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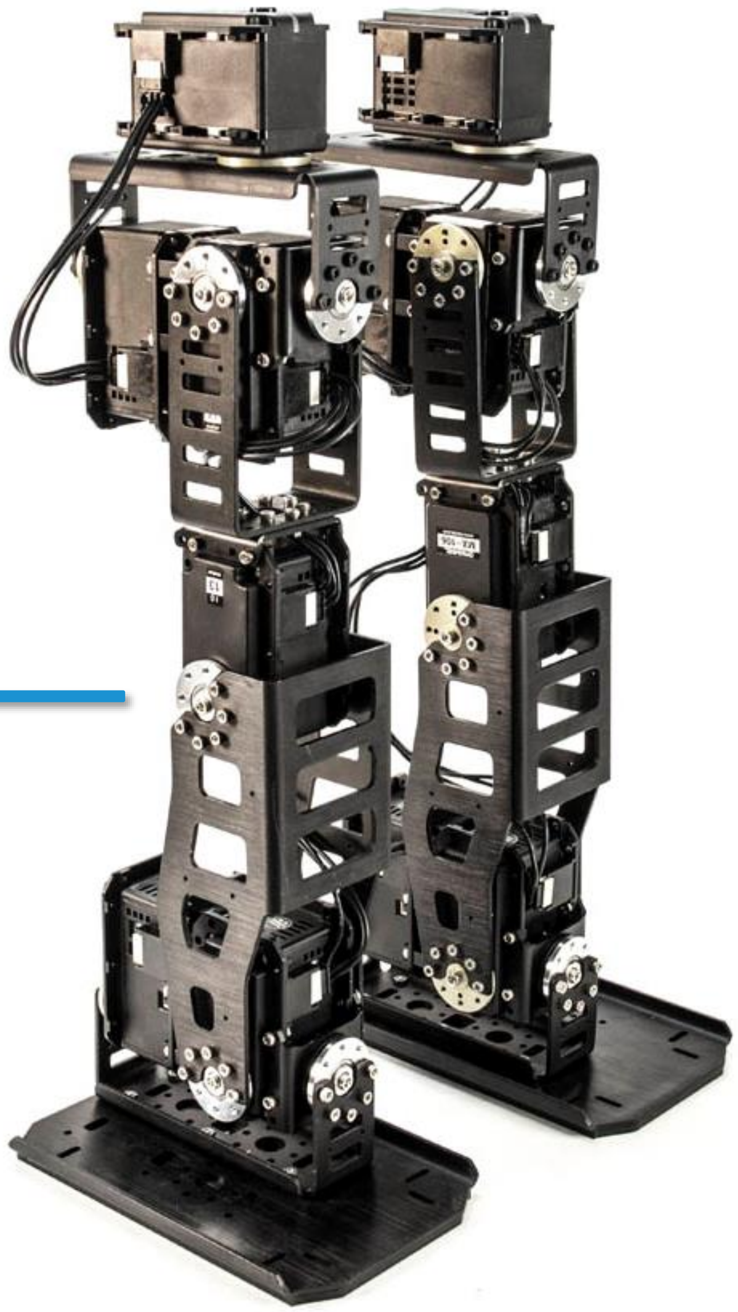
ROCO318 Mobile and Humanoid Robots



Mock-up Exam

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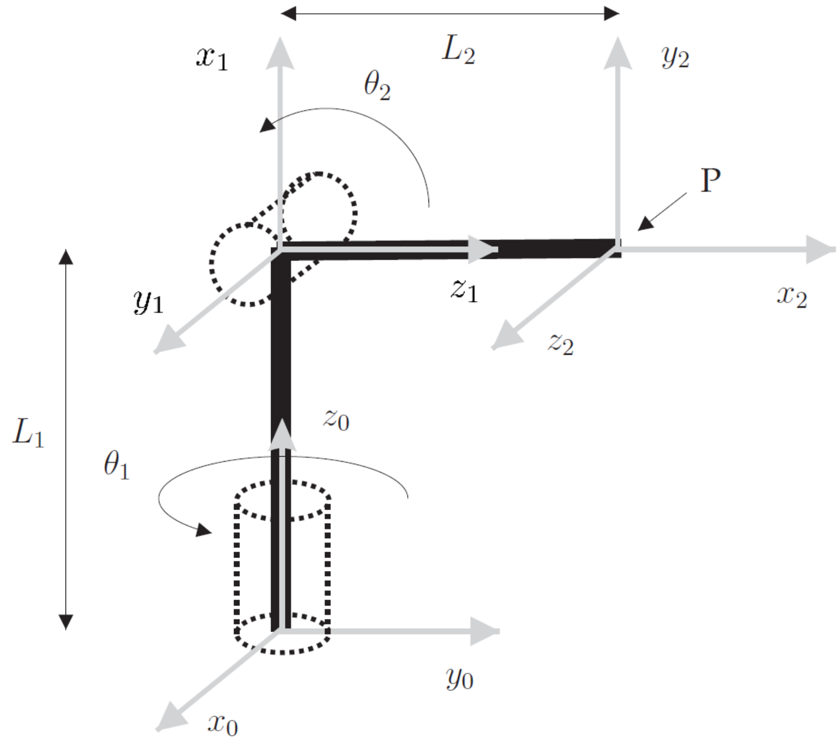
Exercise 1: Kinematics and homogeneous transformations

