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ROCO318 Mobile and Humanoid Robots

Mock-up Exam

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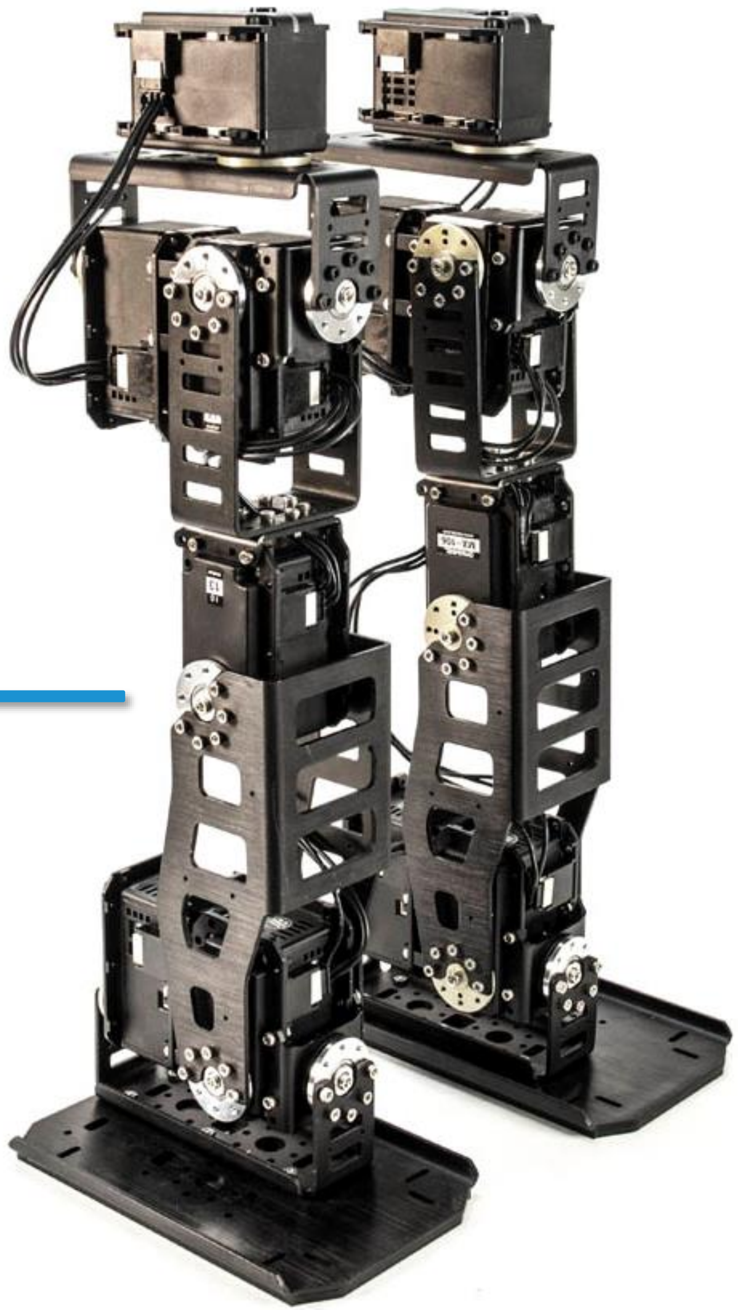

Exercise 2: Jacobian, singular postures and robustness

Consider the direct kinematic model below

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_3 s_1 s_2 + x_2 c_1 + z_2 s_1 - y_3 s_3 \\ x_3 c_2 + x_1 c_3 \\ x_3 c_1 s_2 - x_2 s_1 + z_2 c_1 \end{pmatrix}$$

The model determines the position of the end point of a kinematic structure composed of three joints q_1, q_2 and q_3.

Exercise 2: Jacobian, singular postures and robustness

Question 1: Compute the Jacobian matrix of the given kinematic model.

Hint: you need to compute a matrix where

- The number of rows must be equal to the number of variables representing the position (x, y, z) and the orientation (alpha, beta, gamma) of an end point. Note that your model does not specify the orientation of the end point. This means that your Jacobian will have only three rows.
- The number of columns must be equal to the number of joint variables. Note that from the kinematic model above you have three joints q_1, q_2 and q_3. This means that your Jacobian will have three columns.
- The elements of the columns of the Jacobian are the derivatives with respect to the joint variables associated to the columns, For example, the first element of the first row of the Jacobian is the derivative of the expression of x in the above model with respect to q_1, the first element of the second row of the Jacobian is the derivative of the expression of y in the above model with respect to q_1.
- Make use of the rules of the derivatives for trigonometric functions that you can find below to compute the elements of your Jacobian

$$\frac{d}{dx}[sin(f(x))] = cos(f(x)) \cdot f'(x)$$

$$\frac{d}{dx}[cos(f(x))] = -sin(f(x)) \cdot f'(x)$$

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Exercise 2: Jacobian, singular postures and robustness

