Intro to Probability

February 26, 2019

Data Science CSCI 1951A

Brown University

Instructor: Ellie Pavlick

HTAs: Wennie Zhang, Maulik Dang, Gurnaaz Kaur

Content stolen largely from Dan Potter's 2016 version of this course, which were in turn stolen largely from the 2014 version of MIT's course (18.05)

Announcements

- Projects: You'll be receiving feedback on final project proposals soon. Mentor TA will reach out to schedule an initial check in sometime between March 2 - March 8

- Comments about project scope and such

- MR homework—its okay to change the number of mappers/reducers as long as the output is correct

Today

- Probability Spaces, Probability Functions, Events
- Bayesian Statistics
- Random Variables

Today

- Probability Spaces, Probability Functions, Events
- Bayesian Statistics
- Random Variables

A bit of a
laundry list,
sorry.

Today

- Probability Spaces, Probability Functions, Events
- Bayesian Statistics
- Random Variables

Statistics vs. Prob. Theory

Statistics vs. Prob. Theory

- Probability theory: mathematical theory that describes uncertainty.

Statistics vs. Prob. Theory

- Probability theory: mathematical theory that describes uncertainty.
  - Distributions/parameters are known

Statistics vs. Prob. Theory

- Probability theory: mathematical theory that describes uncertainty.
  - Distributions/parameters are known
- Statistics: techniques for extracting useful information from data.

Statistics vs. Prob. Theory

- Probability theory: mathematical theory that describes uncertainty.
    - Distributions/parameters are known
- Statistics: techniques for extracting useful information from data.
    - Distributions/parameters are generally unknown and need to be estimated from the data

The bigger picture

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

The bigger picture

Whether a coin is heads or tails

What a person will say next

Whether someone will click on an add

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

The bigger picture

You *always* make assumptions about the structure of the process that is generating the data

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

"All models are bad, but some are useful"

The bigger picture

Goal is always to "explain the data"

- Start with real world phenomenon/observations
- Make assumptions about the underlying model
- Fit the parameters of the model based on data

Two typical (different) use cases:

1) understand the underlying model better

2) make predictions

Probability Space

$$\langle \Omega, F, P \rangle$$

Probability Space

$$\langle \Omega, F, P \rangle$$

the set of all
possible
outcomes of
the random
process
modeled

Probability Space

$$\langle \Omega, F, P \rangle$$

A family of sets F representing the allowable events, where each set in F is a subset of the sample space Ω

$$F = \{E_i \subseteq \Omega\}_i$$

Probability Space

$$\langle \Omega, F, P \rangle$$

A family of sets F representing the allowable events, where each set in F is a subset of the sample space Ω

$$F = \{E_i \subseteq \Omega\}_i$$

$$F = 2^{\Omega}$$

Probability Space

$$\langle \Omega, F, P \rangle$$

Probability function which assigns a real number to each event in F

$$P : F \rightarrow \mathbb{R}$$

Probability Space

Valid Probability Function:

$$0 \leq P(E) \leq 1 \ \forall E \in F$$

$$P(\Omega) = 1$$

$$P(\bigcup_i E_i) = \sum_i P(E_i)$$

$$F, P\rangle$$

Probability function which
signs a real number to each
event in F

$$P : F \to \mathbb{R}$$

Over-used Example

Tossing a fair coin once

$$\langle \Omega, F, P \rangle$$

Over-used Example

Tossing a fair coin once

$$\langle \Omega, F, P \rangle$$

{H, T}

Over-used Example

Tossing a fair coin once

$$\langle \Omega, F, P \rangle$$

{H, T} $\quad F = 2^{\Omega}$

Over-used Example

Tossing a fair coin once

$$\langle \Omega, F, P \rangle$$

{H, T}

$$F = 2^{\Omega}$$

{}, {H}, {T}, {H, T}

{} -> 0
{H} -> 0.5
{T} -> 0.5
{H, T} -> ???

sed Example

ng a fair coin once

$$\langle \Omega, F, P \rangle$$

{H, T} $\quad F = 2^{\Omega}$

{}, {H}, {T}, {H, T}

{} -> 0
{H} -> 0.5
{T} -> 0.5
{H, T} -> ???

sed Example

ng a fair coin once

$$\langle \Omega, F, P \rangle$$

{H, T}

$F =$

{}, {H}, {T

Valid Probability Function:

$$0 \leq P(E) \leq 1 \; \forall E \in F$$

$$P(\Omega) = 1$$

$$P(\bigcup_i E_i) = \sum_i P(E_i)$$

{} -> 0
{H} -> 0.5
{T} -> 0.5
{H, T} -> P({H} ∪{T}) = P({H}) + P({T}) = 1

$$\langle \Omega, F, P \rangle$$

{H, T}

$F =$

{}, {H}, {T

Valid Probability Function:

$$0 \leq P(E) \leq 1 \ \forall E \in F$$

$$P(\Omega) = 1$$

$$P(\bigcup_i E_i) = \sum_i P(E_i)$$

Another example

Language modeling: predict what a person will say.

$$\langle \Omega, F, P \rangle$$

Another example

Language modeling: predict what a person will say.

$$\langle \Omega, F, P \rangle$$

English vocabulary

Another example

Language modeling: predict what a person will say.

$$\langle \Omega, F, P \rangle$$

English vocabulary

All sequences of words

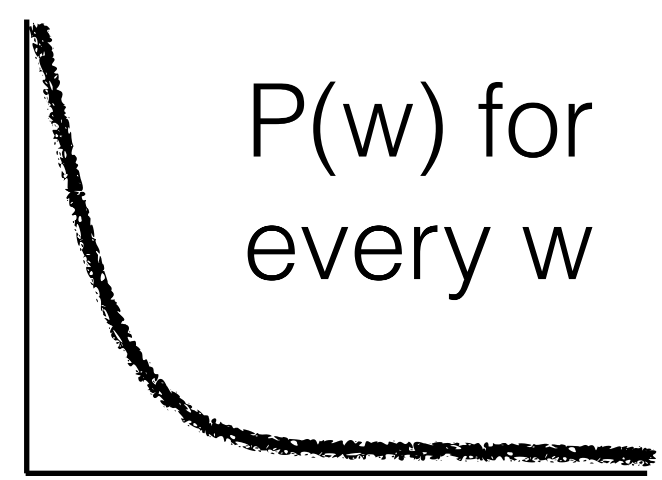
Another example

Language modeling: predict what a person will say.

$$\langle \Omega, F, P \rangle$$

English vocabulary

All sequences of words



P(w) for every w

Events

Events

- Experiment: a repeatable procedure

Events

- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes Ω

Events

- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes Ω
- Event: a subset of the sample space

