

$$\Delta S_{12} = Q_{12}/T_1 = 1.10nR,$$

$$\Delta S_{23} = -\Delta S_{12} - \Delta S_{31} = -1.10nR.$$

P24-5 Note that $T_A = T_B = T_C/4 = T_D$.

Process I: *ABC*

(a) $Q_{AB} = -W_{AB} = nRT_0 \ln(V_B/V_A) = p_0V_0 \ln 2$. $Q_{BC} = \frac{3}{2}nR(T_C - T_B) = \frac{3}{2}(p_CV_C - p_BV_B) = \frac{3}{2}(4p_0V_0 - p_0V_0) = 4.5p_0V_0$.

(b) $W_{AB} = -nRT_0 \ln(V_B/V_A) = -p_0V_0 \ln 2$. $W_{BC} = 0$.

(c) $E_{\mathrm{int}} = \frac{3}{2}nR(T_C - T_A) = \frac{3}{2}(p_CV_C - p_AV_A) = \frac{3}{2}(4p_0V_0 - p_0V_0) = 4.5p_0V_0$.

(d) $\Delta S_{AB} = nR\ln(V_B/V_A) = nR\ln 2$; $\Delta S_{BC} = \frac{3}{2}nR\ln(T_C/T_B) = \frac{3}{2}nR\ln 4 = 3nR\ln 2$. Then $\Delta S_{AC} = 4nR$.

Process II: *ADC*

(a) $Q_{AD} = -W_{AD} = nRT_0 \ln(V_D/V_A) = -p_0V_0 \ln 2$. $Q_{DC} = \frac{5}{2}nR(T_C - T_D) = \frac{5}{2}(p_CV_C - p_DV_D) = \frac{5}{2}(4p_0V_0 - p_0V_0) = 10p_0V_0$.

(b) $W_{AB} = -nRT_0 \ln(V_D/V_A) = p_0V_0 \ln 2$. $W_{DC} = -p\Delta V = -p_0(2V_0 - V_0/2) = -\frac{3}{2}p_0V_0$.

(c) $E_{\mathrm{int}} = \frac{3}{2}nR(T_C - T_A) = \frac{3}{2}(p_CV_C - p_AV_A) = \frac{3}{2}(4p_0V_0 - p_0V_0) = 4.5p_0V_0$.

(d) $\Delta S_{AD} = nR\ln(V_D/V_A) = -nR\ln 2$; $\Delta S_{DC} = \frac{5}{2}nR\ln(T_C/T_D) = \frac{5}{2}nR\ln 4 = 5nR\ln 2$. Then $\Delta S_{AC} = 4nR$.

P24-6 The heat required to melt the ice is

$$\begin{aligned} Q &= m(c_{\mathrm{w}}\Delta T_{23} + L + c_{\mathrm{i}}\Delta T_{12}), \\ &= (0.0126\,\mathrm{kg})[(4190\,\mathrm{J/kg\cdot K})(15\,\mathrm{C}^\circ) + (333\times 10^3\,\mathrm{J/kg}) + (2220\,\mathrm{J/kg\cdot K})(10\,\mathrm{C}^\circ)], \\ &= 5270\,\mathrm{J}. \end{aligned}$$

The change in entropy of the ice is

$$\begin{aligned} \Delta S_{\mathrm{i}} &= m[c_{\mathrm{w}}\ln(T_3/T_2) + L/T_2 + c_{\mathrm{i}}\ln(T_2/T_1)], \\ &= (0.0126\mathrm{kg})[(4190\mathrm{J/kg\cdot K})\ln(288/273) + (333\times 10^3\,\mathrm{J/kg})/(273\mathrm{K}), \\ &\quad +(2220\mathrm{J/kg\cdot K})\ln(273/263)], \\ &= 19.24\,\mathrm{J/K} \end{aligned}$$

The change in entropy of the lake is $\Delta S_{\mathrm{l}} = (-5270\,\mathrm{J})/(288\,\mathrm{K}) = 18.29\,\mathrm{J}/K$. The change in entropy of the system is 0.95 J/kg.

