summary

16 June 2023 11:02 AM

① **How many sol** → for $a*b=c$ where $a,b,c \in \mathbb{Z}$ then a,b should be divisors of c (std result). If a,b are not integers forcefully make them integers by multiplying by 10.

② **Paired cubes** → If $a^x a^y$ is perfect cube (or any perfect power) then $(x+y)\%3=0$

③ **Hard floor** → when numbers are extremely big, we try to observe pattern for certain portion of numbers in generalised way following some property. Here they are being constant for certain portion of numbers. Say new number of $\lfloor \frac{n}{i} \rfloor$ starts at i,

we need to find largest x such that

$$\left\lfloor \frac{n}{i} \right\rfloor = \left\lfloor \frac{n}{x} \right\rfloor$$

(don't fear while you see this type of eqn's, you can solve them!!)

$$\Rightarrow x = \left\lfloor \frac{n}{\left\lfloor \frac{n}{i} \right\rfloor} \right\rfloor$$

some application in fraction rank

④ **Multiply All GCD's**:- very good Q. fraction Rank too We will approach some Q like this (tough Mex in combinatorics too) in this manner. We need to multiply all $\underbrace{\text{GCD's}}_{\text{generalise as Quantity } x}$, then we see $\underbrace{\text{GCD}}_{x} \le 10^5$ (due to some reasons

$x \le 10^5$, then we try to find # of subsequences/subsets that satisfies property $= x$, for $1 \le x \le 10^5$ then we use this to find ans.

⑤ **Multiply all GCD's**:- School level but yet efficient method for finding GCD. Write prime factorisation of each number then take common among all prime factorisations to get ans.

⑥ Inefficient program:- based on property of %

⑥ Inefficient program:- based on property of %
it rounds to same value after m integers. This
property is often exploited in many Q

⑦ Fraction Rank:- It takes motivation from multiply
all GCD's and Hard floor.
Here $y \leq 10^5$. So that generalised Quantity y.
So we start ~~find~~ finding for each y ($1 \leq y \leq n$)
How many possible fractions exist.
↳ Then we take this resultant as potential
number (like multiply GCD's) and get no. of irreducible
fractions.

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How Many Solutions

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Ask Doubt

From <https://www.learning.algozenith.com/problems/How-Many-Solutions-219>

Description

Given the value of integer N how many solutions does the following equation have?

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{N}$$

If x and y are integers there is only a finite number of solutions but if x and y are real numbers then there can be an infinite number of solutions. What if x and y are floating-point numbers with limited size, e.g. x and y are floating point numbers with d digits after the decimal points, how many different solutions will be there?

Input Format

Input file contains at most 2000 lines of input. Each line contains two integers N ($0 < N \le 10000000000$) and d ($0 \le d \le 1000$), here d means that there can be maximum d digits after the decimal point. Input is terminated by a line containing two zeros. This line should not be processed.

Output Format

For each line of input, produce one line of output which contains an integer T. This line contains the number of different solutions the equation has for the given value of

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$$(n - x)(n - y) = n^2$$

$$(\underbrace{n \cdot 10^d}_{N'} - \underbrace{x \cdot 10^d}_{x'})(\underbrace{n \cdot 10^d}_{N'} - \underbrace{y \cdot 10^d}_{y'})$$

$$= (\underbrace{n\, 10^d}_{N'})^2$$

All are integers

now, proceed in std way

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Paired Cubes

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Ask Doubt

From <https://www.learning.algozenith.com/problems/Paired-Cubes-211>

Description

You have given n positive integers $A_1, A_2, \ldots, A_n$. Your task is to find the number of pairs (i, j) such that $A_i * A_j$ is a **cube number**.

Input Format

The first line of input contains n - the size of the array.
The second line of input contains n space-separated integers $A_1, A_2, \ldots, A_n$.

Output Format

Print the answer on a new line.