Events

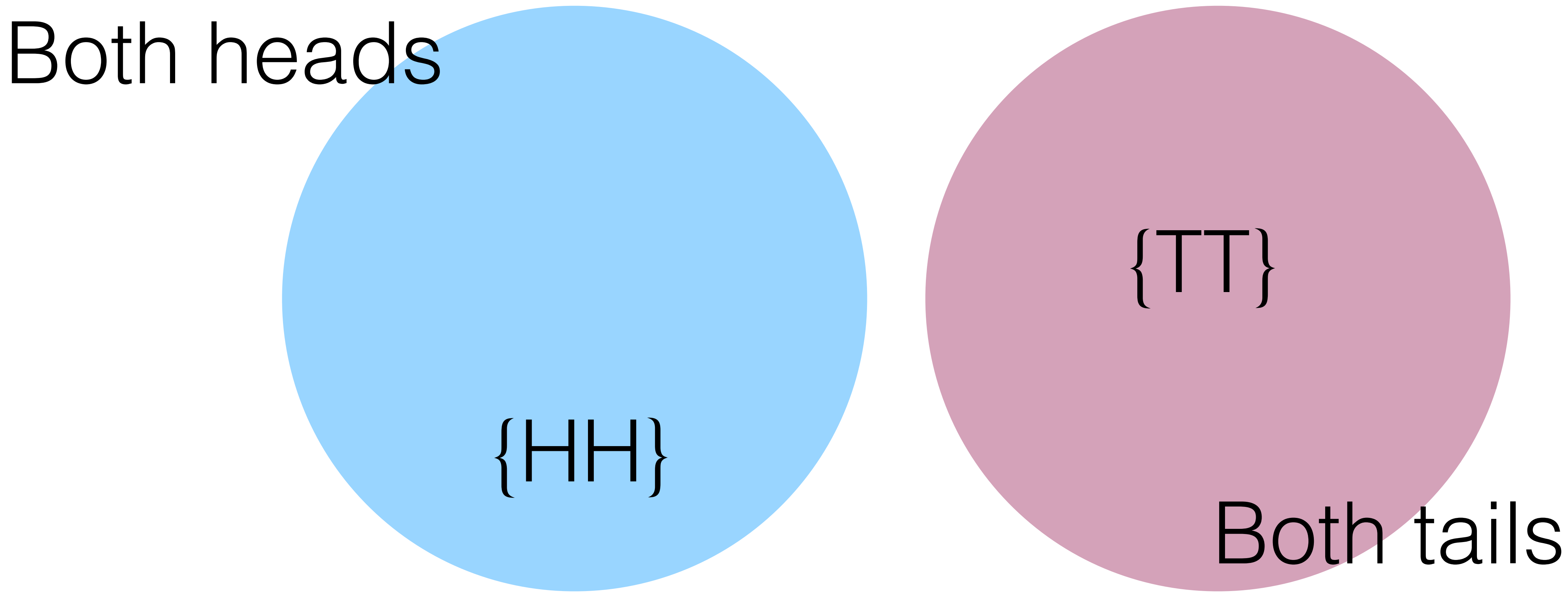
- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes Ω
- Event: a subset of the sample space
- Discrete — listable/countable; can be infinite (e.g. $\{1, 2, 3, 4, \ldots\}$) or finite (e.g. $\{a, b, \ldots z\}$)

Events

- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes Ω
- Event: a subset of the sample space
- Discrete — listable/countable; can be infinite (e.g. $\{1, 2, 3, 4, \ldots\}$) or finite (e.g. $\{a, b, \ldots z\}$)
- Continuous — not discrete :)