$\boxed{\textbf{P24-7}}$ (a) This is a problem where the total internal energy of the two objects doesn't change, but since no work is done during the process, we can start with the simpler expression $Q_1 + Q_2 = 0$. The heat transfers by the two objects are

$$\begin{aligned} Q_1 &= m_1c_1(T_1 - T_{1,\mathrm{i}}), \\ Q_2 &= m_2c_2(T_2 - T_{2,\mathrm{i}}). \end{aligned}$$

Note that we don't call the final temperature T_{f} here, because we *are not* assuming that the two objects are at equilibrium.

We combine these three equations,

$$\begin{aligned} m_2c_2(T_2 - T_{2,\mathrm{i}}) &= -m_1c_1(T_1 - T_{1,\mathrm{i}}), \\ m_2c_2T_2 &= m_2c_2T_{2,\mathrm{i}} + m_1c_1(T_{1,\mathrm{i}} - T_1), \\ T_2 &= T_{2,\mathrm{i}} + \frac{m_1c_1}{m_2c_2}(T_{1,\mathrm{i}} - T_1) \end{aligned}$$

As object 1 "cools down", object 2 "heats up", as expected.

(b) The entropy change of *one* object is given by

$$\Delta S = \int_{T_\mathrm{i}}^{T_\mathrm{f}} \frac{mc\,dT}{T} = mc\ln\frac{T_\mathrm{f}}{T_\mathrm{i}},$$

and the total entropy change for the system will be the sum of the changes for each object, so

$$\Delta S = m_1c_1\ln\frac{T_1}{T_{\mathrm{i},1}} + m_2c_2\ln\frac{T_2}{T_{\mathrm{i},2}}.$$

Into the this last equation we need to substitute the expression for T_2 in as a function of T_1. There's no new physics in doing this, just a mess of algebra.

(c) We want to evaluate $d(\Delta S)/dT_1$. To save on algebra we will work with the last expression, remembering that T_2 is a function, not a variable. Then

$$\frac{d(\Delta S)}{dT_1} = \frac{m_1c_1}{T_1} + \frac{m_2c_2}{T_2}\frac{dT_2}{dT_1}.$$

We've saved on algebra, but now we need to evaluate dT_2/dT_1. Starting with the results from part (a),

$$\begin{aligned} \frac{dT_2}{dT_1} &= \frac{d}{dT_1}\left(T_{2,\mathrm{i}} + \frac{m_1c_1}{m_2c_2}(T_{1,\mathrm{i}} - T_1)\right), \\ &= -\frac{m_1c_1}{m_2c_2}. \end{aligned}$$

Now we collect the two results and write

$$\begin{aligned} \frac{d(\Delta S)}{dT_1} &= \frac{m_1c_1}{T_1} + \frac{m_2c_2}{T_2}\left(-\frac{m_1c_1}{m_2c_2}\right), \\ &= m_1c_1\left(\frac{1}{T_1} - \frac{1}{T_2}\right). \end{aligned}$$

We could consider writing T_2 out in all of its glory, but what would it gain us? Nothing. There is actually considerably more physics in the expression as written, because...

(d) ...we get a maximum for ΔS when $d(\Delta S)/dT_1 = 0$, and this can only occur when $T_1 = T_2$ according to the expression.

P24-8 $T_b = (10.4 \times 1.01\times10^5\mathrm{Pa})(1.22\,\mathrm{m}^3)/(2\ \mathrm{mol})(8.31\,\mathrm{J/mol}\cdot\mathrm{K}) = 7.71\times10^4\mathrm{K}$. Maybe not so realistic? T_a can be found after finding

$$p_c = p_b(V_b/V_c)^\gamma = (10.4 \times 1.01\times10^5\mathrm{Pa})(1.22/9.13)^{1.67} = 3.64\times10^4\mathrm{Pa},$$

Then

$$T_a = T_b(p_a/p_b) = (7.71\times10^4\mathrm{K})(3.64\times10^4/1.05\times10^6) = 2.67\times10^3\mathrm{K}.$$

Similarly,

$$T_c = T_a(V_c/V_a) = (2.67\times10^3\mathrm{K})(9.13/1.22) = 2.00\times10^4\mathrm{K}.$$