Constraints

$1 \le n \le 10^5$
$1 \le A_i \le 10^6$

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technique

Incremental Processing using map to track frequencies

logic

If $n = p_1^{a_1 \% 3} \, p_2^{a_2 \% 3}$

search for $map\left[p_1^{(3 - a_1 \% 3)} p_2^{(3 - a_2 \% 3)}\right]$

as these two numbers form perfect cube

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Hard Floor

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Ask Doubt

From <https://www.learning.algozenith.com/problems/Hard-Floor-217>

Description

Your task is to find the answer to the following sum.

$$\sum_{i=1}^{i=n} \lfloor \frac{n}{i} \rfloor^m \% 1000000007$$

Input Format

The only line of input contains two space-separated integers n and m.

Output Format

Print the answer on a new line.

Constraints

$1 \leq n \leq 10^{12}$
$1 \leq m \leq 10^9$

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Hint 1

Find the largest x such that $\lfloor \frac{n}{i} \rfloor = \lfloor \frac{n}{x} \rfloor$

Solution Approach

The value of $\lfloor \frac{n}{i} \rfloor$ remains same from i to la where

Value of $la = \lfloor \frac{n}{\lfloor \frac{n}{i} \rfloor} \rfloor$

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$\lfloor \frac{n}{i} \rfloor = \lfloor \frac{n}{x} \rfloor$

$x = \lfloor \frac{n}{\lfloor \frac{n}{i} \rfloor} \rfloor$

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Multiply All GCDs

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Ask Doubt

From <https://www.learning.algozenith.com/problems/Multiply-All-GCDs-164>

Description

You are given N integers (not necessarily distinct) => $A_1, A_2, A_3, ..., A_N$. There are 2^N possible subsets (including the empty subset).
The GCD of a subset is defined as the greatest common divisor of all the integers in that subset.
You need to find the product of the GCDs of all the 2^N possible subsets you can construct from A. Since the answer can be large, you need to output the answer modulo 1000000007. Do you think you can solve this question?

Note: The greatest common divisor of an empty subset is 1.

Input Format

The first line of input consists of a single integer T denoting the number of test cases. The description of T test cases follow.
The first line of each test case consists of a single integer N. The second line of each test case consists of N space-separated integers $A_1, A_2, ..., A_N$.

Output Format

For each test case, output a single integer on a separate line denoting the answer for that test case. Note that you need to output all the values modulo 1000000007 (10^9 + 7).

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Multiply all GCD's [Number theory Day 4 std]

(*) Numbers are upto range of 10^5
$\Rightarrow$ gcd's of all subsets $\le 10^5$

(*) Find # of subset with ($gcd = x_i$) $= y_i$

$1 \le x_i \le 10^5$

$\Rightarrow$ $\{x_i, y_i\}$ $\rightarrow$ ans
$\prod x_i^{y_i}$ $\rightarrow$ ans

$\{2, 4, 6, 4, 3\}$

freq(i) $\rightarrow$ # i appear in

0	1	2	3	4	5	6	7	8	9
0	0	1	1	2	0	1	0	0	0

FREQ(i) $\rightarrow$ # of multiples of i in input

0	5	4	2	2	0	1	0	0	0	0

$\rightarrow$ for (int i=1; i≤N; i++)
for (j=i; j≤N; j+=i)
$\rightarrow O(N \log N)$

Freq(i) $\rightarrow (2^{Freq(i)} - 1)$

0	1	2	3	4	5	6					
0	31	15	3	3	0	1	0	0	0	0	0

The value in $2^{Freq(i)} - 1$ is potential answer but we did some overcounting, we need to subtract overcounting

$\rightarrow$ where we did overcounting?
(Eg:) for 3 $\rightarrow$ we considered all the subsets that can be formed by 3, 6
[{3} {6} {3, 6}]

this gcd is 6 which is wrong too, but we subtract these type of subsets in all the overcounting by 6

for 6 $\rightarrow$ we considered subsets that can be formed 6
[{6}]

{6} $\rightarrow$ overcounted

for from freq(i) = [$2^{freq(i)} - 1$] array we need to subtract the no. of subsets starting from 2i, 3i ....

$\Rightarrow$ freq(i) = freq(i) # subsets with gcd (2i)
− exact # subsets with gcd (3i)
− exact # subsets with gcd (4i)
− exact # subsets with gcd (5i) ....

