Assignment # 3 and 4

Linear Algebra

(Absolute 30 Marks)

Student Name:	
Student ID:	
Section:	B
Program: BS (CS/SE/TN/EE):	BS(CS)
Total Marks:	30
Marks obtained:	
Dead Line:	20 May, 2020

Instructions:

1. **Please submit before deadline.**
2. **Use blue ink or blue ball pen to write.**
3. **Copying will result = 0.**
4. **Solve Problems on your papers then take snap from mobile and paste it in the given area (space).**
5. **Write final result at the end of questions as well (in given space)**

Q 1) Determine whether the given vectors are linearly independent or are linearly dependent in R^3; where, S = {(−3,0,4), (5, −1,2), (1,1,3)}?

Solution #1

$S = \{(-3,0,4), (5,-1,2), (1,1,3)\}$

To check its dependence or Independence

Take determinant

$$S = \begin{bmatrix} -3 & 5 & 1 \\ 0 & -1 & 1 \\ 4 & 2 & 3 \end{bmatrix}$$

$$|S| = \begin{vmatrix} -3 & 5 & 1 \\ 0 & -1 & 1 \\ 4 & 2 & 3 \end{vmatrix} = -3(-3-2) - 5(-4) + 1(0-(-4))$$

$$= 15 + 20 + 4$$

$$= 39$$

$|S| = 39 \neq 0$ So vectors are linearly Independent.

Using proper method by Solving vectors through matrix

$\boxed{Ax = 0}$

$$S = \begin{bmatrix} -3 & 5 & 1 \\ 0 & -1 & 1 \\ 4 & 2 & 3 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} -3 & 5 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 4 & 2 & 3 & 0 \end{array}\right]$$

$$\sim \begin{bmatrix} 1 & -5/3 & -1/3 & 0 \\ 0 & -1 & 1 & 0 \\ 4 & 2 & 3 & 0 \end{bmatrix} -\frac{1}{3}R_1 \sim \begin{bmatrix} 1 & -5/3 & -1/3 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 26/3 & 13/3 & 0 \end{bmatrix} R_3-4R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & -5/3 & -1/3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 26/3 & 13/3 & 0 \end{array}\right] \xrightarrow{(-1)R_2} \sim \left[\begin{array}{ccc|c} 1 & -5/3 & -1/3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 13 & 0 \end{array}\right] R_3 - \tfrac{26}{3}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -5/3 & -1/3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \tfrac{1}{13}R_3$$

$\boxed{x_3 = 0}$ Put it for other values

$x_2 - 0 = 0$ $\boxed{x_2 = 0}$

$x_1 - \frac{5}{3}(0) - \frac{1}{3}(0) = 0$

$\boxed{x_1 = 0}$

As there is unique solution for values of x, so vectors are linearly **Independent**

Vectors are (paste your answer here): X1=0 , X2=0 , X3=0 V= (0,0,0)

Q 2) Find out Eigen values and bases for Eigen space for the given matrix A where; A = $\begin{pmatrix} 2 & -1 \\ 10 & -9 \end{pmatrix}$?

Solution #2

$A = \begin{bmatrix} 2 & -1 \\ 10 & -9 \end{bmatrix}$

Solving for eigen values.

$[\lambda I - A]$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 10 & -9 \end{bmatrix}$$

$$\begin{bmatrix} \lambda - 2 & 1 \\ -10 & \lambda + 9 \end{bmatrix} \text{ — } (*)$$

Put determinant of $\lambda I - A = 0$

$$|\lambda I - A| = \begin{vmatrix} \lambda - 2 & 1 \\ -10 & \lambda + 9 \end{vmatrix}$$

