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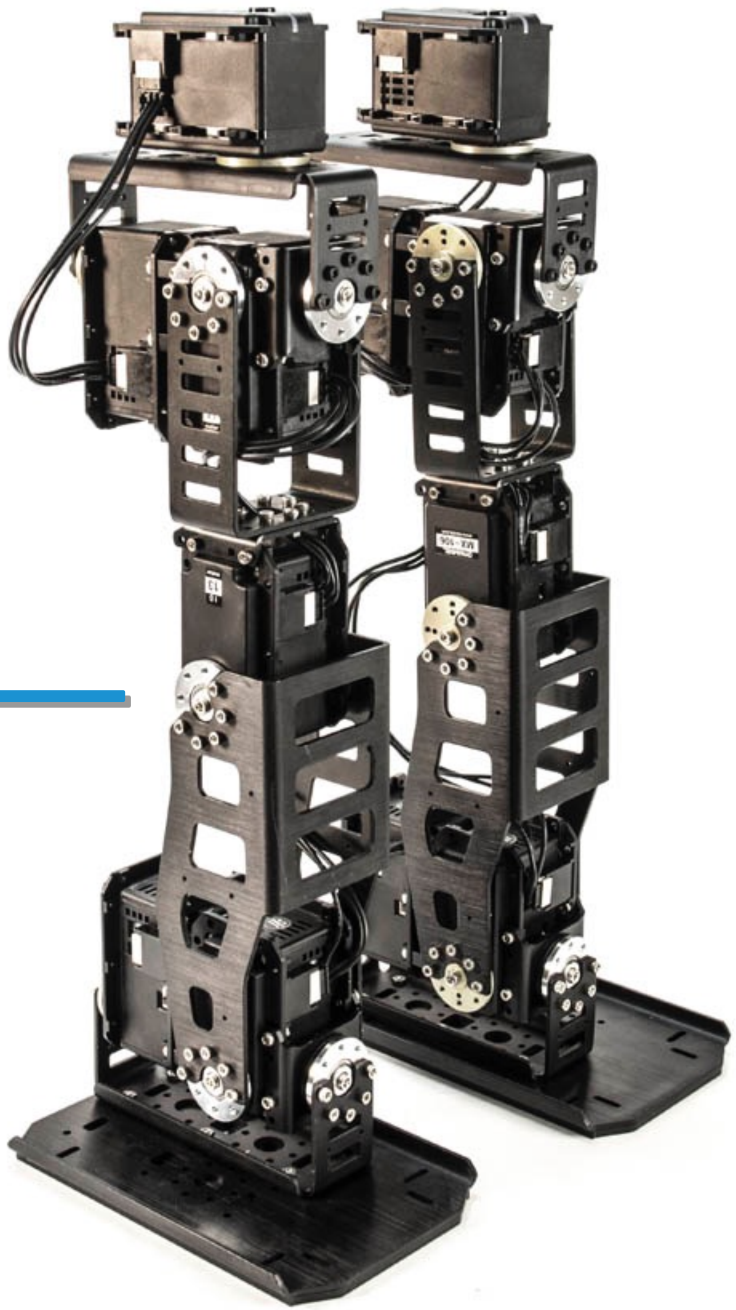
ROCO318 Mobile and Humanoid Robots



ZMP and Humanoid Dynamics

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Lecture Content

- The Support Polygon (SP)
- The Zero-Moment Point (ZMP)
- Relation between COM, ZMP and SP
  - Region of the ZMP
- The humanoid robot foot model
- 6 axis force/torque sensors in ROS
- Measurement of ZMP
  - Single Foot (single and multiple contact points)
  - Right and Left Foot

Main Questions

- How can we say if a motion of the humanoid robot ensure that the contact can be maintained between the sole and the ground?

- How can we plan a motion of the humanoid robot maintaining contact between the sole and the ground?

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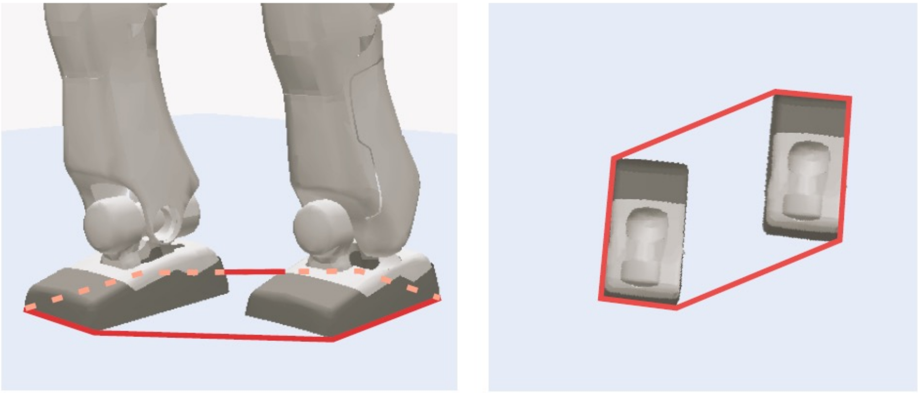
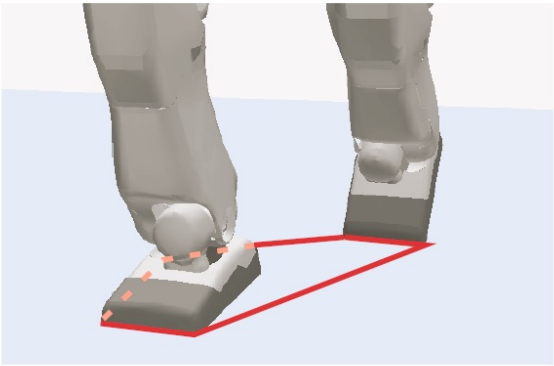
Support Polygon

Region formed by enclosing all the contact points between the robot and the ground by using an elastic cord braid



