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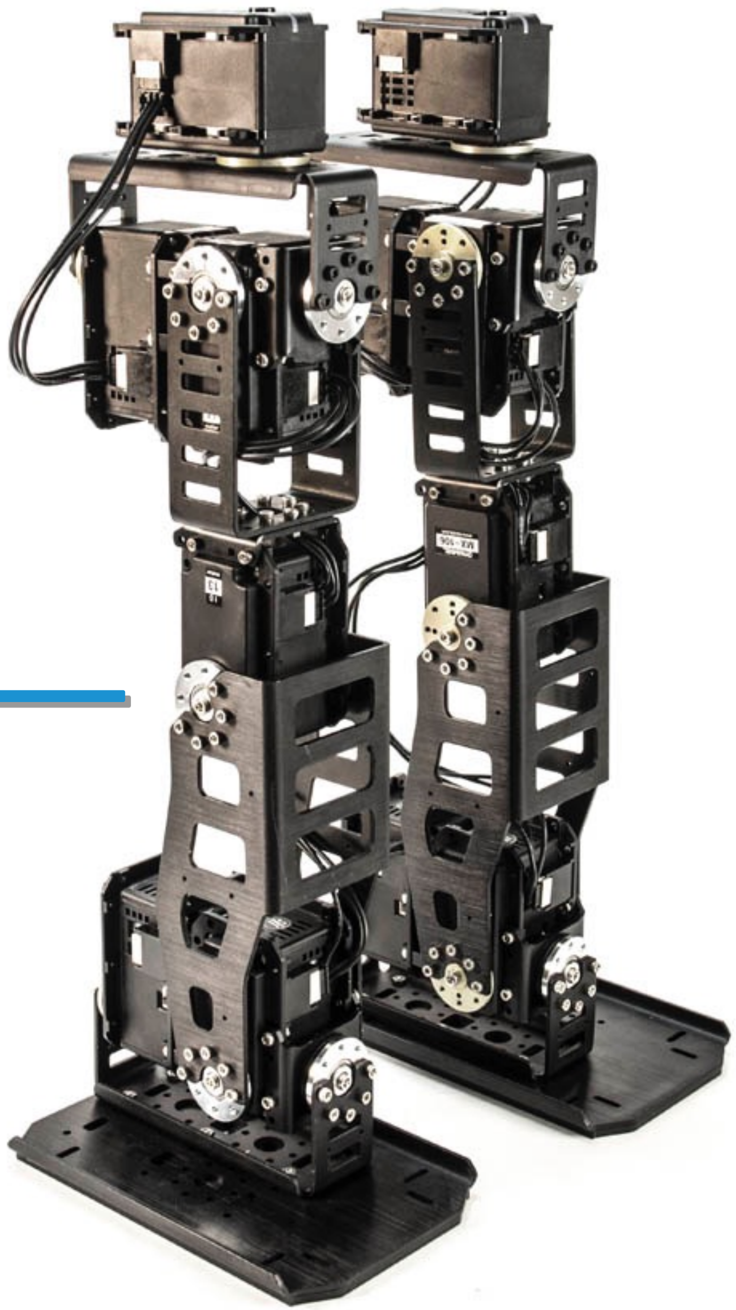
ROCO318 Mobile and Humanoid Robots



Humanoid Dynamics and Motion

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Lecture Content

- Basic physical parameters
- Dynamics of translational motion
- Dynamics of rotational motion

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Humanoid Motion and Ground Reaction Forces

- Physical parameters of the motion

total mass of the robot [kg] M

centre of mass of the robot [m] $\mathbf{c} = \begin{bmatrix} c_x & c_y & c_z \end{bmatrix}^\top$

Linear (translational) momentum [Ns] $\mathcal{P} = \begin{bmatrix} \mathcal{P}_x & \mathcal{P}_y & \mathcal{P}_z \end{bmatrix}^\top$

Angular momentum about the origin [Nms] $\mathcal{L} = \begin{bmatrix} \mathcal{L}_x & \mathcal{L}_y & \mathcal{L}_z \end{bmatrix}^\top$