Events

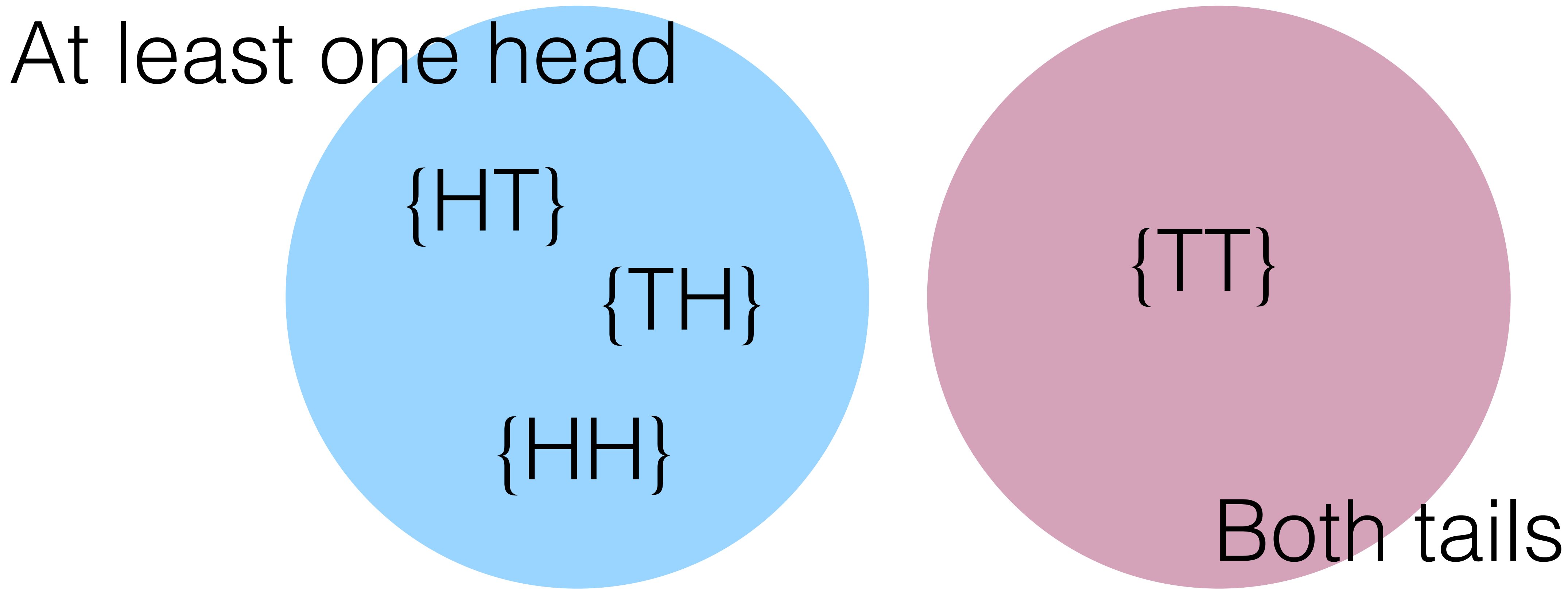
Experiment: Flip a coin once



Disjoint Events

Events

Experiment: Flip a coin once

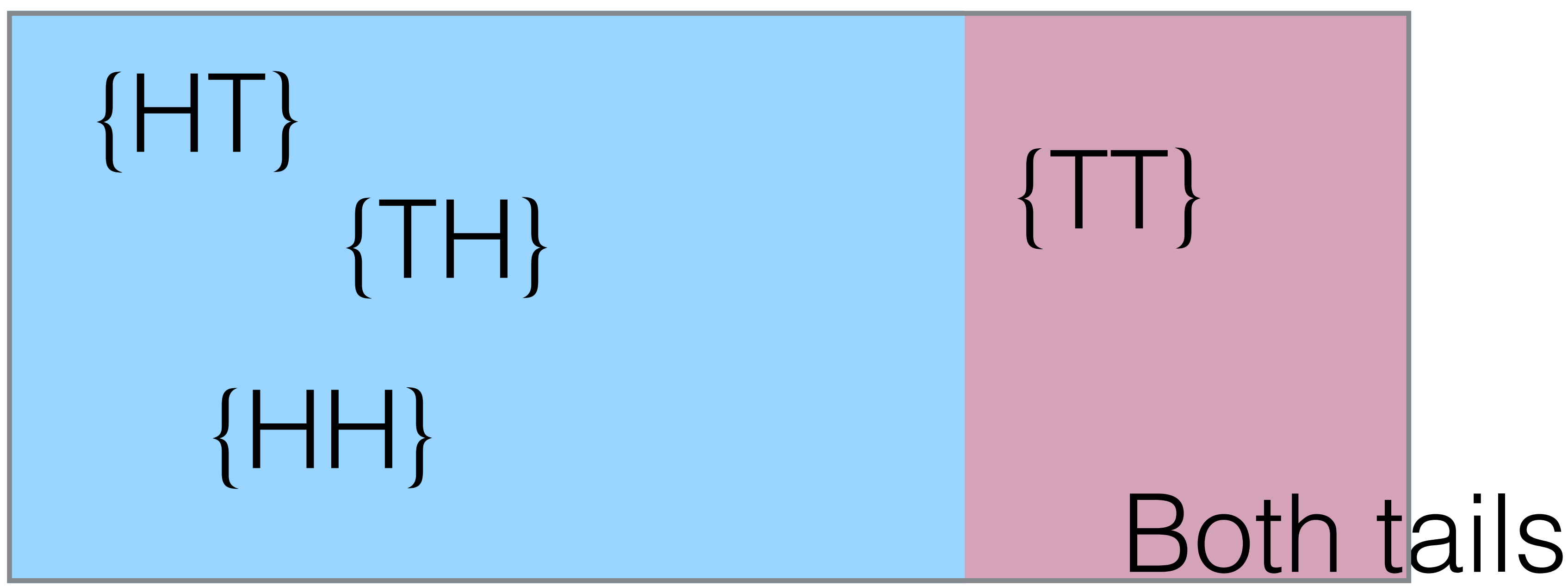


Disjoint Events

Events

Experiment: Flip a coin once

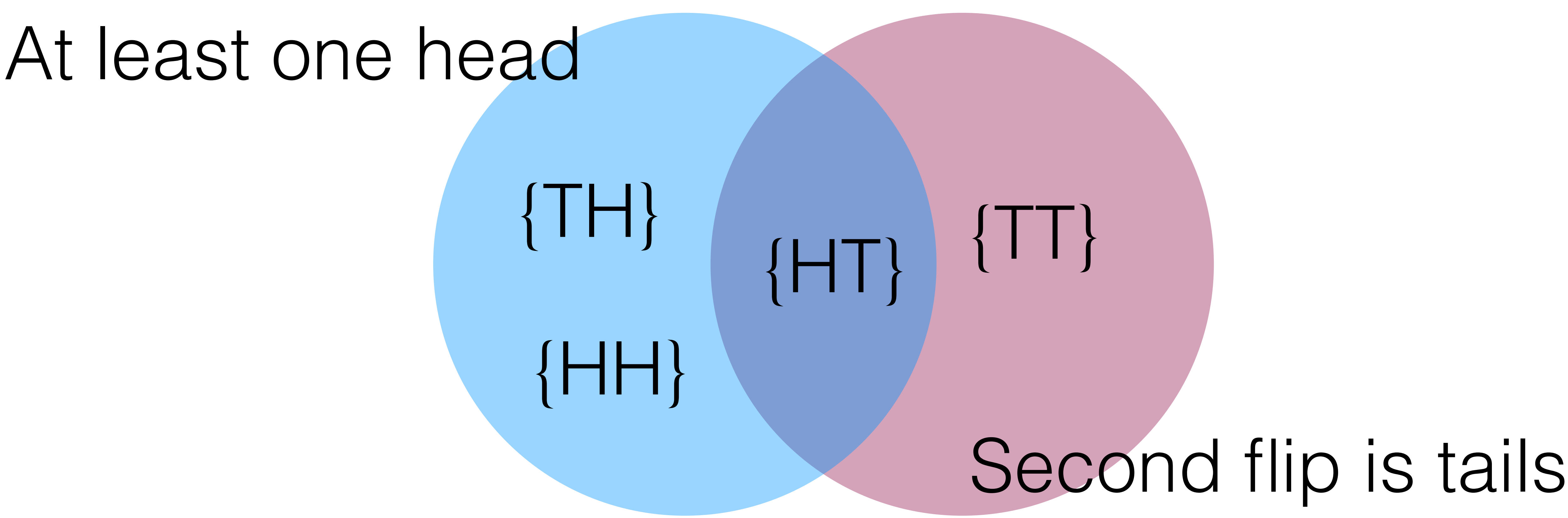
At least one head



Partition

Events

Experiment: Flip a coin once



Inclusion-Exclusion Priciple

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Clicker Question!

Clicker Question!

Experiment: toss a coin 3 times. Which of following equals the event "exactly two heads"?

(a) {THH,HTH,HHT,HHH}
(b) {HHT}
(c) {THH, HTH, HHT}

Clicker Question!

Experiment: toss a coin 3 times. Which of following equals the event "exactly two heads"?

(a) {THH,HTH,HHT,HHH}
(b) {HHT}
(c) {THH, HTH, HHT}

Clicker Question!

We have a class of 50 students. 25 are male and 20 have brown eyes. If we randomly select a student, what bounds can we put on the probability that they are male *or* have brown eyes?

(a) less than 0.4
(b) between 0.4 and 0.5
(c) between 0.4 and 0.9
(d) between 0.5 and 0.9
(e) greater than 0.5

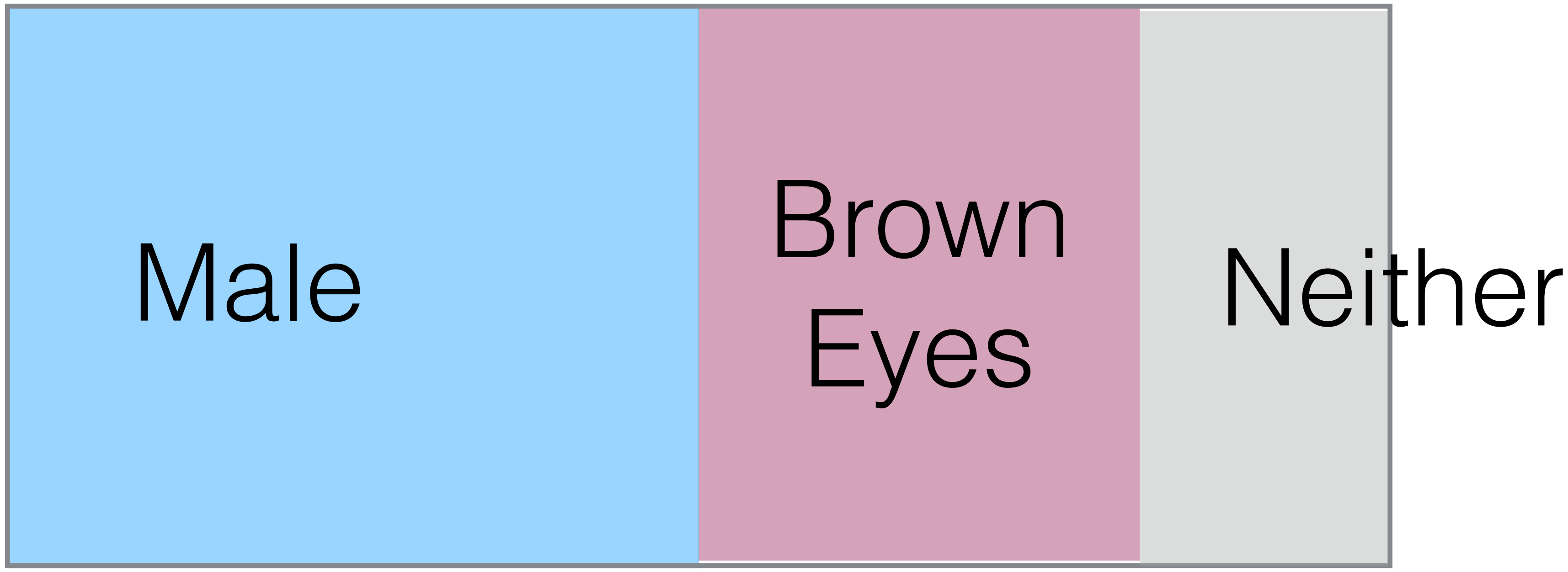
Clicker Question!

We have a class of 50 students. 25 are male and 20 have brown eyes. If we randomly select a student, what bounds can we put on the probability that they are male *or* have brown eyes?