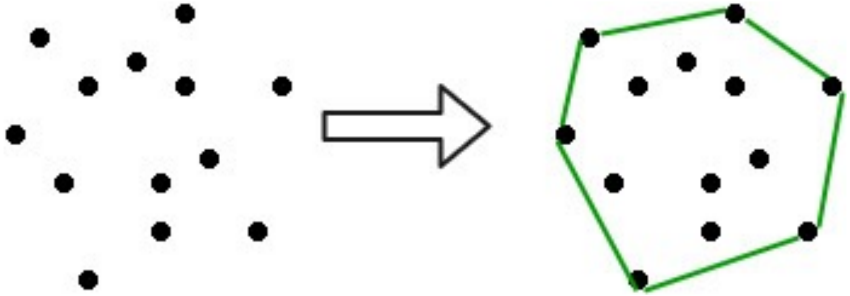
(a) Full contact of both feet

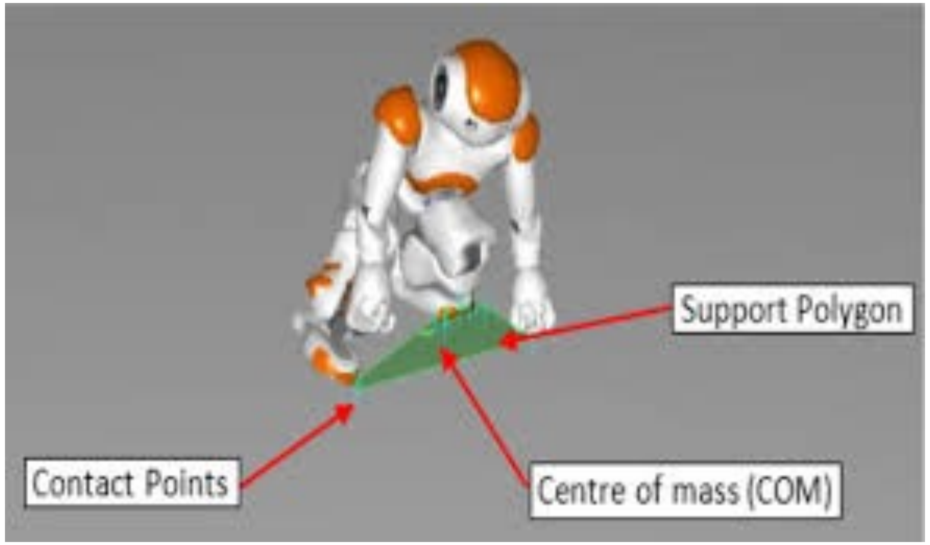


(b) Partial contact

Mathematically:

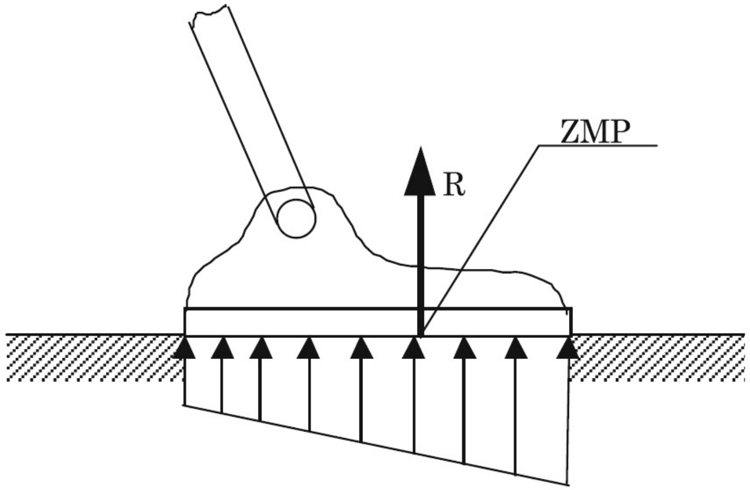
- convex hull,
  - The smallest convex set including all contact points.

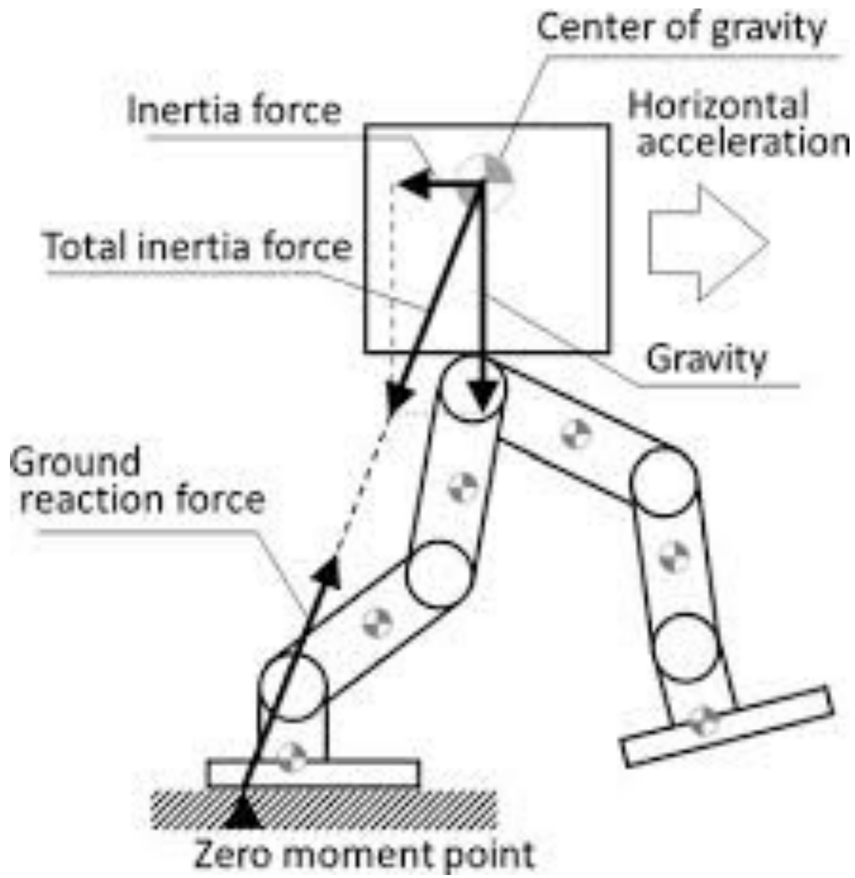




Zero-Moment Point

- Specify the point with respect to which dynamic reaction force at the contact of the foot with the ground does not produce any moment
    - The point on the surface of the foot where the total inertial forces pass and the resultant R (sum of inertia and gravitational forces) is equal to zero
        - Only for the 2D case





- The inertial forces distribution across the foot have the same sign over the surface
    - it can be reduced to the resultant force R
- Zero-Moment Point, or ZMP in short
    - the point on the surface of the foot where the resultant R passed

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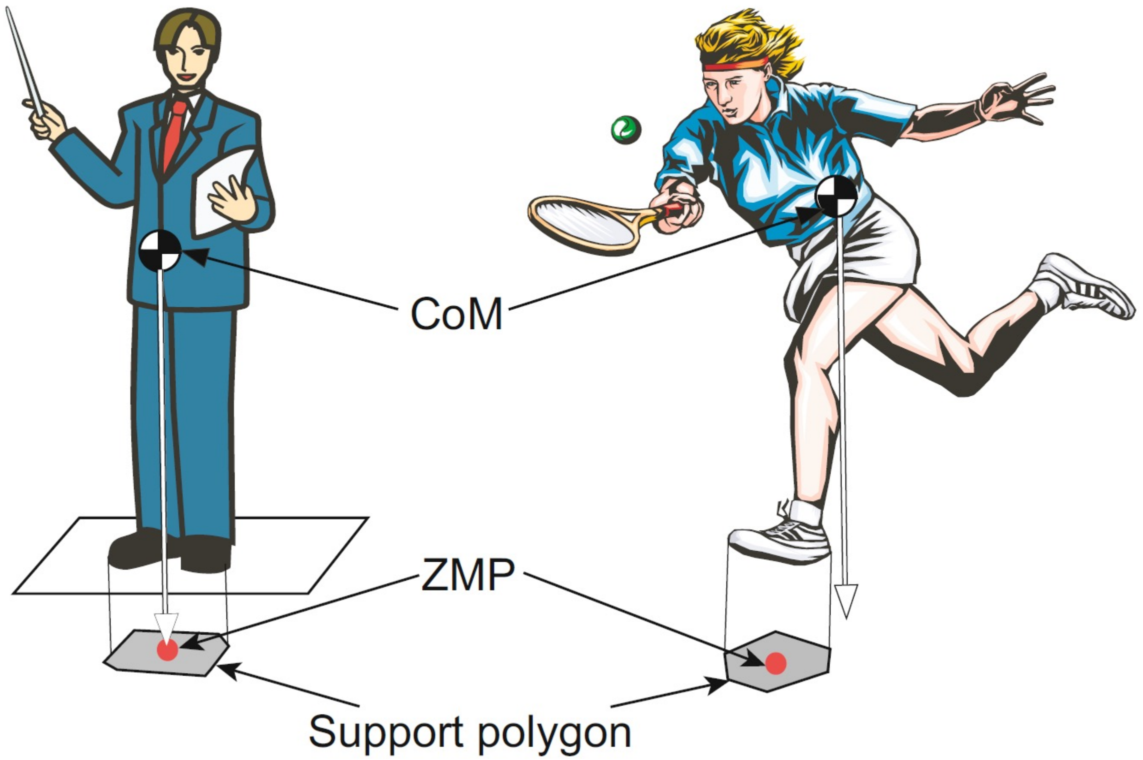
CoM, ZMP and Support Polygon

