Summary

16 June 2023 11:37 AM

1. character Encoding → Binomial th application (JEE)

2. Learn this thing → direct Lucas th application with $p=2$ (as it decides odd/even)

3. count good subsequences → when something (here median) is defined seperately (here even and odd). then you need to solve problem for all possibilities and never use permutation/choice concept in subsequence. Always do by combination concept

4. Tough Mex ⇒ we see $Mex \le N \le 10^5$, so here generalised $x = Mex$ (like Multiply all GCD's). we try to find for each x (mex) ($1 \le x \le N$) how many subsequences/ subsets have that $Mex = x$

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Character Encoding

- Ask Doubt

From <https://www.learning.algozenith.com/problems/Character-Encoding-168>

Description

Find the number of solutions of following equation.

$x_1 + x_2 + x_3 + \ldots + x_n = k$, satisfying that $0 \le x_i < m$, modulo 1000000007.

Input Format

The only line of input contains three space-separated integers n, m, k.

Output Format

Print the number of solutions.

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Sol :- $x_1 + x_2 + x_3 + \text{---} + x_n = k \qquad 0 \le x_i < m$

Coefficient of x^k in $(x^0 + x^1 + \text{---} + x^{m-1})(x^0 + x^1 + \text{---} + x^{m-1})$

$\ldots (x^0 + x^1 + \text{---} + x^{m-1})$ [n times]

Coeff of x^k in $(x^0 + x^1 + \text{---} + x^{m-1})^n$

$$\Rightarrow \left(\frac{1 - x^m}{1 - x}\right)^n$$

Coeff of x^k in $\Rightarrow (1 - x^m)^n (1 - x)^{-n}$

expand by binomial th, See code in Vs code for implementation

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Learn This Thing

-

Ask Doubt

From <https://www.learning.algozenith.com/problems/Learn-This-Thing-189>

Description

For a given **n**, find the number of even and odd numbers among the set, **{ $^nC_0$, $^nC_1$,... ^nC_n }**.

Input Format

First-line contains **T ($1 \le T \le 10^5$)**, the number of test cases. Next T lines contain one integer per line, denoting **n ($0 \le n \le 10^{12}$)**.

Output Format

For each test case, output two space-separated integers specifying the number of even numbers and odd numbers respectively.

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Sol:- Lucas theorem

$$\binom{n}{r} \bmod p = \binom{n_1}{r_1}\binom{n_2}{r_2}\binom{n_3}{r_3}\binom{n_4}{r_4} \% p$$

$n_1 n_2 n_3 n_4 \rightarrow$ Base p of n

$r_1 r_2 r_3 r_4 \rightarrow$ Base p of r

for odd and even p=2 decides

$n \rightarrow$ Binary form $\rightarrow$ 101

$r \rightarrow$ Binary form $\rightarrow$

Note $^0C_0 = 1$ | $^0C_1 = 1$

$^1C_0 = 1$ | $^1C_1 = 1$

If atleast one $\binom{n_1}{r_1} = 0$

then it is 0,

for odd all $\binom{n_i}{r_i} = 1$

$\Rightarrow$ at position of 0, in binary it is always 0

at 1 $\rightarrow$ two choices

0 , 1

both yield one

for odd all $\binom{n_i}{r_i} = 1$ both yield one

$\Rightarrow 2^{\text{no. of set bits in 'n'}}$

$\longrightarrow$ # odd numbers

Total $= n+1$

even no. $= n+1 -$ # odd

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Count Good Subsequences

- Ask Doubt

From <https://www.learning.algozenith.com/problems/Count-Good-Subsequences-185>

You are given a sequence $A_1, A_2, \ldots, A_N$. Let's call a subsequence $A_{i_1}, A_{i_2}, \ldots, A_{i_k}$ (for any $k > 0$, $1 \le i_1 < i_2 < \ldots < i_k \le N$) good if the median of this subsequence is an element of this subsequence.
Find the number of good subsequences. Since this number may be large, compute it modulo 1000000007 (10^9+7).

Note:

1. The median of a set of data is the middlemost number in the set. The median is also the number that is halfway into the set. To find the median, the data should first be arranged in order from least to greatest.
2. For odd length sequence, the median is the middle element in the sorted sequence. While for even length sequence, it is the average of the middle two elements.

Input Format

The first line of input contains T - the number of test cases.
The first line of each test case contains a number N - the size of the array.
The second line of each test case contains N space-separated integers $A_1, A_2, \ldots, A_N$.