(a) less than 0.4
(b) between 0.4 and 0.5
(c) between 0.4 and 0.9
(d) between 0.5 and 0.9
(e) greater than 0.5

Clicker Question!

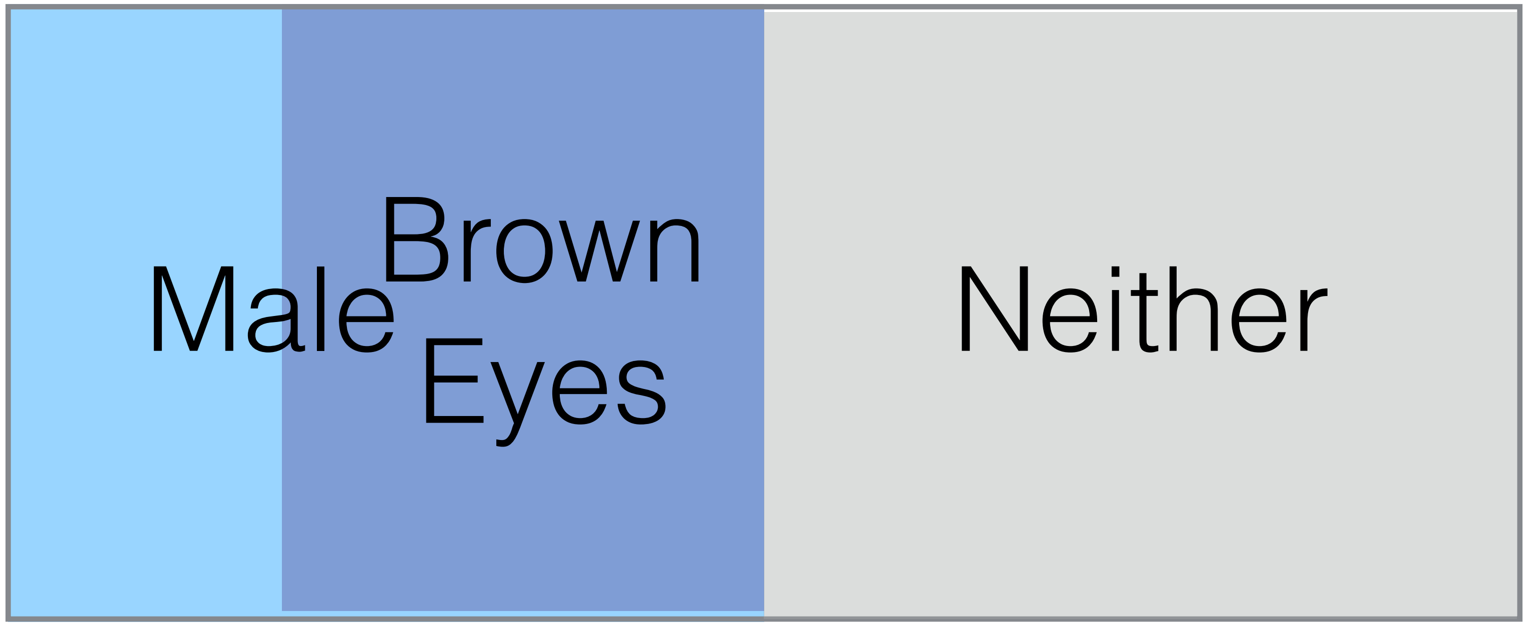
We have a class of 50 students. 25 are male and 20 have brown eyes. If we randomly select a student, what bounds can we put on the probability that they are male *or* have brown eyes?



$p = 45/50 = 90\%$

Clicker Question!

We have a class of 50 students. 25 are male and 20 have brown eyes. If we randomly select a student, what bounds can we put on the probability that they are male *or* have brown eyes?



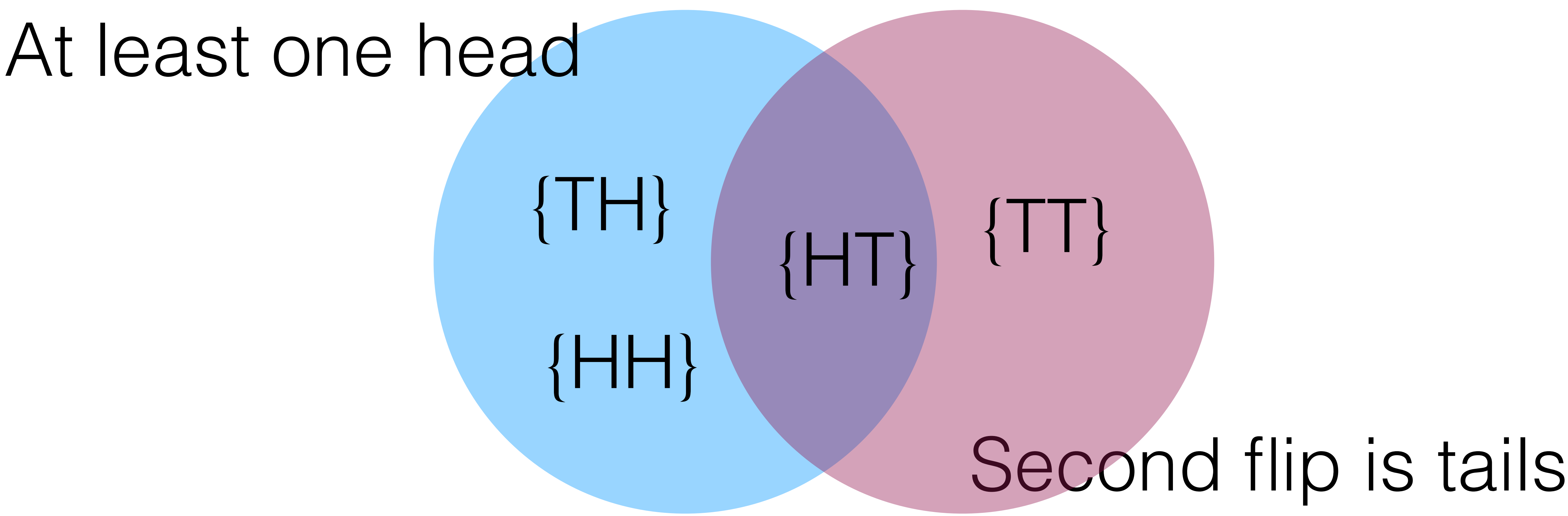
p = 25/50 = 50%

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability

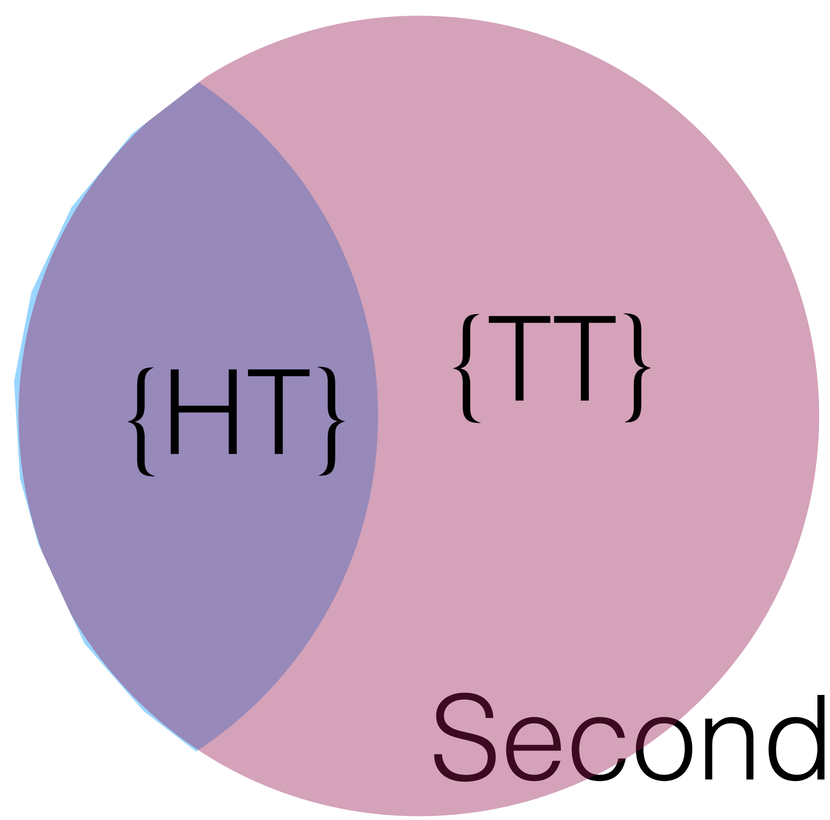
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

At least one head



Second flip is tails

Clicker Question!