(a) Heat is added during process ab only;

$$Q_{ab} = \frac{3}{2}nR(T_b - T_a) = \frac{3}{2}(2\ \mathrm{mol})(8.31\,\mathrm{J/mol\cdot K})(7.71\times10^4\mathrm{K} - 2.67\times10^3\mathrm{K}) = 1.85\times10^6\mathrm{J}.$$

(b) Heat is removed during process ca only;

$$Q_{ca} = \frac{5}{2}nR(T_a - T_c) = \frac{5}{2}(2\ \mathrm{mol})(8.31\,\mathrm{J/mol\cdot K})(2.67\times10^3\mathrm{K} - 2.00\times10^4\mathrm{K}) = -0.721\times10^6\mathrm{J}.$$

(c) $W = |Q_{ab}| - |Q_{ca}| = (1.85\times10^6\mathrm{J}) - (0.721\times10^6\mathrm{J}) = 1.13\times10^6\mathrm{J}.$

(d) $e = W/Q_{ab} = (1.13\times10^6)/(1.85\times10^6) = 0.611.$

P24-9 The pV diagram for this process is Figure 23-21, except the cycle goes clockwise.

(a) Heat is input during the constant volume heating and the isothermal expansion. During heating,

$$Q_1 = \frac{3}{2}nR\Delta T = \frac{3}{2}(1\ \mathrm{mol})(8.31\,\mathrm{J/mol\cdot K})(600\,\mathrm{K} - 300\,\mathrm{K}) = 3740\,\mathrm{J};$$

During isothermal expansion,

$$Q_2 = -W_2 = nRT\ln(V_\mathrm{f}/V_\mathrm{i}) = (1\ \mathrm{mol})(8.31\,\mathrm{J/mol\cdot K})(600\,\mathrm{K})\ln(2) = 3460\,\mathrm{J};$$

so $Q_\mathrm{in} = 7200\,\mathrm{J}$.

(b) Work is only done during the second and third processes; we've already solved the second, $W_2 = -3460\,\mathrm{J}$;

$$W_3 = -p\Delta V = p_aV_c - p_aV_a = nR(T_c - T_a) = (1\ \mathrm{mol})(8.31\,\mathrm{J/mol\cdot K})(600\,\mathrm{K} - 300\,\mathrm{K}) = 2490\,\mathrm{J};$$

So $W = -970\,\mathrm{J}$.

(c) $e = |W|/|Q_\mathrm{in}| = (970\,\mathrm{J})/(7200\,\mathrm{J}) = 0.13.$

P24-10 (a) $T_b = T_a(p_b/p_a) = 3T_a$;

$$T_c = T_b(V_b/V_c)^{\gamma-1} = 3T_a(1/4)^{0.4} = 1.72T_a;$$

$$p_c = p_b(V_b/V_c)^{\gamma} = 3p_a(1/4)^{1.4} = 0.430p_a;$$

$$T_d = T_a(V_a/V_d)^{\gamma-1} = T_a(1/4)^{0.4} = 0.574T_a;$$

$$p_d = p_a(V_a/V_d)^{\gamma} = p_a(1/4)^{1.4} = 0.144p_a.$$

(b) Heat in occurs during process ab, so $Q_\mathrm{i} = \frac{5}{2}nR\Delta T_{ab} = 5nRT_a$; Heat out occurs during process cd, so $Q_\mathrm{o} = \frac{5}{2}nR\Delta T_{cd} = 2.87nRT_a$. Then

$$e = 1 - (2.87nRT_a/5nRT_a) = 0.426.$$

P24-11 (c) $(V_B/V_A) = (p_A/p_B) = (0/0.5) = 2$. The work done on the gas during the isothermal compression is

$$W = -nRT\ln(V_B/V_A) = -(1\ \mathrm{mol})(8.31\,\mathrm{J/mol\cdot K})(300\,\mathrm{K})\ln(2) = -1730\,\mathrm{J}.$$

Since $\Delta E_\mathrm{int} = 0$ along an isotherm $Q_\mathrm{h} = 1730\,\mathrm{J}$.

The cycle has an efficiency of $e = 1 - (100/300) = 2/3$. Then for the cycle,

$$W = eQ_\mathrm{h} = (2/3)(1730\,\mathrm{J}) = 1150\,\mathrm{J}.$$