



دانشگاه شهید باهنر کرمان

SBUK

**Report title:** Comparison of the performance of “**Alamouti code**” as an STBC code with the “**MRRC**” method.

**Producer:** Shahin Majazi

**Field of Study:** Electrical Engineering (Telecommunication-System)

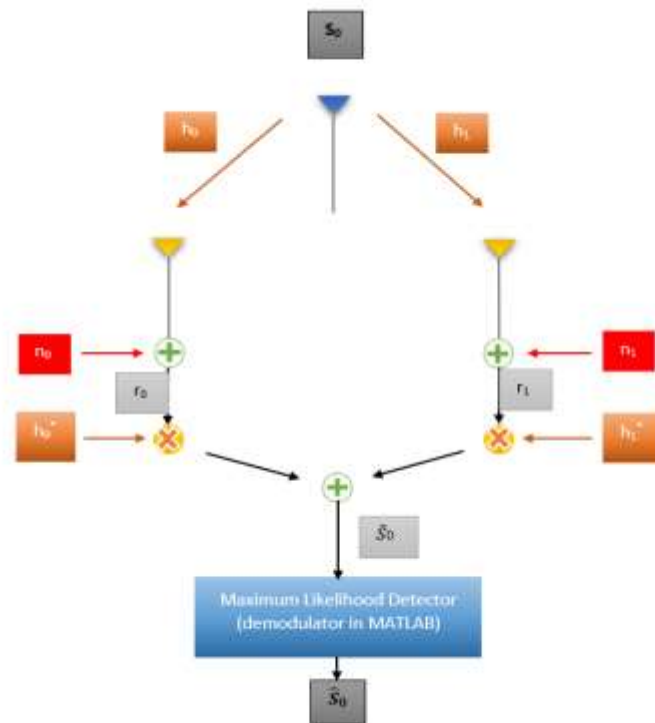
**Degree:** M.Sc

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**Objective:** Implementation of Alamouti code as an **STBC** code(Space Time Block Coding) and comparing its performance with **MRRC**(Maximum Ratio Receiver Combining) Method.

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1. MRRC method with two antennas in receiver (1 TX, 2 RX)



And we have

$$r_0 = h_0 s_0 + n_0$$

$$r_1 = h_1 s_0 + n_1$$

$$\tilde{s}_0 = h_0^* r_0 + h_1^* r_1$$

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$$\tilde{s}_0 = |h_0|^2 s_0 + |h_1|^2 s_0 + f(n_i) \quad (1)$$

Where

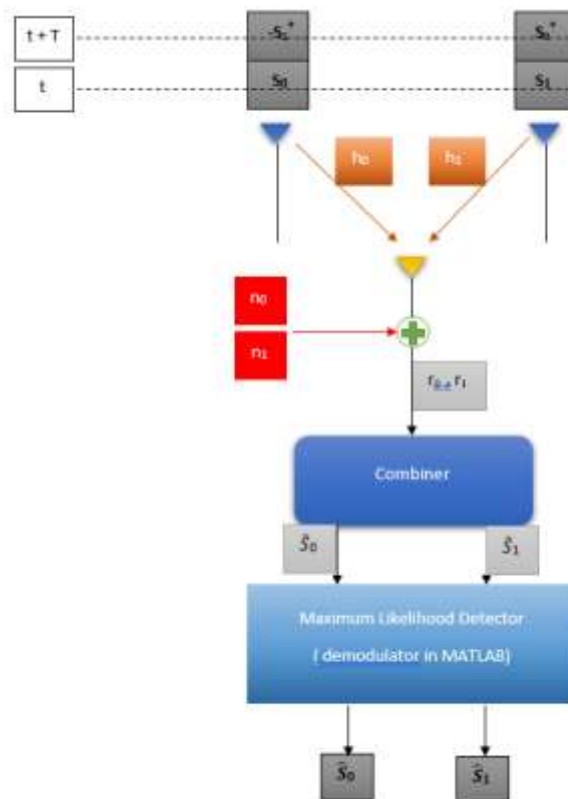
$n_i$  is the Additive White Gaussian Noise(AWGN) of the  $i^{\text{th}}$  path.