Clicker Question!

Toss a coin 4 times.
Let A = ‘at least three heads’.
Let B = ‘first toss is tails’
What is P(A|B)?

(a) 1/16 (c) 1/4
(b) 1/8 (d) 1/5

Clicker Question!

Toss a coin 4 times.
Let A = 'at least three heads'.
Let B = 'first toss is tails'
What is P(A|B)?

(a) 1/16 (c) 1/4
(b) 1/8 (d) 1/5

Clicker Question!

Toss a coin 4 times.
Let A = 'at least three heads'.
Let B = 'first toss is tails'
What is $P(A|B)$?

(a) 1/16 (c) 1/4
(b) 1/8 (d) 1/5

HHHH	HHHT	HTTH	THTT
THHH	TTHH	HTHT	TTHT
HTHH	THTH	HHTT	TTTH
HHTH	THHT	HTTT	TTTT

Clicker Question!

Toss a coin 4 times.

Let A = 'at least three heads'.

Let B = 'first toss is tails'

What is P(A|B)?

(a) 1/16 (c) 1/4

(b) 1/8 (d) 1/5

HHHH	HHHT	HTTH	THTT
THHH	TTHH	HTHT	TTHT
HTHH	THTH	HHTT	TTTH
HHTH	THHT	HTTT	TTTT

Clicker Question!

Toss a coin 4 times.
Let A = ‘at least three heads’.
Let B = ‘first toss is tails’
What is P(B|A)?

(a) 1/16 (c) 1/4
(b) 1/8 (d) 1/5

HHHH	HHHT	HTTH	THTT
THHH	TTHH	HTHT	TTHT
HTHH	THTH	HHTT	TTTH
HHTH	THHT	HTTT	TTTT

Clicker Question!

Toss a coin 4 times.

Let A = 'at least three heads'.

Let B = 'first toss is tails'

What is P(B|A)?

(a)	1/16	(c)	1/4
(b)	1/8	(d)	1/5

HHHH	HHHT	HTTH	THTT
THHH	TTHH	HTHT	TTHT
HTHH	THTH	HHTT	TTTH
HHTH	THHT	HTTT	TTTT

Independence

Events A and B are independent if the probability that one occurred is not affected by knowledge that the other occurred.

$$\Updownarrow$$

$$P(A \cap B) = P(A)P(B)$$

$$\Updownarrow$$

$$P(A|B) = P(A)$$

$$\Updownarrow$$

$$P(B|A) = P(B)$$

Independence

Events A and B are independent if the probability that one occurred is not affected by knowledge that the other occurred.

$$\Updownarrow$$

$$Pr(\bigcap_i E_i) = \prod_i Pr(E_i)$$

Clicker Question!

Clicker Question!

Roll two dice. Are the following events independent?
Let A = ‘first die is 3’.
Let B = ‘sum is 6’

(a) Yes
(b) No
(c) Neither :)

Clicker Question!

Roll two dice. Are the following events independent?

Let A = ‘first die is 3’.

Let B = ‘sum is 6’

(a) Yes
(b) No
(c) Neither :)

Clicker Question!

Roll two dice. Are the following events independent?
Let A = ‘first die is 3’.
Let B = ‘sum is 6’

$P(A) = 1/6$

(a) Yes
(b) No
(c) Neither :)

A: 3,1 3,2 3,3 3,4 3,5 3,6

Clicker Question!

Roll two dice. Are the following events independent?
Let A = ‘first die is 3’.
Let B = ‘sum is 6’

P(A) = 1/6
P(B) = 5/36

(a) Yes
(b) No
(c) Neither :)

A: 3,1 3,2 3,3 3,4 3,5 3,6

B: 1,5 2,4 3,3 4,2 5,1

Clicker Question!

Roll two dice. Are the following events independent?
Let A = 'first die is 3'.
Let B = 'sum is 6'

P(A) = 1/6
P(B) = 5/36
P(B|A) = 1/6 != P(B)

(a) Yes
(b) No
(c) Neither :)

A: 3,1 3,2 3,3 3,4 3,5 3,6
B: 1,5 2,4 3,3 4,2 5,1

Clicker Question!

Stan has two kids. One of his kids is a boy. What is the likelihood that the other one is also a boy?

(a) 0 (b) 1 (c) 1/2 (d) 1/3

Clicker Question!

Stan has two kids. One of his kids is a boy. What is the likelihood that the other one is also a boy?

(a) 0 (b) 1 (c) 1/2 (d) 1/3

Clicker Question!

Stan has two kids. One of his kids is a boy. What is the likelihood that ~~the other one is also a boy~~?

both children are boys given that at least one is a boy

(a) 0 (b) 1 (c) 1/2 (d) 1/3

Clicker Question!

Stan has two kids. One of his kids is a boy. What is the likelihood that ~~the other one is also a boy~~?

both children are boys given that at least one is a boy

(a) 0 (b) 1 (c) 1/2 (d) 1/3

Clicker Question!

Stan has two kids. One of his kids is a boy. What is the likelihood that ~~the other one is also a boy~~?

both children are boys given that at least one is a boy

(a) 0 (b) 1 (c) 1/2 (d) 1/3

BB	GB
BG	GG

Clicker Question!

Stan has two kids. One of his kids is a boy. What is the likelihood that ~~the other one is also a boy~~?

child 2 is a boy given that child 1 is a boy

(a) 0 **(b)** 1 **(c)** 1/2 **(d)** 1/3

Clicker Question!

Stan has two kids. One of his kids is a boy. What is the likelihood that ~~the other one is also a boy~~?

child 2 is a boy given that child 1 is a boy

(a) 0 (b) 1 (c) 1/2 (d) 1/3

BB	GB
BG	GG

Today

- Probability Spaces, Probability Functions, Events
- Bayesian Statistics
- Random Variables

Bayes Rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes Rule

Likelihood Prior

$$\boxed{P(A|B)} = \frac{\boxed{P(B|A)}\boxed{P(A)}}{\boxed{P(B)}}$$

Posterior

Marginal

Bayes Rule

You’re a doctor and are considering adopting a test for a rare disease. The disease is found in 100 of every million patients. The test expensive to administer, but is advertised as 99% accurate! Specifically:

- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

Bayes Rule

- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

	Disease	No Disease	
Positive Test	99	9,999	10,098
Negative Test	1	989,901	989,902
	100	999,900	1,000,000

Bayes Rule

P(correct) = (99 + 989,901) / 1,000,000 = 99%

	Disease	No Disease	
Positive Test	99	9,999	10,098
Negative Test	1	989,901	989,902
	100	999,900	1,000,000

Bayes Rule

P(correct|positive) = 99 / 9,999 = 1%

	Disease	No Disease	
Positive Test	99	9,999	10,098
Negative Test	1	989,901	989,902
	100	999,900	1,000,000