Consider the non-planar kinematic structure in the figure below



- This structure can be similar to a part of a legged robot. It is composed of two non-planar links: Link 1 and Link 2.
- Link 1 rotates of an angle θ_1 around the z_0-axis of the inertial reference frame (frame 0).
- Link 2 rotates of an angle θ_2 around the y_1-axis of the reference frame 1 and it is translated with respect Link 1 of L_1 units along the z_0-axis of the reference frame 0.

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Exercise 1: Kinematics and homogeneous transformations

Knowing that the rotations by an angle θ around the x-, y- and z- axis are given by the following homogeneous transformations

$$\mathrm{RotX}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathrm{RotY}(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{RotZ}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

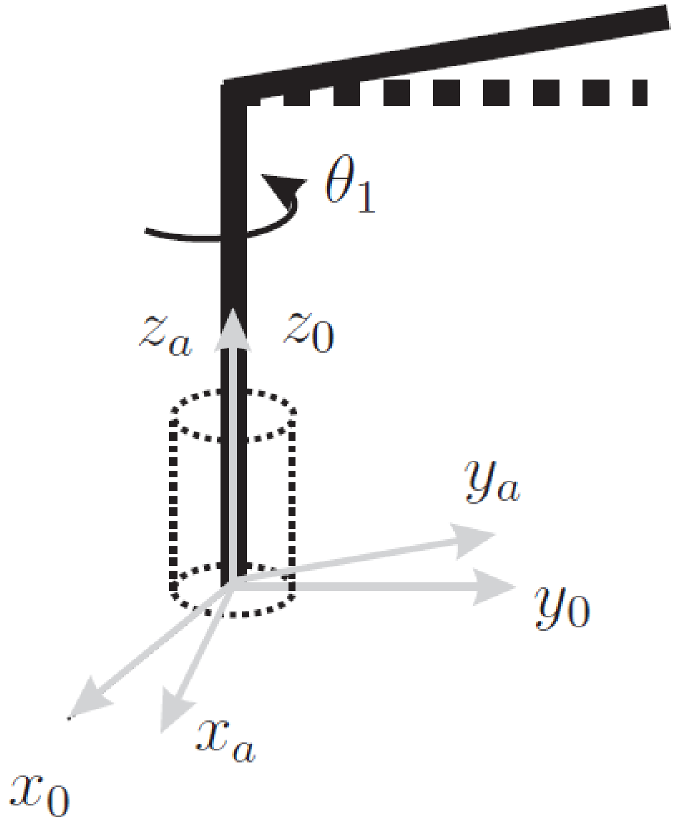
Exercise 1: Kinematics and homogeneous transformations

