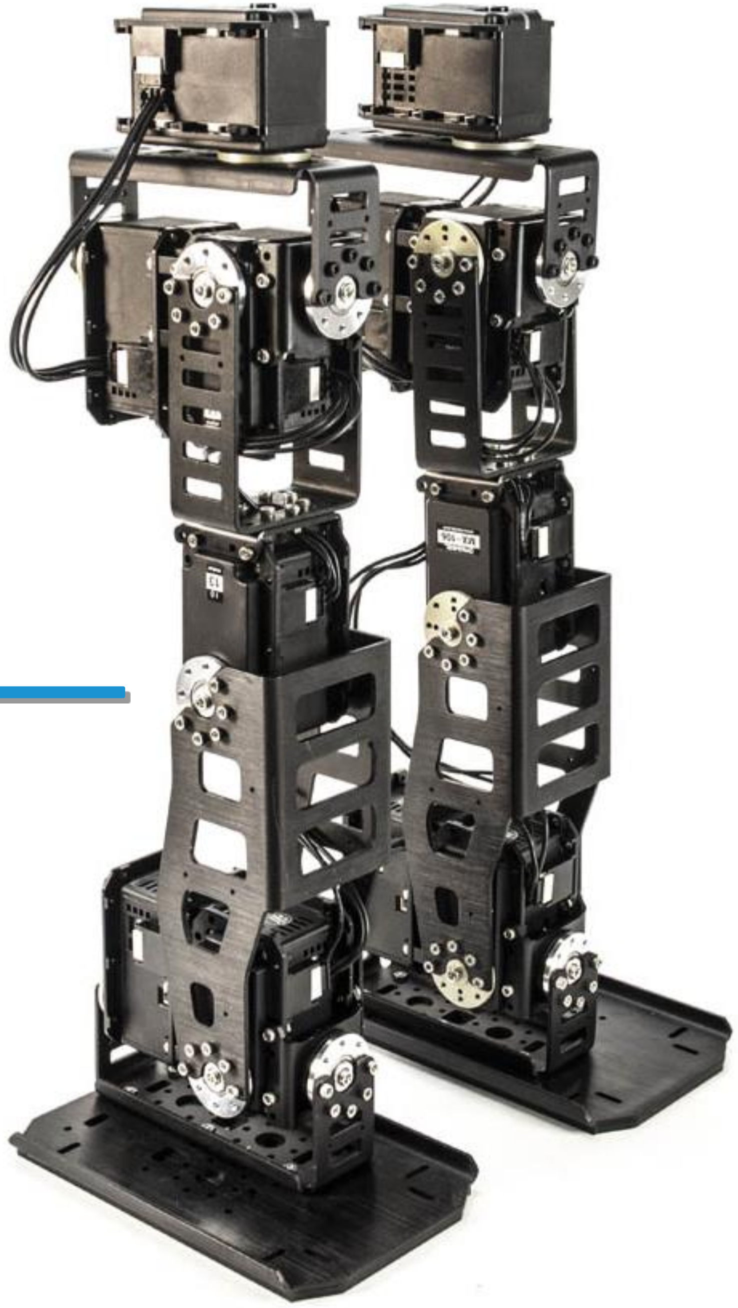
ROCO318
Mobile and Humanoid Robots

Biped Walking



Dr Mario Gianni
School of Engineering, Computing and Mathematics
Faculty of Science and Engineering
University of Plymouth

Lecture Content

- Bipedal Walking Control
- Two-dimensional Walking Pattern Generation
    - 2D Linear Inverted Pendulum
    - Biped Gait
- Three-dimensional Walking Pattern Generation
    - 3D Linear Inverted Pendulum

UNIVERSITY OF PLYMOUTH

Walk

move along at a moderate pace by lifting up and putting down each foot in turn, so that one foot is on the ground while the other is being lifted (Oxford Advanced Learner’s Dictionary, Oxford University Press)

