The Relational Algebra and Relational Calculus

Outline

- Example Database Application (COMPANY)
- Relational Algebra
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  - Relational Algebra Operations From Set Theory
  - Binary Relational Operations
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  - Examples of Queries in Relational Algebra
- Relational Calculus
  - Tuple Relational Calculus
  - Domain Relational Calculus
- Overview of the QBE language

Database State for COMPANY

All examples discussed below refer to the COMPANY database shown here.

Figure 7.5 Schema diagram for the COMPANY relational database schema; the primary keys are underlined.

Relational Algebra

- The basic set of operations for the relational model is known as the relational algebra. These operations enable a user to specify basic retrieval requests.
- The result of a retrieval is a new relation, which may have been formed from one or more relations. The **algebra operations** thus produce new relations, which can be further manipulated using operations of the same algebra.
- A sequence of relational algebra operations forms a **relational algebra expression**, whose result will also be a relation that represents the result of a database query (or retrieval request).

Unary Relational Operations

- **SELECT Operation**

SELECT operation is used to select a *subset* of the tuples from a relation that satisfy a **selection condition**. It is a filter that keeps only those tuples that satisfy a qualifying condition – those satisfying the condition are selected while others are discarded.

Example: To select the EMPLOYEE tuples whose department number is four or those whose salary is greater than $30,000 the following notation is used:

$\sigma_{DNO = 4}$ **(EMPLOYEE)**

$\sigma_{SALARY > 30,000}$ **(EMPLOYEE)**

In general, the select operation is denoted by $\sigma_{<selection\ condition>}(R)$ where the symbol σ (sigma) is used to denote the select operator, and the selection condition is a Boolean expression specified on the attributes of relation R

Unary Relational Operations

SELECT Operation Properties

- The SELECT operation $\sigma_{<selection\ condition>}(R)$ produces a relation S that has the same schema as R
- The SELECT operation σ is **commutative;** i.e.,
  $\sigma_{<condition1>}(\sigma_{<condition2>}(R)) = \sigma_{<condition2>}(\sigma_{<condition1>}(R))$
- A cascaded SELECT operation **may be applied in any order;** i.e.,
  $\sigma_{<condition1>}(\sigma_{<condition2>}(\sigma_{<condition3>}(R))$
  $= \sigma_{<condition2>}(\sigma_{<condition3>}(\sigma_{<condition1>}(R)))$
- A cascaded SELECT operation may be replaced by a single selection with a conjunction of all the conditions; i.e.,
  $\sigma_{<condition1>}(\sigma_{<condition2>}(\sigma_{<condition3>}(R))$
  $= \sigma_{<condition1>\ AND\ <condition2>\ AND\ <condition3>}(R)))$

Unary Relational Operations (cont.)

Figure 7.8 Results of SELECT and PROJECT operations.
(a) $\sigma_{(DNO=4 \text{ AND } SALARY>25000) \text{ OR } (DNO=5 \text{ AND } SALARY>30000)}(EMPLOYEE)$.
(b) $\pi_{LNAME,\ FNAME,\ SALARY}(EMPLOYEE)$. (c) $\pi_{SEX,\ SALARY}(EMPLOYEE)$

(a)

FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
Franklin	T	Wong	333445555	1955-12-08	638 Voss,Houston,TX	M	40000	888665555	5
Jennifer		Wallace	987654321	1941-06-20	291 Berry,Bellaire,TX	F	43000	888665555	4
Ramesh		Narayan	666884444	1962-09-15	975 FireOak,Humble,TX	M	38000	333445555	5

(b)

LNAME	FNAME	SALARY
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

SEX	SALARY
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Unary Relational Operations (cont.)

- **PROJECT Operation**

This operation selects certain *columns* from the table and discards the other columns. The PROJECT creates a vertical partitioning – one with the needed columns (attributes) containing results of the operation and other containing the discarded Columns.

Example: To list each employee's first and last name and salary, the following is used:

$$\pi_{LNAME,\ FNAME,SALARY}(\textbf{EMPLOYEE})$$

The general form of the project operation is π<attribute list>(R) where π (pi) is the symbol used to represent the project operation and <attribute list> is the desired list of attributes from the attributes of relation R.