$h_i$  is fading coefficient of the  $i^{\text{th}}$  path.

( $*$ ) shown the conjugate of arbitrary parameter.

Note that  $\tilde{s}$  is the input symbol of the demodulator and for simulation we have to make it and final symbol will be output of the demodulator(the same of  $\hat{s}$ ).

## 2. Alamouti code method with one antenna in the receiver (2 $T_x$ , 1 $R_x$ )



Where the indices show the chronological order of each of the parameters and T is the time interval between sending two consecutive symbols in the transmitter and we also have:

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0$$

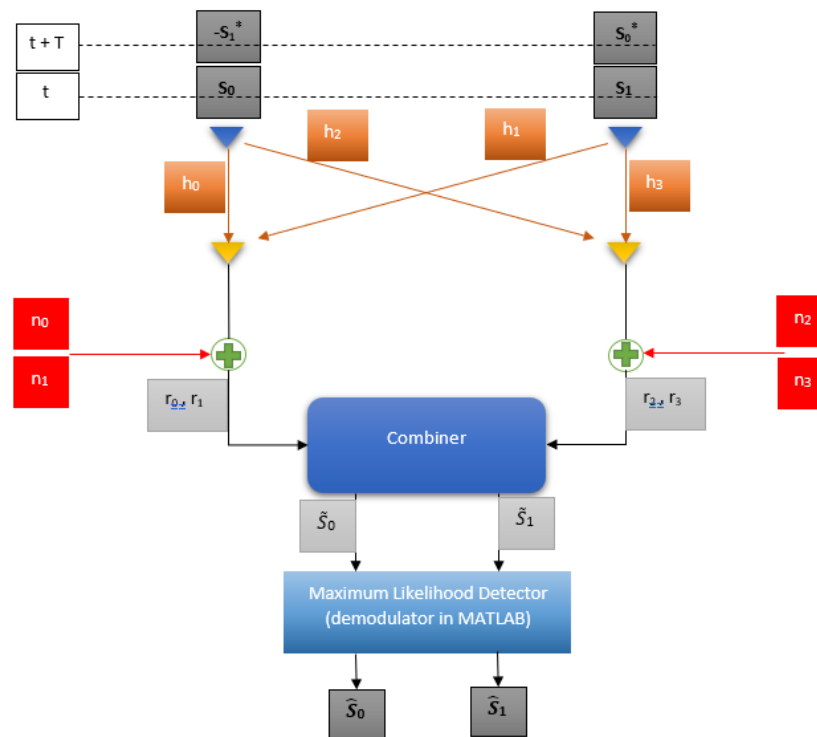
$$r_1 = r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1$$

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^*$$

$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^*$$

$$\tilde{s}_0 = |h_0|^2 s_0 + |h_1|^2 s_0 + f(n_i) \quad (2)$$

### 3. Alamouti code method with two antennas in the receiver (2 T<sub>x</sub>, 2 R<sub>x</sub>)



Where

$$\begin{array}{ll}
 r_0 = h_0 s_0 + h_1 s_1 + n_0 & r_0 = h_0 s_0 + h_1 s_1 + n_0 \\
 r_1 = -h_0 s_1^* + h_1 s_0^* + n_1 & r_1^* = h_1^* s_0 - h_0^* s_1 + n_1^* \\
 r_2 = h_2 s_0 + h_3 s_1 + n_2 & r_2 = h_2 s_0 + h_3 s_1 + n_2 \\
 r_3 = -h_2 s_1^* + h_3 s_0^* + n_3 & r_3^* = h_3^* s_0 - h_2^* s_1 + n_3^*
 \end{array} \longrightarrow$$

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^* + h_2^* r_2 + h_3 r_3^*$$

$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^* + h_3^* r_2 - h_2 r_3^*$$