Bayes Rule

- 100 out of 1,000,000 have the disease
- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

$$P(disease|positive\_test) = \frac{P(positive\_test|disease)P(disease)}{P(positive\_test)}$$

Bayes Rule

- 100 out of 1,000,000 have the disease
- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

$$P(disease|positive\_test) = \frac{P(positive\_test|disease)P(disease)}{P(positive\_test)}$$

Bayes Rule

- 100 out of 1,000,000 have the disease
- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

$$P(disease|positive\_test) = \frac{P(positive\_test|disease)P(disease)}{P(positive\_test)}$$

Bayes Rule

- 100 out of 1,000,000 have the disease
- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

$$P(disease|positive\_test) = \frac{P(positive\_test|disease)P(disease)}{P(positive\_test)}$$

$$P(positive\_test|disease)P(disease) + P(positive\_test|\neg disease)P(\neg disease)$$

Bayes Rule

- 100 out of 1,000,000 have the disease
- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

$$P(disease|positive\_test) = \frac{P(positive\_test|disease)P(disease)}{P(positive\_test)}$$

$$\boxed{P(positive\_test|disease)P(disease) + P(positive\_test|\neg disease)P(\neg disease)}$$

Bayes Rule

- 100 out of 1,000,000 have the disease
- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

$$P(disease|positive\_test) = \frac{\overset{0.99}{P(positive\_test|disease)}\overset{0.0001}{P(disease)}}{P(positive\_test)}$$

$$\boxed{\underset{0.99}{P(positive\_test|disease)}\underset{0.0001}{P(disease)} + \underset{0.01}{P(positive\_test|\neg disease)}\underset{0.9999}{P(\neg disease)}}$$

Bayes Rule

- 100 out of 1,000,000 have the disease
- When the patient has the disease, the test is positive 99% of the time.
- When the patient does not have the disease, the test is positive only 1% of the time.

$$P(disease|positive\_test) = 0.01$$

Clicker Question!

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

What is the probability that the first coin was the biased one?

(a) 1/3 (b) 2/5 (c) 2/3 (d) 1

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

What is the probability that the first coin was the biased one?

(a) 1/3 (b) 2/5 (c) 2/3 (d) 1

E_i = i^{th} coin flip is the biased one
B = observed event (i.e. HHT)

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{\sum_i^3 P(B|E_i)P(E_i)}$$

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

What is the probability that the first coin was the biased one?

(a) 1/3 (b) 2/5 (c) 2/3 (d) 1

E_i = i^{th} coin flip is the biased one
B = observed event (i.e. HHT)

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{\sum_i^3 P(B|E_i)P(E_i)}$$

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

E_i = i^{th} coin flip is the biased one
B = observed event (i.e. HHT)

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{\sum_i^3 P(B|E_i)P(E_i)}$$

P(B|E1) = 2/3 * 1/2 * 1/2 = 1/6

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

E_i = i^{th} coin flip is the biased one
B = observed event (i.e. HHT)

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{\sum_i^3 P(B|E_i)P(E_i)}$$

P(B|E1) = 2/3 * 1/2 * 1/2 = 1/6

P(E1) = 1/3

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

E_i = i^{th} coin flip is the biased one
B = observed event (i.e. HHT)

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{\sum_i^3 P(B|E_i)P(E_i)}$$

P(B|E1) = 2/3 * 1/2 * 1/2 = 1/6

P(E1) = 1/3

P(B|E1) = P(B|E2) = 1/6
P(B|E3) = 1/2 * 1/2 * 1/3 = 1/12

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

E_i = i^{th} coin flip is the biased one
B = observed event (i.e. HHT)

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{\sum_i^3 P(B|E_i)P(E_i)}$$

$$\frac{1/6 \times 1/3}{(1/6 \times 1/3) + (1/6 \times 1/3) + (1/12 \times 1/3)}$$

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

E_i = i^{th} coin flip is the biased one
B = observed event (i.e. HHT)

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{\sum_i^3 P(B|E_i)P(E_i)}$$

$$\frac{1/6 \times 1/3}{5/36}$$

Clicker Question!

We are given three coins, two of them are fair and one is biased (landing heads with probability 2/3). You want to identify the the biased coin. We flip each of the coins once. The first and second come up heads, and the third comes up tails.

E_i = i^{th} coin flip is the biased one
B = observed event (i.e. HHT)

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{\sum_i^3 P(B|E_i)P(E_i)}$$

$$\frac{2}{5}$$

Today

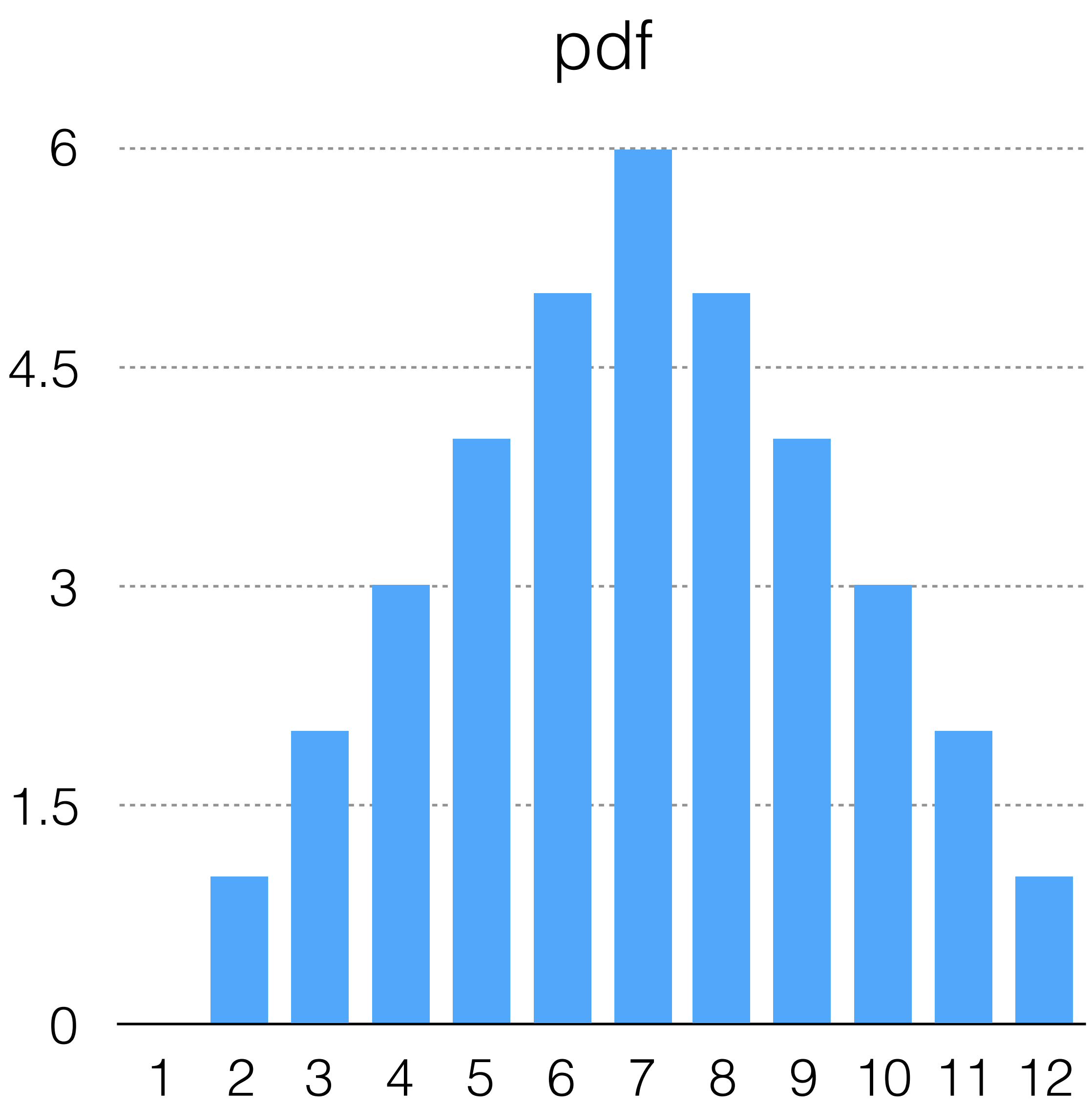
- Probability Spaces, Probability Functions, Events
- Bayesian Statistics
- Random Variables

