MTH793P   **Advanced Machine Learning, Semester B, 2025/26**
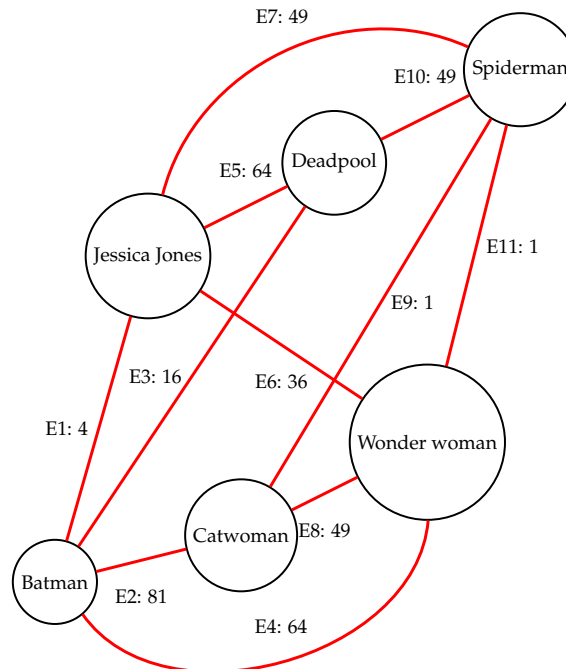**Coursework 1**

Graph-based binary classification

1. Write down the incidence matrix for the following weighted, undirected graph:

   Any missing edges correspond to weights of zero between the two nodes. Order the columns of the incidence matrix alphabetically according to the vertex name and the rows according to the edge numbering (E1, E2, E3, ...).

2. Compute the corresponding graph Laplacian for the incidence matrix in Exercise (1).

3. We want to use the graph from Question (1) to determine whether a node in the graph belongs to the class "Marvel" or the class "DC". Suppose we are in a semi-

supervised setting, where the node "Deadpool" is already labelled $v_{\text{Deadpool}} = 0$ (class "Marvel") and the node "Catwoman" is labelled as $v_{\text{Catwoman}} = 1$ (class "DC"). Determine the labels for all remaining nodes, and classify each node.

4. Repeat the previous exercises in Python by following the steps outlined in the template Jupyter notebook.

5. Compute the second eigenvector, i.e. the eigenvector that corresponds to the second smallest eigenvalue, of the graph-Laplacian in Python. What do you observe?