Independent Mathematics

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Chapter 1

Combinatorics

1.1 Basic Principles

1.1.1 Addition

Theorem 1. *If there are n_1 ways to do the first task, n_2 ways to do the second task, n_3 ways to do the third task, and so on, then there are $n_1+n_2+n_3+\cdots$ ways to do the tasks in succession.*

This can be represented as follows:

$$n\big(A \cup B\big) = n(A) + n(B)$$

provided that A and B are mutually exclusive i.e. $A \cap B = \varnothing$.

1.1.2 Multiplication

Theorem 2. *If there are n_1 ways to do the first task, n_2 ways to do the second task, n_3 ways to do the third task, and so on, then there are $n_1 \times n_2 \times n_3 \times \cdots$ ways to do the tasks in succession.*

This can be represented as follows:

$$n\big(A \cap B\big) = n(A) \times n(B)$$

provided that A and B are independent

1.1.3 Addition or Multiplication ?

Addition is used when the tasks are mutually exclusive.
When we can do a task either by following option 1 or option 2 and let's say we can do the task in n_1 ways following option 1 and n_2 ways following option 2
then we can do the task in $n_1 + n_2$ ways.

Here you can observe that all the steps are independent. So we can use multiplication principle.
Multiplication is used when the tasks are independent. When to do a task, we must follow two steps in order. Let's say we can do the first step in n_1 ways and the second step in n_2 ways. Then we can do the task in $n_1 \times n_2$ ways.

Example: How many 3 digit numbers can be formed using the digits 1,2,3,4,5,6,7,8,9 if repetition is allowed?

Solution:

We can do this task in 3 steps.
Step 1: Choose the first digit. We can do this in 9 ways.
Step 2: Choose the second digit. We can do this in 9 ways.
Step 3: Choose the third digit. We can do this in 9 ways.
By multiplication principle, we can do the task in $9 \times 9 \times 9 = 9^3$ ways.

Here you can observe that all the steps are independent. So we can use multiplication principle.