Homework 1 - 2024-2025 Report

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Underwater Acoustics

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MIR | MARINE & MARITIME INTELLIGENT ROBOTICS

1 Arctic Ocean

The simplest model for the velocity profile in this ocean is an affine function of depth:

$$c(z) = c_0 + \gamma_0 |z|$$

where $c_0 = 1450$ and $\gamma_0 = 1.63e-2$, with seafloor at depth $h = 3.50$.

Question 1:
Let us consider a transmitter at depth z_s. What are the geometrical characteristics of the trajectory of a ray transmitted with an initial angle θ_0 toward the sea bottom (measured from the horizontal axis)? Plot this ray for $\theta_0 = 2°$ and $z_s = 500$ using ray tracing software.

To answer this question, we developed a MATLAB ray tracing code to plot the ray required. We included the code in our submission (file named `arctic_solution_HW1.m`) . To get the required ray, we set the theta initial angle in the code (theta0 variable) to 2 degrees, which is converted to radians in our code to be used in the processing algorithm:

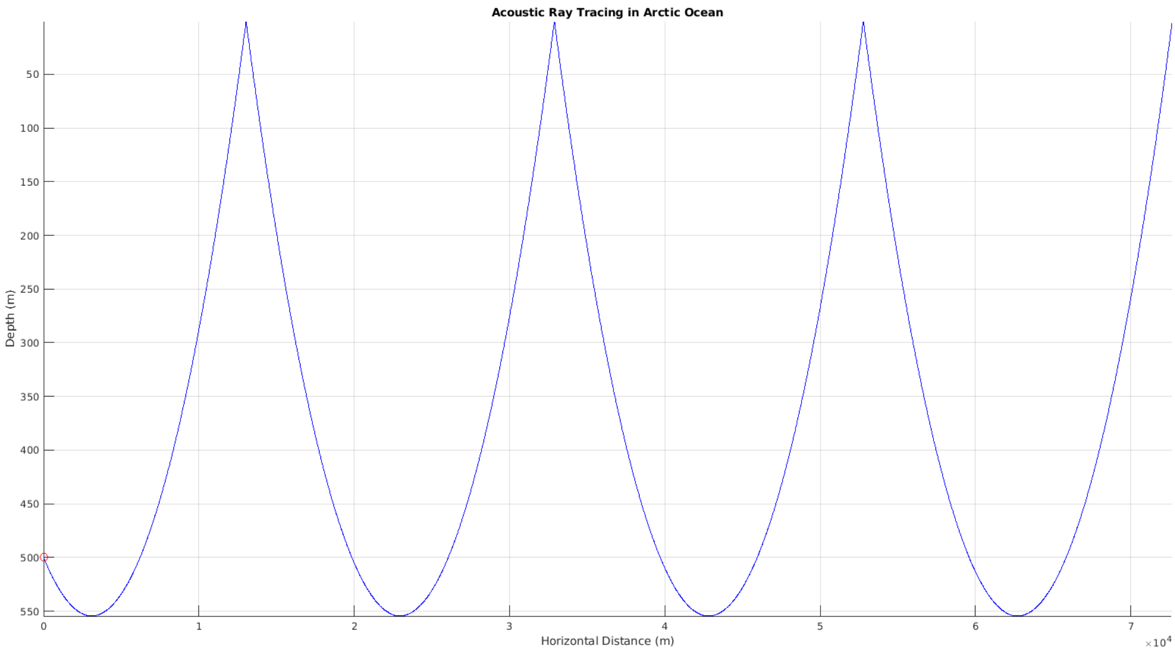

Figure 1: Traced ray with 2 degrees initial angle

Regarding the geometrical characteristics, as seen in figure [1], we can observe that the plotted acoustic ray follows a cyclic oscillatory trajectory in water due to the linear variation of sound speed with depth, which enforces the ray direction to alternate between moving towards the sea bottom and the sea surface.

We can observe cycle characteristics such as the Horizontal Distance Between Peaks as well as the wave amplitude. Also, we observe reflection points due to the direction change of the ray.

Finally, the geometrical characteristics mentioned are dependent on the initial angle with the horizontal axis. The ray will go deeper in depth and its curve will have a steep zig zag shape if the initial angle is large. For small initial angles, the ray will go to a smaller depth and the trajectory will be smoother with fewer reflection points.