So we'll have:

$$\begin{pmatrix} r_0 \\ r_1^* \\ r_2 \\ r_3^* \end{pmatrix}_{4 \times 1} = \begin{pmatrix} h_0 & h_1 \\ h_1^* & -h_0^* \\ h_2 & h_3 \\ h_3^* & -h_2^* \end{pmatrix}_{4 \times 2} \times \begin{pmatrix} s_0 \\ s_1 \end{pmatrix}_{2 \times 1} + \underline{n}$$

$$\begin{pmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{pmatrix}_{2 \times 1} = \begin{pmatrix} h_0^* & h_1 & h_2^* & h_3 \\ h_1^* & -h_0 & h_3^* & -h_2 \end{pmatrix}_{2 \times 4} \times \begin{pmatrix} r_0 \\ r_1^* \\ r_2 \\ r_3^* \end{pmatrix}_{4 \times 1}$$

#### 4. Generalized Alamouti code

If we consider the number of antennas in the receiver equal to  $n_{rx}$ , then we will have:

$$\begin{array}{c} \boxed{\vec{z}} \\ \text{ } \\ \left( \begin{array}{c} r_0 \\ r_1^* \\ \vdots \\ r_{2n_{rx}-1}^* \end{array} \right)_{2n_{rx} \times 1} \end{array} = \begin{array}{c} \boxed{H_A} \\ \text{ } \\ \left( \begin{array}{cc} h_0 & h_1 \\ h_1^* & -h_0^* \\ \vdots & \vdots \\ h_{2n_{rx}-2} & h_{2n_{rx}-1} \\ h_{2n_{rx}-1}^* & -h_{2n_{rx}-2}^* \end{array} \right)_{2n_{rx} \times 2} \end{array} \times \begin{array}{c} \left( \begin{array}{c} s_0 \\ s_1 \end{array} \right)_{2 \times 1} \end{array} + \underline{n}$$

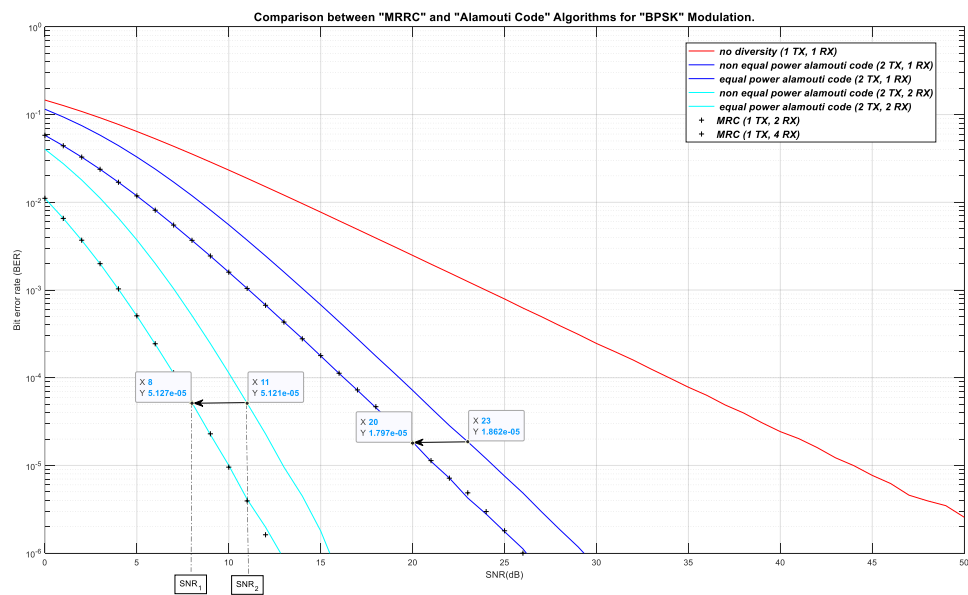
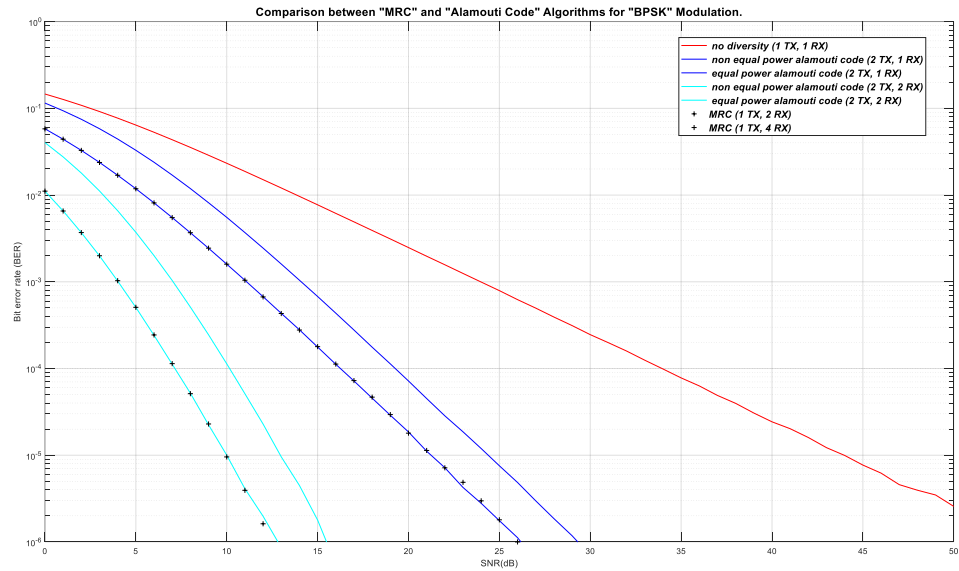
$$\rightarrow \boxed{\vec{z} = H_A \vec{s} + \vec{n}}$$