Question 1: Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame 1

Exercise 1: Kinematics and homogeneous transformations

Solution: Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame 1

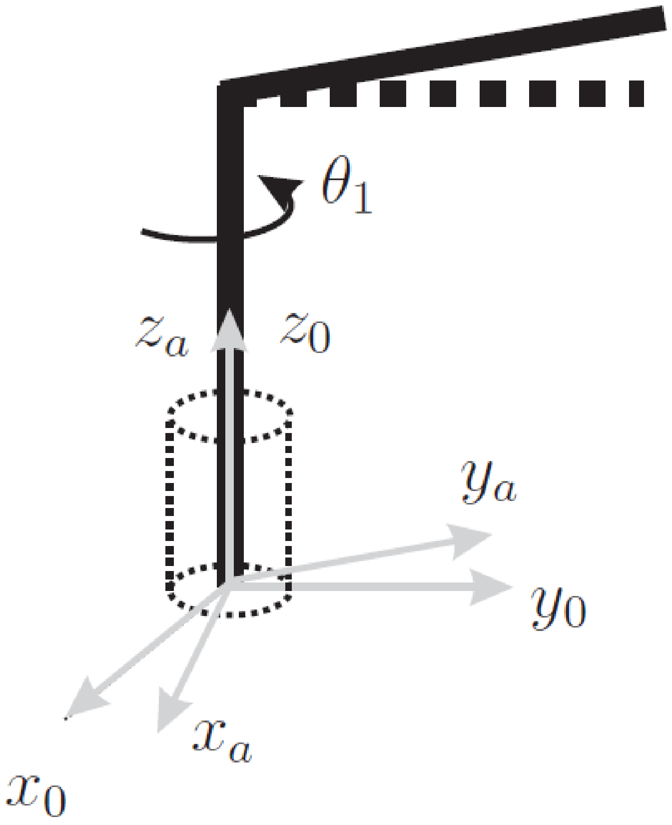
- Introduce a virtual reference frame (frame a) whose centre coincides with the centre of frame 0



Exercise 1: Kinematics and homogeneous transformations

Solution: Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame 1

- Introduce a virtual reference frame (frame a) whose centre coincides with the centre of frame 0
- Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame a

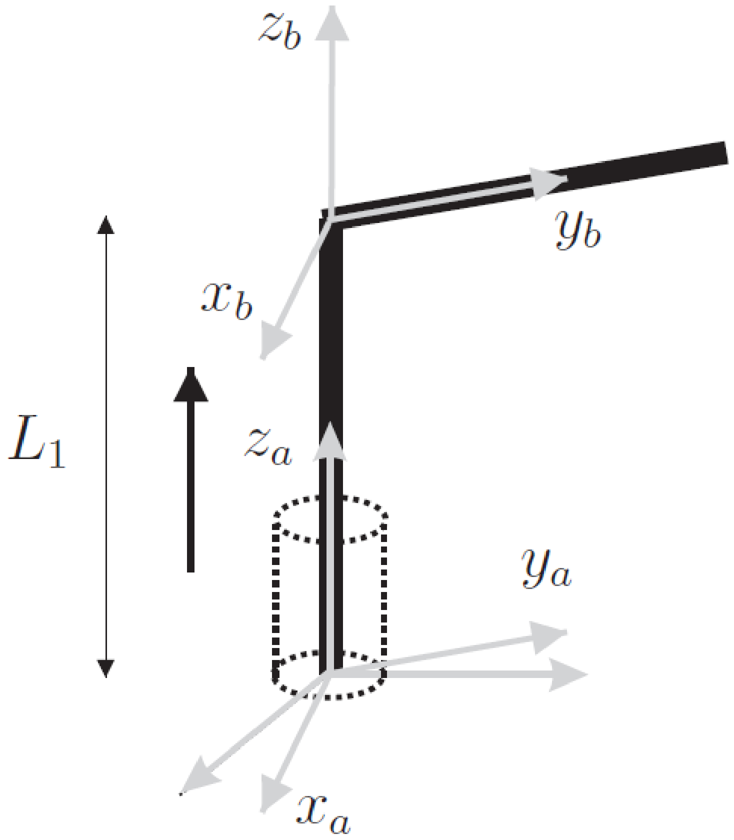


$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_a \\ y_a \\ z_a \\ 1 \end{pmatrix} = \begin{pmatrix} x_a\cos\theta_1 - y_a\sin\theta_1 \\ x_a\sin\theta_1 + y_a\cos\theta_1 \\ z_a \\ 1 \end{pmatrix}.$$

Exercise 1: Kinematics and homogeneous transformations

Solution: Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame 1

- Introduce a virtual reference frame (frame a) whose centre coincides with the centre of frame 0
- Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame a
- Introduce a virtual reference frame (frame b) whose centre coincides with the centre of frame 1