Question 2:
We wish to establish communication between two hydrophones located at $(0, 0, z_s)$ and $(x, 0, z_s)$, respectively. Express, as a function of c_0, γ, x, and z_s, an emission angle θ_0 generating a ray whose path passes through both hydrophones. Verify the result using ray tracing software for $x = 20$ and $z_s = 500$ (to the nearest 0.1° for θ_0). Use the simulator to explore other possibilities. Have you found any? If so, which solution is preferable for operational conditions, and why?

To derive the equation of the emission angle θ_0 , we will use the angle with vertical axis as shown in figure [2], then we will make a subtraction between 90 and the angle with vertical in order to get our desired angle with the horizontal x axis.

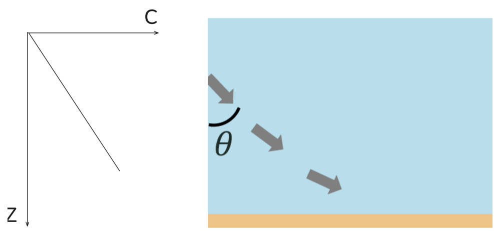

Figure 2: Angle with vertical axis

The radius of the curve is given by:

$$R = \frac{c(z_s)}{\gamma_0 \sin\theta}.$$

The center of the curve, X_c, is defined as:

$$X_c = R\cos\theta.$$

The horizontal distance to the desired point is:

$$X = 2X_c = 2R\cos\theta.$$

Substituting R, we get:

$$X = 2\left(\frac{c(z_s)}{\gamma_0 \sin\theta}\right)\cos\theta.$$

Simplifying further:

$$\frac{X}{1} = \frac{2c(z_s)\cos\theta}{\gamma_0 \sin\theta}.$$

The calculated angle θ is derived as follows:

$$\tan\theta = \frac{2(c_0 + \gamma_0 z_s)}{X \cdot \gamma_0}.$$

Thus:
The desired angle with the horizontal axis is:

$$\boxed{\theta_0 = 90^\circ - \tan^{-1}\left(\frac{2(c_0 + \gamma_0 z_s)}{X \cdot \gamma_0}\right)}.$$

$$\theta = 90^\circ - \tan^{-1}\left(\frac{2(1450 + 0.0163 \cdot 500)}{20000 \cdot 0.0163}\right) = 90^\circ - 83.6^\circ = 6.4^\circ.$$

Note: Our code uses the horizontal angle with x-axis (6.4) to get the following plot:

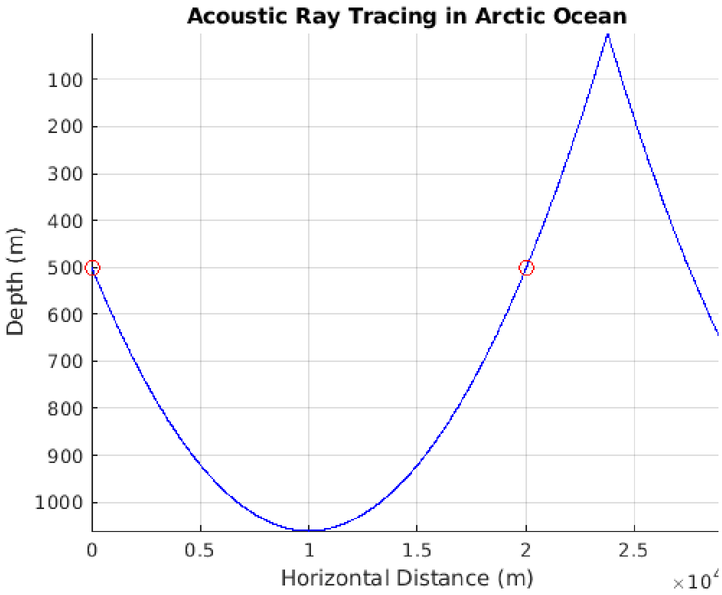

Figure 3: Verification of angle 6.4°

as seen in figure [3], we can observe that the communication between the two hydrophones located at $(0, 0, 500)$ and $(20000, 0, 500)$, respectively are done successfully when the initial angle with the x axis is 6.4°. The two red circles in the plot indicate the locations of the transmitter and receiver. It is shown that the ray passed successfully through the two hydrophones locations.