$(\lambda - 2)(\lambda + 9) + 10 = 0$

$\lambda^2 + 9\lambda - 2\lambda - 18 + 10 = 0$

$\lambda^2 + 7\lambda - 8 = 0$

$\lambda^2 - \lambda + 8\lambda - 8 = 0$

$\lambda(\lambda - 1) + 8(\lambda - 1) = 0$

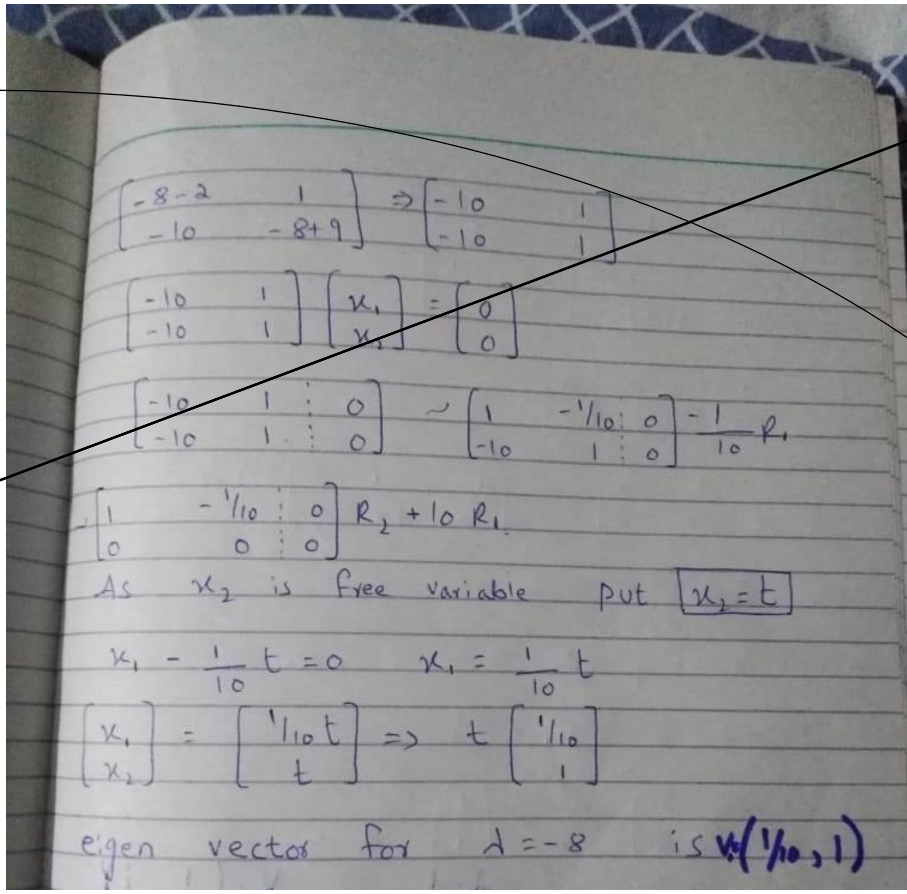
$(\lambda - 1)(\lambda + 8) = 0$

$\boxed{\lambda = -8}$

$\boxed{\lambda = 1}$

Now, Solving for eigen vector when $\boxed{\lambda = -8}$

Put $\lambda = -8$ in eq $(*)$



$$\begin{bmatrix} -8-2 & 1 \\ -10 & -8+9 \end{bmatrix} \Rightarrow \begin{bmatrix} -10 & 1 \\ -10 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -10 & 1 \\ -10 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -10 & 1 & 0 \\ -10 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1/10 & 0 \\ -10 & 1 & 0 \end{array}\right] -\frac{1}{10}R_1$$

$$\sim \left[\begin{array}{cc|c} 1 & -1/10 & 0 \\ 0 & 0 & 0 \end{array}\right] R_2 + 10R_1$$

As x_2 is free variable put $\boxed{x_2 = t}$

$$x_1 - \frac{1}{10}t = 0 \qquad x_1 = \frac{1}{10}t$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/10\, t \\ t \end{bmatrix} \Rightarrow t \begin{bmatrix} 1/10 \\ 1 \end{bmatrix}$$

eigen vector for $\lambda = -8$ is $V_1(1/10, 1)$

Value of Lambda	**Corresponding Eigen Vector**
λ = -8	**V1=(1/10 , 1)**
λ = 1	**V2=(1 ,1)**

Q 3) Find out Eigen values and bases for Eigen space for the given matrix A

where; A = $\begin{pmatrix} \mathbf{5} & \mathbf{1} & \mathbf{3} \\ \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} \end{pmatrix}$?