- Ground projection of CoM
  - The point where the gravity line from the CoM intersects the ground
- Static pose:
  - The ZMP coincides with the ground projection of CoM
  - The human can keep balance if the ground projection of CoM is inside of the support polygon
- Dynamic pose:
  - The ground projection of CoM may fall outside the support polygon



CoM, ZMP and Support Polygon

the ZMP always exists inside the support polygon

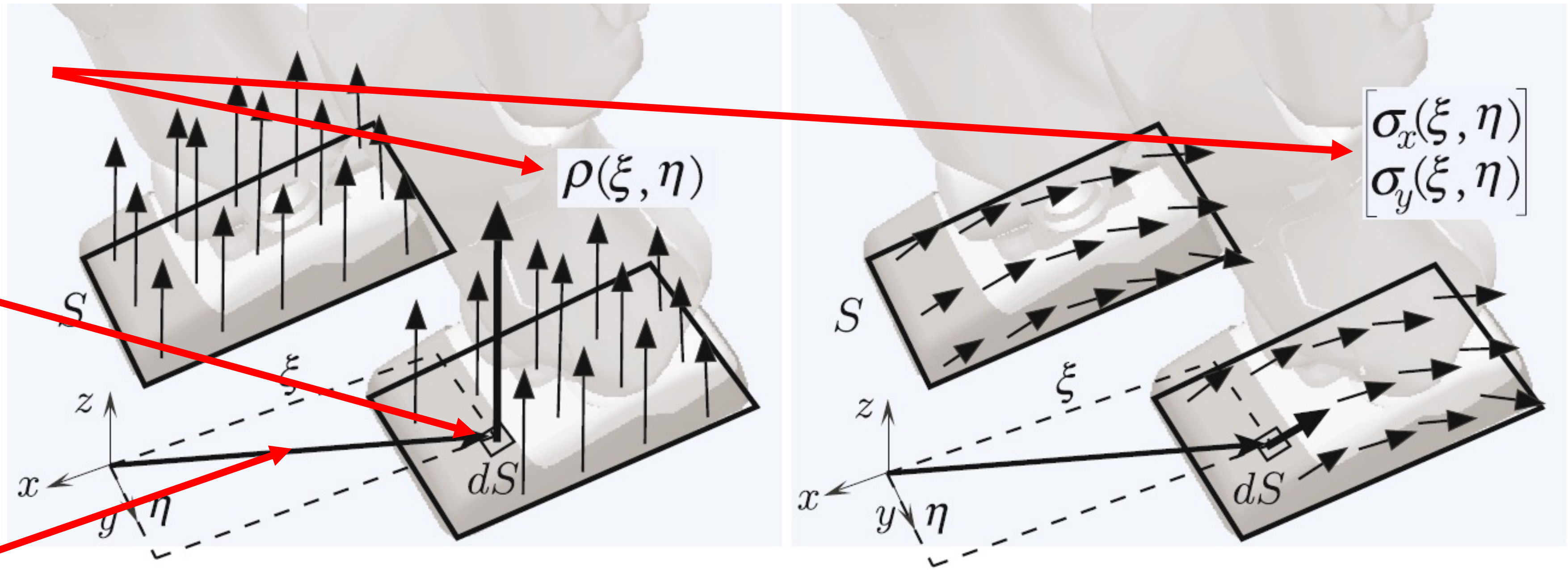
- Consider the ground reaction force applied to the robot moving in 3D space from the flat ground

Forces per unit area

Unit area

$\boldsymbol{r} = [\xi \ \eta \ 0]^T$

position vector defined on the ground



(a) Vertical reaction forces

(b) Horizontal reaction forces

$$f_z = \int_S \rho(\xi,\eta)dS,$$

$$f_x = \int_S \sigma_x(\xi,\eta)dS$$

$$f_y = \int_S \sigma_y(\xi,\eta)dS.$$

Area of contact between the sole and the ground

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The Zero-Moment Point

- The moment of the ground reaction force in a point ***p*** on the ground can be calculated as follows

$$\int_S$$

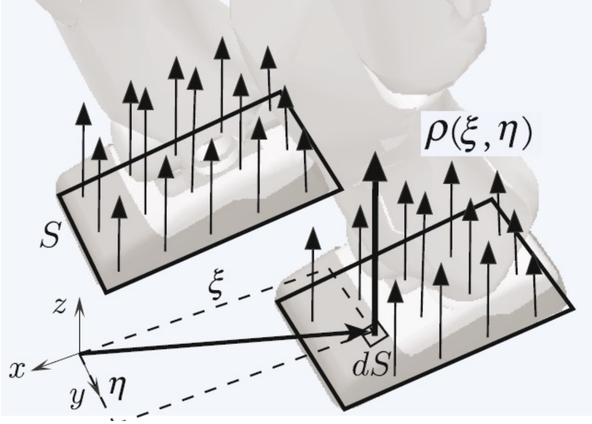
$$\boldsymbol{p} = [p_x \; p_y \; 0]^T$$

$$\rho(\xi, \eta) dS$$

$$\boldsymbol{\tau}(\boldsymbol{p}) = \sum_{i=1}^{N} (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i.$$

$$\boldsymbol{r} = [\xi \; \eta \; 0]^T$$

cross product



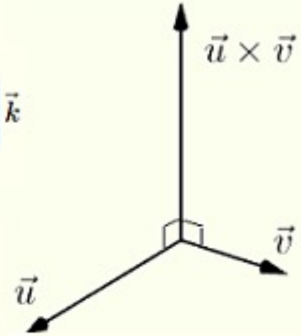
$$\boldsymbol{\tau}_n(\boldsymbol{p}) \equiv [\tau_{nx} \; \tau_{ny} \; \tau_{nz}]^T$$