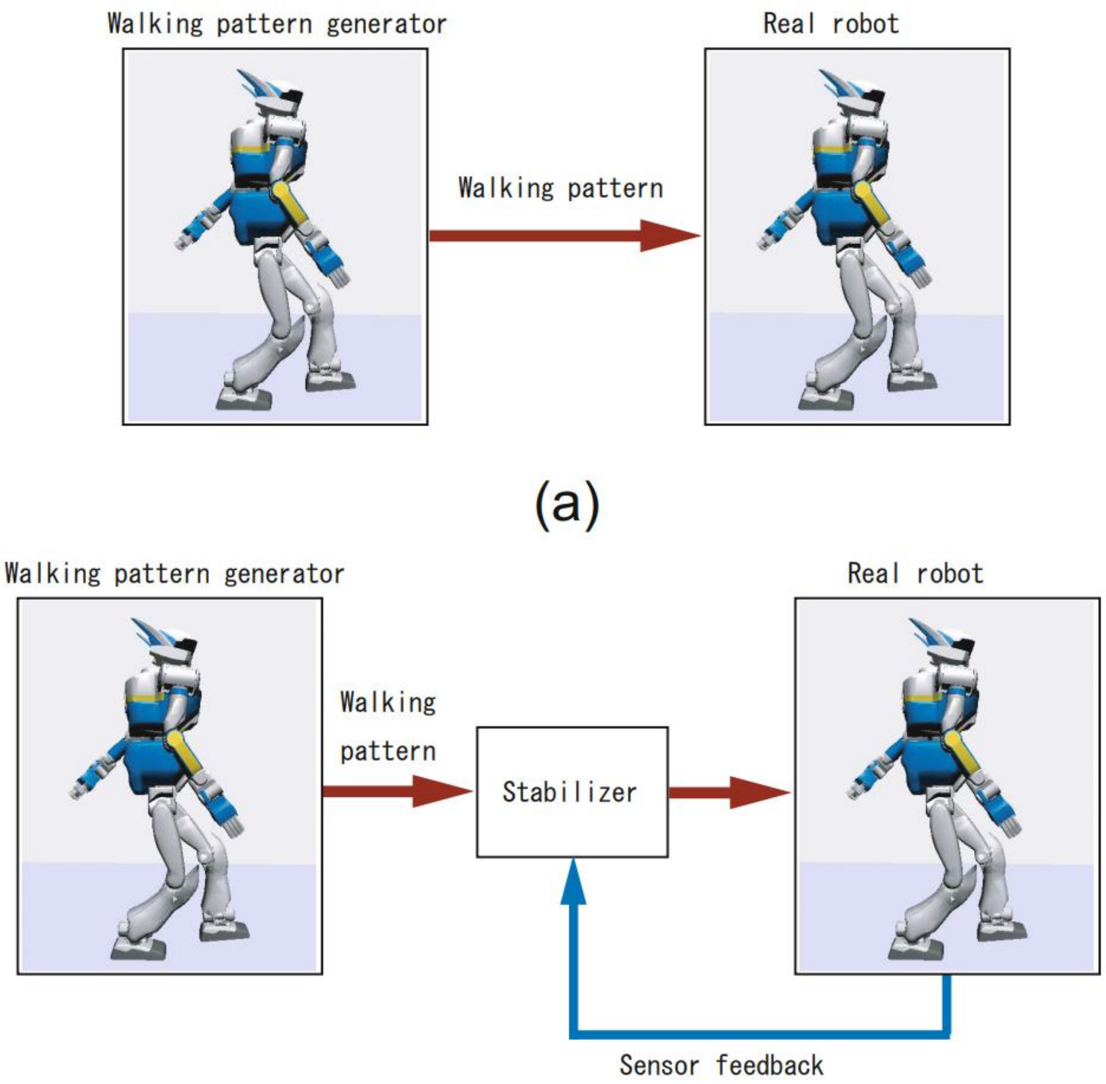
- at least one foot must be in contact with the ground at any moment during walking

- Two kind of walking
  - Static walking
    - the projection of the centre of mass never leaves the support polygon during the walking
  - Dynamic walking
    - there exist periods when the projection of the centre of mass leaves the support polygon

- Humans perform dynamic walking.

  - Why?

Bipedal Walking Control



A set of time series of joint angles for desired walking is called a walking pattern

- to create it, we use a walking pattern generator

In an ideal situation, biped walking can be realized just by giving a walking pattern to an actual robot

- accurate model of the robot
- a stiff mechanism which moves exactly as commanded
- a perfect horizontal floor

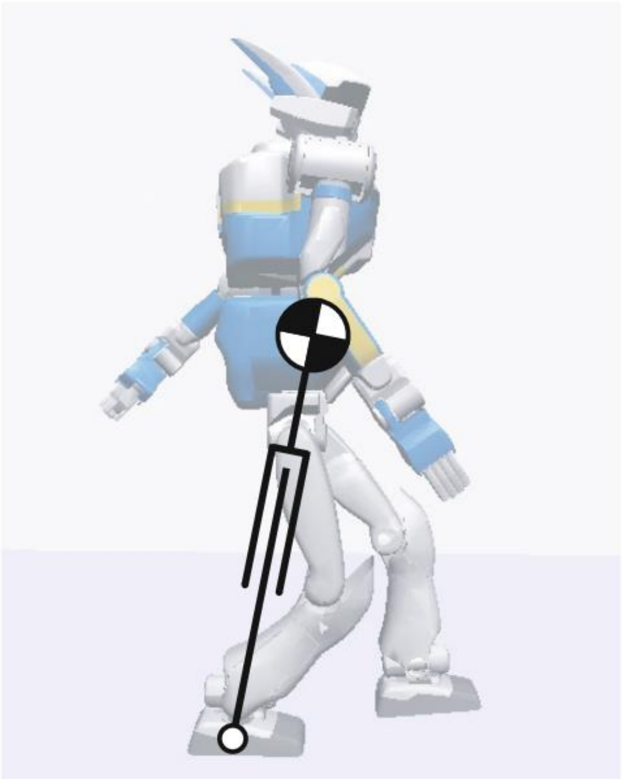
In a real situation, a life size humanoid robot can easily fall down by floor unevenness of only a few millimetres.

