Homework 2 - 2024-2025 Report

Students:
Mohamed Magdy Atta
Shahid Ahamed Hasib

Underwater Acoustics

Instructor:
Prof. Marc Saillard

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Problem: Let us consider a water channel, h in depth, and let us assume that the velocity c is constant and equal to 1500 m/s. A signal is transmitted from a point-like source in the water and measured by a set of 9 receivers located along a vertical line at depths $25 \cdot n$ meters, $1 \leq n \leq 9$. When hitting the (flat) boundaries, the acoustic waves are totally reflected (reflection coefficient $r = -1$).
The file `Received.mat` contains a 9×32000 array which provides the 9 received signals during 6.4 s with sampling path $\mathrm{dt} = 2 \cdot 10^{-4}$ s. Line n corresponds to the receiver at depth $25 \cdot n$ meters.
Plot the received signals and explain how you can get a rough estimation of the depth h and of the location of the transmitter from the various time delays. Improve the accuracy of this first estimate thanks to the simulation of the back propagation of the time-reversed received signals.

Introduction

Acoustic signal propagation in underwater environments provides a robust framework for analyzing water channel characteristics and localizing sources. Here, we analyse signals received by nine vertically aligned receivers spaced at uniform intervals of 25 meters in a water channel of unknown depth. The receivers capture signals transmitted from a point source, which are reflected off flat boundaries with a total reflection coefficient of $r = -1$. The analysis leverages the provided dataset (`Received.mat`), which contains a 9×32000 array representing signals received over a duration of 6.4 seconds, sampled at $\mathrm{dt} = 2 \times 10^{-4}$ seconds.

We can determine the source's location by simulating the backpropagation of the signals received by the receivers and plotting the resulting energy map. This process allows us to estimate the water column depth (h) and the source location. MATLAB is the primary tool for signal analysis, and the corresponding code is attached to this report.

Figure 1 illustrates all 9 signals received by the receiver, and in Figure 2, all received signals are plotted separately.

Solution

From our MATLAB calculations, the signal was received the fastest at the 5th receiver, with a time of 2.246600 s. This indicates that the 5th receiver is horizontally closest to the source. Conversely, the 9th receiver recorded the signal last, with a time of 2.250600 s. This means that the 9th receiver is the farthest one from the source. To simplify depth computations, we can consider a right triangle formed by the source, the 5th receiver, and the 9th receiver.

Using the Pythagorean theorem:

$$s^2 + (\delta h)^2 = (s + \delta s)^2$$

$$\implies s^2 + (\delta h)^2 = (s + c\delta t)^2$$

Here, $\delta h = 125 - 25 = 100\,\mathrm{m}$, and $\delta t = 2.250600 - 2.246600 = 0.00400\,\mathrm{s}$. Substituting these values:

$$s^2 + (\delta h)^2 = s^2 + 2sc\delta t + c^2(\delta t)^2$$

$$\implies (\delta h)^2 - c^2(\delta t)^2 = 2sc\delta t$$

$$\implies s = \frac{(\delta h)^2 - c^2(\delta t)^2}{2c\delta t}$$

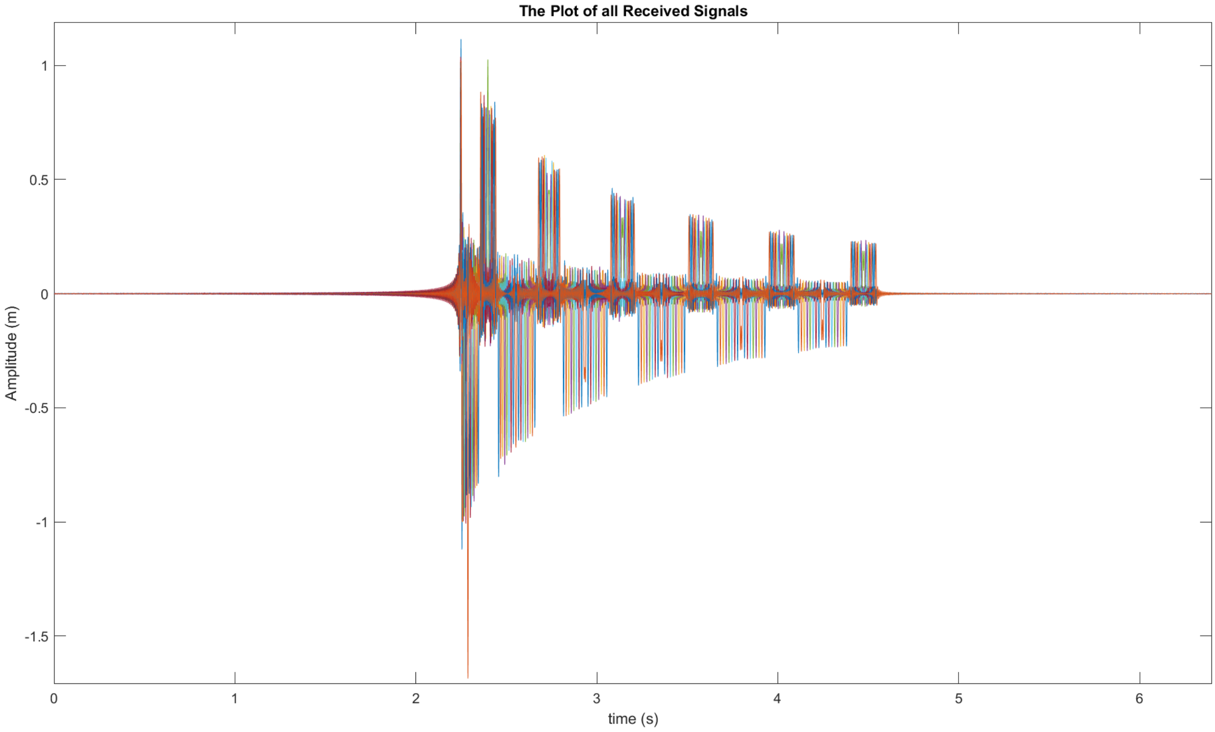