Humanoid Motion and Ground Reaction Forces

- Equations of the dynamics

$$\dot{\mathbf{c}} = \frac{\mathcal{P}}{M}$$ relationship between the velocity of the mass centre and the linear momentum

$$\dot{\mathcal{P}} = \mathbf{f}_{all}$$ how the linear momentum changes according to the external forces

What is $\mathbf{f}_{all}$

- sum of the forces applied to the robot from outside it
  - Gravitational force $M\mathbf{g}$ with $\mathbf{g} = \begin{bmatrix} 0 & 0 & -9.8 \end{bmatrix}^\top$

then

$$\mathbf{f}_{all} = M\mathbf{g} + \mathbf{f}$$ and $$\dot{\mathcal{P}} = M\mathbf{g} + \mathbf{f}$$

ground reaction forces

1. *What is the value of the linear momentum when the robot stands still?*
   a) *Why?*
2. *What's happen if the reaction forces would disappear?*

Humanoid Motion and Ground Reaction Forces

- Equations of the dynamics

$\dot{\mathcal{L}} = \boldsymbol{\tau}_{all}$ how the angular momentum changes according to the moments generated by the forces applied to the robot from outside it

What is $\boldsymbol{\tau}_{all}$

- sum of the moments generated by the forces applied to the robot from outside it
  - moment generated by the gravitational force

$$\boldsymbol{\tau}_g = \mathbf{c} \times M\mathbf{g}$$

then

$$\boldsymbol{\tau}_{all} = \mathbf{c} \times M\mathbf{g} + \boldsymbol{\tau} \quad \text{and} \quad \dot{\mathcal{L}} = \mathbf{c} \times M\mathbf{g} + \boldsymbol{\tau}$$

ground reaction moment

What's happen if the ground reaction moment disappears when the robot stand still?

Centre of Mass

- Humanoid robot composed of N points of mass.

$$M = \sum_{i=1}^{N} m_i$$

total mass of the robot [kg]

Mass of the i-th point

$$\mathbf{c} = \frac{\sum_{i=1}^{N} m_i \mathbf{p}_i}{M}$$

centre of mass of the robot [m]

position of the i-th point

Differentiating

$$\dot{\mathbf{c}} = \frac{\sum_{i=1}^{N} m_i \dot{\mathbf{p}}_i}{M}$$

$$\mathcal{P} = \sum_{i=1}^{N} m_i \dot{\mathbf{p}}_i$$

Linear (translational) momentum [Ns]

$$\dot{\mathbf{c}} = \frac{\mathcal{P}}{M}$$

momentum of the i-th point

Linear Momentum

- Motion of the i-th point

$$m_i \ddot{\mathbf{p}}_i = \sum_{j=1}^{N} \boldsymbol{f}_{ij}^{int} + \boldsymbol{f}_i^{ext}$$

the force applied to the i-th point mass from the outside the robot

the force applied to the i-th point mass from the j-th one and

acceleration of the i-th point

Due to the law of action and reaction

$$\boldsymbol{f}_{ij}^{int} = -\boldsymbol{f}_{ji}^{int} \quad (i \neq j).$$

and

$$\boldsymbol{f}_{ii}^{int} = 0$$ since the force applied by the i-th object from itself is zero

summing for all point masses of the robot it turns out that

$$\dot{\mathcal{P}} = \sum_{i=1}^{N} m_i \ddot{\mathbf{p}}_i \longleftarrow \sum_{i=1}^{N} m_i \ddot{\mathbf{p}}_i = \sum_{i=1}^{N} \boldsymbol{f}_i^{ext} \longrightarrow \mathbf{f}_{all}$$