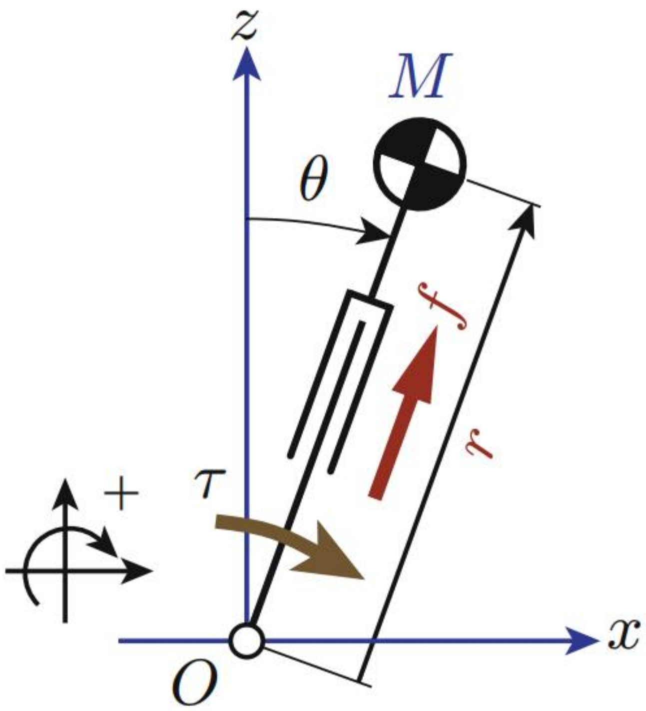
- A humanoid proportion and mass distribution tends to quickly amplify the posture error to an unstable level.
- we need to use gyros, accelerometers, force sensors and other devices to modify the walking pattern (stabilizer)

UNIVERSITY OF PLYMOUTH

Two-dimensional Walking Pattern Generation

- Assumptions
  - All the mass of the robot is concentrated at its centre of mass
  - the robot has massless legs, whose tips contact the ground at single rotating joints
  - the robot motion is constrained to the sagittal plane defined by the axis of walking direction and vertical axis





2D inverted pendulum

2D Inverted Pendulum

- Inputs
  - The torque at the pivot

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - gr\sin\theta = \boxed{\tau}/M$$
$$\ddot{r} - r\dot{\theta}^2 + g\cos\theta = f/M.$$

derived using Lagrange's method

2D Inverted Pendulum

- Inputs
  - The torque at the pivot
  - The kick force at the prismatic joint along the leg

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - gr\sin\theta = \tau/M$$
$$\ddot{r} - r\dot{\theta}^2 + g\cos\theta = \boxed{f}/M.$$

derived using Lagrange's method

2D Inverted Pendulum

- Inputs
  - The torque at the pivot
  - The kick force at the prismatic joint along the leg

$$
\begin{aligned}
r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - gr\sin\theta &= \tau/M \\
\ddot{r} - r\dot{\theta}^2 + g\cos\theta &= f/M.
\end{aligned}
$$

derived using Lagrange's method

Limitations?

2D Inverted Pendulum

- Inputs
  - The torque at the pivot
  - The kick force at the prismatic joint along the leg

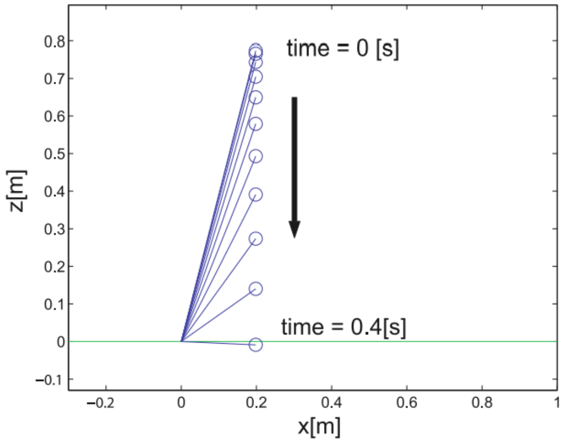
$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - gr\sin\theta = \tau/M$$
$$\ddot{r} - r\dot{\theta}^2 + g\cos\theta = f/M.$$