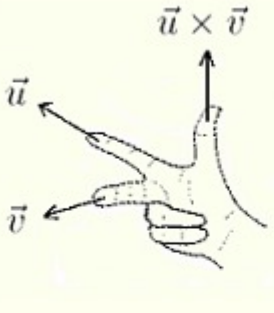
$$\tau_{nx} = \int_S (\eta - p_y)\rho(\xi, \eta) dS$$

$$\tau_{ny} = -\int_S (\xi - p_x)\rho(\xi, \eta) dS$$

$$\tau_{nz} = 0.$$

$$\vec{u} \times \vec{v} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{bmatrix} = \begin{bmatrix} u_y & u_z \\ v_y & v_z \end{bmatrix} \vec{i} - \begin{bmatrix} u_x & u_z \\ v_x & v_z \end{bmatrix} \vec{j} + \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} \vec{k}$$





cross product

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The Zero-Moment Point

- Dynamic reaction force at the contact of the foot with the ground does not produce any moment

$$\boldsymbol{\tau}_n(\boldsymbol{p}) \equiv [\tau_{nx}\ \tau_{ny}\ \tau_{nz}]^T$$
$$\tau_{nx} = \int_S (\eta - p_y)\rho(\xi,\eta)dS$$
$$\tau_{ny} = -\int_S (\xi - p_x)\rho(\xi,\eta)dS$$
$$\tau_{nz} = 0.$$

$$\tau_{nx} = 0$$
$$\tau_{ny} = 0$$

$$p_x = \frac{\int_S \xi\rho(\xi,\eta)dS}{\int_S \rho(\xi,\eta)dS}$$
$$p_y = \frac{\int_S \eta\rho(\xi,\eta)dS}{\int_S \rho(\xi,\eta)dS}$$

To compute the point where the moment of the vertical component of the ground reaction force become zero

$$\boldsymbol{p} = [p_x\ p_y\ 0]^T$$

is the centre of pressure is the **ZMP**

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The Zero-Moment Point

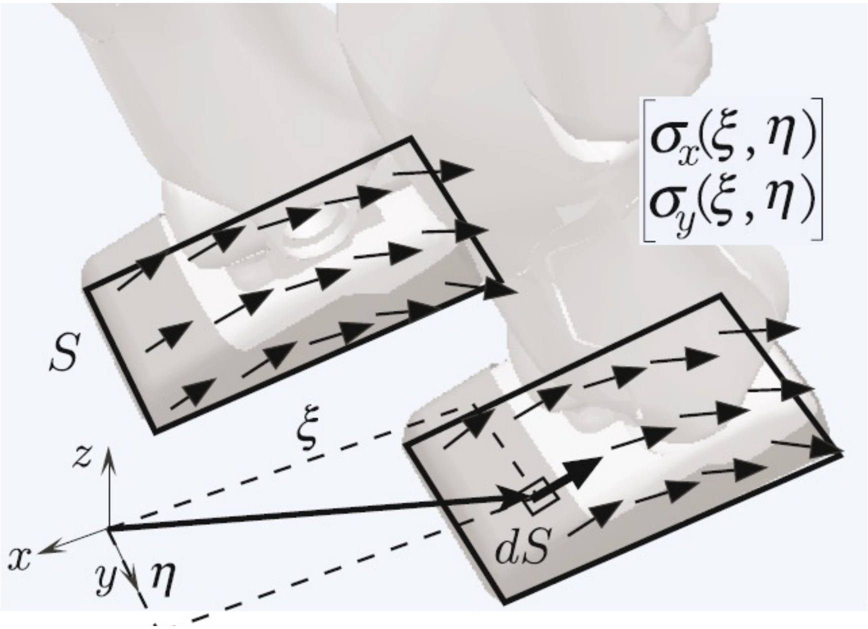
- Dynamic reaction force at the contact of the foot with the ground does not produce any moment

$$\boldsymbol{\tau}_t(\boldsymbol{p}) \equiv [\tau_{tx}\ \tau_{ty}\ \tau_{tz}]^T$$

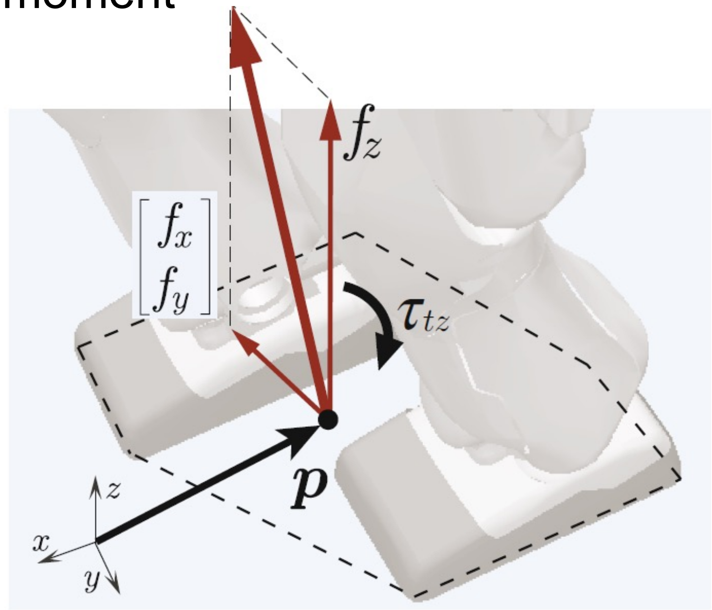
$$\tau_{tx} = 0$$

$$\tau_{ty} = 0$$

$$\tau_{tz} = \int_S \{(\xi - p_x)\sigma_y(\xi,\eta) - (\eta - p_y)\sigma_x(\xi,\eta)\} dS$$



the horizontal ground reaction forces generate the vertical component of the moment



$$\boldsymbol{f} = [f_x\ f_y\ f_z]^T$$

ground reaction forces distributed over the surface of the sole

$$\boldsymbol{\tau}_p = \boldsymbol{\tau}_n(\boldsymbol{p}) + \boldsymbol{\tau}_t(\boldsymbol{p}) = [0\ 0\ \tau_{tz}]^T,$$

moment of the ZMP ***p***

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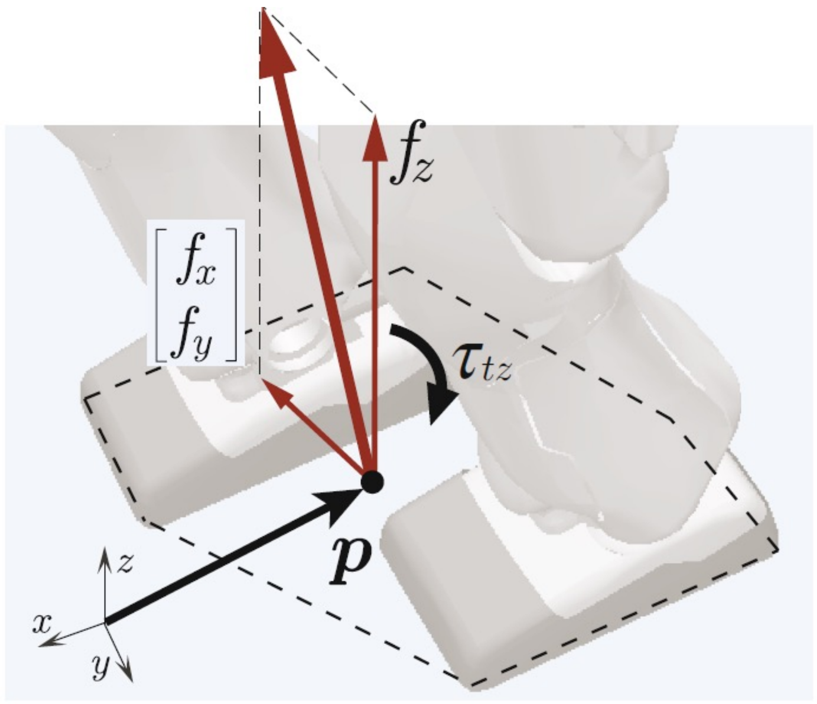
The Zero-Moment Point