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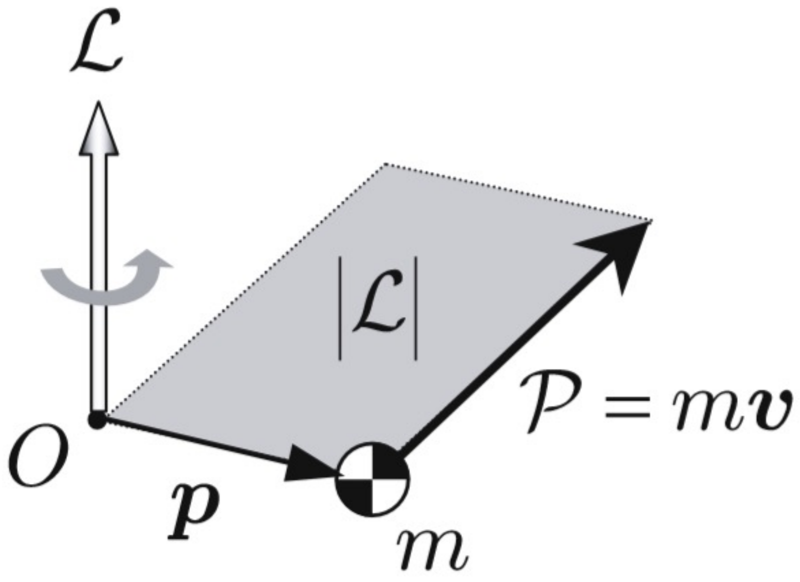
Angular Momentum

- Angular momentum of the i-th point of mass about the origin

$$\mathcal{L}_i = \boldsymbol{p}_i \times \mathcal{P}_i$$

- Angular momentum of the i-th point of mass about a reference point (not the origin)

$$\mathcal{L}_i^{(r)} = (\boldsymbol{p}_i - \boldsymbol{r}) \times \mathcal{P}_i$$



Position of the reference point with respect to the origin

$$= \sum_{i=1}^{N} \boldsymbol{p}_i \times \mathcal{P}_i - \boldsymbol{r} \times \sum_{i=1}^{N} \mathcal{P}_i \longrightarrow \mathcal{L}^{(r)} = \mathcal{L} - \boldsymbol{r} \times \mathcal{P}$$

$$\mathcal{L} = \boldsymbol{p} \times \mathcal{P}$$

Total angular momentum

$$\mathcal{P} = \sum_{i=1}^{N} m_i \dot{\mathbf{p}}_i$$

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Angular Momentum

$$\mathcal{L}_i = \boldsymbol{p}_i \times \mathcal{P}_i$$

Differentiating wrt time

$$\begin{aligned}\dot{\mathcal{L}}_i &= \dot{\boldsymbol{p}}_i \times \mathcal{P}_i + \boldsymbol{p}_i \times \dot{\mathcal{P}}_i \\ &= \dot{\boldsymbol{p}}_i \times (m_i \dot{\boldsymbol{p}}_i) + \boldsymbol{p}_i \times m_i \ddot{\boldsymbol{p}}_i \\ &= \dot{\mathcal{L}}_i = \boldsymbol{p}_i \times m_i \ddot{\boldsymbol{p}}_i\end{aligned}$$

$$m_i \ddot{\boldsymbol{p}}_i = \sum_{j=1}^{N} \boldsymbol{f}_{ij}^{int} + \boldsymbol{f}_i^{ext}$$

then

$$\begin{aligned}\dot{\mathcal{L}}_i &= \boldsymbol{p}_i \times \left(\sum_{j=1}^{N} \boldsymbol{f}_{ij}^{int} + \boldsymbol{f}_i^{ext}\right) \\ &= \sum_{j=1}^{N} \boldsymbol{p}_i \times \boldsymbol{f}_{ij}^{int} + \boldsymbol{p}_i \times \boldsymbol{f}_i^{ext}\end{aligned}$$

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Angular Momentum

- the total angular momentum is the sum of that of point masses

$$\dot{\mathcal{L}} = \sum_{i=1}^{N} \sum_{j=1}^{N} \underbrace{\boldsymbol{p}_i \times \boldsymbol{f}_{ij}^{int}} + \sum_{i=1}^{N} \boldsymbol{p}_i \times \boldsymbol{f}_i^{ext}$$

$$\boldsymbol{p}_i \times \boldsymbol{f}_{ij}^{int} + \boldsymbol{p}_j \times \boldsymbol{f}_{ji}^{int} = (\boldsymbol{p}_i - \boldsymbol{p}_j) \times \boldsymbol{f}_{ij}^{int} = \boldsymbol{r}_{ij} \times \boldsymbol{f}_{ij}^{int}$$

vector from the j-th point mass to the i-th one

$$\boldsymbol{r}_{ij} \times \boldsymbol{f}_{ij}^{int} = 0$$

As action and the reaction forces between two point masses are on the line connecting them

therefore

$$\dot{\mathcal{L}} = \sum_{i=1}^{N} \boldsymbol{p}_i \times \boldsymbol{f}_i^{ext} \longrightarrow \dot{\mathcal{L}} = \boldsymbol{\tau}_{all}$$

