NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
Department of Mathematics
MA2002D Mathematics IV-Tutorial sheet II - Winter Semester 2018-2019
Part B

1. Find the image of the square region with vectors (0,0), (1,0), (1,1) and (0,1) under the transformation $w = 2z - i$.
2. Find the image of the rectangular region bounded by $x = 0$, $y = 0$, $x = 2$, $y = 1$ under the (i) translation $w = z + (1-2i)$ (ii) rotation $w = iz$ (iii) transformation $w = (1 + i)z + (2 - i)$.
3. Find the image of the region $y > 1$ under the transformation $w = iz + 1$.
4. Show that by means of the inversion $w = \frac{1}{z}$, the circle given by $|z-2| = 7$ is mapped into the circle $\left|w + \frac{2}{45}\right| = 7/45$.
5. Find the image of the triangle with vertices i, 1+ i, 1-i in the z – plane under the transformation $w = 3z + 4 - 2i$.
6. Find the image of the following regions under $w = \frac{1}{z}$ (i) the strip $0 < y < \frac{1}{2}$, (ii) the circle, $|z-3i| = 3$.
7. Show that $w = \frac{z-1}{z+1}$ maps the half plane $x \geq 0$ on to the unit circle $|w| \leq 1$. Show also that this transformation maps the half plane $y \geq 0$ on to the half plane $v \geq 0$.
8. Find the region in the w – plane in to which the region $\frac{1}{2} \leq y \leq 1$ is mapped by the transformation $w = z^2$.
9. Under $w = \frac{1}{z}$, find the image of (i) $|z-2i| = 2$ (ii) $\frac{1}{4} \leq y \leq \frac{1}{2}$. Also show the regions graphically.
10. Show that $w = \frac{2z+3}{z-4}$ maps $x^2 + y^2 - 4x = 0$ on to $4u + 3 = 0$.
11. Determine the region of w – plane in to which the first quadrant of z – plane is mapped under the transformation $w = z^2$.
12. Show that $w = \frac{z-i}{z+i}$ maps real axis in z – plane in to $|w| = 1$. What portion of the z – plane corresponds to the interior of the circle in the w – plane.
13. Find the images of $x = 0$, $x = 1$, $y = 0$ and $y = 1$ under $w = z^2$.
14. Find the bilinear transformation which maps

$$\left.\begin{array}{rlll}
\text{i.} & z = 1, -i, -1 & \text{in to} & w = 2, 0, -2 \\
\text{ii.} & z = 1, i, -1 & \text{in to} & w = 0, 1, \infty \\
\text{iii.} & z = -1, 1, \infty & \text{in to} & w = -i, -1, i \\
\text{iv.} & z = 0, 1, i & \text{in to} & w = \frac{-1}{2}, 0, -1+i \\
\text{v.} & z = 0, -1, \infty & \text{in to} & w = -1, -2-i, i \\
\text{vi.} & z = 1+i, -i, 2-i & \text{in to} & w = 0, 1, i \\
\text{vii.} & z = \infty, 1, -1 & \text{in to} & w = 1, \frac{3+2i}{5}, 3-2i \\
\text{viii.} & z = 2, i, -2 & \text{in to} & w = 1, i, -1 \\
\text{ix.} & z = \infty, 0, -1 & \text{in to} & w = 1, 0, \frac{1+i}{2}
\end{array}\right\} \text{respectively}$$

15. Evaluate the following complex line integrals.

(i) $\int_C \operatorname{Re} z\, dz$ where C is the straight line path from 1+ i to 3 + 2i.

(ii) $\int_0^{2+i} \bar{z}^2 dz$ along the following: (a) the line y = x/2 (b) the real axis to 2 and then vertically to (2+i) and

(c) the parabola $2y^2 = x$.

(iii) $\int_C z^2 dz$ where C is the parabola $y = x^2$ from (0, 0) to (2, 4).

(iv) $\int_C (5z^4 - z^3 + 2) dz$ around the square with vertices (0, 0), (3, 0), (3, 3) and (0, 3).

(v) $\int_C (z^2 + 2) dz$ where C is the boundary of the triangle with vertices 0, 2 and (2+i).

(vi) $\int_C \bar{z}^2 dz$ along the circle $|z| = 3$.

16. Verify Cauchy's theorem for the following integrals.

(i) $\int_C 3z^2 - 2z + 4)\, dz$ where C is the boundary of the square with vertices at $2 \pm 2i$, $-2 \pm 2i$,

(ii) $\int_C z^2 - 2z + 3)\, dz$ where C is $|z| = 2$.

(iii) $\int_C z^2 dz$ where C is the boundary of the triangle with vertices 0, 2, 2i.

17. Evaluate the following integrals.

(i) $\int_C \frac{(z^2+1)dz}{z(2z+1)}$, $C: \ |z| = 1$

(ii) $\int_C \frac{(2z+1)dz}{z(z+2)(z-3)}$, $C: \ |z| = 3$

(iii) $\int_C \frac{dz}{(z-3)^3(z-3)(z-4)}$, $C: \ |z-1| = 1$

(iv) $\int_C \frac{e^{-2z}dz}{(z+3)^4}$, $C: |z| = 4$

(v) $\int_C \frac{(5z^2+2z)dz}{(z-2)^3(z-1)(z+4)}$, $C: \ |z| = 3$

(vi) $\int_C \frac{dz}{(z+1)(z-1)^2}$, $C: \ |z-1| = 1$

(vii) $\int_C \frac{(z+1)dz}{z(z-1)(z-2)}$, where C is (a) $\left|z - \frac{1}{2}\right| = \frac{1}{4}$, (b) $|z-2| = 1/2$

(viii) $\int_C \frac{(3z^2+2z-4)dz}{(z^3-4z)}$, $C: \ |z-i| = 3$

(ix) $\int_C \frac{(z^3+1)dz}{(3z+1)^3}$, $C: \ |z| = 1$

4. (i) Obtain Taylor series expansion of $\frac{z}{z-3}$ about z = 1. What is the region of convergence?

(ii) Find the Taylor series expansion of sin z about $z = \frac{\pi}{4}$. Also find the Maclaurin series expansion of sin z.

(iii) Find Taylor or Laurent series expansions of $\frac{z+3}{z(z^2-z-2)}$ when (a) $|z| < 1$ (b) $1 < |z| < 2$ and (iii) $|z| > 2$.

(iv) Find Taylor or Laurent series expansions of $\frac{z^2-z+5}{(z+2)\,(z+3)}$ that are valid when

(a) $|z| < 2$ (b) $2 < |z| < 3$ and (c) $|z| > 2$.

(v) Expand $(z^2-1)(z^2+5z+6)^{-1}$ as a Laurent series in $2 < |z| < 3$.