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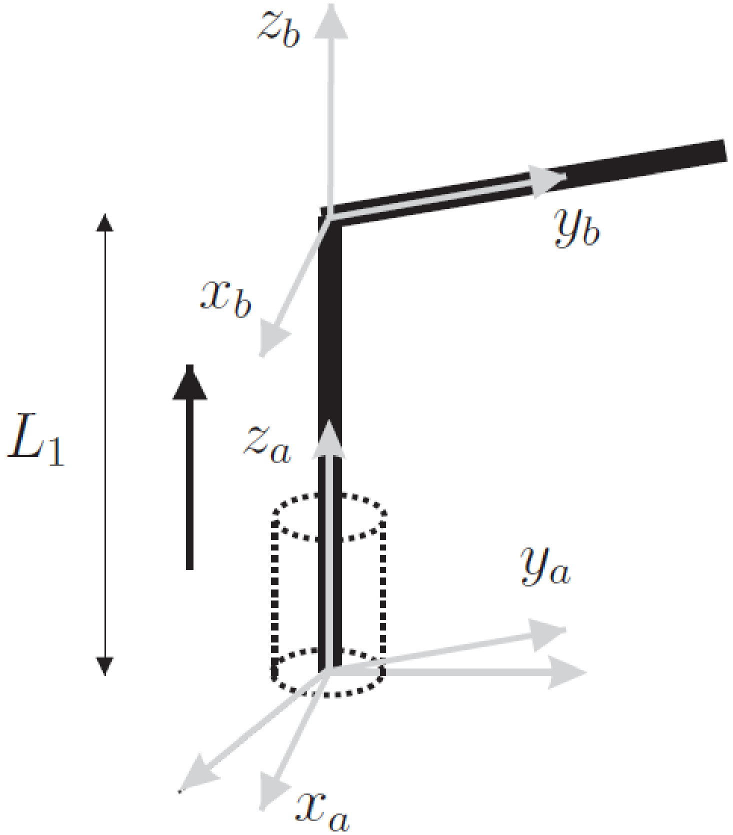
Exercise 1: Kinematics and homogeneous transformations

Solution: Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame 1

- Introduce a virtual reference frame (frame a) whose centre coincides with the centre of frame 0
- Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame a
- Introduce a virtual reference frame (frame b) whose centre coincides with the centre of frame 1
- Compute the homogeneous transformation representing the translation of L_1 length units of frame b along the z_a axis

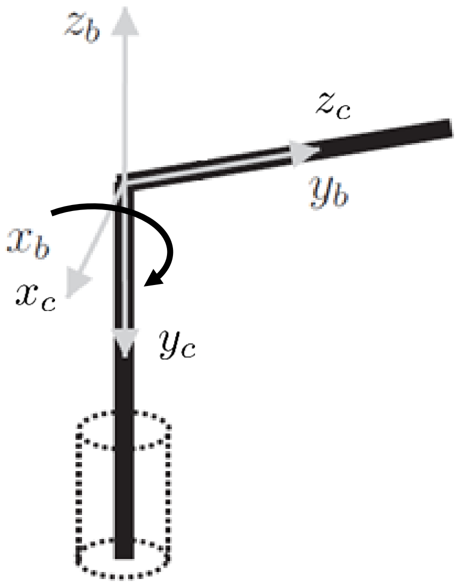


$$\begin{pmatrix} x_a \\ y_a \\ z_a \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \\ 1 \end{pmatrix} = \begin{pmatrix} x_b \\ y_b \\ z_b + L_1 \\ 1 \end{pmatrix}$$

Exercise 1: Kinematics and homogeneous transformations

Solution: Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame 1

- Introduce a virtual reference frame (frame a) whose centre coincides with the centre of frame 0
- Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame a
- Introduce a virtual reference frame (frame b) whose centre coincides with the centre of frame 1
- Compute the homogeneous transformation representing the translation of L_1 length units of frame b along the z_a axis
- Introduce a virtual reference frame (frame c) whose centre coincides with the centre of frame b
    - Rotate frame c of 90 degrees around the x_b axis (note the direction of rotation).



$$\begin{pmatrix} x_b \\ y_b \\ z_b \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = \begin{pmatrix} x_c \\ z_c \\ -y_c \\ 1 \end{pmatrix}$$