Figure 1: All 9 Received Signals by receiver

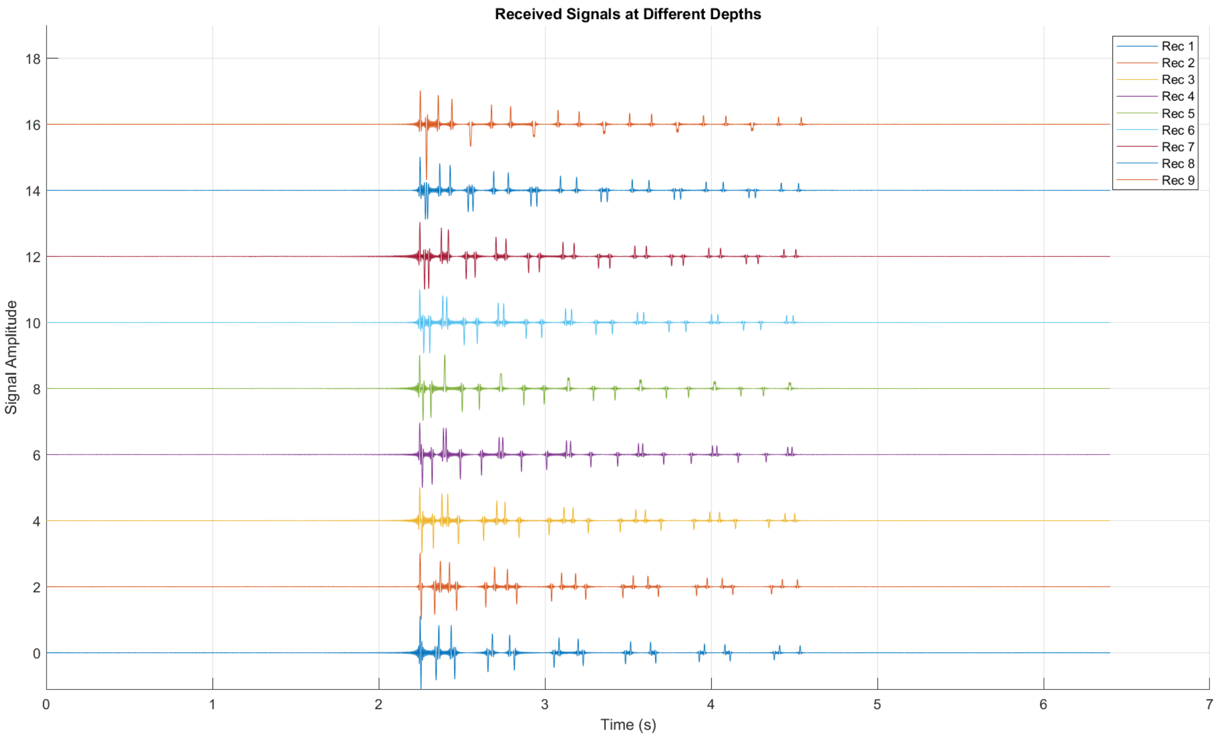

Figure 2: All 9 received signal separately.

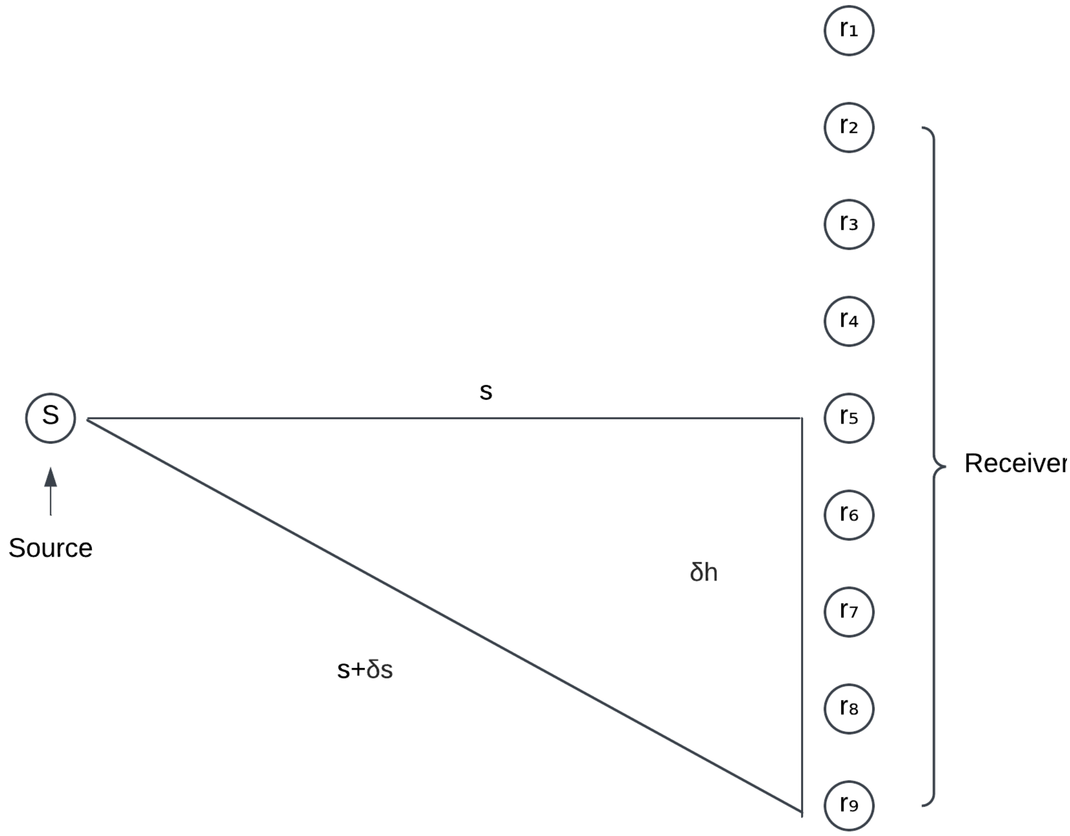

Figure 3: Configuration between source and receivers.

