

SHRI SHANTI C HANDS

B110076CS

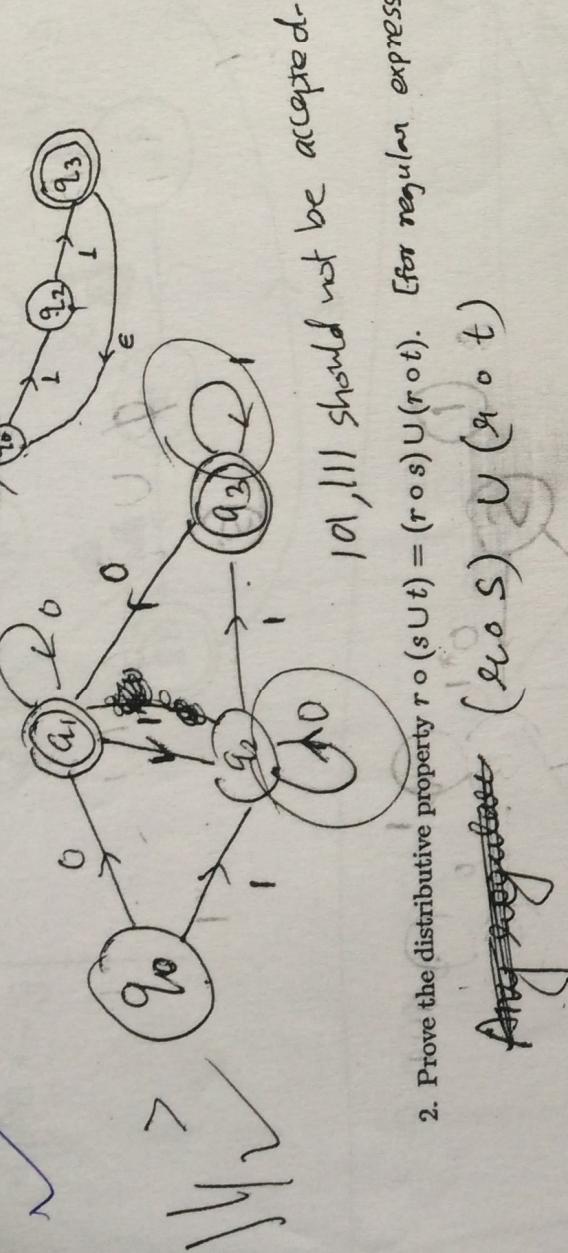
Mid I: September 2013

Theory of Computation

Part II

1. Construct the equivalent DFA for the given NFA.

[3]

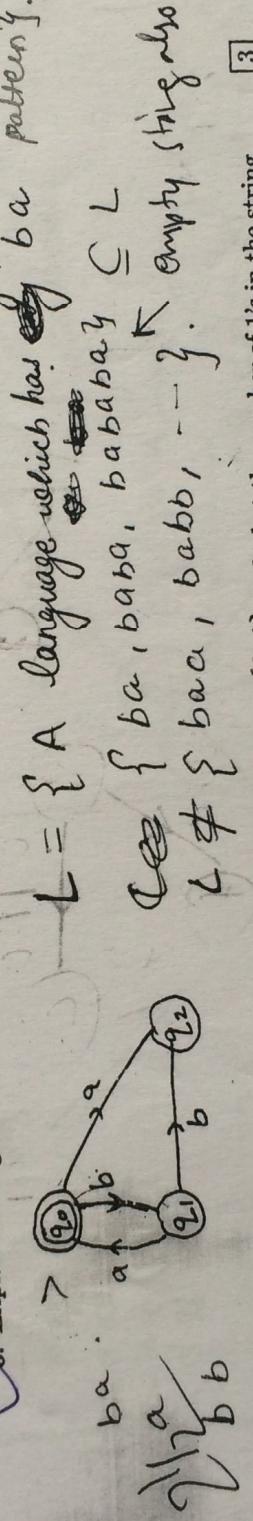


2. Prove the distributive property $r \circ (s \cup t) = (r \circ s) \cup (r \circ t)$. [For regular expressions, set]

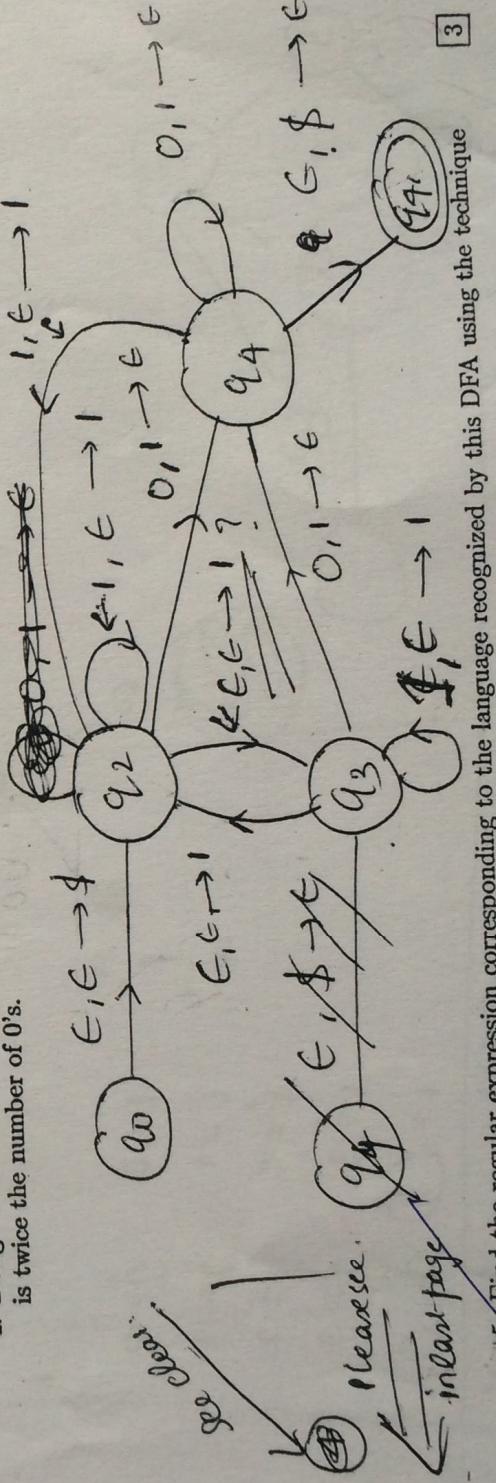
~~$$(r \circ s) \cup (r \circ t)$$~~

[3]

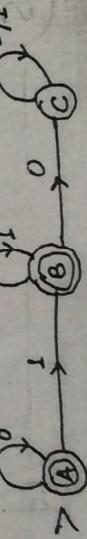
3. Explain in English what is the language recognized by the given finite automata.



4. Design a PDA that accepts all strings over the alphabet {0, 1} such that the number of 1's in the string is twice the number of 0's.



5. Find the regular expression corresponding to the language recognized by this DFA using the technique discussed in the class. (Use backside).



$$0^* 0 0^* 11^* \cup 0 0^* 11^* 0 (001)^*$$

$$0^* + 0^* 11^*$$

Second Exam Part II

CS3001 Theory of Computation

Marks:20

November 2015, S5 B.Tech Max: 3:00Hrs

Name and Roll No.: _____

_____ Name and Roll No.: _____ Explain the (2+1+2)

1. Design a turing machine to compute $\lceil \log r \rceil$ if binary representation of n is given as input. Explain the (2+1+2)
steps clearly in English (state diagram not necessary). What would be the output if the input is 1101?
What is the complexity of this machine?
2. Define the language L_d (and justify why it is not turing recognizable), assuming that strings and machines
can be numbered as w_1, w_2, \dots and M_1, M_2, \dots respectively. Does this numbering need to be unique
(i.e., a machine/string is uniquely represented by a single number)? Why? (3)
3. A vertex cover of G is a set of vertices such that every edge of G is incident on at least one vertex in
this set. VC problem is, given a graph G and a number k , does G have a vertex cover of size k ? Show
that this problem is in NP . (3)
4. $coNP$ is defined as the complexity class that contains complements of all languages that belong to NP .
Show that $P \subseteq NP \cap coNP$. (4)
5. Consider two questions. A : Given a boolean formula ψ , is ψ satisfiable? B : Given a boolean formula
 ψ , find a satisfying truth assignment for ψ . Show that A is NP -hard if B is NP -hard.

November 2015, S5 B.Tech Max: 3:00Hrs

Name and Roll No.: Prabhav Adhikari B130056CS

1. Design a turing machine to compute $\lceil \log n \rceil$ if binary representation of n is given as input. Explain the steps clearly in English (state diagram not necessary). What would be the output if the input is 1101? What is the complexity of this machine? (2+1+2)
2. Define the language L_d (and justify why it is not turing recognizable), assuming that strings and machines can be numbered as w_1, w_2, \dots and M_1, M_2, \dots respectively. Does this numbering need to be unique (i.e., a machine/string is uniquely represented by a single number)? Why? (3+2)
3. A vertex cover of G is a set of vertices such that every edge of G is incident on at least one vertex in this set. VC problem is, given a graph G and a number k , does G have a vertex cover of size k ? Show that this problem is in NP . (3)
4. $coNP$ is defined as the complexity class that contains complements of all languages that belong to NP . Show that $P \subset NP \cap coNP$. (3)
5. Consider two questions. A : Given a boolean formula ψ , is ψ satisfiable? B : Given a boolean formula ψ , find a satisfying truth assignment for ψ . Show that A is NP -hard if B is NP -hard. (4)

(Note: This is an open book test. Sharing of materials and discussions are not permitted during the test)

I Answer True/False for the following eight questions

1. There exists a language that is recursive but not recursively enumerable.(TRUE/FALSE) False
2. There exists a language that is not recursively enumerable. (TRUE/FALSE) True
3. The language $\overline{A_{TM}}$ is not recursively enumerable.(TRUE/FALSE) True
4. For any language L that is recursive, its complement \bar{L} is also recursive.(TRUE/FALSE) True
5. For any language L that is recursively enumerable, its complement \bar{L} is also recursively enumerable . (TRUE/FALSE) False
6. The set Σ^* is uncountable. (TRUE/FALSE) false
7. Every context free language is decidable. (TRUE/FALSE) True
8. The set of all infinite sequences over {0,1} is uncountable. (TRUE/FALSE) True

II Classify each of the following languages as Decidable/ Undecidable.

1. $L_1 = \{ < R, w > / R \text{ is a regular expression that generates string } w \}$.

Ans: Decidable

2. $L_2 = \{ < G > / G \text{ is a CFG and } L(G) = \emptyset \}$.

Ans: Decidable

3. $L_3 = \{ < M_1, M_2 > / M_1 \text{ and } M_2 \text{ are DFAs and } L(M_1) = L(M_2) \}$.

Ans: Decidable

4. $L_4 = \{ < M > / M \text{ is an NFA and } L(M) = \emptyset \}$.

Ans: Decidable

III Given $A \leq_m B$, $B \leq_m C$ and C is decidable. Is A decidable (Yes/No) ? Justify your answer.

Ans: $B \leq_m C$ and C is decidable, then B is decidable. (1 Mark)

And $A \leq_m B$ and B is decidable, then A is also decidable. (1 Mark)
 Let M be the decider for C and f be the reduction from B to C . Then a decider for B can also be determined by combining $f(w)$ on input w and run M on input $f(w)$ and output whatever M outputs. So, if $w \in B$ then $f(w) \in C$ ~~because by defn. of mapping~~ whenever $f(w)$ is accepted M accepts $f(w)$. Thus M accepts $f(w)$. So, B is decidable when $w \in B$.
 IV Let $L_1 = \{ < M > / M \text{ is a TM and } L(M) = \emptyset \}$.
 Let $L_2 = \{ < M_1, M_2 > / M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2) \}$. Similarly, A ..

Is $L_1 \leq_m L_2$ (Yes/No)? Justify your answer.

Ans: (Yes).

Here L_1 is undecidable and L_2 is also undecidable as we have proved L_2 has been proved undecidable with the help of undecidability of L_1 .
 So, whenever L_1 is undecidable, L_2 is undecidable.
 A mapping reduction from L_1 to L_2 lies in the proof of undecidability of L_2 . So, $L_1 \leq_m L_2$.

Mid I Part II

Theory of Computation

Total: 15 Marks

Name and Roll No.: Adilika Kumar (B120329CS)

1. Is the language L , consisting of all binary strings of length greater than or equal to 100 regular? Answer yes/no and then justify. If L is regular, how many states are required for a minimal DFA to accept L ? Justify your answer.

~~NO.~~ Suppose $L = \{0^0, 0^1, 0^2, \dots, 0^{99}, 1^0, 1^1, 1^2, \dots, 1^{99}\}$. Suppose $L_{100} = \{0^{100}\}$
 But their union, $[not even] L_{100} = \{0^{100}\}$ and so $L = \{0^n\mid n \geq 100\}$ [less than or equal to 100] on. All of L_{100}, L_{101}, \dots are regular as they can be captured using a fixed no. of states.

2. Let M_1 and M_2 be finite state machines over $\{0, 1\}$ with state sets Q_1, Q_2 and F_1, F_2 as the set of final states. Suppose you want the product automaton $M_1 \times M_2$ to accept $L(M_1) \cup L(M_2)$, what must be the final states? Justify your answer in one sentence.

~~$f \in F_1 \cup F_2$~~

~~Reason \Rightarrow we should accept any string which is accepted by any one of the machines. That is why we should accept $f(x, y)$ tuples of form (x, y) where $x \in Q_1$, $y \in Q_2$.~~

3. Consider the language L of all binary strings which has no "zeroes" before the occurrence of the first "one". How many Myhill-Nerode equivalence classes are there for the language? Write regular expressions for specifying strings in each equivalence class.

There are ~~those~~ 3 Myhill-Nerode equivalence classes

① One having "no zeroes" before the first ~~1~~ $\boxed{1 \cdot (0+1)^*}$

② One having atleast one 0 before first ~~1~~ $\rightarrow \boxed{0 \cdot 1 \cdot (0+1)^*}$

③ Empty string $\rightarrow \boxed{\epsilon}$

4. Why is it impossible to prove that the language $L = \{a^i b^j c^k : i = j = k\}$ non-regular using the pumping lemma? Is the language actually non-regular? Justify your answer.

~~Reason \Rightarrow Language is not regular~~

~~Suppose $w = a^k b^k c^k$~~

~~It is not impossible to prove this using pumping lemma provided we are ready to make some changes to the lemma (known as Generalized pumping lemma)~~

~~pumping lemma and $w \in L$, we can find~~

~~original \Rightarrow If L is regular, $|w| \geq n$, and $|xy| \leq n$~~

~~① $w = xyz$ such that~~

~~② $xy^iz \in L$~~

Name:

2

Roll No:

National Institute of Technology Calicut

Department of Computer Science and Engineering

First Midterm Examination, August 2012

CS3001 Theory Of Computation

Time: 1 Hour

Maximum Marks: 15

1. Let r be the regular expression $(aa+bb)^*ab$. Write the minimum length string in $L(r)$. (1 Mark)
2. Draw the state transition diagram of a DFA that will accept all the binary strings that are multiples of four (over the alphabet $\Sigma = \{0, 1\}$). (e.g. 0, 000, 100, 0100, 1100). (2 Marks)
3. Draw the transition diagram of a Finite Automata recognizing $L(\varepsilon+\phi)$. (2 Marks)
4. List all the strings in $L((a+b)(\varepsilon+c))$. (1 Mark)
5. Let $L = \{w / w \text{ contains at least one } a\}$. Draw the DFA for \bar{L} . $\bar{L} = \{a, b\}$ (2 Marks)
6. Is the following language regular(Yes/No)? Justify your answer.
 $L = \{\sigma^n / 1 \leq n \leq 100\}$ (2 Marks)
7. $L = \{\tilde{a}\tilde{b}/j=2^k\}$ is the language i. regular(Yes/No)? Justify your answer. (3 Marks)
8. Let L be a regular language over $\{a, b\}$. Let $L^R = \{w^R / w \in L\}$, where w^R denotes the reversal of w , for example if $abb \in L$ then $bab \in L^R$. Is L^R regular (Yes/No)? Justify. (2 Marks)

How to draw DFA for reversal of string 2^n

3/4

Name: Bandna Kumar

Roll No: B100475CS

CS3001 Theory Of Computation

Assignment Test 1

02-07-2012

Time: 30 Minutes

1. List all strings of length 2 over the alphabet {a,b}

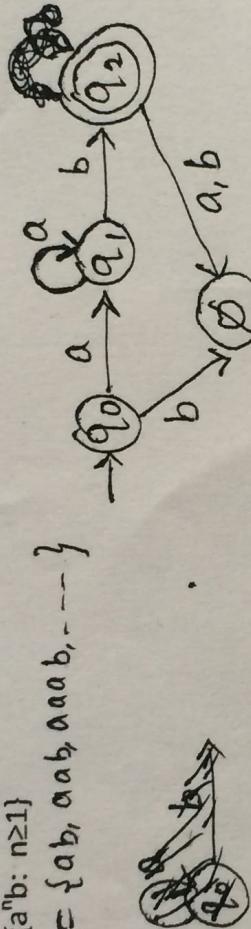
$$\text{Ans: } \{aa, bb, ab, ba\}$$

(1 Mark)

2. Draw the state transition diagram of a Deterministic FA(DFA) recognizing the following language over the alphabet {a,b}

$$L_1 = \{a^n b : n \geq 1\}$$

$$\text{Ans: } L_1 = \{ab, aab, aaaab, \dots\}$$



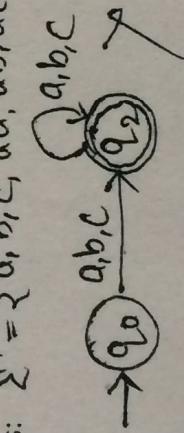
3. Suppose the set of final states, F of a Finite Automata M1 is empty. What is L(M1)?

$$\text{Ans: } L(M1) = L_1 \text{ where } L_1 = \{\}$$

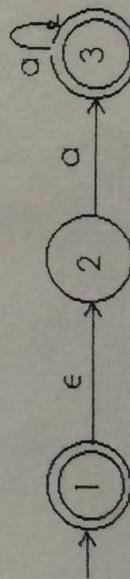
(1 Mark)

4. Draw the transition diagram of a Nondeterministic FA with minimum number of states recognizing the language Σ^* , where $\Sigma = \{a, b, c\}$. (Σ^* is the set consisting of all the strings that can be formed out of the alphabet Σ).

$$\text{Ans: } \Sigma^* = \{a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, \dots\}$$



5. What is the language recognized by the following NFA?



1/2

(1 Mark)

Ans: $L = \{w / w \text{ is either empty or ends with } 'a'\}$

Mid I

Theory of Computation

Total: 30 Marks

Name and Roll No.: SHRI SHTY CHANDRA B110076CS

1. Suppose L_1, L_2, \dots is a countable infinite family of regular languages over alphabet $\{0, 1\}$. Can we say that $\bigcup_{i=1}^{\infty} L_i$ is regular? (Answer Yes/No. If Yes, Prove. If No, Give counter-example.)

YES, it is regular.

Union of Languages is closed under union operators

2. Let M_1 and M_2 be finite state machines over $\{0, 1\}$ with state sets Q_1, Q_2 and F_1, F_2 as the set of final states. Suppose you want the product automaton $M_1 \times M_2$ to accept $L(M_1) - L(M_2)$, what must be the final states? Justify your answer in one sentence.

(Accept, Reject)

~~$f_1 \times f_2 = f_1 \times (Q_2 \setminus F_2)$~~

3. In the proof of the pumping lemma, it was shown that for every CFL L , there exists an n such that for all $z \in L$ with $|z| \geq n$, $\exists u, x, w, y, v$ such that $z = uxwv$ and $ux^iwy^jz \in L \forall i \geq 0$. Is it true that $|uxwiy| \leq n$; i.e. the repetition pattern must be spotted before the first n symbols of z ? (Answer YES/NO first. If Yes, give Proof. If No, give counter-example.)

NO, ~~as $|uxwiy| \leq n$~~

Counter Example:

~~in $a^i b^i$ take $x = a^n, y = b^n$~~

~~then $a^i b^i$ will be accepted
 $u = ab \quad x = c^i \quad y = c^k \quad z = c^l$
in actual pumping lemma.
and may not be accepted by above definition.~~

4. Is the language $L = \{a^i b^j : i = j \text{ if } i \leq 3\}$ regular? (Answer YES or NO. If the answer is Yes, give a DFA. Otherwise give a proof that L is not regular by giving an infinite number of Myhill-Nerode inequivalent strings.)

NO,

Class ①	$i=j$	$\Rightarrow a^i b^j$	and $i < j$	Class 1: $i=1 \quad j=1$
Class ②	$i=0, j=1$	\Rightarrow	b	$i=2 \quad j=2$
Class 3	$i=1, j=1$	\Rightarrow	a	$i=3 \quad j=3$
		\Rightarrow	a	$i=4 \quad j=4$
		\Rightarrow	b	$i=4 \quad j=6$

5. Consider the grammar $G : S \rightarrow Aa|Bb|Sab, A \rightarrow Aa, B \rightarrow Bbb$. Construct a finite state machine that accepts $L^R(G)$, the language consisting of all strings in $L(G)$ written in reverse. (Hint: What is the grammar for $L_R^R(G)$? Answer on the reverse side.)

~~S → aA | bB | Sab~~

~~A → aA | a~~

~~B → bB | b~~

~~Left~~

~~Right~~

~~See back~~