In addition, we used our code to test with other values of the initial angle. We found that communication can be also done if we set the initial angle to 2.2° as shown in the following figure [4]:

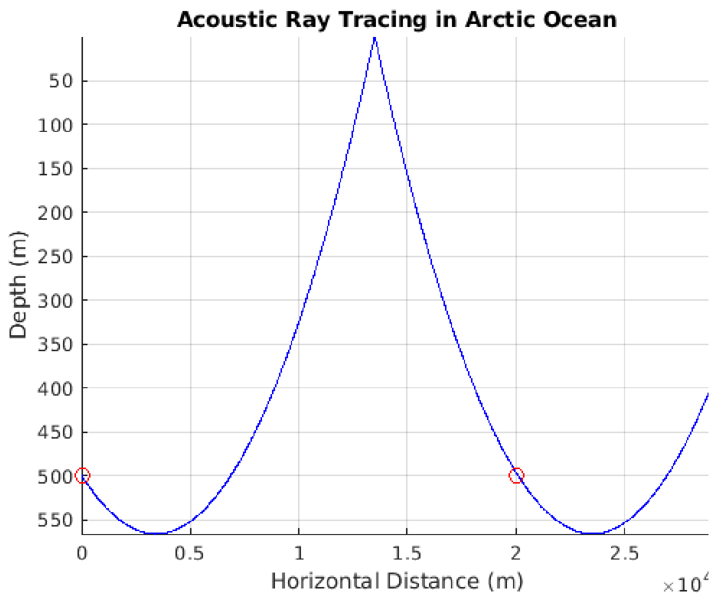

Figure 4: Verification of angle 2.2°

For operational conditions, we choose the solution of setting the initial angle to 6.4° because this solution has fewer reflections and this will ensure that the energy loss of the signal will be reduced and the signal integrity will be maintained in a more better way.

2 Mediterranean Sea

Due to higher surface water temperature, the sound velocity decreases over 700 hundred meters with a gradient of -0.026, before returning to pressure-driven behavior (as in the Arctic Ocean, $\gamma_0 = +1.63e-2$).

Plotting the rays using our MATLAB code

Here we test our developed MATLAB Ray tracing code for Mediterranean Sea by plotting a number of rays with different initial angles. We included the code in our submission (file named `Mediterranean_Sea_solution_HW1.m`) . To plot the rays, we set the initial angle in the code (theta0 variable) to an array from -2 to +2 degrees to form 20 rays:

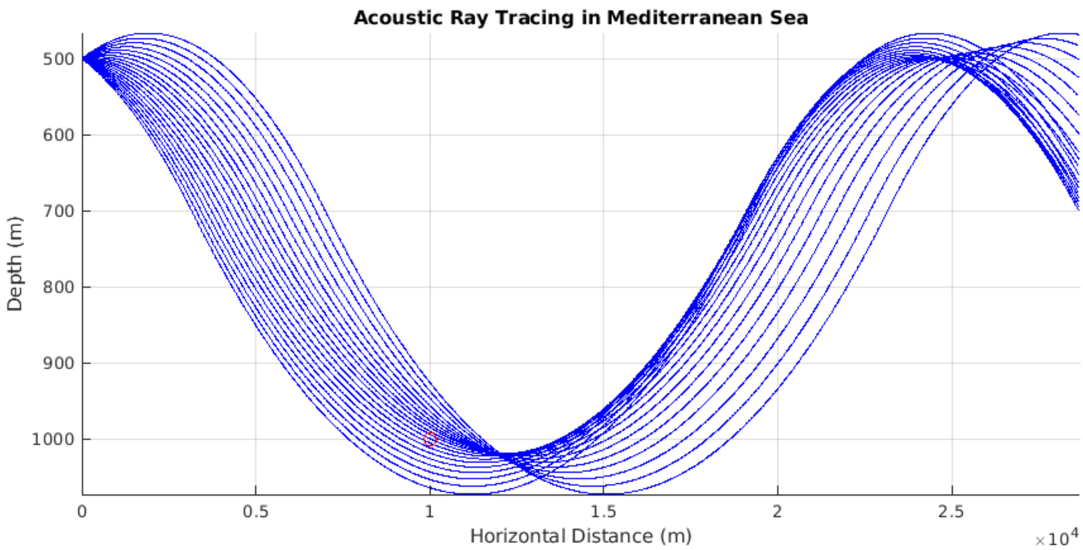

Figure 5: Verification of our code algorithm

Question 3:
We plan to establish communication with a receiver located 10 km away at a depth of 1000 m. At what depth z_s and emission angle θ_0 would you suggest transmitting? Is there flexibility in the transmitter depth? If operational conditions require rapid solution-finding, suggest a strategy.