Distance (s) between source and receiver:
Considering the 5th and 9th receivers:

$$s = \frac{100^2 - 36}{2 \cdot 1500 \cdot 0.00400} = 830.33\,\mathrm{m}$$

Considering the 1st and 5th receivers:

$$s = \frac{100^2 - 17.64}{2 \cdot 1500 \cdot 0.0028} = 1188.376\,\mathrm{m}$$

Figure 4 refers to the plot of signals 1, 5, and 9 providing a clear visualization of the signal reception by these receivers. The amplitude of these signals around the peak is observed. Using this observation, the δt between the reception times of the signals is calculated.

In Figure 5, the successive peaks of the 5th signal are illustrated. This indicates that the signal behaves according to its reception before any others. This observation shows the value of δt between the first peak and its reflected first peak.

To estimate the distance between the transmitter and the receiver, the calculated distances are averaged as follows:

$$s_{\text{avg}} = \frac{(830.33 + 1188.376)\,\mathrm{m}}{2} = 1009.353\,\mathrm{m}$$

Thus, the distance between the transmitter and the receiver is approximated to be around 1000 m, and the source will be considered within this range. Next, the depth of the water column,

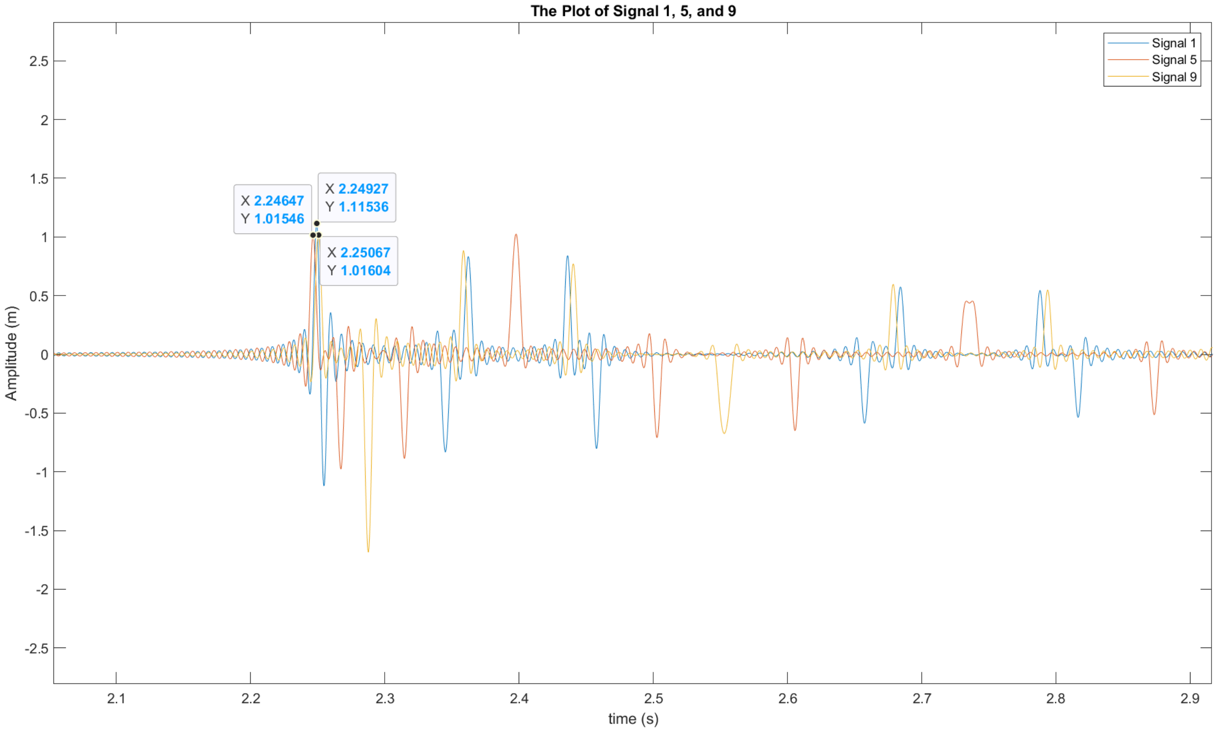

Figure 4: Signals at 1st, 5th and 9th receiver.