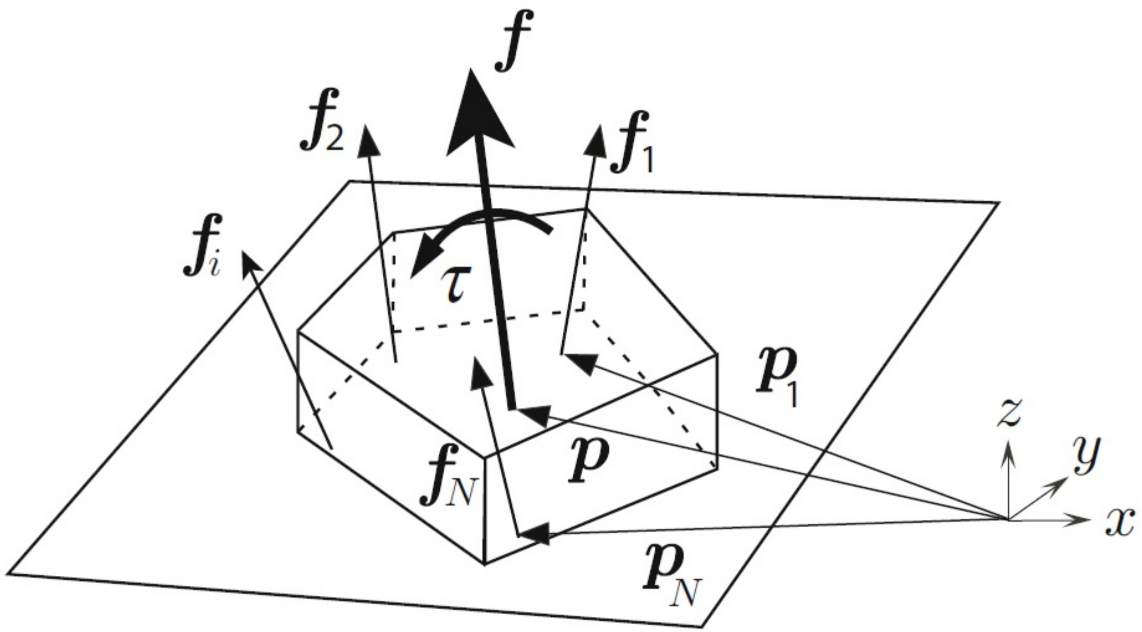
$$
\begin{aligned}
\boldsymbol{\tau}_p &= \boldsymbol{\tau}_n(\boldsymbol{p}) + \boldsymbol{\tau}_t(\boldsymbol{p}) \\
&= [0\ 0\ \tau_{tz}]^T,
\end{aligned}
$$

moment of the ZMP ***p***

Not always equal to zero
when the robot moves.

The ZMP is defined as the point where the horizontal component of the moment of the ground reaction forces becomes zero for 3D cases

The Zero-Moment Point



$$\boldsymbol{f}_i = [f_{ix}\ f_{iy}\ f_{iz}]^T$$ ground reaction forces

acting at the discretized points

$$\boldsymbol{p}_i \in S\ (i = 1, \cdots, N)$$

On a point ***p***

$$\boldsymbol{f} = \sum_{i=1}^{N} \boldsymbol{f}_i$$

$$\boldsymbol{\tau}(\boldsymbol{p}) = \sum_{i=1}^{N} (\boldsymbol{p}_i - \boldsymbol{p}) \times \boldsymbol{f}_i.$$

Position of the ZMP

- set the first and the second elements to zero

$$\tau_{nx} = 0$$
$$\tau_{ny} = 0$$

$$\boldsymbol{p} = \frac{\sum_{i=1}^{N} \boldsymbol{p}_i f_{iz}}{\sum_{i=1}^{N} f_{iz}}$$

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Region of the ZMP

Given

$$\boldsymbol{f}_i = [f_{ix} \; f_{iy} \; f_{iz}]^T$$ ground reaction forces

acting at the discretized points

$$\boldsymbol{p}_i \in S \; (i = 1, \cdots, N)$$

and

$$f_{iz} \geq 0 \qquad (i = 1, \cdots, N)$$

introduce a new variable

$$\alpha_i = f_{iz} / \textstyle\sum_{j=1}^{N} f_{jz}$$

$$\begin{cases} \alpha_i \geq 0 & (i = 1, \cdots, N) \\ \sum_{i=1}^{N} \alpha_i = 1. \end{cases}$$

Region of the ZMP

$$\boldsymbol{p} \in \left\{ \sum_{i=1}^{N} \alpha_i \boldsymbol{p}_i \,\middle|\, \boldsymbol{p}_i \in S \; (i = 1, \cdots, N) \right\}$$

Can you see why the ZMP is included in this region?

This definition is equal to the definition of convex hull.