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Subsequence good
⇓
median of subsequence is element of subsequence

Find
good subsequences

Constraints
$1 \le T \le 30$
$1 \le N \le 1000$
$1 \le A_i \le 2N$

From <https://www.learning.algozenith.com/problems/Count-Good-Subsequences-185>

$\rightarrow \cancel{O(N^3)}$ ✗
$O(N^2\sqrt{N})$
$O(N^2)$
$O(N^2\log N)$

Sol: Brute Find all subsequences $[2^n]$
median for every seq and check
$O(N \cdot 2^n)$ → won't pass $\rightarrow O(N)$

optimise

Obs: ① Subsequences of odd length are always good

$n + n(n-1)(n-2)$

② for even length subsequence to be good the middle two elements should be same
↳ why?, avg of two diff middle values cannot give a number in that subsequence? → No

→ Sorted order (by def of median)

if two no. are different, then the resulting avg is present in the middle of those two in sorted order, but no element is present in between
⇒ If ele are diff → not good.

Now let's find No. of such sequences

odd
↳ Sorted is only condition.
↳ of course in order → for subsequence property to get satisfied

1 → — } → N

2 → — — — }

91

Solution

If subsequence is odd length, then median definitely lies inside it ⇒ good

No. of odd length subsequences $= 2^{N-1}$

proof:

— — — — — — — — , ⊜

form all subsequences $N-1$

2^{N-1} subsequences

nth element → attach to even length ⇒ results in odd
→ don't attach to odd ⇒ odd

2^{n-1} subsequences

⇒ results in odd

don't attach to odd ⇒ odd

only ways for forming odd subsequence

$ans += 2^{n-1}$

Now array is sorted
for even length subsequences

[__________ ⓐ aaaa. ______]

for even length subsequence to be good the middle elements where we take average should be equal

assume b and c are distnict

sorted subsequence → abcd   median = $\frac{b+c}{2}$

this number is >b, <c

exist in between b and c but there is no number between b and c!!!

⇒ b and c must be equal

Say sorted array is like this

we work by fixing left side number

$\underbrace{[\_\_\_\_\_]}_{l \text{ ele.}}$ ⓐ $\overbrace{a\,a\,a\,a\,a}^{k \text{ same excluding fixed}}$ [____] r ele.

for above subsequence to be good and of even length

from l you need to take j ele

from r ele you need to take $j+1$ elements in which atleast 1 a need to be present to make it good

k → you can find by upperbound or by maintaining frequency map carefully.

→ # of such ways

for $j = 0$ to $\min(l, r-1)$

$$ans += {}^{l}C_{j} \times \left({}^{r}C_{j+1} - {}^{r-k}C_{j+1}\right)$$

these don't yield good subsequences as we are not selecting a

So subtract them from all possible

i.e., total possible − # of those not giving good.

this simple concept has some fancy name

good

this simple concept has some fancy name

inclusion - exclusion principle

Note: for subsequences questions never use permutation or choices concept

Always use combination concept as subsequence means order is fixed!

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Tough Mex

- Ask Doubt

From <https://www.learning.algozenith.com/problems/Tough-Mex-186>

Description

For a (possibly empty) sequence of positive integers S, mex is defined as $f(S)$ as the smallest positive integer that does not appear in S. For example, $f([]) = 1$, $f([3,4,3,5]) = 1$, $f([2,5,1,1,2,3]) = 4$.

You have given a sequence of N integers $A_1, A_2, \ldots, A_N$. Your task is to find the sum of $f(S)$ over all 2^N possible subsequences S of this sequence.

Since the resulting sum can be very big, compute it modulo 998244353.

Input Format

The first line of the input contains a single integer T denoting the number of test cases. The description of T test cases follows.

The first line of each test case contains a single integer N.

The second line contains N space-separated integers $A_1, A_2, \ldots, A_N$.

Output Format

For each test case, print a single line containing one integer — the sum of $f(S)$ over all subsequences modulo 998244353.

Constraints

$1 \le T \le 10$

$1 \le N \le 10^5$

$1 \le A_i \le 10^9$ for each valid i

Sample Input 1

```
2
2
1 1
3
1 2 1
```

Sample Output 1

```
7
17
```

Sequence → +ve integers → can be empty too

mex → $f(S)$ = smallest +ve int that don't appear in S.

[] → 1

3, 4, 3, 5 → 1

2, 5, 1, 1, 2, 3 → 4

Sequence → $A_1, A_2 \ldots A_N$ → find $f(S)$ over all 2^N subsequences → subsets

sum % 998244353

Sol

mex → order don't matter

$N \le 10^5$

$A_1, A_2, A_3 \ldots A_N$ → sort

Smallest int that can if we don't consider this

Incrementation Technique?