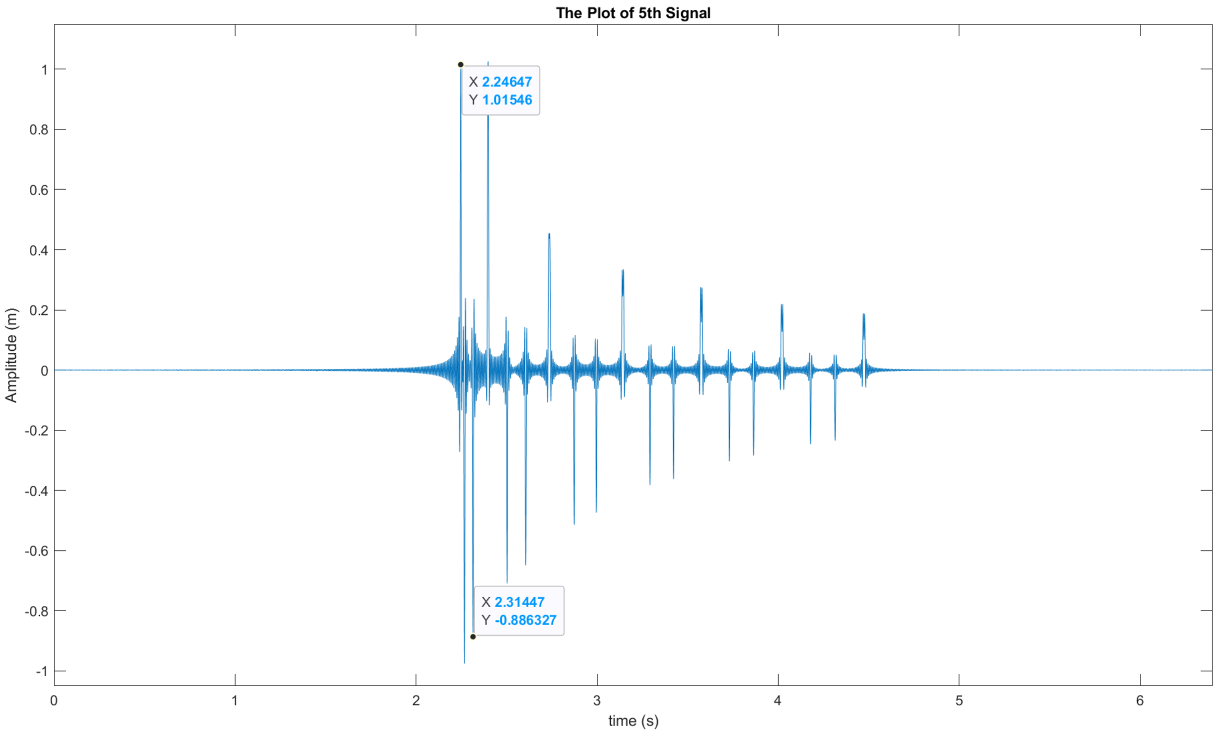

Figure 5: 5th received signal, successive peak.

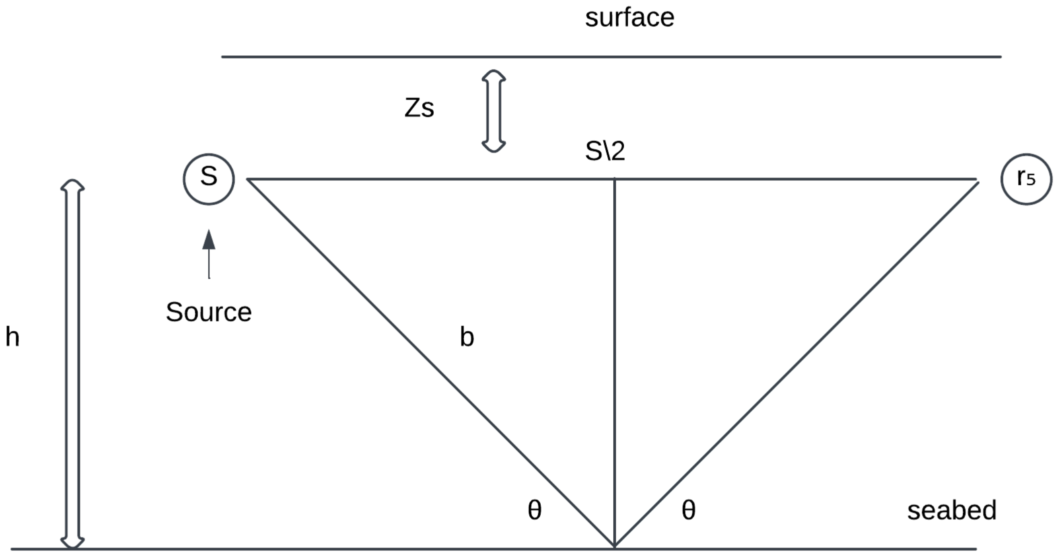

Figure 6: Depth "h" calculation.

which is the sum of the transmitter depth h_s from the seabed and the transmitter depth z_s from the surface, is calculated. Considering total internal reflection in the flat seabed and equal incoming-reflected angles θ, the result can be derived from Figure 7.

$$\begin{aligned} 2b - s &= c\delta t \\ \implies b - \frac{s}{2} &= \frac{c\delta t}{2} \\ \implies b &= \frac{s}{2} + \frac{c\delta t}{2} \end{aligned} \tag{1}$$

Applying the Pythagoras theorem:

$$b^2 = (h_{sb})^2 + \left(\frac{s}{2}\right)^2 \tag{2}$$

From equation 1, placing the value of b in equation 2, we get:

$$(h_{sb})^2 + \left(\frac{s}{2}\right)^2 = \frac{1}{4}(s + c\delta t)^2$$

$$\implies (h_{sb})^2 = \frac{1}{4}(s + c\delta t)^2 - \left(\frac{s}{2}\right)^2$$

$$\implies (h_{sb})^2 = \frac{1}{4}\left[(s^2 + 2sc\delta t + (c\delta t)^2) - s^2\right]$$

$$(h_{sb})^2 = \frac{1}{4}\left[(2sc\delta t + (c\delta t)^2)\right]$$

$$\implies (h_{sb}) = \sqrt{\frac{1}{2}(sc\delta t) + \left(\frac{c\delta t}{2}\right)^2}$$

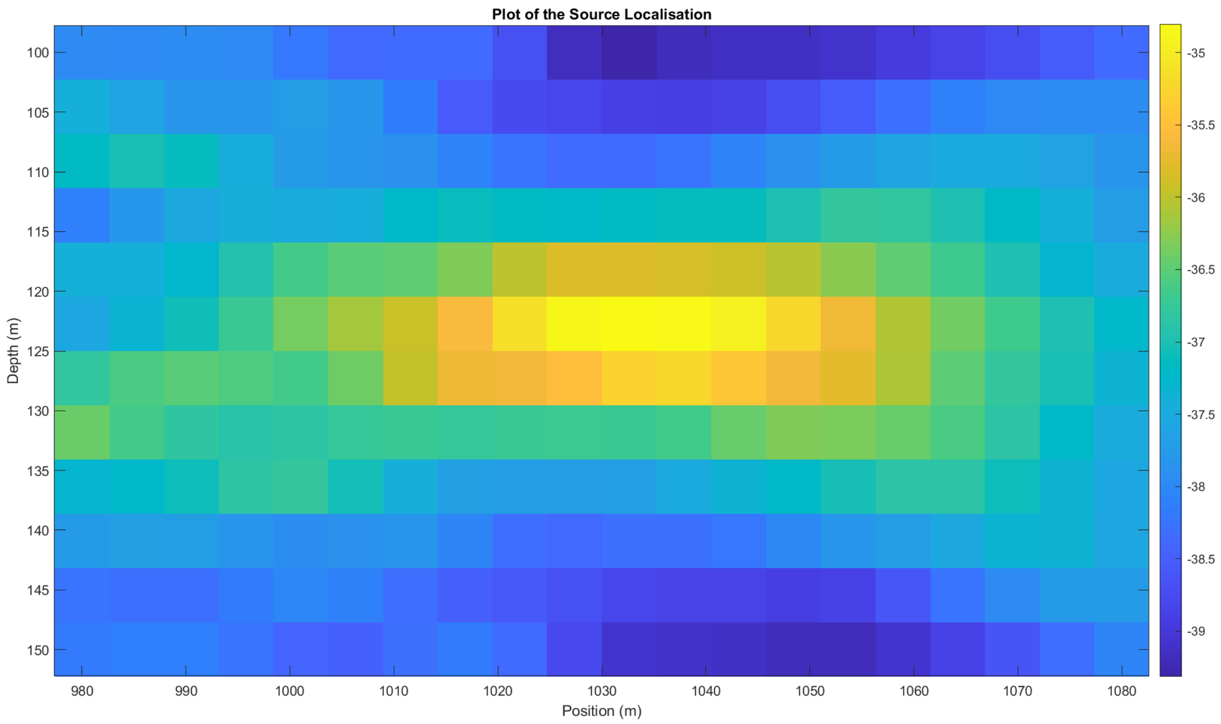

Figure 7: Expected source localisation

So the h_{total} will be:

$$h_{\text{total}} = \sqrt{\frac{1}{2}(sc\delta t) + \left(\frac{c\delta t}{2}\right)^2} + z_s \tag{3}$$

Now for getting the value of the δt, which is the time difference between the first peak and the first reflected peak of the signal by the 5th receiver, from figure-6 we get:

$$\delta t = (2.31447 - 2.24647)s = 0.068s$$

Putting the values in equation 3, we get the h_{total} or depth h:

$$h_{\text{total}} = [\sqrt{\frac{1009.353 \cdot 1500 \cdot 0.068}{2} + \left(\frac{1500 \cdot 0.068}{2}\right)^2} + 125\,]\text{m}$$

$$\implies h_{\text{total}} = \left[\sqrt{51477.003 + 2601} + 125\right]\ \text{m}$$

$$\implies h_{\text{total}} = [233.3 + 125]\,\text{m}$$

$$\implies h_{\text{total}} = 358.3\,\text{m}$$

Enhancing the Solution through Backpropagation of Time-Reversed Received Signals

The Green's Function was applied to estimate the x and z coordinates of the signal source. To achieve this, an initial guess for the x and z values of the signal source was assumed, followed by the application of Green's Function in MATLAB. Based on this initial location estimate, a detailed search was performed over a grid of possible coordinates to refine the estimate further.

The initial estimate's accuracy can be enhanced by applying backpropagation of time-reversed signals received by the receivers.

A simulation was conducted to model the backpropagation within a specified area represented as a search grid using the estimated depth and distance values. The objective of the simulation was to generate a plot that accurately visualizes the results. To achieve greater precision, the search range on the x-axis was refined to a narrower interval, specifically between 980 and 1080 meters.

This adjustment allowed for a more focused investigation of a smaller region, thereby reducing potential errors and enhancing the clarity of the results. The refined plot generated through this approach is illustrated in the accompanying figure. Additionally, a MATLAB script used to implement the simulation is provided to facilitate understanding of the methodology and ensure reproducibility of the results.

From Figure 7, the improved values obtained are: $S = 1035\,\mathrm{m}$ and source depth is between $120\,\mathrm{m}$ and $125\,\mathrm{m}$.

Extensive computations were carried out over a large grid, leading to significant simulation time. However, by refining the initial estimates and narrowing the search grid, more accurate results can be obtained. Figure 7 illustrates the improved 7.