convex hull = support polygon

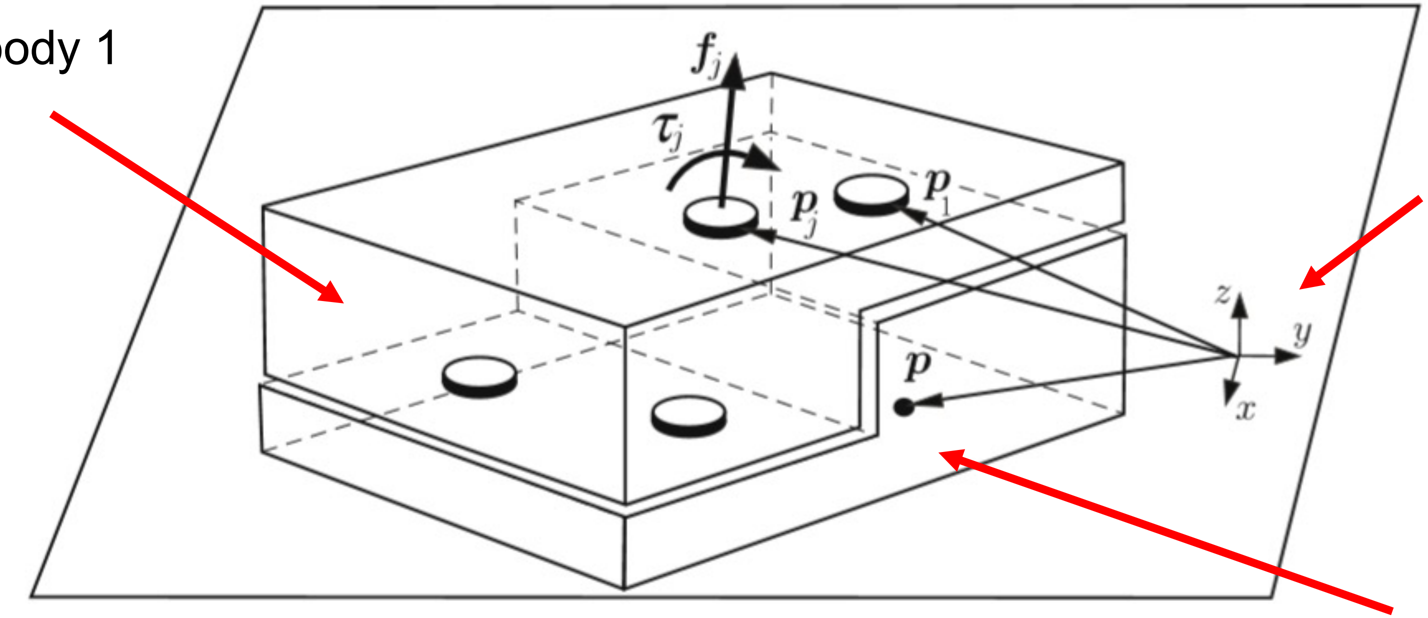
the ZMP always exists inside the support polygon

Humanoid Robot Foot Model

- Two rigid bodies, one of them also in contact with the ground
- Assume that the forces and moments applied by one rigid body to another are measured at multiple points

rigid body 1

reference coordinate system



rigid body 2

- For each point

  $p_j \ (j = 1, \cdots, N)$ you can measure, during the motions of the two rigid bodies (e.g., imagine when the robot moves and the foot is forced on the ground)

  - Forces $\mathbf{f}_j$
  - Momentums τ_j

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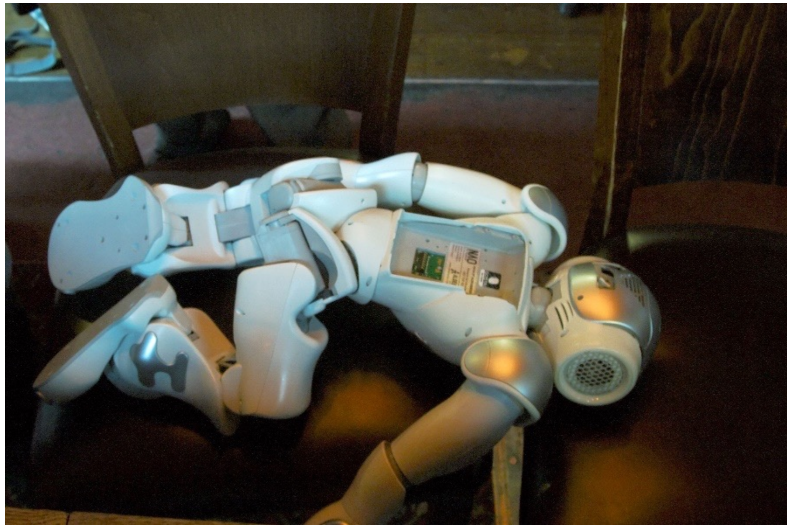
Humanoid Robot Foot Model

- Do you know how to compute the components of the ZMP?
- Given

  points $\boldsymbol{p}_j \; (j = 1, \cdots, N)$ forces $\mathbf{f}_j$ momentums $\boldsymbol{\tau}_j$

- You must consider a point $\boldsymbol{p} = [p_x \; p_y \; p_z]^T$
- Then you need to compute the moment in $\boldsymbol{p} = [p_x \; p_y \; p_z]^T$



$$\boldsymbol{\tau}(\boldsymbol{p}) = \sum_{j=1}^{N} (\boldsymbol{p}_j - \boldsymbol{p}) \times \boldsymbol{f}_j + \boldsymbol{\tau}_j$$

Then you need to
- set the x and y components of the moment to zero
- solve for p_x and p_y

$$p_x = \frac{\sum_{j=1}^{N} \{-\tau_{jy} - (p_{jz} - p_z) f_{jx} + p_{jx} f_{jz}\}}{\sum_{j=1}^{N} f_{jz}}$$

$$p_y = \frac{\sum_{j=1}^{N} \{\tau_{jx} - (p_{jz} - p_z) f_{jy} + p_{jy} f_{jz}\}}{\sum_{j=1}^{N} f_{jz}}$$

where

$\boldsymbol{f}_j = [f_{jx} \; f_{jy} \; f_{jz}]^T$ $\quad \boldsymbol{\tau}_j = [\tau_{jx} \; \tau_{jy} \; \tau_{jz}]^T$ $\quad \boldsymbol{p}_j = [p_{jx} \; p_{jy} \; p_{jz}]^T$ are known