The project operation *removes any duplicate tuples,* so the result of the project operation is a set of tuples and hence a valid relation.

Unary Relational Operations (cont.)

PROJECT Operation Properties

- The number of tuples in the result of projection $\pi_{<list>}(R)$ is always less or equal to the number of tuples in R.
- If the list of attributes includes a key of R, then the number of tuples is equal to the number of tuples in R.
- $\pi_{<list1>}(\pi_{<list2>}(R)) = \pi_{<list1>}(R)$ as long as <list2> contains the attributes in <list2>

Unary Relational Operations (cont.)

Figure 7.8 Results of SELECT and PROJECT operations.
(a) $\sigma_{(DNO=4 \text{ AND } SALARY>25000) \text{ OR } (DNO=5 \text{ AND } SALARY>30000)}(EMPLOYEE)$.
(b) $\pi_{LNAME, FNAME, SALARY}(EMPLOYEE)$. (c) $\pi_{SEX, SALARY}(EMPLOYEE)$

(a)

FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
Franklin	T	Wong	333445555	1955-12-08	638 Voss,Houston,TX	M	40000	888665555	5
Jennifer		Wallace	987654321	1941-06-20	291 Berry,Bellaire,TX	F	43000	888665555	4
Ramesh		Narayan	666884444	1962-09-15	975 FireOak,Humble,TX	M	38000	333445555	5

(b)

LNAME	FNAME	SALARY
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

SEX	SALARY
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Unary Relational Operations (cont.)

- **Rename Operation**

We may want to apply several relational algebra operations one after the other. Either we can write the operations as a single **relational algebra expression** by nesting the operations, or we can apply one operation at a time and create **intermediate result relations**. In the latter case, we must give names to the relations that hold the intermediate results.

Example: To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation. We can write a single relational algebra expression as follows:

$\pi_{FNAME, LNAME, SALARY}(\sigma_{DNO=5}(EMPLOYEE))$

OR We can explicitly show the sequence of operations, giving a name to each intermediate relation:

$DEP5\_EMPS \leftarrow \sigma_{DNO=5}(EMPLOYEE)$

$RESULT \leftarrow \pi_{FNAME, LNAME, SALARY}(DEP5\_EMPS)$

Unary Relational Operations (cont.)

- **Rename Operation (cont.)**

The rename operator is ρ

The general Rename operation can be expressed by any of the following forms:

- $\rho_{S(B_1, B_2, ..., B_n)}(R)$ is a renamed relation S based on R with column names $B_1, B_1, ...B_n$.
- $\rho_S(R)$ is a renamed relation S based on R (which does not specify column names).
- $\rho_{(B_1, B_2, ..., B_n)}(R)$ is a renamed relation with column names $B_1, B_1, ...B_n$ which does not specify a new relation name.

Unary Relational Operations (cont.)

Figure 7.9 Results of relational algebra expressions.
(a) $\pi_{\text{LNAME, FNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))$. (b) The same expression using intermediate relations and renaming of attributes.

(a)

FNAME	LNAME	SALARY
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b) TEMP

FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
John	B	Smith	123456789	1965-01-09	731 Fondren,Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss,Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak,Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice,Houston,TX	F	25000	333445555	5

FIRSTNAME	LASTNAME	SALARY
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Relational Algebra Operations From Set Theory

- **UNION Operation**

The result of this operation, denoted by $R \cup S$, is a relation that includes all tuples that are either in R or in S or in both R and S. Duplicate tuples are eliminated.

Example: To retrieve the social security numbers of all employees who either work in department 5 or directly supervise an employee who works in department 5, we can use the union operation as follows:

DEP5_EMPS ← $\sigma_{\text{DNO=5}}$ (EMPLOYEE)

RESULT1 ← π_{SSN}(DEP5_EMPS)

RESULT2(SSN) ← π_{SUPERSSN}(DEP5_EMPS)

RESULT ← RESULT1 ∪ RESULT2

The union operation produces the tuples that are in either RESULT1 or RESULT2 or both. The two operands must be "type compatible".

