Name:________________________ Roll number:________________ Semester and Section:___

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
MID SEMESTER TEST-I WINTER 2018

CS 2005 DATA STRUCTURES AND ALGORITHMS

**MAXIMUM MARKS: 20** **TIME : 1 HOUR** **DATE 29/1/2018**

## PART-A

| QUESTION NUMBER | I | II | III | IV | V | TOTAL MARKS |
|---|---|---|---|---|---|---|
| MARKS OBTAINED | | | | | | |

1. WRITE ONLY IN THE SPACE PROVIDED. ADDITIONAL SHEETS AND ROUGH WORK SPACE WOULD NOT BE VALUED
2. PROOFS SHOULD BE VALID, SOUND, LOGICAL AND CLEAR TO GET CREDITS
3. Conversing, exchanging documents and gadgets and all other forms of suspicable behavior would be appropriately penalized. Academic Integrity violations would lead to a zero for the test, and further penalty after subsequent enquiries by committees specially constituted for the purpose
4. Maximum marks for each question are given in the boxes adjacent to the questions

1. Is $3^n = O(2^n)$? Justify your answer. [1 Mark]

False.

Proof by contradiction.

Assume that $3^n = O(2^n)$. Then $\exists c : 3^n \leq c \cdot 2^n \; \forall n \geq n_0$

ie $(3/2)^n \leq c \quad \forall n \geq n_0$

$\Rightarrow n \leq \log_{3/2} c \quad \forall n > n_0$

$\Rightarrow \log_{3/2} c + n_0 \leq \log_{3/2} c$

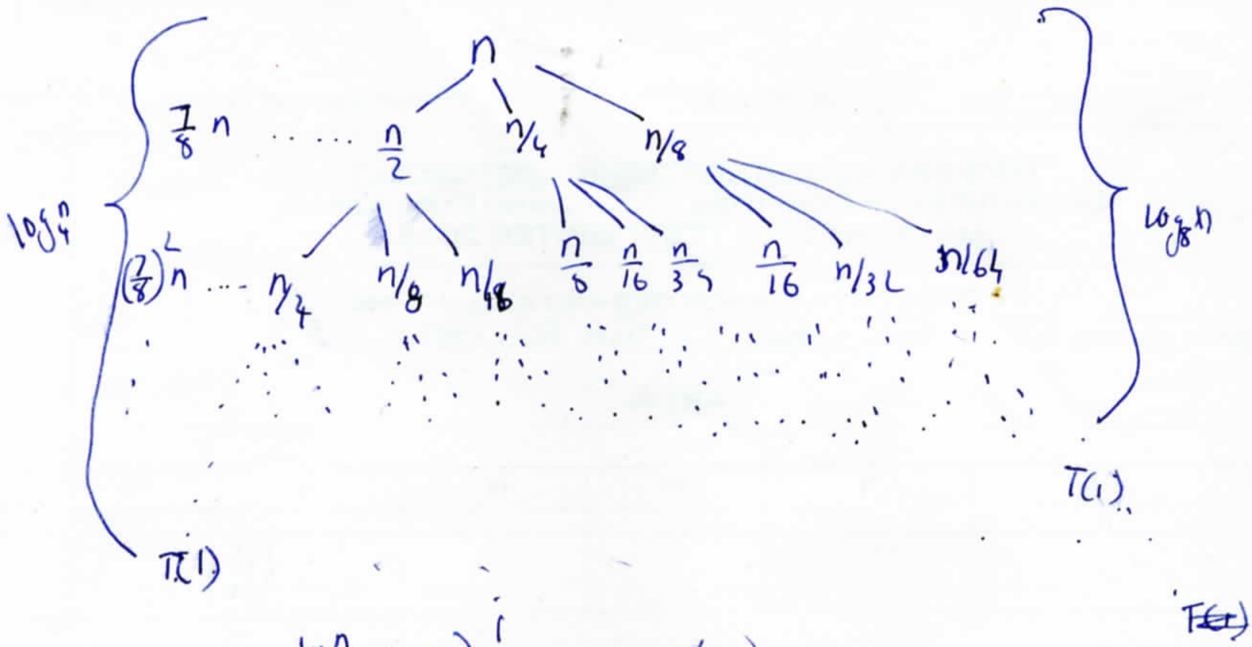
↳ Contradiction. Hence the assumption is not true

2.Give asymptotical upper and lower bounds for the following recurrence [1 Mark]

ii) T(n)= T(n/2)+T(n/4)+T(n/8)+n

(Use the other side of this sheet to answer this question)