In order for us to choose an optimal depth and emission angle for the transmitter, we will base our choice on three criteria. Firstly, we will select a depth that can be reached by our instruments and sensors and we will make sure first that our hydrophone can practically work on that depth. Secondly, after choosing a number of depths to make experiments on them, we will test by our software a number of different angle values at each depth and we will filter them first based on our defined accepted error between the actual depth reached by the ray at 10 km and the receiver location. Thirdly, after we make the first filtration process, we will now do a second filtration step for the filtered angles and depths from the first filtration by selecting one condition that took the shortest time to reach our receiver location. By this method, we managed to determine the optimal depth and initial angle that has the shortest arrival time, the minimum depth error, and is reachable given the state of our systems and hydrophones. For our case, we assumed that the transmitter hydrophone can reach a depth of 1800 meters, then we selected a number of depth values up to 1800 m ,then we tested a number of angles on each depth and selected the best values based on our mentioned method. We implemented our method using our developed code and included the code in our submission (file named `Mediterranean_Sea_solution_version2_HW1.m`). After performing a number of tests using our software, we found that the optimal depth is 1200 meters and the optimal emission angle is $2.11°$. Using these values, the ray will take 6.96 seconds to reach the receiver location, which is the shortest possible time. The following plot verifies our findings:

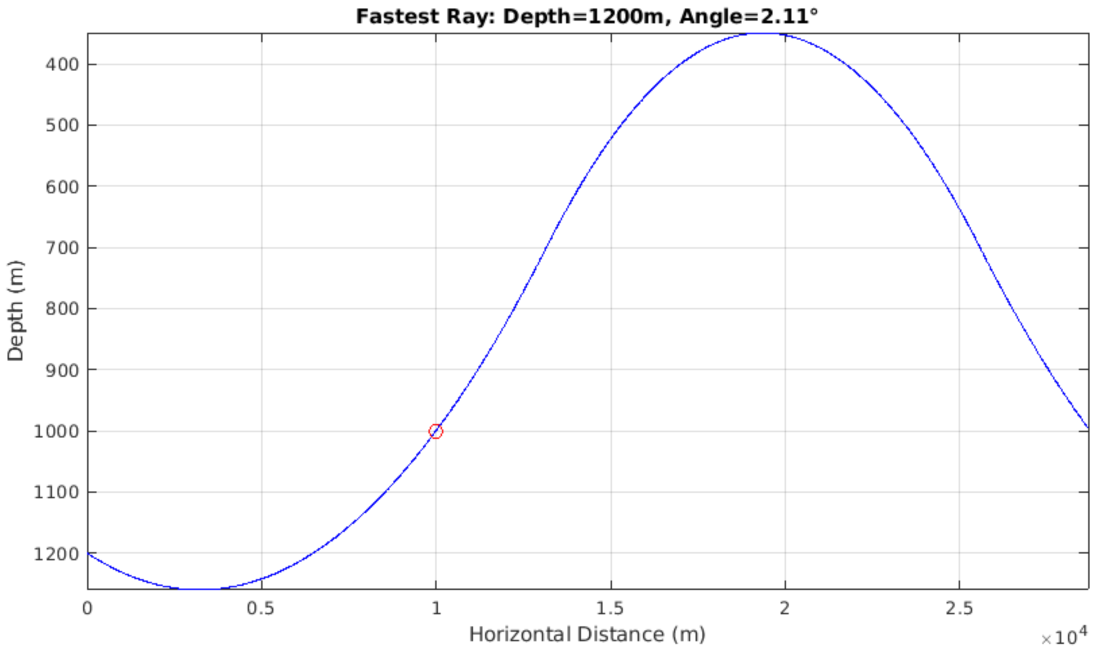

Figure 6: Verification of our solution

as seen in figure [6], we can observe that the communication between the two hydrophones located at $(0, 0, 1200)$ and $(10000, 0, 1000)$, respectively are done successfully when the initial angle with the x axis is $2.11°$. The red circle in the plot indicate the location of the receiver.

Regarding the choice for transmitter depth, we observed that we can have a choice of that, but based on the acceptable depth error for a user. The more accuracy in the depth we want, the fewer transmitter depths we can choose from.

If operational conditions require to find rapidly a solution, we suggest to use the strategy we used and implemented in our code. Our strategy will find the optimal depth and initial angle that has the shortest arrival time, the minimum depth error, and is reachable given the state of our systems and hydrophones. In such a case, we can use our code in the file named (`Mediterranean_Sea_solution_version2_HW1.m`) to test and verify the results with different depths and angles, then choose the optimal values quickly.