Relational Algebra Operations From Set Theory

- **Type Compatibility**
  - The operand relations $R_1(A_1, A_2, ..., A_n)$ and $R_2(B_1, B_2, ..., B_n)$ must have the same number of attributes, and the domains of corresponding attributes must be compatible; that is, $dom(A_i)=dom(B_i)$ for $i=1, 2, ..., n$.
  - The resulting relation for $R_1 \cup R_2$, $R_1 \cap R_2$, or $R_1 - R_2$ has the same attribute names as the *first* operand relation R1 (by convention).

Relational Algebra Operations From Set Theory

- **UNION Example**

STUDENT

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

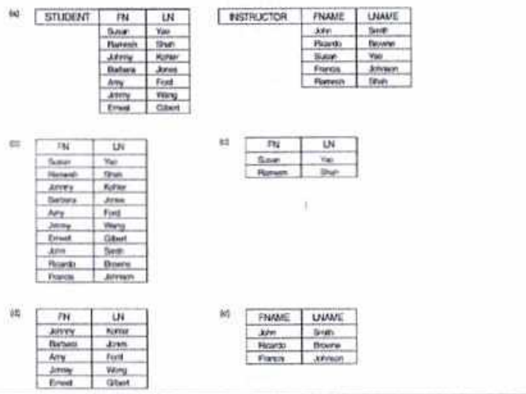
FNAME	LNAME
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

STUDENT $\cup$ INSTRUCTOR

Relational Algebra Operations From Set Theory (cont.) – use Fig. 6.4

Figure 7.11 Illustrating the set operations union, intersection, and difference. (a) Two union compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∪ INSTRUCTOR. (d) STUDENT − INSTRUCTOR. (e) INSTRUCTOR − STUDENT.



(a) STUDENT

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

FNAME	LNAME
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

FN	LN
Susan	Yao
Ramesh	Shah

(d)

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

FNAME	LNAME
John	Smith
Ricardo	Browne
Francis	Johnson

Relational Algebra Operations From Set Theory (cont.)

- **INTERSECTION OPERATION**

The result of this operation, denoted by $R \cap S$, is a relation that includes all tuples that are in both R and S. The two operands must be "type compatible"

Example: The result of the intersection operation (figure below) includes only those who are both students and instructors.

FN	LN
Susan	Yao
Ramesh	Shah

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

STUDENT ∩ INSTRUCTOR

Relational Algebra Operations From Set Theory (cont.)

- **Set Difference (or MINUS) Operation**

The result of this operation, denoted by R - S, is a relation that includes all tuples that are in R but not in S. The two operands must be "type compatible".

Example: The figure shows the names of students who are not instructors, and the names of instructors who are not students.

STUDENT

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

STUDENT-INSTRUCTOR

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR-STUDENT

FNAME	LNAME
John	Smith
Ricardo	Browne
Francis	Johnson

Relational Algebra Operations From Set Theory (cont.)

- Notice that both union and intersection are *commutative operations;* that is

  $$R \cup S = S \cup R, \text{ and } R \cap S = S \cap R$$

- Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative operations;* that is

  $$R \cup (S \cup T) = (R \cup S) \cup T, \text{ and } (R \cap S) \cap T = R \cap (S \cap T)$$

- The minus operation is *not commutative;* that is, in general

  $$R - S \neq S - R$$

Relational Algebra Operations From Set Theory (cont.)

- **CARTESIAN (or cross product) Operation**
  - This operation is used to combine tuples from two relations in a combinatorial fashion. In general, the result of $R(A_1, A_2, \ldots, A_n) \times S(B_1, B_2, \ldots, B_m)$ is a relation Q with degree n + m attributes $Q(A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_m)$, in that order. The resulting relation Q has one tuple for each combination of tuples—one from R and one from S.
  - Hence, if R has n_R tuples (denoted as $|R| = n_R$), and S has n_S tuples, then $| R \times S |$ will have $n_R * n_S$ tuples.
  - The two operands do NOT have to be "type compatible"

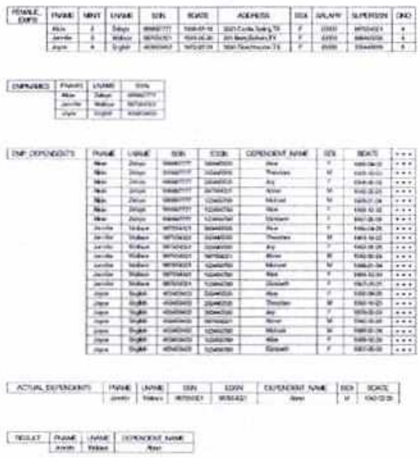
Example:

FEMALE_EMPS ← $\sigma_{SEX='F'}$(EMPLOYEE)
EMPNAMES ← $\pi_{FNAME, LNAME, SSN}$ (FEMALE_EMPS)

EMP_DEPENDENTS ← EMPNAMES x DEPENDENT

Relational Algebra Operations From Set Theory (cont.)