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GCD of Products

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Ask Doubt

From <https://www.learning.algozenith.com/problems/GCD-of-Products-92>

Description

You have to calculate the GCD of N numbers. Since the numbers are too large to be written as numbers, they are written as products of smaller numbers. Find their GCD.

Input Format

The first line of the input contains one integer N - the number of numbers.
Each of the next N lines contains an integer M followed by M space-separated integers, $A_1, A_2, ..., A_M$ - the numbers whose product is the i-th number.
It is guaranteed that the sum of M over all N numbers doesn't exceed 10^6.

Output Format

Print the GCD on a single line. Since this number might be pretty big, output it modulo 10^9+7.

Constraints

$2 \le N \le 10^6$
$1 \le M \le 10^6$
$1 \le A_i \le 10^7$

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Sample Input 1

```
3
4 2 3 1 2
3 3 6 2
3 2 4 7
```

Sample Output 1

```
4
```

Note

The first number will be A_1 = 2*3*1*2 = 12
The second number will be A_2 = 3*6*2 = 36
The third number will be A_3 = 2*4*7 = 56

The GCD of these three numbers will be GCD(12, 36, 56) = 4

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School gcd method, write prime factorisation then take common

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Inefficient Program

- Ask Doubt

From <https://www.learning.algozenith.com/problems/Inefficient-Program-78>

Description

Consider the following function.

```
long long get(long long L, long long R, long long m) {
    long long sum = 0;
    for(long long i = L; i <= R; i++) {
        sum += (i % m);
        sum %= 1000000007;
    }
    return sum;
}
```

The above program is very inefficient for larger values of *L, R, m*.
Your challenge is to write an efficient program, that will find the value of **get** function in very less time.

Input Format

The first line of input contains one integer *T* — the number of test cases. Then *T* test cases follow.
The only line of each test case contains three space-separated numbers *L, R,* and *m*.

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Output Format

For each test case, print a number on a new line denoting the answer return by **get** function in the above program.

Constraints

$1 \le T \le 10^5$
$1 \le L \le R \le 10^{18}$
$1 \le m \le 10^{18}$

Sample Input 1

Copy

```
3
4 5 6
4 10 9
1 1000000000000000000 93464156618
```

Sample Output 1

Copy

```
9
```

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$$\boxed{\begin{aligned} &1\%m + 2\%m + \ldots + R\%m \\ &\left\lfloor \frac{R}{m} \right\rfloor \times (1+2+\ldots+m-1) + (1+2+\ldots+(R\%m)) \end{aligned}}$$

eg

$$\underline{1\%3 + 2\%3} + \underset{0}{\cancel{3\%3}} + \underline{4\%3 + 5\%3} + \underset{0}{\cancel{6\%3}} + 7\%3$$

$$\left\lfloor \frac{7}{3} \right\rfloor \Rightarrow 2 \Rightarrow 2(1+2) + \underline{(1)}$$

$$O(1) \sim \left\lfloor \frac{R}{m} \right\rfloor \times \left(\frac{(m-1)(m)}{2}\right) + \left(\frac{(R\%m)(R\%m+1)}{2}\right)$$

$$\boxed{L \text{ to } R \rightarrow (1 \text{ to } R) - (1 \text{ to } L-1)}$$

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Fraction Rank

From <https://www.learning.algozenith.com/problems/Fraction-Rank-77>

Let us consider a set of fractions x / y, such that $0 \le x/y \le 1$, $y \le n$ and *gcd* (x, y) = 1.

For example, say $n = 5$. Then we have the following set in increasing order : 0/1, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, 4/5, 1/1

You are given *n*, *a* and *b*. The task is to find the rank of *a* / *b* in a set of fractions as stated above which is in increasing order.

Note: Fractions should be finite.

Input Format

The first line of contains a number T ($1 \le T \le 20$) - the number of testcases. Then T lines follow.
In each of T lines you are given *n*, *a*, *b* ($1 \le n \le 100000$, $a \le b \le n$) such that *gcd*(*a*, *b*) = 1.