Solution #3

$$A = \begin{bmatrix} 5 & 1 & 3 \\ 0 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Solving for eigen values

$[\lambda I - A]$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 5 & 1 & 3 \\ 0 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\lambda I - A = \begin{bmatrix} \lambda - 5 & -1 & -3 \\ 0 & \lambda + 1 & 0 \\ 0 & -1 & \lambda - 2 \end{bmatrix}$$

put determinant of $\lambda I - A = 0$

$$|\lambda I - A| = \begin{vmatrix} \lambda - 5 & -1 & -3 \\ 0 & \lambda + 1 & 0 \\ 0 & -1 & \lambda - 2 \end{vmatrix}$$

R_1 $= (\lambda - 5)\,[(\lambda + 1)(\lambda - 2)] = 0$

$= (\lambda - 5)\,[\lambda^2 - 2\lambda + \lambda - 2] = 0$

$= (\lambda - 5)\,(\lambda^2 - \lambda - 2) = 0$

$= (\lambda - 5)\,[\lambda^2 + \lambda - 2\lambda - 2] = 0$

$= (\lambda - 5)\,[\lambda(\lambda + 1) - 2(\lambda + 1)] = 0$

$(\lambda - 5)(\lambda - 2)(\lambda + 1) = 0$

$\boxed{\lambda = 5}$ $\boxed{\lambda = 2}$ $\boxed{\lambda = -1}$

Synthetic division gives same solution $\boxed{\lambda = 5}$

Solving for eigen Vector whe $\boxed{\lambda = 2}$

put $\lambda = 2$ in eq ②

$$\begin{bmatrix} 2-5 & -1 & -3 \\ 0 & 2+1 & 0 \\ 0 & -1 & 2-2 \end{bmatrix} \Rightarrow \begin{bmatrix} -3 & -1 & -3 \\ 0 & 3 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] -\frac{1}{3}R_1 \sim \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \frac{1}{3}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3 + R_2$$

x_3 is free variable put $\boxed{x_3 = s}$

$\boxed{x_3 = s}$

$\boxed{x_2 = 0}$

$x_1 + \frac{1}{3}(0) + s = 0$

$\boxed{x_1 = -s}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -s \\ 0 \\ s \end{pmatrix} \Rightarrow s \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

eigen vector for $\lambda = 2$ is **$V_2 = (-1, 0, 1)$**

Now Solving for eigen vector when $\boxed{\lambda = 5}$ put $\lambda = 5$ in eq (*)

$$\begin{bmatrix} 5-5 & -1 & -3 \\ 0 & 5+1 & 0 \\ 0 & -1 & 5-2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & -1 & -3 \\ 0 & 6 & 0 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & 3 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & -1 & 3 & 0 \end{array}\right] (-1) R_1 \sim \left[\begin{array}{ccc|c} 0 & 1 & 3 & 0 \\ 0 & 0 & -18 & 0 \\ 0 & 0 & 6 & 0 \end{array}\right] \begin{array}{l} R_2 - 6R_1 \\ R_3 + R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 \end{array}\right] -\frac{1}{18} R_2 \sim \left[\begin{array}{ccc|c} 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_1 - 6R_2$$

As $C_3 = R_3 = 0$ and x_1 is free variable

put $x_1 = t$ where $t \in (-\infty, +\infty)$

$\boxed{x_1 = t}$

$\boxed{x_3 = 0}$

$x_2 + 3(0) = 0$

$\boxed{x_2 = 0}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix} \Rightarrow t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

eigen vector for $\lambda = 5$ is $v_1 = (1, 0, 0)$

Solving for eigen vector when $\boxed{\lambda = -1}$

put $\lambda = -1$ in eq ⊛

$$\begin{bmatrix} -1-5 & -1 & -3 \\ 0 & -1+1 & 0 \\ 0 & -1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} -6 & -1 & -3 \\ 0 & 0 & 0 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -6 & -1 & -3 & \\ 0 & -1 & -3 & \\ 0 & 0 & 0 & \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1/6 & 1/2 & 0 \\ 0 & -1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] -\frac{1}{6}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/6 & 1/2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] (-1) R_2$$

x_3 is free variable put $\boxed{x_3 = w}$

$\boxed{x_3 = w}$

$x_2 + 3w = 0$

$\boxed{x_2 = -3w}$

$x_1 + \frac{1}{6}(-3w) + \frac{1}{2}(w) = 0$

$x_1 = \frac{w}{2} - \frac{w}{2}$

$\boxed{x_1 = 0}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -3w \\ w \end{pmatrix} \Rightarrow w \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$$

eigen vector for $\lambda = -1$ is $V_3 = (0, -3, 1)$

Value of Lambda	Corresponding Eigen Vector
$\lambda = 2$	V1=(-1 , 0 , 1)
$\lambda = 5$	V2=(1 , 0 , 0)
$\lambda = -1$	V3=(0 , -3 , 1)