Figure 7.12 An illustration of the CARTESIAN PRODUCT operation.



Binary Relational Operations

- **JOIN Operation**
  - The sequence of cartesian product followed by select is used quite commonly to identify and select related tuples from two relations, a special operation, called **JOIN**. It is denoted by a $\bowtie$
  - This operation is very important for any relational database with more than a single relation, because it allows us to process relationships among relations.
  - The general form of a join operation on two relations $R(A_1, A_2, \ldots, A_n)$ and $S(B_1, B_2, \ldots, B_m)$ is:

    $R \bowtie_{\text{<join condition>}} S$

    where R and S can be any relations that result from general *relational algebra expressions.*

Binary Relational Operations (cont.)

Example: Suppose that we want to retrieve the name of the manager of each department. To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple. We do this by using the join $\bowtie$ operation.

DEPT_MGR ← DEPARTMENT $\bowtie_{\text{MGRSSN=SSN}}$ EMPLOYEE

DEPT_MGR	DNAME	DNUMBER	MGRSSN	...	FNAME	MINIT	LNAME	SSN	...
	Research	5	333445555	...	Franklin	T	Wong	333445555	...
	Administration	4	987654321	...	Jennifer	S	Wallace	987654321	...
	Headquarters	1	888665555	...	James	E	Borg	888665555	...

FIGURE 6.6 Result of the JOIN operation DEPT_MGR ← DEPARTMENT $\bowtie_{\text{MGRSSN=SSN}}$ EMPLOYEE.

Binary Relational Operations (cont.)

- **EQUIJOIN Operation**

  The most common use of join involves join conditions with equality comparisons only. Such a join, where the only comparison operator used is =, is called an EQUIJOIN. In the result of an EQUIJOIN we always have one or more pairs of attributes (whose names need not be identical) that have *identical values* in every tuple.

  The JOIN seen in the previous example was EQUIJOIN.

- **NATURAL JOIN Operation**

  Because one of each pair of attributes with identical values is superfluous, a new operation called natural join—denoted by *—was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.

  $\bowtie$The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, have the **same name** in both relations. If this is not the case, a renaming operation is applied first.

Binary Relational Operations (cont.)

Example: To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT_LOCATIONS, it is sufficient to write:

DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS

(a)

PROJ_DEPT	PNAME	PNUMBER	PLOCATION	DNUM	DNAME	MGRSSN	MGRSTARTDATE
	ProductX	1	Bellaire	5	Research	333445555	1988-05-22
	ProductY	2	Sugarland	5	Research	333445555	1988-05-22
	ProductZ	3	Houston	5	Research	333445555	1988-05-22
	Computerization	10	Stafford	4	Administration	987654321	1995-01-01
	Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
	Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

(b)

DEPT_LOCS	DNAME	DNUMBER	MGRSSN	MGRSTARTDATE	LOCATION
	Headquarters	1	888665555	1981-06-19	Houston
	Administration	4	987654321	1995-01-01	Stafford
	Research	5	333445555	1988-05-22	Bellaire
	Research	5	333445555	1988-05-22	Sugarland
	Research	5	333445555	1988-05-22	Houston

FIGURE 6.7 Results of two NATURAL JOIN operations. (a) PROJ_DEPT ← PROJECT * DEPT. (b) DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS.

Complete Set of Relational Operations

- The set of operations including **select σ, project π , union $\cup$, set difference - , and cartesian product X** is called a complete set because any other relational algebra expression can be expressed by a combination of these five operations.

- For example:

  $R \cap S = (R \cup S) - ((R - S) \cup (S - R))$

  $R \bowtie_{\text{<join condition>}} S = \sigma_{\text{<join condition>}} (R \times S)$

Binary Relational Operations (cont.)

- **DIVISION Operation**
  - The division operation is applied to two relations $R(Z) \div S(X)$, where X subset Z. Let $Y = Z - X$ (and hence $Z = X \cup Y$); that is, let Y be the set of attributes of R that are not attributes of S.
  - The result of DIVISION is a relation T(Y) that includes a tuple t if tuples t_R appear in R with $t_R [Y] = t$, and with $t_R [X] = t_s$ *for every tuple* t_s in S.
  - For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with *every* tuple in S.

Binary Relational Operations (cont.)

(a)

SSN_PNOS	ESSN	PNO
	123456789	1
	123456789	2
	666884444	3
	453453453	1
	453453453	2
	333445555	2
	333445555	3
	333445555	10
	333445555	20
	999887777	30
	999887777	10
	987987987	10
	987987987	30
	987654321	30
	987654321	20
	888665555	20

SMITH_PNOS	PNO
	1
	2

SSNS	SSN
	123456789
	453453453

(b)

R	A	B
	a1	b1
	a2	b1
	a3	b1
	a4	b1
	a1	b2
	a3	b2
	a2	b3
	a3	b3
	a4	b3
	a1	b4
	a2	b4
	a3	b4

S	A
	a1
	a2
	a3

T	B
	b1
	b4

Recap of Relational Algebra Operations

TABLE 6.1 OPERATIONS OF RELATIONAL ALGEBRA

Operation	Purpose	Notation
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{\langle\text{selection condition}\rangle}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{\langle\text{attribute list}\rangle}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\langle\text{join condition}\rangle} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\langle\text{join condition}\rangle} R_2$, OR $R_1 \bowtie_{(\langle\text{join attributes 1}\rangle),\ (\langle\text{join attributes 2}\rangle)} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\langle\text{join condition}\rangle} R_2$, OR $R_1 *_{(\langle\text{join attributes 1}\rangle),\ (\langle\text{join attributes 2}\rangle)} R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
$\bowtie$ INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Additional Relational Operations

- **Aggregate Functions and Grouping**
  - A type of request that cannot be expressed in the basic relational algebra is to specify mathematical **aggregate functions** on collections of values from the database.
  - Examples of such functions include retrieving the average or total salary of all employees or the total number of employee tuples. These functions are used in simple statistical queries that summarize information from the database tuples.
  - Common functions applied to collections of numeric values include SUM, AVERAGE, MAXIMUM, and MINIMUM. The COUNT function is used for counting tuples or values.

Additional Relational Operations (cont.)

(a)

R	DNO	NO_OF_EMPLOYEES	AVERAGE_SAL
	5	4	33250
	4	3	31000
	1	1	55000

(b)

DNO	COUNT_SSN	AVERAGE_SALARY
5	4	33250
4	3	31000
1	1	55000

(c)

COUNT_SSN	AVERAGE_SALARY
8	35125

Additional Relational Operations (cont.)

Use of the Functional operator $\mathcal{F}$

$\mathcal{F}_{\text{MAX } \textit{Salary}}$ **(Employee)** retrieves the maximum salary value from the Employee relation

$\mathcal{F}_{\text{MIN } \textit{Salary}}$ **(Employee)** retrieves the minimum Salary value from the Employee relation

$\mathcal{F}_{\text{SUM } \textit{Salary}}$ **(Employee)** retrieves the sum of the Salary from the Employee relation

${}_{\text{DNO}}\,\mathcal{F}_{\textit{COUNT SSN, AVERAGE Salary}}$ **(Employee)** groups employees by DNO (department number) and computes the count of employees and average salary per department.[Note: count just counts the number of rows, without removing duplicates]

Additional Relational Operations (cont.)

- **The OUTER JOIN Operation**
  - In NATURAL JOIN tuples without a *matching* (or *related*) tuple are eliminated from the join result. Tuples with null in the join attributes are also eliminated. This amounts to loss of information.
  - A set of operations, called outer joins, can be used when we want to keep all the tuples in R, or all those in S, or all those in both relations in the result of the join, regardless of whether or not they have matching tuples in the other relation.
  - The left outer join operation keeps every tuple in the *first* or *left* relation R in R ⟕ S; if no matching tuple is found in S, then the attributes of S in the join result are filled or "padded" with null values.
  - A similar operation, right outer join, keeps every tuple in the *second* or right relation S in the result of R ⟖ S.
  - A third operation, full outer join, denoted by ⟗ keeps all tuples in both the left and the right relations when no matching tuples are found, padding them with null values as needed.

Additional Relational Operations (cont.)

RESULT	FNAME	MINIT	LNAME	DNAME
	John	B	Smith	null
	Franklin	T	Wong	Research
	Alicia	J	Zelaya	null
	Jennifer	S	Wallace	Administration
	Ramesh	K	Narayan	null
	Joyce	A	English	null
	Ahmad	V	Jabbar	null
	James	E	Borg	Headquarters

Additional Relational Operations (cont.)

- **OUTER UNION Operations**
  - The outer union operation was developed to take the union of tuples from two relations if the relations are *not union compatible.*
  - This operation will take the union of tuples in two relations R(X, Y) and S(X, Z) that are **partially compatible**, meaning that only some of their attributes, say X, are union compatible.
  - The attributes that are union compatible are represented only once in the result, and those attributes that are not union compatible from either relation are also kept in the result relation T(X, Y, Z).
  - **Example:** An outer union can be applied to two relations whose schemas are STUDENT(Name, SSN, Department, Advisor) and INSTRUCTOR(Name, SSN, Department, Rank). Tuples from the two relations are matched based on having the same combination of values of the shared attributes—Name, SSN, Department. If a student is also an instructor, both Advisor and Rank will have a value; otherwise, one of these two attributes will be null.

    The result relation STUDENT_OR_INSTRUCTOR will have the following attributes:

    STUDENT_OR_INSTRUCTOR (Name, SSN, Department, Advisor, Rank)

Examples of Queries in Relational Algebra

- **Retrieve the name and address of all employees who work for the 'Research' department.**

RESEARCH_DEPT ← σ DNAME='Research' (DEPARTMENT)

RESEARCH_EMPS ← (RESEARCH_DEPT ⋈ DNUMBER=DNOEMPLOYEE EMPLOYEE)

RESULT ← π FNAME, LNAME, ADDRESS (RESEARCH_EMPS)

- **Retrieve the names of employees who have no dependents.**

ALL_EMPS ← π SSN(EMPLOYEE)

⋈

EMPS_WITH_DEPS(SSN) ← π ESSN(DEPENDENT)

EMPS_WITHOUT_DEPS ← (ALL_EMPS - EMPS_WITH_DEPS)

RESULT ← π LNAME, FNAME (EMPS_WITHOUT_DEPS * EMPLOYEE)

Questions

- **For every project located at 'Bangalore', list the project number, the controlling department number, and the department manager's last name and address.**

⋈

Questions

- **Retrieve the names of employees who work on all the projects controlled by department number 5.**

$\bowtie$

Relational Calculus

- A **relational calculus** expression creates a new relation, which is specified in terms of variables that range over rows of the stored database relations (in **tuple calculus**) or over columns of the stored relations (in **domain calculus**).
- In a calculus expression, there is *no order of operations* to specify how to retrieve the query result—a calculus expression specifies only what information the result should contain. This is the main distinguishing feature between relational algebra and relational calculus.
- Relational calculus is considered to be a **nonprocedural** language. This differs from relational algebra, where we must write a *sequence of operations* to specify a retrieval request; hence relational algebra can be considered as a **procedural** way of stating a query.

Tuple Relational Calculus

- The tuple relational calculus is based on specifying a number of **tuple variables.** Each tuple variable usually *ranges over* a particular database relation, meaning that the variable may take as its value any individual tuple from that relation.
- A simple tuple relational calculus query is of the form
  {t | COND(t)}
  where t is a tuple variable and COND (t) is a conditional expression involving t. The result of such a query is the set of all tuples t that satisfy COND (t).

  Example: To find the first and last names of all employees whose salary is above $50,000, we can write the following tuple calculus expression:

  {t.FNAME, t.LNAME | EMPLOYEE(t) AND t.SALARY>50000}

  The condition EMPLOYEE(t) specifies that the **range relation** of tuple variable t is EMPLOYEE. The first and last name (PROJECTION $\pi_{FNAME, LNAME}$) of each EMPLOYEE tuple t that satisfies the condition t.SALARY>50000 (SELECTION $\sigma_{SALARY>50000}$) will be retrieved.

The Existential and Universal Quantifiers

- Two special symbols called **quantifiers** can appear in formulas; these are the **universal quantifier** ($\forall$) and the **existential quantifier** ($\exists$).
- Informally, a tuple variable t is bound if it is quantified, meaning that it appears in an ($\forall$ t) or ($\exists$ t) clause; otherwise, it is **free.**
- If F is a formula, then so is ($\exists$ t)(F), where t is a tuple variable. The formula ($\exists$ t)(F) is true if the formula F evaluates to true for *some* (at least one) tuple assigned to free occurrences of t in F; otherwise ($\exists$ t)(F) is **false.**
- If F is a formula, then so is ($\forall$ t)(F), where t is a tuple variable. The formula ($\forall$ t)(F) is true if the formula F evaluates to true for *every tuple* (in the universe) assigned to free occurrences of t in F; otherwise ($\forall$ t)(F) is **false.** It is called the universal or "for all" quantifier because every tuple in "the universe of" tuples must make F true to make the quantified formula true.

Example Query Using Existential Quantifier

- Retrieve the name and address of all employees who work for the 'Research' department.

  Query :

  {t.FNAME, t.LNAME, t.ADDRESS | EMPLOYEE(t) and (∃ d) (DEPARTMENT(d) and d.DNAME='Research' and d.DNUMBER=t.DNO) }

- The *only free tuple variables* in a relational calculus expression should be those that appear to the left of the bar (|). In above query, t is the only free variable; it is then *bound successively* to each tuple. If a tuple *satisfies the conditions* specified in the query, the attributes FNAME, LNAME, and ADDRESS are retrieved for each such tuple.

$\bowtie$

- The conditions EMPLOYEE (t) and DEPARTMENT(d) specify the range relations for t and d. The condition d.DNAME = 'Research' is a selection condition and corresponds to a SELECT operation in the relational algebra, whereas the condition d.DNUMBER = t.DNO is a JOIN condition.

Languages Based on Tuple Relational Calculus

- The language **SQL** is based on tuple calculus. It uses the basic

  SELECT <list of attributes>

  FROM <list of relations>

  WHERE <conditions>

  block structure to express the queries in tuple calculus where the SELECT clause mentions the attributes being projected, the FROM clause mentions the relations needed in the query, and the WHERE clause mentions the selection as well as the join conditions.

  SQL syntax is expanded further to accommodate other operations.

The Domain Relational Calculus

- Another variation of relational calculus called the domain relational calculus, or simply, **domain calculus** is equivalent to tuple calculus and to relational algebra.
- The language called QBE (Query-By-Example) that is related to domain calculus was developed almost concurrently to SQL at IBM Research, Yorktown Heights, New York. Domain calculus was thought of as a way to explain what QBE does.
- Domain calculus differs from tuple calculus in the *type of variables* used in formulas: rather than having variables range over tuples, the variables range over single values from domains of attributes. To form a relation of degree n for a query result, we must have n of these **domain variables**—one for each attribute.
- An expression of the domain calculus is of the form
  {x1, x2, . . ., xn | COND(x1, x2, . . ., xn, xn+1, xn+2, . . ., xn+m)}

⋈ where x1, x2, . . ., xn, xn+1, xn+2, . . ., xn+m are domain variables that range over domains (of attributes) and COND is a **condition** or **formula** of the domain relational calculus.

Example Query Using Domain Calculus

- Retrieve the birthdate and address of the employee whose name is 'John B. Smith'.

Query :

{uv | (∃ q) (∃ r) (∃ s)
(EMPLOYEE(qrstuvwxyz) and q='John' and r='B' and s='Smith')}

- Ten variables for the employee relation are needed, one to range over the domain of each attribute in order. Of the ten variables q, r, s, . . ., z, only u and v are free.
- Specify the *requested attributes,* BDATE and ADDRESS, by the free domain ⋈ variables u for BDATE and v for ADDRESS.
- Specify the condition for selecting a tuple following the bar (|)—namely, that the sequence of values assigned to the variables qrstuvwxyz be a tuple of the employee relation and that the values for q (FNAME), r (MINIT), and s (LNAME) be 'John', 'B', and 'Smith', respectively.