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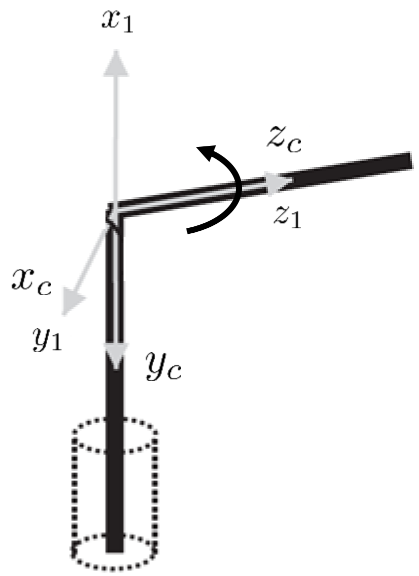
Exercise 1: Kinematics and homogeneous transformations

Solution: Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame 1

- Introduce a virtual reference frame (frame a) whose centre coincides with the centre of frame 0
- Compute the homogeneous transformation relating frame 0 (the inertial frame) to frame a
- Introduce a virtual reference frame (frame b) whose centre coincides with the centre of frame 1
- Compute the homogeneous transformation representing the translation of L_1 length units of frame b along the z_a axis
- Introduce a virtual reference frame (frame c) whose centre coincides with the centre of frame b
    - Rotate frame c of 90 degrees around the x_b axis (note the direction of rotation).
- Rotate frame 1 0f 90 degrees around the z_c axis (note the direction of rotation).



$$\begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} = \begin{pmatrix} y_1 \\ -x_1 \\ z_1 \\ 1 \end{pmatrix}$$

- Put all together to complete the transformation relating frame 1 to frame 0

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Exercise 1: Kinematics and homogeneous transformations

Question 2: Compute the homogeneous transformation relating frame 1 to frame 2

Exercise 1: Kinematics and homogeneous transformations

Solution: Compute the homogeneous transformation relating frame 1 to frame 2

- Introduce a virtual reference frame (frame d) whose centre coincides with the centre of frame 1
- Compute the homogeneous transformation relating frame d to frame 1

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_d \\ y_d \\ z_d \\ 1 \end{pmatrix} = \begin{pmatrix} x_d\cos\theta_2 + z_d\sin\theta_2 \\ y_d \\ -x_d\sin\theta_2 + z_d\cos\theta_2 \\ 1 \end{pmatrix}$$

- Introduce a virtual reference frame (frame e) whose centre coincides with the centre of frame 2
- Compute the homogeneous transformation representing the translation of L_2 length units of frame e along the y_e axis

$$\begin{pmatrix} x_d \\ y_d \\ z_d \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_e \\ y_e \\ z_e \\ 1 \end{pmatrix} = \begin{pmatrix} x_e \\ y_e + L_2 \\ z_e \\ 1 \end{pmatrix}$$

- Find the combination of rotations which relate frame e to frame 2
- Put all together to complete the transformation relating frame 2 to frame 1
- Put all together to complete the transformation relating frame 2 to frame 0

Exercise 2: Jacobian, singular postures and robustness

Consider the direct kinematic model below

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_3 c_1 - y_3 s_1 c_2 - x_2 c_1 \\ x_3 s_1 + y_3 c_1 c_2 - x_2 s_1 + y_1 \\ y_3 s_2 - z_2 \end{bmatrix}$$

The model determines the position of the end point of a kinematic structure composed of three joints q_1, q_2 and q_3.

Exercise 2: Jacobian, singular postures and robustness

Question 1: Compute the Jacobian matrix of the given kinematic model.

Hint: you need to compute a matrix where

- The number of rows must be equal to the number of variables representing the position (x, y, z) and the orientation (alpha, beta, gamma) of an end point. Note that your model does not specify the orientation of the end point. This means that your Jacobian will have only three rows.
- The number of columns must be equal to the number of joint variables. Note that from the kinematic model above you have three joints q_1, q_2 and q_3. This means that your Jacobian will have three columns.
- The elements of the columns of the Jacobian are the derivatives with respect to the joint variables associated to the columns, For example, the first element of the first row of the Jacobian is the derivative of the expression of x in the above model with respect to q_1, the first element of the second row of the Jacobian is the derivative of the expression of y in the above model with respect to q_1.
- Make use of the rules of the derivatives for trigonometric functions that you can find below to compute the elements of your Jacobian

$$\frac{d}{dx}[sin(f(x))] = cos(f(x)) \cdot f'(x)$$

$$\frac{d}{dx}[cos(f(x))] = -sin(f(x)) \cdot f'(x)$$

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Exercise 2: Jacobian, singular postures and robustness

Solution: Compute the Jacobian matrix of the given kinematic model.

$$J(q) = \begin{bmatrix} -x_3 s_1 - y_3 c_1 c_2 + x_2 s_1 & y_3 s_1 s_2 & 0 \\ x_3 c_1 - y_3 s_1 c_2 - x_2 c_1 & -y_3 c_1 s_2 & 0 \\ 0 & y_3 c_2 & 0 \end{bmatrix}$$

Exercise 2: Jacobian, singular postures and robustness

Question 2: Identify a configuration of the joint angles q_1, q_2 and q_3 such that the kinematic structure is in a singular configuration. Explain also why.