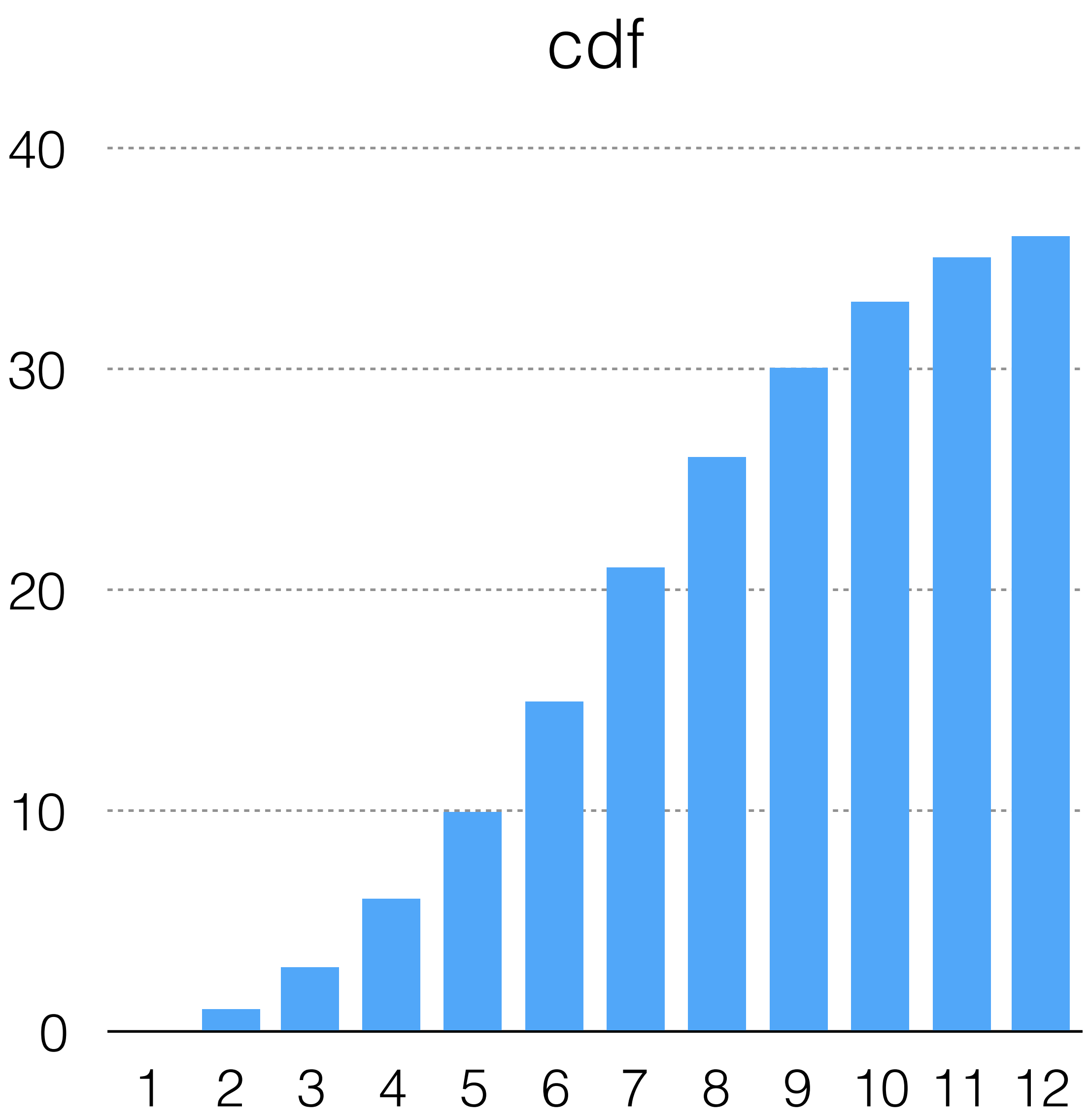
Random Variables

- Random variable X assigns a number to each outcome: $X : \Omega \to \mathbb{R}$
- Use $X = a$ to mean the event $\{\omega | X(\omega) = a\}$
- Probability mass function (pmf) gives probability that X takes the value a: $p(a) = Pr(X = a)$
- Cumulative distribution function (cdf) gives probability that X takes any value up to a: $F(a) = Pr(X \leq a)$

Random Variables

X = sum of two dice





Clicker Question!

Clicker Question!

X is a random variable with the below cdf.

X	1	2	3	4
cdf F(a)	0.5	0.75	0.9	1

What is $P(X \leq 3)$?

(a) 0 (b) 0.15 (c) 0.9 (d) 1

Clicker Question!

X is a random variable with the below cdf.

X	1	2	3	4
cdf F(a)	0.5	0.75	0.9	1

What is $P(X \leq 3)$?

(a) 0 (b) 0.15 (c) 0.9 (d) 1

Clicker Question!

X is a random variable with the below cdf.

X	1	2	3	4
cdf F(a)	0.5	0.75	0.9	1

What is P(X=3)?

(a) 0 (b) 0.15 (c) 0.9 (d) 1

Clicker Question!

X is a random variable with the below cdf.

X	1	2	3	4
cdf F(a)	0.5	0.75	0.9	1

What is P(X=3)?

(a) 0 (b) 0.15 (c) 0.9 (d) 1

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

$$\downarrow\uparrow$$

$$E(X) = \int_i x_i Pr(x_i)$$

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1

0.5

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1

0.5 + 0.5

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1

0.5 + 0.5 + 0.45

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1

0.5 + 0.5 + 0.45 + 0.4

Expected Value

$$E(X) = \sum_i x_i Pr(x_i)$$

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1

0.5 + 0.5 + 0.45 + 0.4 = 1.85

Variance

$$Var(X) = E((X - E(X))^2)$$

Variance

$$Var(X) = E((X - E(X))^2)$$

E(X) = 1.85

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1

Variance

$$Var(X) = E((X - E(X))^2)$$

E(X) = 1.85

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15

Variance

$$Var(X) = E((X - E(X))^2)$$

E(X) = 1.85

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

Variance

$$Var(X) = E((X - E(X))^2)$$

E(X) = 1.85

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

0.361

Variance

$$Var(X) = E((X - E(X))^2)$$

E(X) = 1.85

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

0.361 + .006

Variance

$$Var(X) = E((X - E(X))^2)$$

E(X) = 1.85

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

0.361 + .006 + 0.198

Variance

$$Var(X) = E((X - E(X))^2)$$

E(X) = 1.85

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

0.361 + .006 + 0.198 + 0.462

Variance

$$Var(X) = E((X - E(X))^2)$$

E(X) = 1.85

X	1	2	3	4
pdf	0.5	0.25	0.15	0.1
X - E(X)	-0.85	0.15	1.15	2.15
$(X - E(X))^2$	0.722	0.023	1.32	4.62

0.361 + .006 + 0.198 + 0.462 = 1.027

Interpreting Expectation

Would you accept a gamble that offers a 10% chance to win $95 and a 90% chance of losing $5?