derived using Lagrange’s method

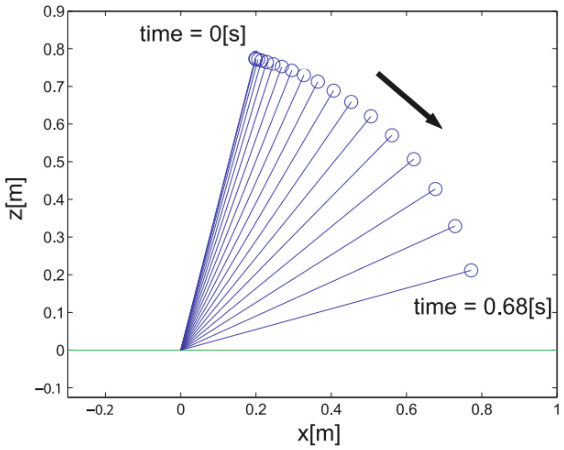
- Limitations
  - we cannot use big torque since the feet of biped robot is very small
  - If a walking robot has a point contact like a stilt, we must set the torque equal to zero
    - In this case, the pendulum will almost always fall down, unless the CoM is located precisely above the pivot

Falling Patterns

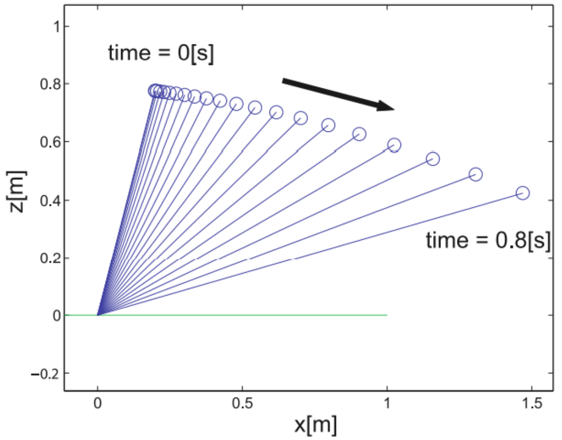
- different kick forces



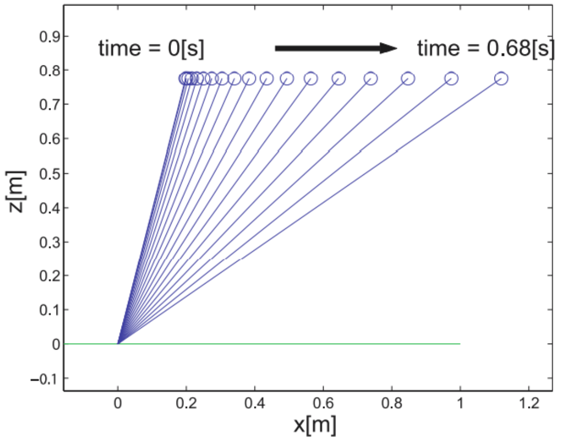
(a) $f = 0$: Free fall of CoM



(b) $f = Mg\cos\theta - Mr\dot{\theta}^2$: Fall down with constant leg length



(c) $f = Mg$: Fall down and acceleration



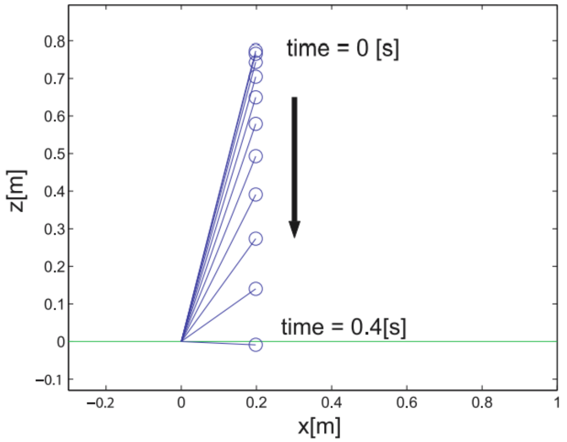
(d) $f = Mg/\cos\theta$: CoM accelerates while keeping the initial height

