MTAT.05.024 Quantum Crypto

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Homework # 1

Handed out:	Tue Feb. 27
Due:	Tue March 5, 10:00
As PDF by email to	`shahla.novruzova@ut.ee`
subject:	`QCRY-HW1-`*lastname*

1 Orthonormal bases

Definition. The "computational (orthonormal) basis" of the Hilbert space of an n-qubit quantum register consists of the states $|x\rangle$, where x ranges over all elements of $\{0,1\}^n$, i.e., length-n bit strings. For example, for a single qubit, we get the familiar ONB $|0\rangle, |1\rangle$.

(a) Verify that the following four 2-qubit states form an ONB:

- $(|00\rangle + |11\rangle)/\sqrt{2}$

  (Your solution here.)

- $(|00\rangle - |11\rangle)/\sqrt{2}$

  (Your solution here.)

- $(|01\rangle + |10\rangle)/\sqrt{2}$

  (Your solution here.)

- $(|01\rangle - |10\rangle)/\sqrt{2}$

  (Your solution here.)

(b) Write the following 2-qubit states as superposition of the basis defined in the (a):

- $|00\rangle$

  (Your solution here.)

- $|01\rangle$

  (Your solution here.)

- $|11\rangle$

  (Your solution here.)

(c) ....

2 Measurement I

A computational basis measurement of a single qubit has possible outcomes $0, 1$. If the single qubit is in state

$$\psi = \alpha_0 \left|0\right\rangle + \alpha_1 \left|1\right\rangle ,$$

then the probability of outcome 0 is $|\alpha_0|^2$ and the probability of outcome 1 is is $|\alpha_1|^2 = 1 - |\alpha_0|^2$.

For each of the following states, give the measurement probabilities of the outcomes:

(a) $\left|0\right\rangle$ ....

(b) $\left|1\right\rangle$ ....

(c) $\left|+\right\rangle$ ....

(d) $\left|-\right\rangle$ ....

(e) $\left|\circlearrowleft\right\rangle$ ....

(f) $\left|\circlearrowright\right\rangle$ ....

3 Measurement II

A computational basis measurement of the *left* one of two qubits has possible outcomes $0, 1$. If the two qubits are in the state

$$\psi = \alpha_{00} \left|00\right\rangle + \alpha_{01} \left|01\right\rangle + \alpha_{10} \left|10\right\rangle + \alpha_{11} \left|11\right\rangle ,$$

then the probability of outcome 0 is

$$|\alpha_{00}|^2 + |\alpha_{01}|^2$$

and the probability of outcome 1 is is

$$|\alpha_{10}|^2 + |\alpha_{11}|^2 = 1 - \left(|\alpha_{00}|^2 + |\alpha_{01}|^2\right) .$$

For each of states in item (a) of exercise 1, give the measurement probabilities of the outcome of measuring the left qubit.

- $(\left|00\right\rangle + \left|11\right\rangle)/\sqrt{2}$ ......
- $(\left|00\right\rangle - \left|11\right\rangle)/\sqrt{2}$ ......
- $(\left|01\right\rangle + \left|10\right\rangle)/\sqrt{2}$ ......
- $(\left|01\right\rangle - \left|10\right\rangle)/\sqrt{2}$ ......

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