Output Format

Print T lines. The *i*th line contains the rank of a fraction *a* / *b* for a given *n*, *a* and *b* in the (*i* + 1)th line of input.

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Ask Doubt

x/y ① gcd(x, y) =
② $0 \le \left(\frac{x}{y}\right) \le 1$ (Proper) | $y \le n$

= 5

$\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \boxed{\frac{1}{2}}$ (*)
$\frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1}$

(n, a, b) rank of (a/b)

Hint 1

One way to approach this problem is to count the number of fractions which are less than or equal to the given fraction $\frac{a}{b}$ in the given set. Once we know the count of such fractions, we can easily find the rank of $\frac{a}{b}$ in the set by adding 1 to this count.

To count the number of fractions less than or equal to $\frac{a}{b}$, we can make use of the following observation:

For any given denominator y, the fractions $\frac{x}{y}$ which satisfy the condition $gcd(x, y) = 1$ and $0 \le \frac{x}{y} \le 1$ form a Farey sequence of order y, denoted as F_y. The set of all fractions satisfying the given conditions can be obtained by taking the union of all Farey sequences of order y, where $1 \le y \le n$.

Now, we can count the number of fractions less than or equal to $\frac{a}{b}$ in each Farey sequence of order y, and add them up to get the total count of fractions less than or equal to $\frac{a}{b}$ in the given set. To count the number of fractions less than or equal to $\frac{a}{b}$ in a Farey sequence of order y, we can make use of the following property:

If $\frac{a}{b}$ is a fraction in a Farey sequence of order y, then the fractions $\frac{2a}{2b}, \frac{3a}{3b}, \dots, \frac{(y-1)a}{(y-1)b}$ are not in the sequence.

If $\frac{a}{b}$ is a fraction in a Farey sequence of order y, then the fractions $\frac{2a}{2b}, \frac{3a}{3b}, \dots, \frac{(y-1)a}{(y-1)b}$ are not in the sequence.

Using this property, we can count the number of fractions less than or equal to $\frac{a}{b}$ in a Farey sequence of order y, by first calculating the number of fractions $\frac{x}{y}$ in the sequence for each $y \le n$, and then subtracting the number of fractions which are not in the sequence using the property mentioned above.

gcd like question

Hard floor.

n = 5 a = 3 b = 4

~~y = 0~~

y = 1 → 0

y = 2 → for y = 2. $\frac{x}{y} \le \frac{a}{b}$

$\frac{x}{y} \le \frac{a}{b} \Rightarrow \frac{x}{2} \le \frac{3}{4} \cdot 2$

$x \le 1.5$

y = 3 → x should be $x \le \frac{3}{4}$ less than or equal to $\frac{3}{4}$

x = 1, x = 2

y = 4 But x is an integer

y = 5 No. of possible int : 0 ⇒ floor working.

x = 1, x = 2, x = 3

x = 1, x = 2, x = 3

$\frac{1}{3}, \frac{1}{2}$ $\frac{x}{5} \le \frac{3}{4}$ is (3 $\frac{x}{3} \le \frac{3}{4}$ $x \le \frac{9}{4}$

$\frac{x}{4} \le \frac{3}{4}$ 4) 9 (2.2
8
10

$\frac{0 \times 2}{1 \times 2}$.

y = 0 — ✗

x = 0 ✓ y = 1 — $\frac{0}{1}$

x = 1 ✓ y = 2 → $\cancel{\frac{0}{2}}, \frac{1}{2}$

x = 2 ✓ y = 3 → $\cancel{\frac{0}{3}}, \frac{1}{3}, \frac{2}{3}$

x = 3 ✓ y = 4 → $\cancel{\frac{0}{4}}, \frac{1}{4}, \cancel{\frac{2}{4}}, \frac{3}{4}$

x = 3 ✓ y = 5 → $\cancel{\frac{0}{5}}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$,

Solution

(*) mutiply all gcd's like question, we can find all possible fraction for each y [1 ≤ y ≤ N] $\le \frac{a}{b}$, since we counted all possible fractions, we overcounted fractions which are reducible (like in $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$, all reducible

fractions $\frac{2}{4}, \frac{3}{6}$ ... are ...

possible fractions, we overcounted fractions which are reducible (like in $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$, all reducible fractions $\frac{2}{4}, \frac{3}{6} \cdots$ are overcounted. we will subsequently subtract them to get answer.