Falling inverted pendulum under the various kick force

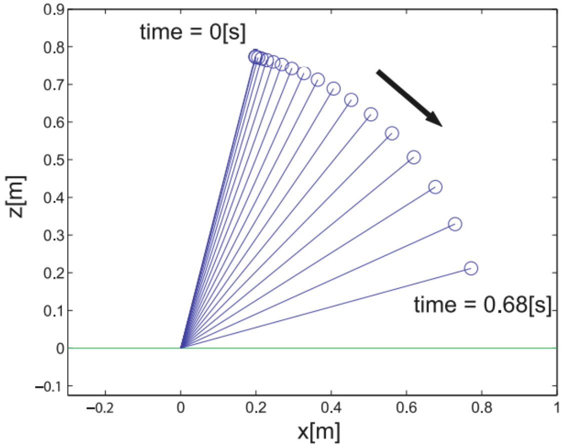
- The pivot torque is kept zero

Falling Patterns

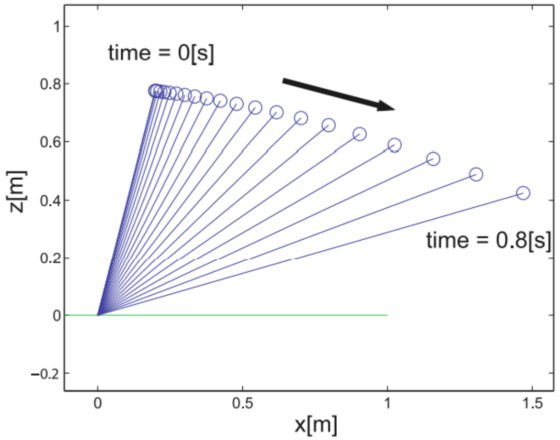
- different kick forces



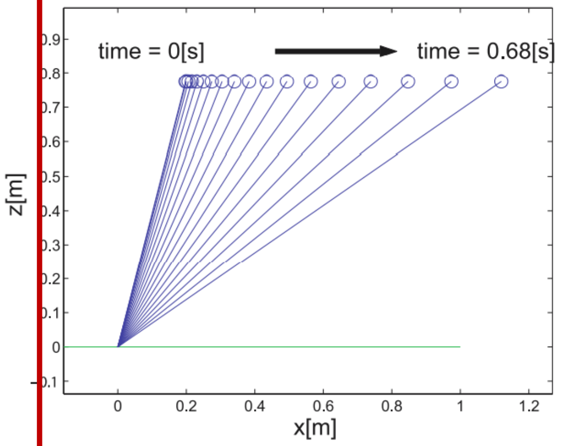
(a) $f = 0$: Free fall of CoM



(b) $f = Mg\cos\theta - Mr\dot{\theta}^2$: Fall down with constant leg length



(c) $f = Mg$: Fall down and acceleration



(d) $f = Mg/\cos\theta$: CoM accelerates while keeping the initial height

Falling inverted pendulum under the various kick force

- The pivot torque is kept zero

Most interesting case

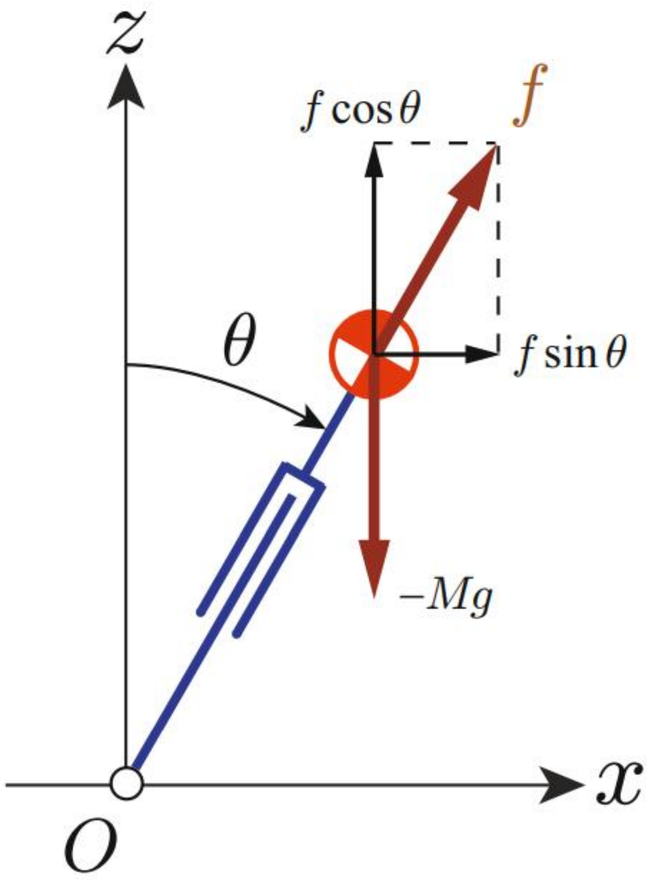
- The CoM moves horizontally under the kick force

$$f = \frac{Mg}{\cos\theta}.$$

- Intuitively, we can say that the pendulum is keeping the CoM height by extending its leg as fast as it is falling.
  - Linear Inverted Pendulum

2D Linear Inverted Pendulum

- Horizontal motion



- the horizontal component of the kick force remains while the vertical component is cancelled by gravity

- The horizontal component accelerates the CoM horizontally

$$M\ddot{x} = f \sin\theta.$$

2D Linear Inverted Pendulum

$$f = \frac{Mg}{\cos\theta}. \qquad\qquad M\ddot{x} = f\sin\theta.$$

$$M\ddot{x} = \frac{Mg}{\cos\theta}\sin\theta = Mg\tan\theta = Mg\frac{x}{z}$$

- where, *x*, *z* gives the CoM of the inverted pendulum.
- By rewriting the above equation, we obtain a differential equation for the horizontal dynamics of the CoM

$$\ddot{x} = \frac{g}{z}x.$$

- Since we have constant *z* in a Linear Inverted Pendulum

$$x(t) = x(0)\cosh(t/T_c) + T_c\dot{x}(0)\sinh(t/T_c)$$
$$\dot{x}(t) = x(0)/T_c\sinh(t/T_c) + \dot{x}(0)\cosh(t/T_c)$$
$$T_c \equiv \sqrt{z/g}$$

where T_c is the time constant depending the height of the CoM and gravity acceleration.