Angular momentum about the origin

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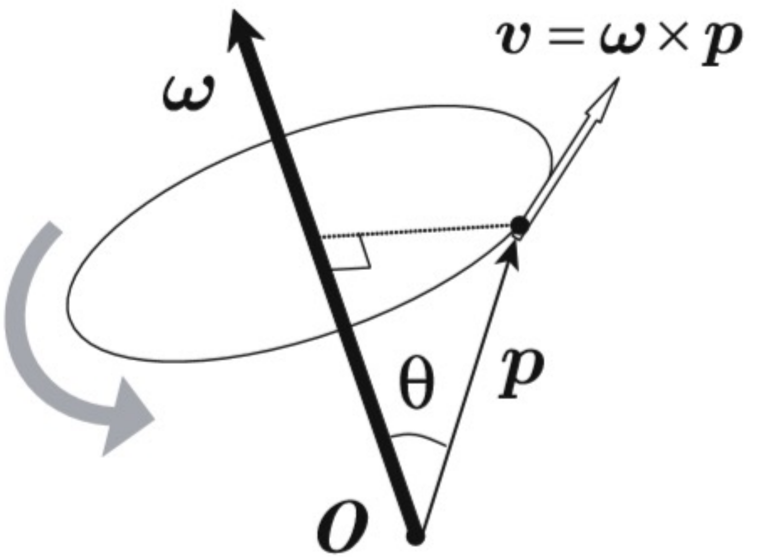
Angular Momentum and Inertia

- Assume that the origin of the reference coordinate system coincides with its centre of mass
  - Then the velocity at a point can be expressed wrt the angular velocity vector $\boldsymbol{\omega}$

$$\dot{\mathbf{p}}_i = \boldsymbol{\omega} \times \mathbf{p}_i$$

  - and the linear momentum of the i-th point of mass

$$\mathcal{P}_i = m_i \dot{\mathbf{p}}_i = m_i \boldsymbol{\omega} \times \mathbf{p}_i$$



- Based on the above, the total linear momentum (the sum of that of point masses) can be re-written as follows

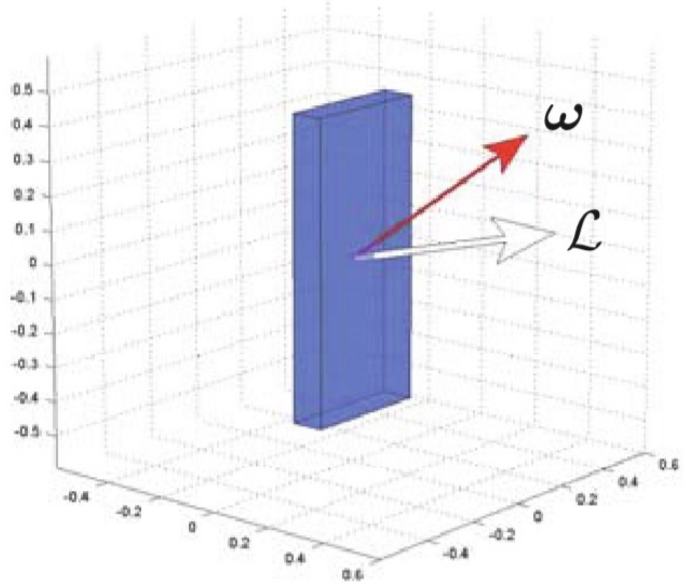
$$\mathcal{L} = \sum_{i=1}^{N} \mathbf{p}_i \times \mathcal{P}_i = \sum_{i=1}^{N} \mathbf{p}_i \times (m_i \boldsymbol{\omega} \times \mathbf{p}_i)$$

$$= \sum_{i=1}^{N} m_i \mathbf{p}_i \times (-\mathbf{p}_i \times \boldsymbol{\omega})$$

$$= \left(\sum_{i=1}^{N} m_i \hat{\mathbf{p}}_i \hat{\mathbf{p}}_i^\top \right) \omega = \mathbf{I} \omega$$