$A_1 \to 4$ if $A_1 = 1 \to$ then 2 else → 1 = −

+ A_2

exponential

fix mex → mex = 1

≥ 1

1 2 3 mex = 4

1 2 ~~10~~ 4 (N+1) → mex = 3

if $a[i] > N$, $a[i] = N+1$

1 2 10 → mex = 3   1 2 4 → mex = 3

1 3 ~~10~~ 4 → 2

for (mex = 1; mex <= N+1; mex++)

1 1 2   1 1   1 1 · 2

1 1 2   11   11·2

- { }   1   m   12

                       1   12

$\rightarrow$ $1 \times 2 \times 0 =$ $\underbrace{C}_{(2^2-1)}$ 8 $3 \rightarrow \underbrace{(2^2-1)}_{3} \times \underbrace{(2^1-1)}_{1}$

1 1 2 2 3 3   mex = 1

$(2^2-1)(2^2 \cancel{\ })$ $\rightarrow$ mex = 2   - { }

$\Rightarrow 3 \times 3 = 9$ 1 $\cancel{2}$

1   3   1 3   mex = 3

1 1   3   1 3 $(2^2-1)(2^2-1)$

                  3 3   1 3 3

1   { }   1 3

2   1 1 3

                  1 1 3 $\underline{1 1 3 3}$ 1 3 3

$a_1 \quad a_2 \quad a_3 \quad \underset{\text{mex}}{a_4} \quad \underbrace{(a_5 \quad a_6 \quad)}_{\text{rem}}$

$(2^{a_1}-1)(2^{a_2}-1)(2^{a_3}-1) \ldots \times (1 << \text{rem})$

1 1

{ } $\rightarrow$?   mex = 2

```cpp
ll ans=1; //including empty subset
    ll run_pro=1,run_sum=0;
    for(int mex=2;mex<=f;mex++){
        run_pro=mul(run_pro,(1<<m[mex-1])
);
        run_sum+=m[mex-1];
        ans=add(ans,mul(mex,run_pro*(1<<(
n-run_sum-m[mex]))));
    }
    cout<<ans<<"\n";
```

run_pro =

ans = 1 $\rightarrow$ { }

$\cancel{1, 1, 2}$ $\cancel{1, 2, 1}$

```cpp
    ll ans=1; //including empty
subset
    ll run_pro=1,run_sum=0;
    for(int mex=2;mex<=f;mex++)
{
        run_pro=mul(run_pro,((1<
<m[mex-1])-1));
        run_sum+=m[mex-1];
        ans=add(ans,mul(mex,run_
pro*(1<<(n-run_sum-m[mex]))));
    }
    cout<<ans<<"\n";
  }
```

run_pro = $2^2 - 1 \Rightarrow 3$

run_sum = 2

ans = $1 + (3 \times 2^0) \times 2$

$1 + 3 \times 2$

$\Rightarrow 7$ $\left. \begin{array}{l} n = 3 \\ \text{run\_sum} = 2 \\ m[2] = 1 \end{array} \right\}$

run_pro = $(3)(2^1-1) \Rightarrow \underline{3}$ n = 3

run_sum = 3   run_sum = 3

m[3] = 0

ans = $7 + (\underline{3 \times 1}) \times 3$

run_sum = 0

ans = 7 + (3 × 1) × 3

3 × 3 = 9

$$\begin{array}{r} 9 \\ 7 \\ \hline 16 \end{array}$$

Solution: Sorting causes no problem as in calculation of mex order don't matter

Why sort? mex → lowest possible non present integer, if we find upto where series is continous with lowest integers (1, 2, 3 ...) and where series is breaking it will be used

$1, 2, 3 \ldots f-1, x_1, x_2, x_3 \ldots$

$x_1 \neq f$ in sorted array

then for all subsequences formed by $x_1, x_2, x_3 \ldots$

mex = f

we found mex for $x_1, x_2, x_3 \ldots$ and we know mex ≤ f

Now we will find each mex ($1 \le mex \le f$) has how many subsequences.

for that we maintain frequency map

$1 \to f_1$

$2 \to f_2$

$3 \to f_3$

⋮

No. of subsequences with mex = 1

⇒ Consider all subsequences that don't have 1

2^{n-f_1}

No. of subsequences with mex = 2

1 should be there, 2 shouldn't be there, from 3 it is optional

$$(2^{f_1}-1)(2^{n-f_1-f_2})$$

similarly for mex = 3

$$(2^{f_1}-1)(2^{f_2}-1)(2^{n-f_1-f_2-f_3})$$

⋮

carry on till mex = f

carry on till $mex = f$