Solution: Compute the Jacobian matrix of the given kinematic model.

$$J(\mathbf{q}) = \begin{pmatrix} x_3 c_1 s_2 - x_2 s_1 + z_2 c_1 & x_3 s_1 c_2 & -y_3 c_3 \\ 0 & -x_3 s_2 & -x_1 s_3 \\ -x_3 s_1 s_2 - x_2 c_1 - z_2 s_1 & x_3 c_1 c_2 & 0 \end{pmatrix}$$

Exercise 2: Jacobian, singular postures and robustness

Question 2: Determine for which of the following values of the angles of the joints, the kinematic structure is in a singular configuration. Also state why these configurations are singular or not.

1) $q_1 = 0$, $q_2 = 0$, $q_3 = 0$
2) $q_1 = 0$, $q_2 = 0$, $q_3 = \pi/2$
3) $q_1 = 0$, $q_2 = \pi/2$, $q_3 = 0$
4) $q_1 = 0$, $q_2 = \pi/2$, $q_3 = \pi/2$
5) $q_1 = \pi/2$, $q_2 = 0$, $q_3 = 0$
6) $q_1 = \pi/2$, $q_2 = 0$, $q_3 = \pi/2$
7) $q_1 = \pi/2$, $q_2 = \pi/2$, $q_3 = 0$
8) $q_1 = \pi/2$, $q_2 = \pi/2$, $q_3 = \pi/2$

Hint: you could use either the concept of number of linearly independent rows/columns of a matrix or the determinant to determine the singularities.

Exercise 2: Jacobian, singular postures and robustness

Solution: Determine for which of the following values of the angles of the joints, the kinematic structure is in a singular configuration. Also state why these configurations are singular or not.

1) $q_1 = 0$, $q_2 = 0$, $q_3 = 0$ – Yes. The second row of the Jacobian is zero. As a consequence the rank is 2 and not 3.
2) $q_1 = 0$, $q_2 = 0$, $q_3 = \pi/2$ - No. Three linear independent rows in the Jacobian. The rank is equal to 3
3) $q_1 = 0$, $q_2 = \pi/2$, $q_3 = 0$ - No. Three linear independent rows in the Jacobian. The rank is equal to 3
4) $q_1 = 0$, $q_2 = \pi/2$, $q_3 = \pi/2$ - Yes. The determinant of the Jacobian is equal to zero
5) $q_1 = \pi/2$, $q_2 = 0$, $q_3 = 0$ - Yes. The second row of the Jacobian is zero. As a consequence the rank is 2 and not 3.
6) $q_1 = \pi/2$, $q_2 = 0$, $q_3 = \pi/2$ - No. Three linear independent rows in the Jacobian. The rank is equal to 3
7) $q_1 = \pi/2$, $q_2 = \pi/2$, $q_3 = 0$ - No. Three linear independent rows in the Jacobian. The rank is equal to 3
8) $q_1 = \pi/2$, $q_2 = \pi/2$, $q_3 = \pi/2$ - Yes. The determinant of the Jacobian is equal to zero

Exercise 3

Demonstrate that the dynamics of the angular momentum depends by the contribution of the moment applied from the outside the robot

Hint: For this demonstration you need to differentiate wrt time the equation of the angular momentum of the i-th point of mass about the origin. When differentiating make use of the rule of the product of two functions. Use the equation of the linear moment in the resulting equation. Then use the properties of the cross product to simplify the result. After that, you need to use the equation of the dynamics of the i-th point mass. Calculate the total angular momentum. Use the action reaction law to simplify the result.

Exercise 3

Solution

$\mathcal{L}_i = \boldsymbol{p}_i \times \mathcal{P}_i$ Angular momentum of the i-th point of mass about the origin

Differentiating wrt time

$$\dot{\mathcal{L}}_i = \dot{\boldsymbol{p}}_i \times \mathcal{P}_i + \boldsymbol{p}_i \times \dot{\mathcal{P}}_i$$
$$= \dot{\boldsymbol{p}}_i \times (m_i \dot{\boldsymbol{p}}_i) + \boldsymbol{p}_i \times m_i \ddot{\boldsymbol{p}}_i$$