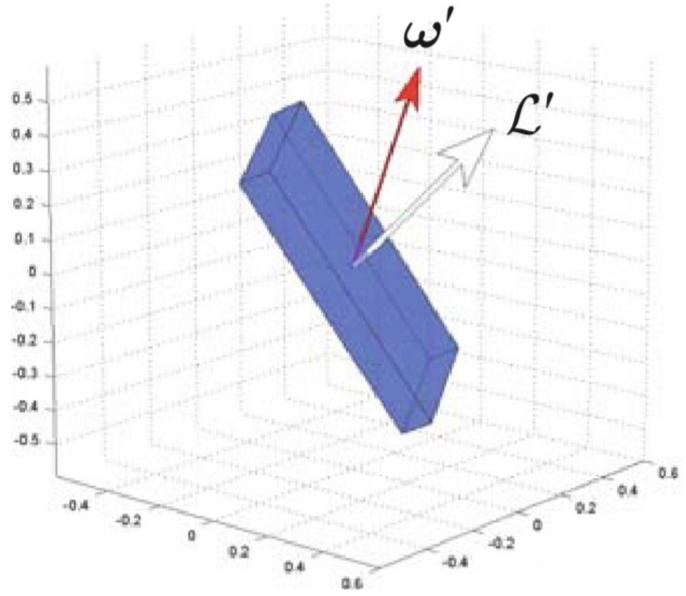
$\mathbf{I} = \sum_{i=1}^{N} m_i \hat{\mathbf{p}}_i \hat{\mathbf{p}}_i^\top$ 3 x 3 symmetric matrix named **Inertial Tensor**

Angular Momentum and Inertia: General Case



Reference Posture

$$\mathcal{L} = \mathbf{I}\omega$$



Rotation of Rigid Body
(position does not change)

The body is rotated my multiplying for a rotation matrix

$$\mathbf{R}$$

$$\omega' = \mathbf{R}\omega \longrightarrow \mathbf{R}^{-1}\omega' = \mathbf{R}^{-1}\mathbf{R}\omega$$

$$\mathbf{R}^{-1} = \mathbf{R}^{\top}$$ Only for rotation matrices

$$\omega = \mathbf{R}^{\top}\omega'$$

$$\mathcal{L}' = \mathbf{R}\mathcal{L} = \mathbf{R}\mathbf{I}\omega$$

inertia tensor of a rigid body in any attitude $\longrightarrow$ $$= \left(\mathbf{R}\mathbf{I}\mathbf{R}^{\top}\right)\omega'$$

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ZMP and Robot Motion

- Dynamics of the angular momentum according to the moments generated by the forces applied to the robot from outside it

$$\dot{\mathcal{L}} = \mathbf{c} \times M\mathbf{g} + \boldsymbol{\tau}$$

ground reaction moment

- Dynamics of the linear momentum according to the moments generated by the forces applied to the robot from outside it

$$\dot{\mathcal{P}} = M\mathbf{g} + \mathbf{f}$$

ground reaction forces

- Moment of the ZMP about the vertical line including the ZMP

$$\boldsymbol{\tau}_p = \boldsymbol{\tau}_n(\boldsymbol{p}) + \boldsymbol{\tau}_t(\boldsymbol{p})$$

ZMP and Robot Motion

- Ground reaction moment

$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{f} + \boldsymbol{\tau}_p \qquad \begin{cases} \dot{\mathcal{L}} = \mathbf{c} \times M\mathbf{g} + \boldsymbol{\tau} \\ \dot{\mathcal{P}} = M\mathbf{g} + \mathbf{f} \\ \boldsymbol{\tau}_p = \boldsymbol{\tau}_n(\boldsymbol{p}) + \boldsymbol{\tau}_t(\boldsymbol{p}) \end{cases}$$

- then

$$\boldsymbol{\tau}_p = \dot{\mathcal{L}} - \boldsymbol{c} \times M\boldsymbol{g} + (\dot{\mathcal{P}} - M\boldsymbol{g}) \times \boldsymbol{p}$$