Exercise 2: Jacobian, singular postures and robustness

Solution: Identify a configuration of the joint angles q_1, q_2 and q_3 such that the kinematic structure is in a singular configuration. Explain also why.

The rank of the Jacobian matrix is equal to <u>two</u> even though the Jacobian has three rows and three columns. This is clear by observing the last column of the matrix. A configuration of the joint angles for which the kinematic structure is in a singular configuration is the one where <u>the rank of the Jacobian is less than two</u>. To determine the singularities the concept of number of linearly independent rows/columns of a matrix must be used. Based on this concept, the configuration **q_1 = 0 and q_2 = π/2** results in a singular posture. Under this configuration, the Jacobian contains only <u>one</u> linearly independent row, therefore its <u>rank becomes equal to one</u>.

Exercise 3

Demonstrate that the dynamics of the linear momentum depends by the contribution of the external forces

Hint: For this demonstration you need to use the Newton's Second Law and use it to define the dynamics of a point. This formula relates the acceleration (second derivative of the position vector) and the forces acting on the point. Please remind that the forces include the contribution of the internal and external ones. you need to make few considerations about the internal to derive the demonstration.

Exercise 3

Solution:

Newton's Second Law defining the dynamics of the i-th point.

$$m_i \ddot{\mathbf{p}}_i = \sum_{j=1}^{N} \boldsymbol{f}_{ij}^{int} + \boldsymbol{f}_i^{ext}$$

Due to the law of action and reaction

$$\boldsymbol{f}_{ij}^{int} = -\boldsymbol{f}_{ji}^{int} \quad (i \neq j). \qquad \text{and} \qquad \boldsymbol{f}_{ii}^{int} = 0$$

since the force applied by the i-th object from itself is zero

Finally, by substituting the above equations and summing over all the points

$$\sum_{i=1}^{N} m_i \ddot{\mathbf{p}}_i = \sum_{i=1}^{N} \boldsymbol{f}_i^{ext}$$

The left part of the above equation is exactly the momentum

$$\dot{\mathcal{P}} = \sum_{i=1}^{N} m_i \ddot{\mathbf{p}}_i$$

While the right part of the equation is the resultant of the contribution of the external forces only. The resultant is exactly $\boldsymbol{f}_{all}$.

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Exercise 4

Explain the main difference between the definition of the Zero-Moment-Point in the bi-dimensional and three-dimensional case (no more than 150 words).

Exercise 4

Answer:

While in the bi-dimensional case the Zero-Moment-Point is the point on the surface of the foot where the total inertial forces pass and the resultant R (sum of inertia and gravitational forces) is equal to zero, in the three-dimensional case the Zero-Moment-Point is defined as the point where the horizontal component of the moment of the ground reaction forces becomes zero.

Exercise 5

Briefly explain the main purpose of the `urdf` model in ROS (no more than 100 words).

Exercise 5

Answer:

The `urdf` model specifies the visual model of a robot that you can view in Rviz. It defines the static and dynamic joints of your robot including physical properties of the links. It defines the tree of transformations (homogeneous transformations) between the links of your robot.

Exercise 6

Derive the differential equation for the horizontal dynamics of the Centre of Mass (CoM) of a humanoid robot, using the model of the two-dimensional inverted pendulum

Exercise 6

Answer:

Now that

$$M\ddot{x} = f\sin\theta.$$ the horizontal component accelerates the CoM horizontally

and that the CoM moves horizontally under the kick force $$f = \frac{Mg}{\cos\theta}.$$

We have

$$M\ddot{x} = \frac{Mg}{\cos\theta}\sin\theta = Mg\tan\theta = Mg\frac{x}{z}$$

$$\ddot{x} = \frac{g}{z}x.$$