2D Linear Inverted Pendulum

$$f = \frac{Mg}{\cos\theta}.$$

$$M\ddot{x} = f\sin\theta.$$

$$M\ddot{x} = \frac{Mg}{\cos\theta}\sin\theta = Mg\tan\theta = Mg\frac{x}{z}$$

- where, *x*, *z* gives the CoM of the inverted pendulum.
- By rewriting the above equation, we obtain a differential equation for the horizontal dynamics of the CoM

$$\ddot{x} = \frac{g}{z}x.$$

- Since we have constant *z* in a Linear Inverted Pendulum

$$\begin{aligned} x(t) &= \boxed{x(0)}\cosh(t/T_c) + T_c\boxed{\dot{x}(0)}\sinh(t/T_c) \\ \dot{x}(t) &= \boxed{x(0)}/T_c\sinh(t/T_c) + \boxed{\dot{x}(0)}\cosh(t/T_c) \\ T_c &\equiv \sqrt{z/g} \end{aligned}$$

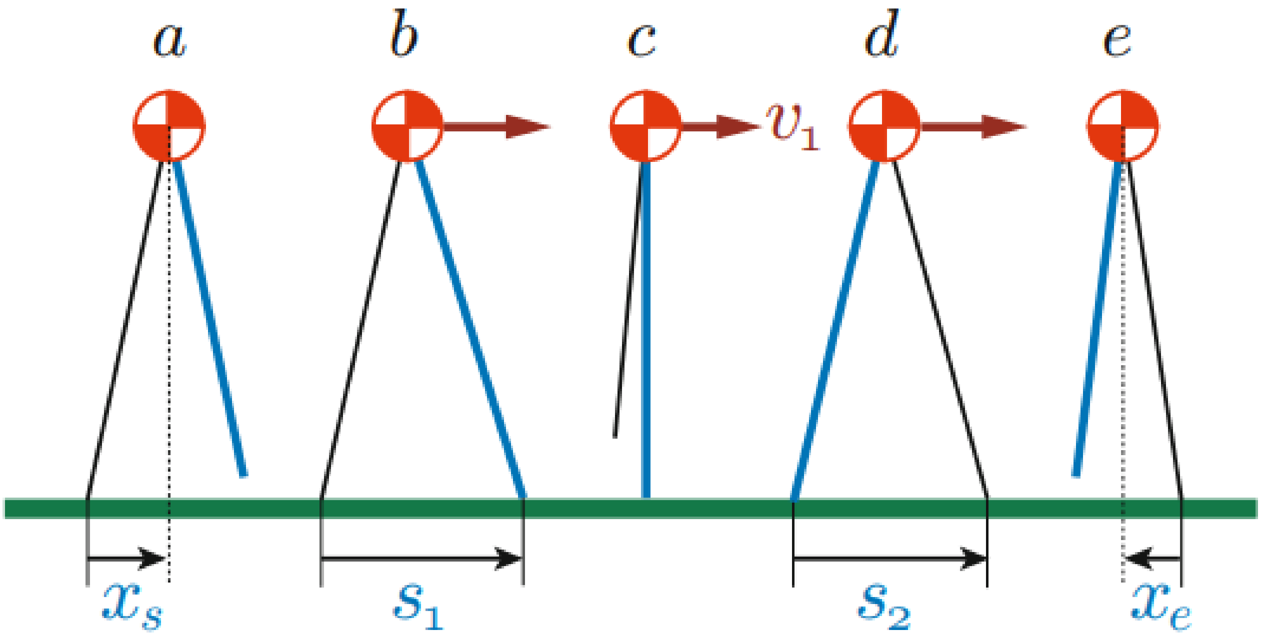
initial conditions

where *Tc* is the time constant depending the height of the CoM and gravity acceleration.