(*) Finding all possible fractions $\le \frac{a}{b}$, for each y

(y-fixed) $\frac{x}{y} \le \frac{a}{b}$

$$x \le \frac{(a*y)}{b}$$

the possible values x can take to satisfy the above equation are 0 to $\lfloor \frac{ay}{b} \rfloor$

for each y we can calculate the no. of possible fractions

$y=1 \Rightarrow \lfloor \frac{a}{b} \rfloor \Rightarrow$ say f_1

$y=2 \Rightarrow \lfloor \frac{2a}{b} \rfloor \Rightarrow$ say $f_2 \longrightarrow$ ($\frac{1}{2}$ can be counted)

$y=3 = \lfloor \frac{3a}{b} \rfloor = f_3$

$y=4 = \lfloor \frac{4a}{b} \rfloor = f_4 \longrightarrow$ ($\frac{2}{4}$ can be counted which is same $\frac{1}{2}$)

$\vdots$

$y=n \Rightarrow \lfloor \frac{na}{b} \rfloor = f_n$

If $\frac{1}{2}$ is counted for particular $\frac{a}{b}$, then definetly $\frac{2}{4}$ will be counted as $\frac{1}{2} = \frac{2}{4}$ and similarly multiples of other fractions

$\Rightarrow$ Whatever calculated in f_1, it's equivalent fractions will be $f_2, f_3 \cdots f_{K(1)} \cdots f_n$

$\Rightarrow$ whatever calculated in f_2, its equivalent fractions will be in $f_4, f_6, f_8 \cdots f_{2 \times k} \cdots f_n$ and similarly others

as we are considering all the possible denominator multiples [all the possible, stressing!!] the numerator for equivalent fraction definetly exist, by the same argument, that if $\frac{1}{2} \le \frac{a}{b}$ then $\frac{2}{4}, \frac{3}{6}, \frac{4}{8} \cdots$ also $\le \frac{a}{b}$ and they present in the respective denominators

$\frac{1}{2} \frac{(\cdot 5)}{(\times 3 \cdot 5)} \frac{?}{7}$

$\frac{7}{14}$

So we can remove overcounting by sieve like technique.

it is nothing but $O(n \log n) \leftarrow$

```
for(int i=1; i<=n; i++)
    for(j = 2*i; j<=n; j+=i)
```

you might ask i will multiply denominator with some fraction and will make denominator equal to some number which is not multiple and accordingly adjust the numerator, but it is not possib

whole analysis says since num and deno are integers you are allowed to allow multiply num and deno by integers

If you multiply by some fraction $\frac{p}{q}$, which on simplification [forcing num and den to be integers]

will yield as num and den multiplied by some integer

$\frac{x \times \frac{p}{q}}{y \times \frac{p}{q}}$ $\frac{y \times p}{q}$ = no. which is not multiple of y

$\Rightarrow \frac{xp}{yp}$ (upon simplification)

$\Rightarrow$ we got all unique fractions $\le \frac{a}{b}$

$\Rightarrow$ answer = rank $\left(\frac{a}{b}\right)$ = Sum of array element

(i.e., all unique (irreducible) fractions) (i.e., all unique (irreducible) fractions)

$\Rightarrow \frac{xp}{yp}$ (upon simplification)

$\Rightarrow$ since num, deno are integers we need to convert like this and it happens you will get fraction in $\underline{\underline{g = yp}}$

$\underline{eg}$: $\frac{1 \times 35}{2 \times 35} \Rightarrow \frac{35}{70} = \frac{\cancel{35}^{1}}{\cancel{10}} \Rightarrow \frac{35}{2(35)}$

multiple of $\underline{\underline{2}}$

you will get this!!