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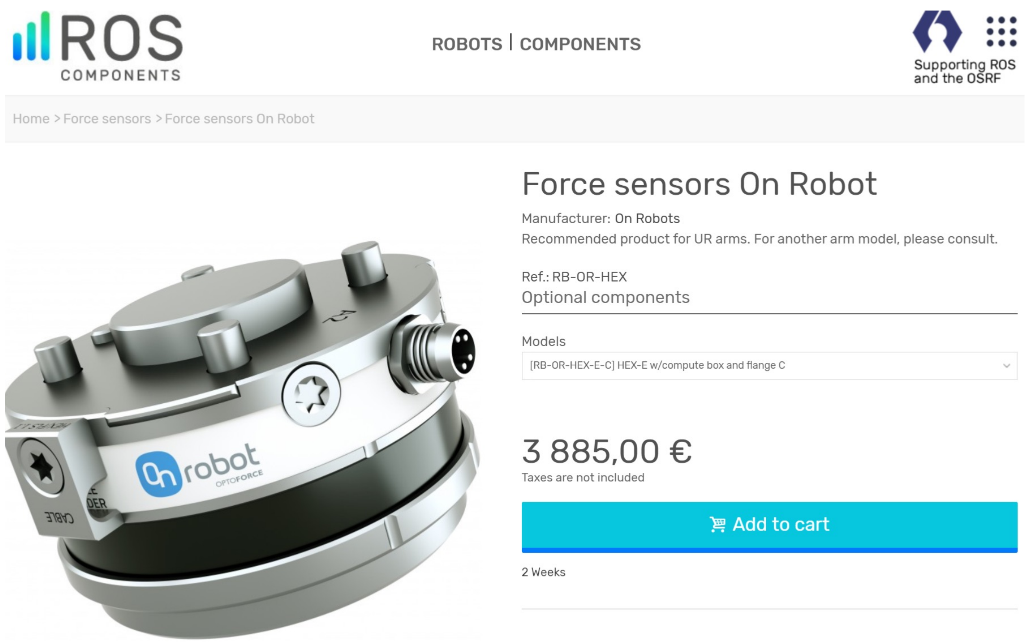
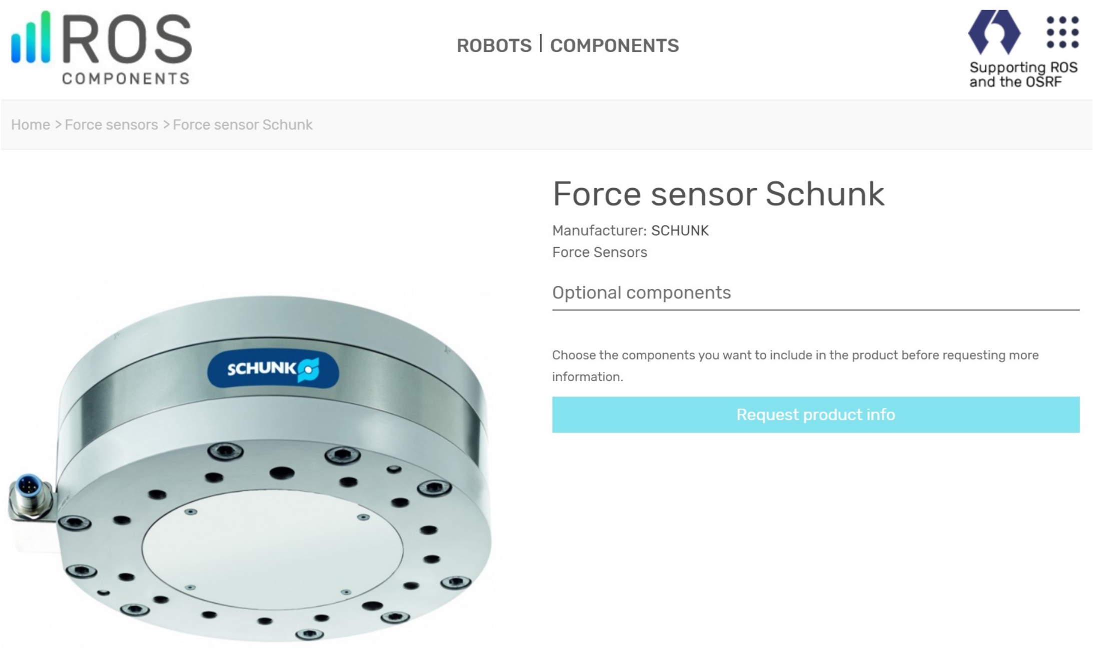
6 Axis Force/Torque Sensors





Main features:

- Optical based technologies
- Keep contact forces while moving
- Integrated temperature compensation
- electro-magnetic noise reduction

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Measurement of ZMP - Single Foot – Trivial Case

when the centre of measurement of the sensor lies on the z axis of the reference coordinate system

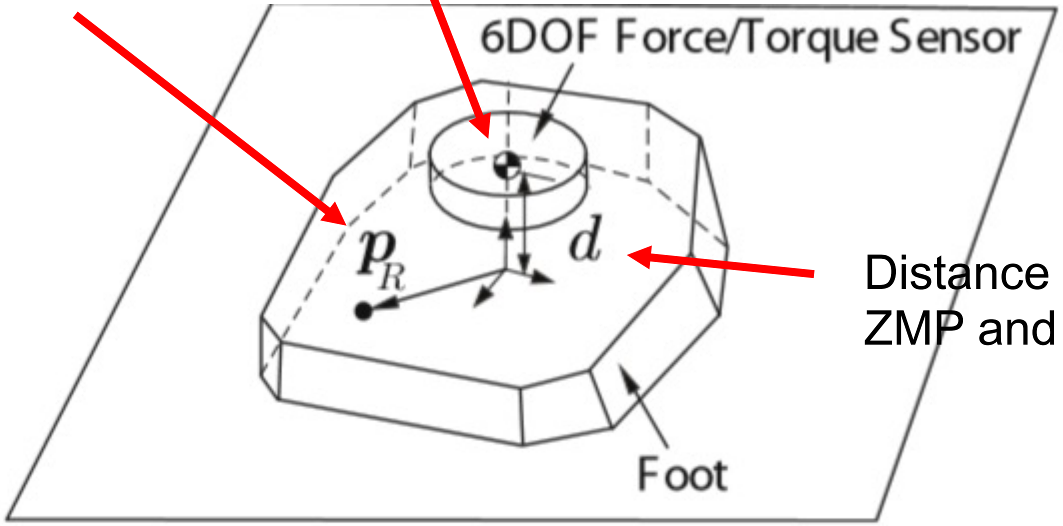
6 axis force/torque sensor

- Measure the force and the momentum applied from outside the robot

$$\boldsymbol{f} = [f_x,\ f_y,\ f_z]$$

$$\boldsymbol{\tau} = [\tau_x\ \tau_y\ \tau_z]$$

position of the ZMP in the right foot



Distance between ZMP and COM

To measure the ZMP (*N* = *1*)

$$\boldsymbol{\tau}(\boldsymbol{p}) = \sum_{j=1}^{N} (\boldsymbol{p}_j - \boldsymbol{p}) \times \boldsymbol{f}_j + \boldsymbol{\tau}_j$$

$$p_x = \frac{\sum_{j=1}^{N} \{-\tau_{jy} - (p_{jz} - p_z) f_{jx} + p_{jx} f_{jz}\}}{\sum_{j=1}^{N} f_{jz}}$$

$$p_y = \frac{\sum_{j=1}^{N} \{\tau_{jx} - (p_{jz} - p_z) f_{jy} + p_{jy} f_{jz}\}}{\sum_{j=1}^{N} f_{jz}}$$