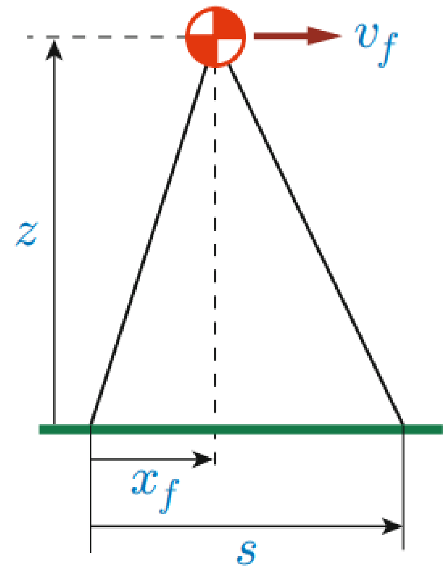
UNIVERSITY OF PLYMOUTH

Biped Gait

- Assumption
  - ideal biped robot on a level plane walks just one step and stops



we need support leg exchanges twice



state of a support leg exchange

- The robot exchanges its support twice and three **orbital energies** must be specified
  - sum of the kinetic energy and the imaginary potential energy
  - In mechanics, this value is called constant of motion

Biped Gait

- From *a* to *b* the orbital energy is specified by the initial position of the CoM

$$E_0 = -\frac{g}{2z} x_s^2. \quad \text{imaginary potential energy}$$

- For the step (b→c→d), the orbital energy is specified by the speed v_1 at the moment that CoM passes over the supporting point

$$E_1 = \frac{1}{2} v_1^2. \quad \text{kinetic energy}$$

- For the walk finish (d→e), the orbital energy is specified by the final position of the CoM

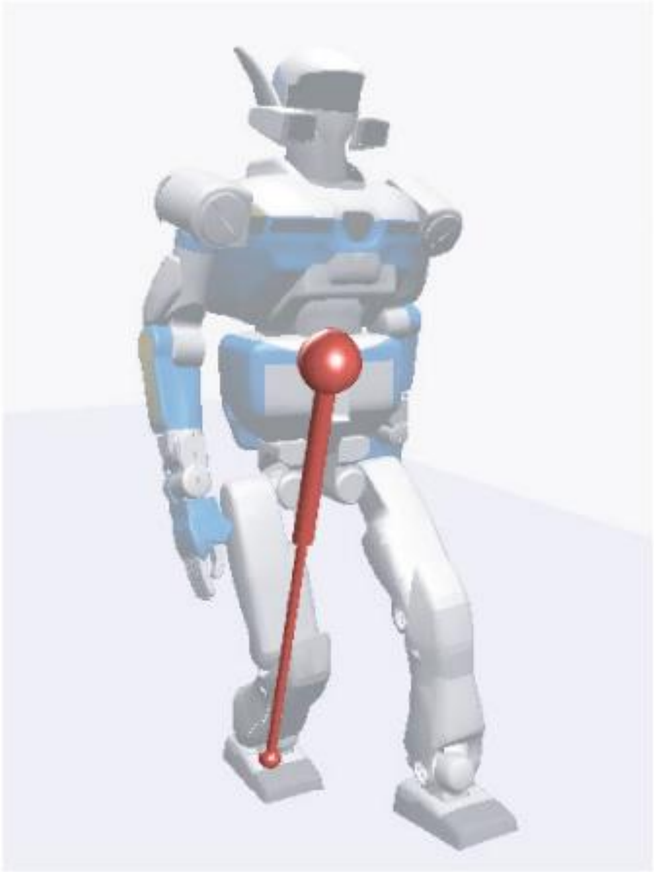
$$E_2 = -\frac{g}{2z} x_e^2. \quad \text{imaginary potential energy}$$

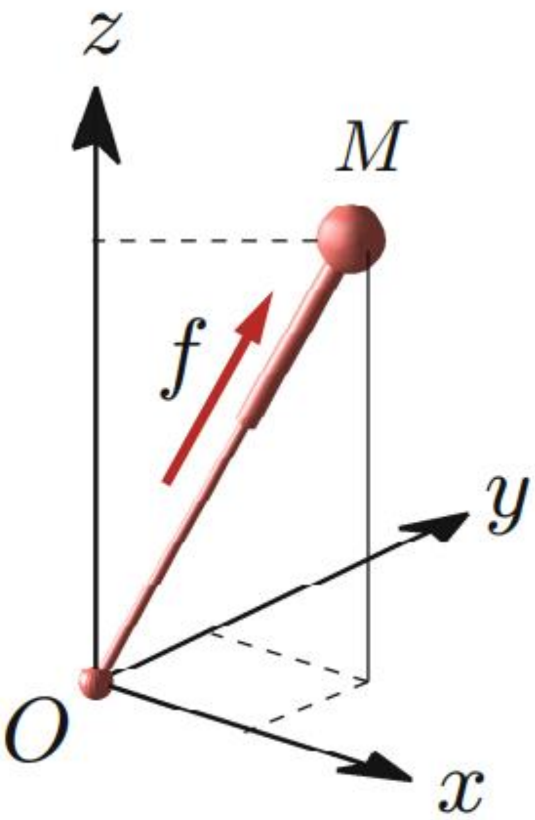
- The first support exchange condition is obtained from E_0 and E_1
- The second support exchange condition is obtained from E_1 and E_2
- Desired walking motion is realized by controlling the swing leg so that those exchanges occur at the right time
- Speed at the moment of the exchange

$$v_f = \sqrt{2E_1 + \frac{g}{z} x_f^2}.$$

- By calculating the speed at each exchange the complete trajectory of each step can be calculated