Knowing that the cross product between parallel vectors is equal to zero

$$\dot{\mathcal{L}}_i = \dot{\boldsymbol{p}}_i \times \mathcal{P}_i + \boldsymbol{p}_i \times \dot{\mathcal{P}}_i$$
$$= \cancel{\dot{\boldsymbol{p}}_i \times (m_i \dot{\boldsymbol{p}}_i)} + \boldsymbol{p}_i \times m_i \ddot{\boldsymbol{p}}_i$$

$$\dot{\mathcal{L}}_i = \boldsymbol{p}_i \times m_i \ddot{\boldsymbol{p}}_i$$

Using the equation of the dynamics of the i-th point mass $m_i \ddot{\boldsymbol{p}}_i = \sum_{j=1}^{N} \boldsymbol{f}_{ij}^{int} + \boldsymbol{f}_i^{ext}$

$$\dot{\mathcal{L}}_i = \boldsymbol{p}_i \times (\sum_{j=1}^{N} \boldsymbol{f}_{ij}^{int} + \boldsymbol{f}_i^{ext})$$

$$= \sum_{j=1}^{N} \boldsymbol{p}_i \times \boldsymbol{f}_{ij}^{int} + \boldsymbol{p}_i \times \boldsymbol{f}_i^{ext}$$

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Exercise 3

Solution

Sum over all the points

$$\dot{\mathcal{L}} = \sum_{i=1}^{N}\sum_{j=1}^{N} \underbrace{\boldsymbol{p}_i \times \boldsymbol{f}_{ij}^{int}} + \sum_{i=1}^{N} \boldsymbol{p}_i \times \boldsymbol{f}_i^{ext}$$

$$\boldsymbol{p}_i \times \boldsymbol{f}_{ij}^{int} + \boldsymbol{p}_j \times \boldsymbol{f}_{ji}^{int} = (\boldsymbol{p}_i - \boldsymbol{p}_j) \times \boldsymbol{f}_{ij}^{int} = \boldsymbol{r}_{ij} \times \boldsymbol{f}_{ij}^{int}$$

vector from the j-th point mass to the i-th one

Since action and the reaction forces between two point masses are on the line connecting them

$$\boldsymbol{r}_{ij} \times \boldsymbol{f}_{ij}^{int} = 0$$

Therefore

$$\dot{\mathcal{L}} = \cancel{\sum_{i=1}^{N}\sum_{j=1}^{N} \boldsymbol{p}_i \times \boldsymbol{f}_{ij}^{int}} + \sum_{i=1}^{N} \boldsymbol{p}_i \times \boldsymbol{f}_i^{ext} \longrightarrow \dot{\mathcal{L}} = \sum_{i=1}^{N} \boldsymbol{p}_i \times \boldsymbol{f}_i^{ext}$$

$$\dot{\mathcal{L}} = \boldsymbol{\tau}_{all}$$

Exercise 4 (Part 1)

Explain the main reason why sometimes it is not possible to compute the inverse of the Jacobian (no more than 50 words)

Exercise 4 (Part 1)

Answer:

Because the determinant of the Jacobian can be zero meaning that the robot is in a singular posture.

Exercise 4 (Part 2)

Explain how you can solve the problem of computing the inverse of the Jacobian when the robot is in a singular posture (no more than 50 words)

Exercise 4 (Part 2)

Answer:

You can either use the pseudo-inverse or the solution provided by the Levenberg-Marquart's method. The latter is also called regularized pseudo-inverse.

Exercise 5

Briefly explain the main steps to create a catkin workspace in ROS and a rospackage

Exercise 5

Answer:

- `mkdir -p ~/catkin_ws/src`
  - This creates a folder called `catkin_ws`, inside of which is another folder called `src`
- Switch to the `src` folder by using the command `cd catkin_ws/src`
- Initialise the workspace using the command `catkin_init_workspace`
- In the `src` folder use the command `catkin_create_pkg ros_package_name package_dependencies` to create a rospackage.
  - For example `cd ~/catkin_ws/src catkin_create_pkg hello_world roscpp rospy std_msgs`

Exercise 6

Derive the differential equation for the horizontal dynamics of the Centre of Mass (CoM) of a humanoid robot, using the model of the three-dimensional inverted pendulum

Exercise 6

Answer:

Now that $M\ddot{x} = (x/r)f$ and $M\ddot{y} = (y/r)f$

are the horizontal motion equations of CoM

and that the CoM moves under the force $f = \dfrac{Mgr}{z_c}.$

which is derived taking into account the constraint plane for CoM $z = k_x x + k_y y + z_c,$

We have

$$\ddot{x} = \frac{g}{z_c}x,$$
$$\ddot{y} = \frac{g}{z_c}y.$$