$$p_{Rx} = (-\tau_{1y} - f_{1x} d)/f_{1z}$$

$$p_{Ry} = (\tau_{1x} - f_{1y} d)/f_{1z}$$

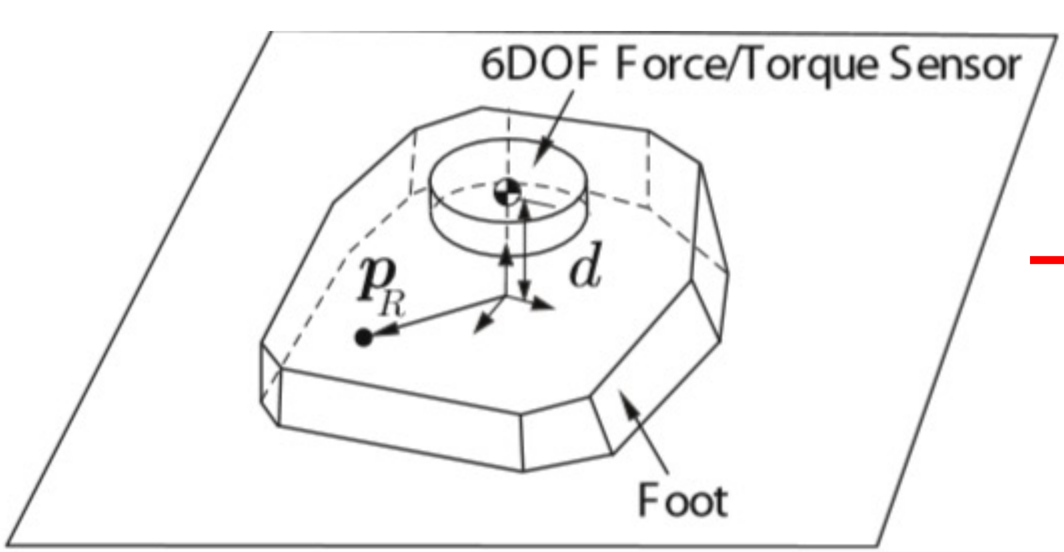
where

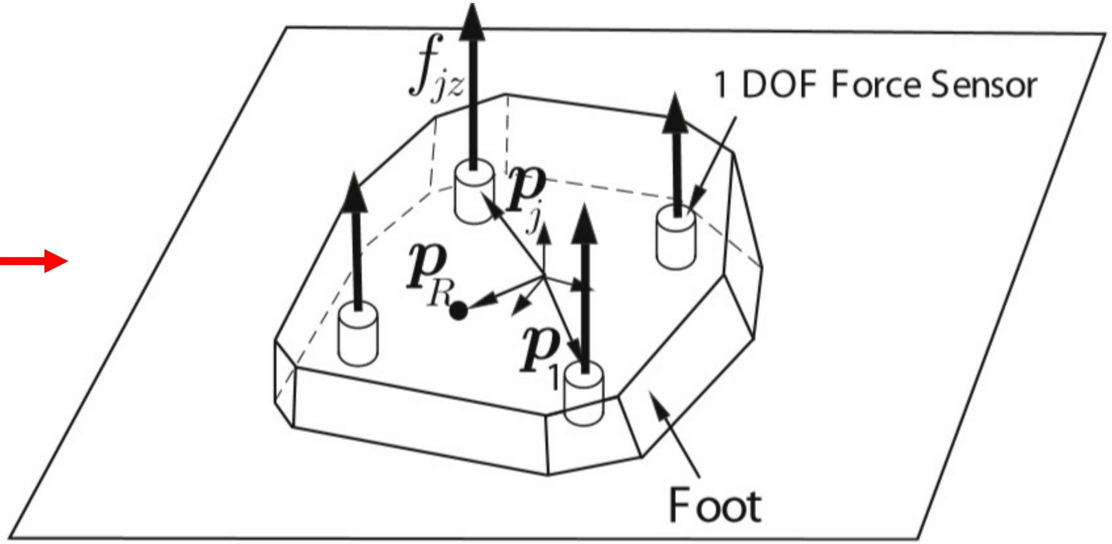
$$\boldsymbol{p}_R = [p_{Rx}\ p_{Ry}\ p_{Rz}]^T$$

$$\boldsymbol{p}_1 = [0\ \ 0\ \ d]^T.$$

Measurement of ZMP - Single Foot – Trivial Case

from single to multiple points of contacts





To measure the ZMP (N can be any number, depending by the force/torque sensor)

$$p_x = \frac{\sum_{j=1}^{N} p_{jx} f_{jz}}{\sum_{j=1}^{N} f_{jz}} \qquad p_y = \frac{\sum_{j=1}^{N} p_{jy} f_{jz}}{\sum_{j=1}^{N} f_{jz}}$$

How did you compute the above values?

What is the main problem with this approach for measuring the ZMP?

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Measurement of ZMP - Both Feet

How to compute the ZMP in the case where both feet are in contact with the ground?

- You must measure the ZMP of the left and right foot individually
  - You need to use the Single Foot Model – Single Contact Point

ZMP in the Right foot

$$\mathbf{p}_R = \begin{bmatrix} p_{Rx} & p_{Ry} & p_{Rz} \end{bmatrix}^\top$$

$$p_{Rx} = -\frac{\tau_{Ry} + f_{Rx} d}{f_{Rz}}$$

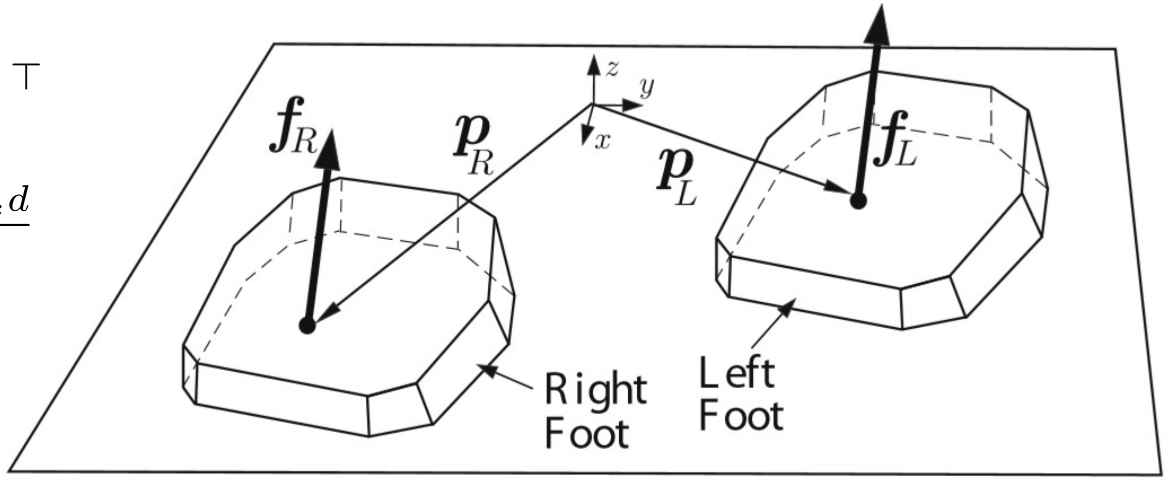
$$p_{Ry} = \frac{\tau_{Rx} - f_{Ry} d}{f_{Rz}}$$

ZMP in the Left foot

$$\mathbf{p}_L = \begin{bmatrix} p_{Lx} & p_{Ly} & p_{Lz} \end{bmatrix}^\top$$

$$p_{Lx} = -\frac{\tau_{Ly} + f_{Lx} d}{f_{Lz}}$$

$$p_{Ly} = \frac{\tau_{Lx} - f_{Ly} d}{f_{Lz}}$$



- Then you must measure the ZMP both feet
  - You need to use the Single Foot Model – Multiple Contact Points

$$\mathbf{p} = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix}^\top$$

j iterates over left and right

$$p_x = \frac{\sum_{j=1}^{N} p_{jx} f_{jz}}{\sum_{j=1}^{N} f_{jz}} \qquad p_y = \frac{\sum_{j=1}^{N} p_{jy} f_{jz}}{\sum_{j=1}^{N} f_{jz}}$$

$$p_x = \frac{p_{Rx} f_{Rz} + p_{Lx} f_{Lz}}{f_{Rz} + f_{Lz}}$$

$$p_y = \frac{p_{Ry} f_{Rz} + p_{Ly} f_{Lz}}{f_{Rz} + f_{Lz}}$$

Useful Resources

- How do biped robots walk
  - https://scaron.info/teaching/how-do-biped-robots-walk.html

- Zero-tilting moment point
  - https://scaron.info/teaching/zero-tilting-moment-point.html

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