3D Linear Inverted Pendulum





- 3D inverted pendulum as an approximated walking robot
  - The supporting point is a spherical joint which allows free rotation
  - The leg can change its length by generating a kick force *f*

3D Linear Inverted Pendulum

- The kick force can be decomposed into three components

$$f_x = (x/r)f$$
$$f_y = (y/r)f$$
$$f_z = (z/r)f$$

r is the distance between the supporting point and the CoM

- Since only the kick force and gravity act on the CoM, the motion equation of the CoM is given by

$$M\ddot{x} = (x/r)f$$
$$M\ddot{y} = (y/r)f$$
$$M\ddot{z} = (z/r)f - Mg.$$

- Constraint plane for the CoM

$$z = k_x x + k_y y + z_c,$$

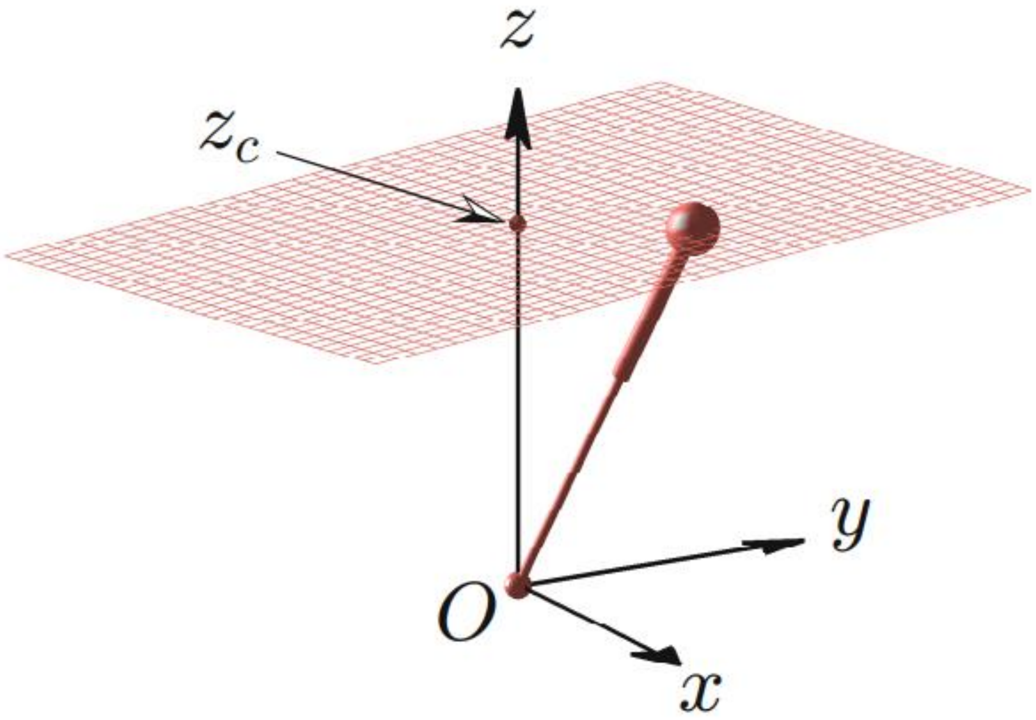
k_x, k_y determine the slope and z_c determines the height of the constraint plane

- To let the CoM move along this plane, we need its acceleration be orthogonal to the normal vector of the constraint

UNIVERSITY OF PLYMOUTH

3D Linear Inverted Pendulum

- To let the CoM move along this plane, we need its acceleration be orthogonal to the normal vector of the constraint



$$z = k_x x + k_y y + z_c,$$

$$\left[f\left(\frac{x}{r}\right) \quad f\left(\frac{y}{r}\right) \quad f\left(\frac{z}{r}\right) - Mg \right] \begin{bmatrix} -k_x \\ -k_y \\ 1 \end{bmatrix} = 0.$$

solving this equation for *f* and substituting into the equation of the constraint plane

$$f = \frac{Mgr}{z_c}.$$

The centre of mass moves on the constraint plane by applying the kick force *f* in proportion to the leg length *r*

UNIVERSITY OF PLYMOUTH

3D Linear Inverted Pendulum

- Horizontal dynamics

$$M\ddot{x} = (x/r)f \qquad M\ddot{y} = (y/r)f \qquad f = \frac{Mgr}{z_c}.$$

$$\ddot{x} = \frac{g}{z_c}x,$$
$$\ddot{y} = \frac{g}{z_c}y.$$

linear equations

- The intersection of the constraint plane z_c, is the only parameter
- The inclination parameters k_x, k_y of the constraint plane do not affect the horizontal motion of the CoM

- The 3D linear inverted pendulum is a concatenation of two 2D linear inverted pendula