Height of the ground. For walking on the flat ground is equal to zero

Then you need to

- set the x and y components of the moment to zero
- solve for p_x and p_y

$$\tau_{px} = \dot{\mathcal{L}}_x + Mgy + \dot{\mathcal{P}}_y p_z - (\dot{\mathcal{P}}_z + Mg)p_y = 0$$
$$\tau_{py} = \dot{\mathcal{L}}_y - Mgx - \dot{\mathcal{P}}_x p_z + (\dot{\mathcal{P}}_z + Mg)p_x = 0$$

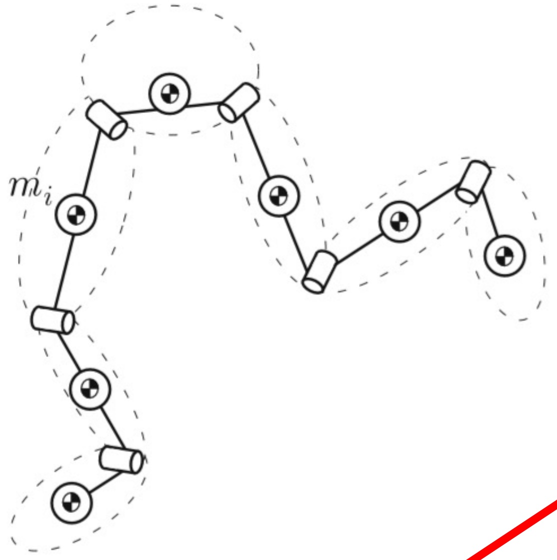
$$\longrightarrow$$

$$p_x = \frac{Mgx + p_z\dot{\mathcal{P}}_x - \dot{\mathcal{L}}_y}{Mg + \dot{\mathcal{P}}_z}$$
$$p_y = \frac{Mgy + p_z\dot{\mathcal{P}}_y + \dot{\mathcal{L}}_x}{Mg + \dot{\mathcal{P}}_z}$$

When the robot stand still the ZMP coincides with the ground projection of the centre of mass. Why?

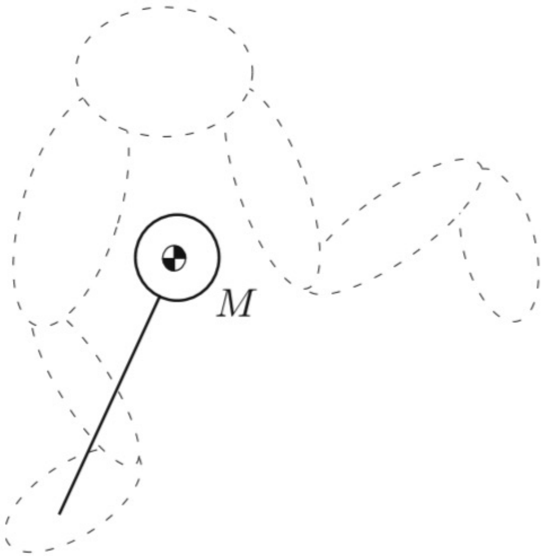
Approximations

$$\mathcal{L} = \sum_{i=1}^{N} \boldsymbol{c}_i \times \mathcal{P}_i$$



$$\mathcal{P} = M\dot{\boldsymbol{c}}$$

$$\mathcal{L} = \boldsymbol{c} \times M\dot{\boldsymbol{c}}$$



$$p_x = \frac{Mgx + p_z\dot{\mathcal{P}}_x - \dot{\mathcal{L}}_y}{Mg + \dot{\mathcal{P}}_z}$$

$$p_y = \frac{Mgy + p_z\dot{\mathcal{P}}_y + \dot{\mathcal{L}}_x}{Mg + \dot{\mathcal{P}}_z}$$

$$p_x = \frac{\sum_{i=1}^{N} m_i\{(\ddot{z}_i + g)x_i - (z_i - p_z)\ddot{x}_i\}}{\sum_{i=1}^{N} m_i(\ddot{z}_i + g)}$$

$$p_y = \frac{\sum_{i=1}^{N} m_i\{(\ddot{z}_i + g)y_i - (z_i - p_z)\ddot{y}_i\}}{\sum_{i=1}^{N} m_i(\ddot{z}_i + g)}$$

$$p_x = x - \frac{(z - p_z)\ddot{x}}{\ddot{z} + g}$$

$$p_y = y - \frac{(z - p_z)\ddot{y}}{\ddot{z} + g}$$