Interpreting Expectation

Would you accept a gamble that offers a 10% chance to win $95 and a 90% chance of losing $5?

$$E(\text{Payoff}) = (95 \times 0.10) - (5 \times 0.9)$$

Interpreting Expectation

Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance of losing \$5?

$$E(\text{Payoff}) = (95 \times 0.10) - (5 \times 0.9)$$

$$E(\text{Payoff}) = (9.5) - (4.5)$$

Interpreting Expectation

Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance of losing \$5?

$$E(\text{Payoff}) = (95 \times 0.10) - (5 \times 0.9)$$

$$E(\text{Payoff}) = (9.5) - (4.5)$$

$$E(\text{Payoff}) = 5$$

Clicker Question!

Clicker Question!

How much would you pay for a lottery ticket that offers a 10% percent chance of winning $100 and a 90% chance of winning nothing?

(a) $0
(b) no more than $2
(c) no more than $5
(d) no more than $10

Clicker Question!

How much would you pay for a lottery ticket that offers a 10% percent chance of winning \$100 and a 90% chance of winning nothing?

(a) \$0
(b) no more than \$2
(c) no more than \$5
(d) no more than \$10

Clicker Question!

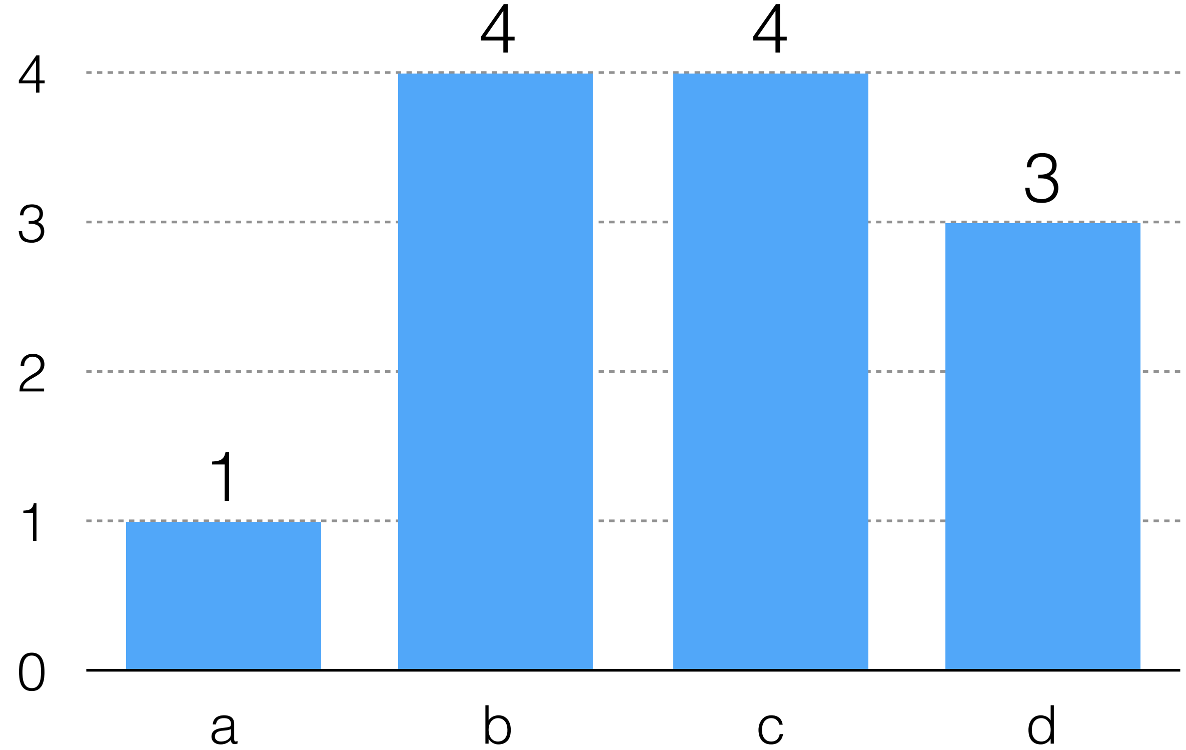
How much would you pay for a lottery ticket that offers a 10% percent chance of winning $100 and a 90% chance of winning nothing?

(a) $0
(b) no more than $2
(c) no more than $5
(d) no more than $10

$$0 = 0.1(100 - \text{cost}) - 0.9(\text{cost})$$
$$0 = 10 - \text{cost}$$
$$\text{cost} = 10$$

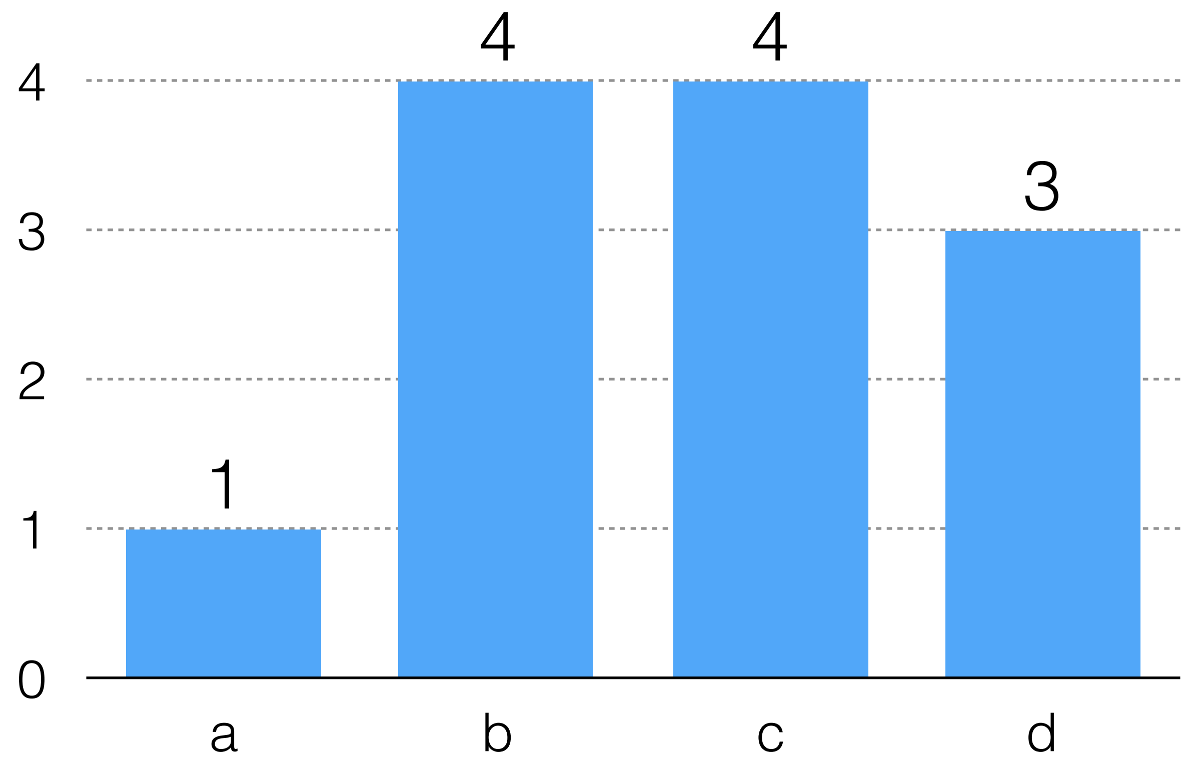
Gaming Clicker Questions!

Are the answers to my clicker questions random?



Gaming Clicker Questions!

Are the answers to my clicker questions random?



…to be continued…

Okay then