$$\begin{array}{c} \left( \begin{array}{c} \tilde{s}_0 \\ \tilde{s}_1 \end{array} \right)_{2 \times 1} \end{array} = \begin{array}{c} \left( \begin{array}{ccccc} h_0^* & h_1 & \cdots & h_{2n_{rx}-2}^* & h_{2n_{rx}-1} \\ h_1^* & -h_0 & \cdots & h_{2n_{rx}-1}^* & -h_{2n_{rx}-2} \end{array} \right)_{2 \times 2n_{rx}} \end{array} \times \begin{array}{c} \left( \begin{array}{c} r_0 \\ r_1^* \\ \vdots \\ r_{2n_{rx}-1}^* \end{array} \right)_{2n_{rx} \times 1} \end{array}$$

$$\rightarrow \boxed{\vec{\tilde{s}} = H_A^H \vec{z}}$$

## 5. Simulation results

After simulation by monte carlo method we'll have:



## 6. Conclusion:

As we have seen in the theoretical parameters (relationship 1 and 2) the Alamouti code is theoretically quite similar to the MRRC method, but in the diagrams above we see that it has weaker performance, in other words, the Alamouti code algorithm for 3dB SNR more, reaches the bit error rate equivalent to the MRRC method.

$$\text{BER}(\text{SNR}_1) = \text{BER}(\text{SNR}_2)$$

$$\text{SNR}_2 - \text{SNR}_1 = 3_{\text{dB}}$$

So to achieve the same performance with the MRRC algorithm, it is necessary to increase the SNR in the Alamouti code by 3dB, and because we have:

$$10 \log_{10} \left( \frac{2Ps}{N} \right) = 10 \log_{10}^{(2)} + 10 \log_{10} \left( \frac{Ps}{N} \right) \simeq 3_{\text{dB}} + \text{SNR}_{\text{dB}}$$

Therefore, we have to double the power of the transmitted signals, by doing so, as shown in the figure, the diagrams related to the Alamouti code are transferred to the left by 3dB and are placed on the MRRC diagrams, so by establishing the hypotheses in Theory (twice the transmission power in Alamouti code compared to MRRC) The results of the theory (the same performance of both methods) are also obtained.

## Reference:



[1] **Siavash M. Alamouti**, "A Simple Transmit Diversity Technique for Wireless Communications", IEEE JOURNAL ON SELECT AREAS IN COMMUNICATIONS, VOL. 16, NO. 8, OCTOBER 1998.