Assume $T(1) = 1$.




$$\sum_{i=1}^{\log n} \left(\frac{7}{8}\right)^i n \quad \cancel{= \Theta(n)}.$$

Upper bound :

$$\sum_{i=0}^{\log_2 n} (7/8)^i n \le \sum_{i=1}^{\infty} (7/8)^i n = \frac{1}{1-7/8} n = 8n = O(n).$$

Lower bound.

$$\sum_{i=0}^{\log_8 n} \left(\frac{7}{8}\right)^i n = n \sum_{i=1}^{\log_8 n} \left(\frac{7}{8}\right)^i = n \cdot \frac{1-(7/8)^{\log_8 n}}{1/8}$$

$$= 8n\left(1 - \frac{7^{\log_8 n}}{n}\right) = \Theta(n)$$

Hence, $T(n) = \Theta(n)$

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3. Use recursion tree method to find a good asymptotic upper bound for the following recurrence.
T(n)=2T(n-2)+ 2 [1 Mark]

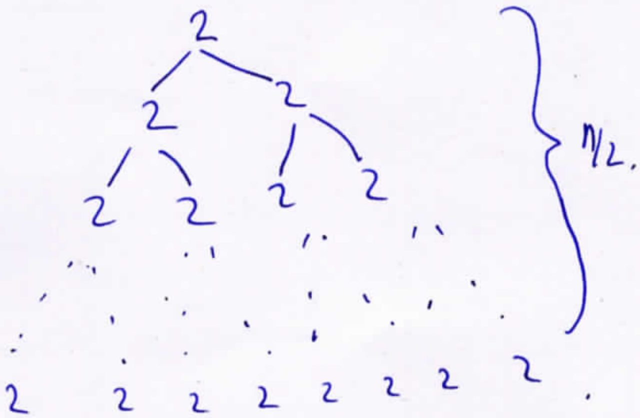
Assume $T(1) = 2$.
$T(0) = 2$

$2^0 \times 2$

$2^1 \times 2$

$2^2 \times 2$

$2^{n/2} \times 2$.




$$T(n) = \sum_{i=0}^{\lfloor n/2 \rfloor} 2 \cdot 2^i$$

$$= \sum_{i=0}^{\lfloor n/2 \rfloor} 2^{i+1}$$

$$= 2^{\lfloor n/2 \rfloor + 1} - 2$$

$$= O(2^{n/2})$$

i)Prove that your answer also works as an asymptotic lower bound using substitution method.

[2 Marks]

Guess : $T(n) = \Omega(2^{n/2})$

Assume $T(i) \geq 2T(i-2) + 2$

for all i upto n-2

Then $T(n-2) \geq d\, 2^{\frac{n-2}{2}}$

Substituting

$$T(n) = 2T(n-2) + 2 \geq 2d\, 2^{\frac{n-2}{2}} + 2$$

$$\geq d\, 2^{n/2} + 2$$

$$\geq d\, 2^{n/2}$$

Hence Proved.

Let $T(0) = 2$ $\quad T(2) = 6$

$T(1) = 2$ $\quad T(3) = 6$

$T(2) \geq d\, 2^{0/2} \geq d$

$T(3) \geq d\, 2^{1/2} \geq \sqrt{2}\, d$.

Let d be a value $< \frac{6}{\sqrt{2}}$ which satisfies the given condition.

4. Illustrate the operation of Counting sort on the array < 2,5, 3, 1, 4, 6, 2,4> [2 Marks]

( Use the other side of this sheet to answer this question)

$A = \langle 2\ 5\ 3\ 1\ 4\ 6\ 2\ 4 \rangle$

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|
| C = | 1 | 2 | 1 | 2 | 1 | 1 | } Freq. |

| | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|
| C = | 1 | 3 | 4 | 6 | 7 | 8 | } Cumulative. |

For i = 1 to 8

| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| i = 1. | B : | | | | | | 4 | | |
| i = 2 | | | | 2 | | | 4 | | |
| i = 3 | | | | 2 | | | 4 | | 6 |
| i = 4 | | | | 2 | | 4 | 4 | | 6 |
| i = 5 | | 1 | | 2 | | 4 | 4 | | 6 |
| i = 6 | | 1 | | 2 | 3 | 4 | 4 | | 6 |
| i = 7 | | 1 | | 2 | 3 | 4 | 4 | 5 | 6 |
| i = 8 | | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |