# **Accepted Manuscript**

Multi-level multi-secret sharing scheme for decentralized e-voting in cloud computing

Jing Li, Xianmin Wang, Zhengan Huang, Licheng Wang, Yang Xiang

PII: S0743-7315(19)30262-X

DOI: https://doi.org/10.1016/j.jpdc.2019.04.003

Reference: YJPDC 4041

To appear in: J. Parallel Distrib. Comput.

Received date: 6 March 2018 Revised date: 1 February 2019 Accepted date: 1 April 2019



Please cite this article as: J. Li, X. Wang, Z. Huang et al., Multi-level multi-secret sharing scheme for decentralized e-voting in cloud computing, *Journal of Parallel and Distributed Computing* (2019), https://doi.org/10.1016/j.jpdc.2019.04.003

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# Highlights

- 1. A multi-secret sharing scheme with multi-level access structure was proposed, where the scheme does not need any trust party as a dealer.
- 2. The secret sharing scheme can share multiple secrets and each *r* arty only keeps a short share.
- 3. A decentralized multi-role e-voting protocol was designed based on the multi-level access structures.
- 4. The e-voting system achieves fast verification for th final election results and also does not need the authority center.

# Multi-level multi-secret sharing scheme for decentralized e-voting in cloud computing

Jing Li $^{1*},\ {\rm Xianmin\ Wang}^1,\ {\rm Zhengan\ Huang}^1,\ {\rm Li\ neng\ Vang}^2$  and Yang  ${\rm Xiang}^{3,4}$ 

- 1. School of Computer Science, Guangzhou University, Cangzhou, China.
- 2. State Key Laboratory of Networking and Switching Technic ogy Beij ig University of Posts and Telecommunications, Beijing, Cuina.
- 3. State Key Laboratory of Integrated Service Networks (ISN), National Oniversity, Xian 710071, China.
  4. School of Software and Electrical Engineering, Swinburne University of Technology, Australia

#### Abstract

The cryptosystem-based data privary preserving methods employ high computing power of cloud servers, where the main feature is to allow resource sharing and provide multi-user mach indent services. Therefore, to achieve the rapid allocation and release of resource sharing in cloud computing, decentralized cryptographic reactions need to be proposed for multi-user consensus systems. In this work, we first present a multi-secret sharing scheme with multi-level access structure, where the secret reconstruction algorithm satisfies the additive for and rephism. The secret sharing scheme needs no trusted third parties and are user can play the role of dealer. In the designing, multiple targe secrets are independently shared, where each subset of users forms a sub-access structure and shares one target secret only with a short secret share. This scheme is efficient and unconditionally secure.

Furthermore, assed on the multi-level access structures, a decentralized multi-role e-voting protocol is designed using Chinese Remainder Theorem, where each is a section is associated with one sub-access structure. The voters employ a snared parameter to blind the sum of ballot values. Meanwhile, the e-voting scheme supports a public verification for the final election results. Compared with the existing e-voting protocols, our e-voting system does not require any authority center and the cloud server runs vote counting. And our e-voting scheme does not need any high-complexity computational comperation such as module exponential operation, etc. Finally, the control of Blockchain and Ad Hoc networks is decentralized. Thus the main idea of this protocol without a trusted third party can be used to

achieve a secure consensus among multiple nodes in Blocken, in and Ad Hoc network, meanwhile, the consensus results can be verified.

# Keywords:

Multi-secret sharing; Multi-role e-voting; Decentralized system; Cloud computing

#### 1. Introduction

In 2006, the concept of cloud computing was proposed by Google at the Search Engine conference. The development of cloud computing quickly established a prairie "fire" and triggered the cloud computing quickly established a prairie "fire" and triggered the cloud wave of the information technology revolution. The computing model can provide usable, convenient, on-demand network access and according configurable computing resource sharing pools [15, 26, 32], especially and Mobile Devices [9]. In outsourcing computing, the user's data and can cloud are transplanted into an external, virtual cloud, then the computing and storage model simplifies the maintenance of information and reduce the cost of the user [22, 23]. To guarantee the security of sensitive data in such a semi-trusted model, the cryptosystems become effective techniques, which may require a trusted third party (or authority center). To reading ray id allocation and release of resource sharing, we mainly consider decentralized consensus mechanisms in multi-user cloud environment, such as mobile Ad Hoc network and Blockchain consensus [1, 19, 31].

Secret sharing is one primitive of multi-party computations, which is to distribute a secret among multiple participants and any authorized subset can recover the secret data. The collection of all authorized subsets is called an access structure, which supports the monotone ascending property [11]. The past there were designed by Sharing schemes (SSSs): In 1979, (t.n) threshold SSSs were designed by Shamir and Blakley based on Lagrange polynomial interpolation and projective geometry theory, respectively. In 1989, Brickell designed an ideal SSS based on vector space [4], realizing a general access structure [3, 14]. After then, verifiable SSSs were given in [6, 7, 8] for checking the validity of each participant. To improve the function of secret sharing schemes, dynamic SSSs [5, 21] were constructed, in which any member can join and leave the group and doesn't reveal any information about the secret. Besides, for the scheme efficiency, multiple secrets sharing schemes are designed within multipartite access structures [5, 11].

In particular, Hsu proposed a multi-secret sharing [11] based on monotone span programs (MSP) for general access structures, vine e each subset of participants shares a corresponding secret, called target scret. That is, the scheme is designed with respect to a family of access structures associated with multiple secrets. We note that the scheme can share n secrets at most, since the vector  $\vec{r}$  hiding secrets only meets n linearly independent equations in the secret distribution phase [11], which can be used in attribute-based cryptosystems [33].

Most existing secret sharing schemes require rusted third parties (TTPs) to distribute secrets. However, considering the relability and cost of TTPs in practical applications, it is desired a construct SSSs without TTPs [27, 10, 25, 20]. In 1991, the first threshold exert sharing scheme without the assistance of any TTP was proposed by Pedersen [27]. And then a strong verifiable (n, t, n) SSS [10] was design on Uarn, in which participants check whether the verification polynomial is 'degree or not to decide the validity of the shares. Based on these SSSs Lippresented a more efficient verifiable (n,t,n) SSS [25] that reduces the number of verification polynomials compared with Harn's SSS. In these schemes, each participant plays the role as a TTP and participates to generate a master secret, where the sub-secret is randomly chosen by each partic pant. Then the participant generates subshares and distributes its su. sec. et to others participants based on Shamir's SSS. Using the proper y c additive homomorphism [2], each participant is able to combine all received sub-shares into a master share. Therefore, the master secret can be recovered with knowledge of any t or more than t master shares. Note that a cheme realizes the (t,n) threshold structure, with n participants also ac ing as dealers simultaneously, so it is called (n, t, n) SSS. It is interesting to construct a more practical SSS for general access structure with no TTPs.

Considering some application scenarios, a SSS without TTPs can used into a decentralized consensus mechanisms. We will build an electronic voting (e-voting) protected without any center authority (CA). Our goal is to make vote counting process more transparent and efficient, while allow voters to verify their election results. Furthermore, based on the multi-access structure SSSs without TTPs, we show the following situation for leader elections in an organization: One e-voting will be used to generate multiple leaders for managing different affairs. For example, a company needs to campaign three roles, a chairman, a general manager and two deputy general managers. It is unpractical to employ the direct sum of three parallel e-voting schemes for

solving this problem, since such a method requires heavy computation cost and communication cost, meanwhile each voter will keep too much information. In this paper, we present a decentralized mult. ro's e-voting system based on a multi-secret sharing scheme (MSSS).

To solve this problem, we will construct a secure model for multi-role election without any authorized center. Based on a multi-access structure secret sharing scheme, each sub-access structure car be utilized to finish the election for one role, where each participant access as a voter  $V_i$  and cloud server is the teller. Thus, the new e-voting model provides a multi-role election for different sets of candidates. In Figure 1, there are two sub-access structures— $\{V_1, V_2, V_3\}$  and  $\{V_3, V_4, V_5\}$ . Each sub-access structure shares a secret and  $V_i$  only carries one share. In the vote and sends the blinded result to the cloud server. The cloud server aggregates all blinded results and returns the aggregated results to the corresponding sub-access structure. Finally, the parties recover their shared secret and get rid of blindness from the aggregated result, then each sub-access structure will obtain their own voting result.

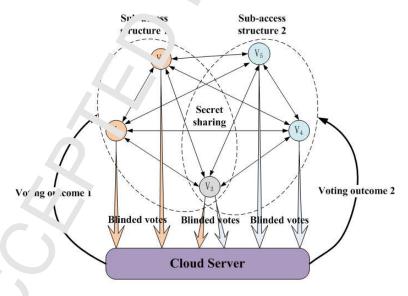


Figure 1: Multi-role E-voting based on MSSS

Car contributions. In this paper, we propose a multi-level multi-secret sharn g scheme that realizes a family of access structures based on multitarget MSP. Due to the linear model of vector space, the secret reconstruction

algorithm has a property of additive homomorphism. Thus, the participants can play the role of dealer to yield the shared secrets. We anwhile, a subaccess structure has its target secret, thus the scheme our ports multi-secret sharing and each participant keeps only one master than. Then, we prove that the scheme is unconditionally secure, where any subsets have no access to the secrets beyond its legal authority.

Furthermore, a multi-role e-voting system in decisived based on the family of sub-access structures in the proposed Modes. In the Setup phase, we use the Chinese Remainder Theorem (CRT) to a lect different prime module for each different candidate. Using this a sthoot the e-voting system can achieve multi-role election among one condition of candidates, especially among disjoint collections of candidates. In the coting phase, the voters use a shared parameter to blind the sum of bank values. Meanwhile, the e-voting scheme supports a public verification to the final election results, where any voter can check whether the sum of bank values and the number of voters satisfy some fixed relation. Compared with the current e-voting protocols, our e-voting system supports public varification and does not need authority centers, that is, this scheme is decentralized.

The rest of this work is organized as follows: In Section 2, some basic definitions are reviewed. Section 3 presents an MSSS with no trusted third party. In Section 4, a decentralized multi-role e-voting system is proposed. Finally, conclusions are provided in Section 5.

#### 2. Preliminaries

#### 2.1. Access stru vu e

Let  $\mathcal{P} = \{I_1, ..., P_n\}$  be a set of participants. An access structure  $\Gamma$  is defined on  $\mathcal{P}$  and  $\Gamma$  is a collection of all authorized subsets of  $\mathcal{P}$  that satisfies monotone are dirg property, then we only need to consider the minimum access structure.

# 2.2. Mon tone span programs

Monotone span program (MSP)  $\mathcal{M}(\mathcal{P}, \mathcal{F}, M, \psi)$  [16] involves a set of parties  $\mathcal{F}$  a first field  $\mathcal{F}$ , a matrix M over this field and a labeling map  $\psi$ , where M is a  $d \times l$  matrix and  $\psi$  is a surjection from  $\{1, \ldots, d\}$  to  $\{P_1, \ldots, P_n\}$ . Actually, MSP is a model for computing Monotone Boolean Function. Suppose that  $\mathcal{F}$  is a target vector, if  $\vec{v}$  can be linearly combined by the rows of  $M_A$  for

 $A \subset \mathcal{P}$ , where the row numbers of  $M_A$  are the preimages of  $\mathcal{L}^1$   $P_i \in A$ . Then Boolean function value f(A) = 1.

# 2.3. linear secret sharing

For a given an access structure  $\Gamma$ , constructing a linear secret sharing scheme (LSSS) on  $\Gamma$  is equivalent to obtaining an M.P. Generally, the target vector  $\vec{v}$  is  $(1,0,\ldots,0)$ . If  $A\in\Gamma$  and  $P_i\in\Lambda$  for some i, then v can be linearly combined by  $M_i$ , that is,  $v=\sum_{P_i\in\Lambda}a_i$ ,  $M_i$  for  $a_i\in\mathcal{F}$ . Let s be the shared secret. Randomly select a vector  $w=(s,w_1,w_2,\ldots,w_{l-1})$ , each  $P_i$  will get his share  $y_i$ , where  $y_i$  is the inner product of  $M_i$  and  $\vec{w}$ . Thus, any authorized subset A can recover the sorter  $s=\sum_{P_i\in A}a_i\cdot y_i$ . Thus, a linear multi-secret sharing scheme [11] can be obtained based on multi-target vectors the corresponding m-tuple  $\vec{\Gamma}=(1,\ldots,\Gamma_m)$  of access structures.

# 3. Multi-secret sharing scheme

Inspired by the multi-secret sharing scheme (MSSS) in [11], we present the construction of our MSSS with no trusted third party.

#### 3.1. Description of MSSS

Setup: Let  $\mathcal{P} = \{P_1, P_2, \dots, P_l\}$  be the participant set and  $\Upsilon(\mathcal{P})$  be the power set of  $\mathcal{P}$ . Here,  $|\Upsilon(\mathcal{P})|$  is  $2^n$ . Then, the cardinality of the collection of the near pty subsets of  $\mathcal{P}$  is  $2^n - 1$ . Let  $m = 2^n - 1$  and an m-tuple access  $|\Upsilon|$  cture be  $\vec{\Gamma} = \{\Gamma_1, \Gamma_2, \dots, \Gamma_m\}$ , where  $\Gamma_l$  only contains the  $|\Omega|$  nonempty subset (we consider the minimum access structure). Let  $S_l$  denote the target secret associated with a sub-access structure  $|\Upsilon|$  ( $|\leqslant l \leqslant m|$ ). Thus, a multi-level access structure is well-defined.

**Multi-target MS?** : Let  $\mathcal{F} = F_p^n$  for prime p. Randomly choose an  $n \times n$  invertible partix M. Let  $u_i = M_i$ , then each set of at most n vectors in  $\{u_1, \ldots, u_n\}$  is linearly independent, where  $u_i$  is the adjoint vector of participant  $P_i$  and  $V_i = \operatorname{span}\{\vec{u}_i\}$ .

Furthermore, let  $\vec{v}_l = \sum_{\substack{P_i \in \Gamma_l \\ x_{i,l} \in F_p}} x_i \cdot \vec{u}_i \ (l = 1, \dots, m)$  be m target vectors, where coefficients  $x_{i,l}$  are randomly chosen from field  $F_p$ . Then we can becain a multi-target MSP  $\mathcal{M}(\mathcal{F}, F_p, M, \psi)$ , where  $\psi(i) = P_i \ (1 \leq i \leq n)$ .

Note that, the matrix M, all target vectors and  $x_{i,l}$  are public.

Secret sharing: The algorithm is composed of three stage.

- Master secret generation. Each participant  $P_i$  ( $i \le i \le n$ ) selects a random vector  $\vec{r_i} \in F_p^n$  and computes  $S_{i,j} = r_i \cdot \vec{v_l}$  for sub-access structure  $\Gamma_l$  (l = 1, ..., m). Here,  $S_{i,l}$  (l = 1, ..., m) are sub-secrets determined by  $P_i$ . And m master  $\vec{r_i}$  crets can be determined as  $S_l = \sum_{i=1}^n S_{i,l}$ , for l = 1, ..., m.
- Sub-share generation. Each participant  $P_i$   $(1 \le i \le n)$  computes and sends the inner product  $s_{ij} = r_i \quad \vec{\tau}_j = \vec{r}_i \cdot M_j$  to  $P_j$ , for  $j = 1, \ldots, n$ . Meanwhile,  $P_i$  obtains a sub-states,  $s_{ji}$   $(j = 1, \ldots, n)$ .
- Master share generation. Participant  $P_i$  calculates the corresponding master share  $s_i = \sum_{j=1}^n c_j$ .

**Secret reconstruction**: For any cumplified subset  $A \in \Gamma_l$   $(1 \le l \le m)$ , since  $\vec{v}_l = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot \vec{u}_i$ , then the participants in A can reconstruct secret  $S_l = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot s_i$   $(1 \le \iota \le m)$  (see the correctness proof of Theorem 1).

Note that in such an N.SSS, each participant  $P_i$   $(1 \le i \le n)$  only needs to carry one master share  $s_i$  to be able to reconstruct multiple secrets by linearly combining the corresponding master shares. Table 1 and Table 2 present master secrets an i master shares generation.

Table 1: Master secrets generation

	-71	•••	$P_n$	Master secret
$\Gamma_1$	$\overline{\mathcal{C}}_{1,1}$	• • •	$S_{n,1}$	$S_1 = \sum_{i=1}^n S_{i,1}$
$\Gamma_2$	$S_{1,2}$	• • •	$S_{n,2}$	$S_2 = \sum_{i=1}^n S_{i,2}$
	( //:	٠	:	:
1 m	$\overline{S_{1,m}}$	• • •	$S_{n,m}$	$S_m = \sum_{i=1}^n S_{i,m}$

# 3.2. Corre tness and Security

in this section, from the information theory, we will prove that our scheme is a perfect SSS, where Theorem 1 shows the correctness and Theorem 2 presents the privacy [11, 29].

Table 2: Master shares generation

	$P_1$		$P_n$	Master sha
$P_1$	$s_{11}$	• • •	$s_{n1}$	$s_1 = \sum_{j=1}^n s_{j1}$
$P_2$	$s_{12}$		$s_{n2}$	$s_2 \leftarrow \sum_{j=1}^n s_{j2}$
:	:	٠	:	
$P_n$	$s_{1n}$	• • •	$s_{nn}$	$s_n = \sum_{j=1}^n s_{jn}$

**Theorem 1.** For any authorized subset  $A \subset \Gamma_l$  ( $^{\circ} \leq l \leq m$ ), A can recover the shared secret.

**Proof.** Since  $A \in \Gamma_l$ , and the target  $v_l = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot \vec{u}_i$ . Then we have the following equations:

$$\begin{split} S_{1,l} &= \vec{r}_1 \cdot \vec{v}_l = \vec{r}_1 \cdot \left( \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot \vec{u}_i \right) - \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i (\vec{r}_1 \cdot \vec{u}_i) = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot s_{1i}, \\ S_{2,l} &= \vec{r}_2 \cdot \vec{v}_l = \vec{r}_2 \cdot \left( \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot u_i \right) - \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i (\vec{r}_2 \cdot \vec{u}_i) = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot s_{2i}, \end{split}$$

 $S_{n,l} = \vec{r}_n \cdot \vec{v}_l = \vec{r}_n \cdot (\sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot \bar{u}_i) = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i (\vec{r}_n \cdot \vec{u}_i) = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot s_{ni}.$  Thus, the master secret S can be determined as

$$S_{i} = \sum_{1}^{n} S_{i,l} = S_{1,l} + \ldots + S_{n,l}$$
 (1)

$$= \sum_{\substack{\gamma_i \in \Gamma_l \\ \alpha_i \in F}} x_i \cdot s_{1i} + \ldots + \sum_{\substack{P_i \in \Gamma_l \\ \alpha_i \in F}} x_i \cdot s_{ni}$$
 (2)

$$= \sum_{\substack{P_i \in \Gamma_l \\ r_i \in F_n}} x_i \cdot \left(\sum_{j=1}^n s_{ji}\right) \tag{3}$$

$$S_{i} = \sum_{1}^{n} S_{i,l} = S_{1,l} + \ldots + S_{n,l}$$

$$= \sum_{\substack{s_{i} \in \Gamma_{l} \\ x_{i} \in F_{p}}} x_{i} \cdot s_{1i} + \ldots + \sum_{\substack{P_{i} \in \Gamma_{l} \\ x_{i} \in F_{p}}} x_{i} \cdot s_{ni}$$

$$= \sum_{\substack{P_{i} \in \Gamma_{l} \\ x_{i} \in F_{p}}} x_{i} \cdot (\sum_{j=1}^{n} s_{ji})$$

$$= \sum_{\substack{P_{i} \in \Gamma_{l} \\ x_{i} \in F_{p}}} x_{i} \cdot s_{i}.$$
(4)
It ws from the property of additive homomorphism. Thus

where Eq. (3) allows from the property of additive homomorphism. Thus, any authorized subset  $A \in \Gamma_l$  can recover the master secret  $S_l$  by calculating a linear cor pir itio of participants' master shares.

**Theorem 2.** In any unauthorized subset  $B \notin \Gamma_l$  and  $A_l \not\subseteq B$  for  $1 \le l \le m$ , where  $\Gamma = \{ \mathcal{N}_i \}$ . Then, B fails to get any information about the shared secret  $S_l$ .

**Proof.** Suppose that  $\Gamma = \{\Gamma_1, \Gamma_2, \dots, \Gamma_m\}, \vec{u}_i$  and  $\vec{v}_l$  are defined in section 3.1, Nince the vectors  $\vec{u}_1, \ldots, \vec{u}_n$  are linearly independent and  $A_l \not\subseteq B$ . Then,  $\vec{v}$  does not lie in  $span\{u_j\}$  for  $P_j$  belongs to the unauthorized subset B. Otherwise, we have  $\vec{v}_l = \sum_{\substack{P_j \in B \\ y_j \in F_p}} y_j \cdot \vec{u}_j$ . Since  $\vec{v}_l = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot \vec{u}_i$ . Then,  $\sum_{\substack{P_j \in J \\ y_j \in F_p}} y_j \cdot \vec{u}_j = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot \vec{u}_i$ . This implies that  $\vec{u}_1, \ldots, \vec{u}_n$  are linearly

dependent, which is a contradiction. Thus, any unauthorize  $^1$  subset B fails to get any information about the shared secret  $S_l$ .

On the other hand, we want to illustrate the security from the information theory. We compute the information entropy H(S) of our aining secret  $S_l$  with knowing its space  $S_l$  and the entropy  $H(S_l|B)$  of getting  $S_l$  with knowing  $S_l$  and shares for B. Suppose that  $\vec{r}$  is an n-dimensional vector and there is a random number  $S'_l \in S_l$ . For any unauthorize T, ubset  $B \notin \Gamma_l$ , we can construct the following equations:

$$\begin{cases}
\vec{v}_l \cdot \vec{r} = S', \\
\vec{u}_1 \cdot \vec{r} = s_1, \\
\dots, \\
\vec{u}_i \cdot \vec{r} = s_i, \\
\dots, \\
\vec{u}_{|B|} \quad \vec{r} = s_{|B|}
\end{cases}$$
(5)

In the master shares generation phase, we have that  $\vec{u}_i \cdot (\vec{r}_1 + \cdots + \vec{r}_n) = s_i$ , for  $i = 1, \ldots, n$ . Thus, the following equations

$$\begin{cases}
\vec{u}_1 \cdot \vec{r} = s_1 \\
\cdots \\
\vec{u}_i \cdot \vec{r} = s_i \\
\cdots \\
\vec{u}_{|B|} \cdot \vec{r} = s_{|B|}
\end{cases}$$
(6)

have solutions. The vis,  $\vec{r} = \sum_{i=1}^{n} \vec{r_i}$ . Meanwhile, since  $\vec{v_l} \notin \bigcup_{B \in (\mathcal{A}_l)_{\text{max}}} \sum_{P_i \in B} V_i$ , we have that  $Rank(\{\vec{u}_1, \ldots, \vec{u}_{|B|}\}) + 1 = Rank \ (\{\vec{u}_1, \ldots, \vec{u}_{|B|}, \vec{v_l}\})$  and the rank of coefficient natrix is equal to that of augmented matrix. Eqs.(5) always have solutions with respect to any element  $S'_l \in \mathcal{S}_l$ . Therefore,

$$H(\mathcal{S}_l|B) = \sum_{S_l \in \mathcal{S}_l} Pr[S_l|B] \cdot \log Pr[S_l|B] = \sum_{i=1}^p \frac{1}{p} \log p = \log p = H(\mathcal{S}_l).$$

We see that the security of our scheme is proved from the aspect of informatio, theory. That is, the scheme is unconditionally-secure without no accumption of intractable problem.

# 3.3 Efficiency analysis

The efficiency of our proposal is summarized in Table 3.  $NMO_D$  denotes the number of module multiplication required by each participant in the dis-

tribution phase and  $NMO_R$  denotes the number of multiplication required in the reconstruction phase. NTS indicates the number of maked target secrets, NMS is the number of master shares for every particapara, and NSS is the number of sub-shares sent from one participant to others.

- NMO<sub>D</sub>: Since each one needs to perform n inner product operations between two n-dimension vectors, then each participant requires  $n^2$  times multiplication.
- NMO<sub>R</sub>: For l-th sub-access structure, secret reconstruction needs  $|\Gamma_l| (\leq n)$  times multiplication (see Section 3.2).
- NTS: There are  $m = 2^n 1$  master secret, shared in the scheme (see Table 1).
- NMS: From Table 2, we see that each participant carries only one share that is combined by n sub-shares.
- NSS: In the distribution phase, each corresponding participants.

Table 3: Committee ve analysis on SSS

	$NMO_D$	$N_{N}O_{R}$	NTS	NMS	NSS
Number	$n^2$	$ \Gamma_l $	$2^{n}-1$	1	n-1

In summary, each participant only carries one master share for sharing multiple master (target) regrets. Furthermore, the scheme needs no trusted third party and it can be used to construct a decentralized e-voting protocol.

T ble : Performance Comparison on SSS without TTPs

	SSS [10]	SSS [25]	SSS [20]	Our SSS
Acce s Structure	One-level	One-level	One-level	Multi-level
Mu. sec et	No	No	Yes	Yes
Exporential computation	No	No	Yes	No

#### 4. A ....'.i-role e-voting system

Yow we present a decentralized e-voting protocol for multi-role election.

4.1. Description of the e-voting system

Setup: Let  $\mathcal{P} = \{P_1, P_2, \dots, P_n\}$  be a voter set,  $m = 2^r - 1$  be the number of sub-voting protocols. Here, we continue to use the access structure and MSP constructed in Section 3. With respect to Lab-access structure  $\Gamma_l$   $(1 \leq l \leq m)$ , a sub-voting protocol can be correspondingly designed. Let  $\mathcal{C}_l = \{C_{l,1}, C_{l,2}, \dots, C_{l,k_l}\}$  be the set of  $k_l$  and addess for the l-th sub-protocol for  $l \in \{1, \dots, m\}$ .

The cloud server broadcasts the values  $v_y$ ,  $v_n$  and  $v_0$ , such that  $n^2v_0 < nv_{no} < v_{yes}$ , where  $v_{yes}$ ,  $v_{no}$  and  $v_0$  are assigned to the yes (Y) vote, the no (N) vote and the abstention (A) vete, respectively. Suppose that  $k = \max\{k_l | l = 1, \ldots, m\}$ . Then the voud chooses a prime sequence  $p_1, p_2, \ldots, p_k$ , such that  $p_{\lambda} > n \cdot v_y$  and  $\prod_{\lambda=1}^k p_{\lambda} < p$  (here, p is defined in above SSS).

Ballot Generation: This algorith, uses Chinese Remainder Theorem and the proposed secret sharing scheme.

• Each voter  $P_i$   $(1 \leq ... \leq n)$  selects a random vector  $\vec{r_i} \in F_p^n$  and computes  $S_{i,l} = \vec{r_i} \cdot v_l$ , for l = 1, ..., m. Then he generates his votes  $b_{i,l,\lambda}$  for calcidate  $C_{l,\lambda}$   $(C_{l,\lambda} \in C_l)$ , where  $b_{i,l,\lambda} \in \{v_{yes}, v_{no}, v_0\}$  and  $\lambda = 1, ..., k_l$ .  $P_i$  computes  $B_{i,l}$  by using Chinese Remainder Theorem to solve  $k_l$  equations:

$$B_{i,l} \equiv b_{i,l,\lambda} \mod p_{\lambda}, C_{l,\lambda} \in \mathcal{C}$$

- Then v ter  $P_i$  broadcasts ballots  $T_{i,l} = S_{i,l} + B_{i,l}$ , where  $S_{i,l}$  are blinding factors, for l = 1, ..., m (see Table 5).
- Each voler  $P_i$   $(1 \le i \le n)$  sends the inner product  $s_{ij} = \vec{r_i} \cdot \vec{u_j}$  to  $P_i$  for  $j = 1, \ldots, n$ . Then each voter  $P_i$  receives  $s_{ji}$  and computes  $s_{ji} = \sum_{j=1}^{n} s_{ji}$ .

Vote Counting: This algorithm mainly employs the secret reconstruction algorithm. At the same time, there is no need of an authority center for vote tallying.

• For the l-th sub-protocol,  $P_i$  ( $P_i \in \Gamma_l$ ,  $1 \le l \le m$ ) sends  $s_i$  to the cloud server without secret channels. Since  $\vec{v}_l = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot \vec{u}_i$ , then the cloud recovers  $S_l = \sum_{\substack{P_i \in \Gamma_l \\ x_i \in F_p}} x_i \cdot s_i$  and computes  $B_l = T_l - S_l$ , where  $T_l = \sum_{i=1}^n T_{i,l}$ ,  $B_l = \sum_{i=1}^n B_{i,l}$ .

• The numbers of Y, N and A votes, denoted by  $x_{l,\lambda}$ ,  $u_{l,\lambda}$  and  $z_{l,\lambda}$  for candidate  $C_{l,\lambda}$  ( $1 \le \lambda \le k_l$ ), can be computed by solving equation:

$$x_{l,\lambda} \cdot v_{yes} + y_{l,\lambda} \cdot v_{no} + z_{l,\lambda} \cdot v_0 \equiv B_l \pmod{p_{\lambda}},$$

where  $n^2v_0 < nv_{no} < v_{yes}$  and  $x_{l,\lambda} + y_{-\lambda} + z_{l-\lambda} = n$ . Then the solution is

$$\hat{x}_{l,\lambda} \leftarrow B_l \mod p_{\lambda}, \qquad x_{l,\lambda} - \lfloor \hat{z}_{l,\lambda}/v_{yes} \rfloor;$$

$$\hat{y}_{l,\lambda} \leftarrow \hat{x}_{l,\lambda} \mod v_{yes}. \qquad y_{l,\lambda} \leftarrow \lfloor \hat{y}_{l,\lambda}/v_{no} \rfloor;$$

$$\hat{z}_{l,\lambda} \leftarrow \hat{y}_{l,\lambda} \mod v_{no}, \qquad z_{l,\lambda} \leftarrow \lfloor \hat{z}_{l,\lambda}/v_0 \rfloor.$$

• Finally, the voting outcome  $x_{l,\lambda}$ ,  $y_{l,\lambda}$ ,  $z_{l,\lambda}$  and  $s_i$  is public. Each additional Y vote is 2 points, additional A vote is 1 point and each additional N vote is 0 point. Hence, all parties can choose the winner according to the voting results. Furthermore, any one can check its validitation if  $x_{l,\lambda} + y_{l,\lambda} + z_{l,\lambda} = n$  and  $x_{l,\lambda}$ ,  $y_{l,\lambda}$ ,  $z_{l,\lambda}$  are integers, then there is no fraud.

Table 5. Or intitative analysis on SSS

	$P_1$	7	$P_n$	Masked votes	
$\Gamma_1$	$T_{1,}$	.,.	$T_{n,1}$	$T_1 = \sum_{i=1}^n T_{i,1}$	
$\Gamma_2$	T		$T_{n,2}$	$T_2 = \sum_{i=1}^n T_{i,2}$	
		· · .	:	<u>:</u>	
$\Gamma_m$	$T_1$ , $m$		$T_{n,m}$	$T_m = \sum_{i=1}^n T_{i,m}$	

To illustrate the e-voting protocol better, we give an example in appendix.

#### 4.2. Dis ussion and analysis

Now we analyze some important properties for this new e-voting protocol.

- Correctness. The correctness holds based on Theorem 1 and Chinese Remando. Theorem.
- VIU : role. An e-voting protocol is said to be multi-use, if it can be used to simultaneously finish multiple roles in one election. In our e-voting system, each role can be chosen based on the corresponding sub-access structure.

Thus, on the basis of the family of access structures define in MSSS, the e-voting model achieves multi-role election.

- **Decentralization.** The e-voting protocol does not need any trusted third party. The vote tallying is finished by the cloud server.
- Anonymity. Any unauthorized voter set or cloud servers cannot link a voter's identity to the corresponding vote. For one case, all voters blind their real votes. For another, the anonymity vas  $\epsilon$  and used by homomorphic property. The cloud servers can only compute the final votes summation  $B_l = T_l S_l$ , without the knowledge of single these  $B_{i,l}$  ( $1 \le i \le n$ ). And Theorem 3 below shows that the published parameters won't reveal the real votes. Thus, the proposed protocol supports the anonymity.
- Uniqueness. Each voter can throw only one vote. If there exists one malicious voter  $P_i$ , giving multiple votes, then  $x_{l,\lambda} + y_{l,\lambda} + z_{l,\lambda} \neq n$ . Thus, no dishonest voter can perform the election find by throwing multiple votes.
- Public verifiability. Since the ballo ,  $T_{i,l}$  and parameters  $s_i$  are published for verification. Any party can check the validity of the outcome based on the Uniqueness.

**Theorem 3.** Suppose that parameters  $T_{i,l}$ ,  $s_i$  (i = 1, ..., n, l = 1, ..., m) published are defined as above, it holds that no one can obtain blinding factors  $S_{i,l}$  (i = 1, ..., n) = 1, ..., m from  $s_i$  and the real votes  $B_{i,l}$  are unknown.

**Proof.** From Theore in 1, we see that the system of equations about  $S_{i,l}$  (i = 1, ..., n, l = 1, ..., n, l) a follows:

$$\begin{cases} S_{1,1} \cdot S_{2,1} + \dots + S_{n,1} = \sum_{\substack{P_i \in \Gamma_1 \\ x_{i1} \in F_p}} x_{i1} \cdot s_i \\ \dots \\ S_{1,l} + S_{2,l} + \dots + S_{n,l} = \sum_{\substack{P_i \in \Gamma_1 \\ x_{il} \in F_p}} x_{il} \cdot s_i \\ \dots \\ S_{m} + S_{2,m} + \dots + S_{n,m} = \sum_{\substack{P_i \in \Gamma_m \\ x_{im} \in F_p}} x_{im} \cdot s_i \end{cases}$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7)$$

$$(7$$

where  $x_l$   $(x_{il} \in F_p)$  are known. Because there are m equations and mn unknown. (m < mn), no one can solve  $S_{i,l}$  (i = 1, ..., n, l = 1, ..., m). Thus, the public parameters does not reveal the real votes  $B_{i,l}$ .  $\square$ 

1 'rfor nance Comparison. The most existing e-voting protocols [17, 1°, 28, 30] were proposed based on digital signature schemes, and those schemes used discrete logarithm and bilinear pairing operations, which need high-omplexity computational cost. Compared with the most existing e-

voting protocols, our protocol is constructed based on linear multi-secret sharing scheme (MSSS) and it only needs some additions and multiplications in the given finite field  $F_p$ . For example, the vote pasting phase for one role election only requires 2n module multiplications, where n is the number of voters. Assume that p is about 160-bit for resisting whaustive attacks and let n < 100, then the computational cost of the phase can be counted in Microseconds ( $\mu s$ ). In [17], the running time of the vote casting phase was counted in Millisecond (ms). Secondly, based on the MSSS and Chinese Remainder Theorem, our scheme achieves multi-role e-voting in one election, thus it has a higher utilization. In addition, compared with the traditional e-voting schemes, our scheme are not need an authority center. In other words, the tally phase can be cuttouted to a cloud server for saving computational resources.

#### 5. Conclusions

In this work, we construct a multi-secret sharing scheme based on multi-target MSP. The scheme does not require any trusted center, where each participant is also a dealer to generate master secrets together. Then, the scheme can be applied into a decentralized consensus system. Furthermore, based on the multi-target MSP, we design a multi-role e-voting model based on Chinese Remainder Theorem that can simultaneously achieve multiple roles in one election, which but employing an authority center. Meanwhile, the protocol supports publicly verification for the voting outcome. In the future work, we will design more efficient e-voting protocols under different security models [12, 13] and apply the protocols to IoTs [24].

#### Acknowled emenus

This work var supported by National Natural Science Foundation of China (Nos. C1472091, 61370194), Natural Science Foundation of Guangdong Prevince for Distinguished Young Scholars (No. 2014A030306020), Guarganou scholars project for universities of Guangahou (No. 1201561613), Science and Technology Planning Project of Guangahou Province, China (No. 2015B010129015), National Natural Science Foundation for Outstanding Young Foundation (No. 61722203), National Key R&D Program of China (No. 2016YFN0800602), Shandong provincial Key R&D Program of China (No. 2018CXGC0701) and JSPS KAKENHI Grant Number JP15K00028.

- [1] M. Ambrosin, P. Braca, M. Conti, et al. ODIN: Obfuscat, n-based privacy preserving consensus algorithm for Decentralized Information tusion in smart device Networks. 2017.
- [2] J.C. Benaloh. Secret sharing homomorphisms: keeping somes of a secret, in: Advances in Cryptology, Proceedings of the Crypto86, 11-15 August, Santa Barbara, California, USA, LNCS, vol. 263, Springe -Verlage, Berlin, 1987: 251-260.
- [3] J. Benaloh, J. Leichter, Generalized secret sharing and monotone functions, in: S. Goldwasser (Ed.), Advances in Crypto. 3y, CRYPTO88, in: Lecture Notes in Computer Science, 1989, 403: 27-5.
- [4] E.F. Brickell. Some Ideal Secret Sharing Scheme. Journal of Combinatorial Mathematics & Combinatorial Computing, 1980, 434: 468-475.
- [5] A. Das, A. Adhikari, An efficient multi-use multi-us
- [6] M.H. Dehkordi, S. Mashhadi. An eminant threshold verifiable multi-secret sharing. Computer Standards & Interfaces, 2008, 30(3): 187-190.
- [7] M.H. Dehkordi, S. Mashhadi. New chicient and practical verifiable multisecret sharing schemes. Information Sciences, 2008, 178(9): 2262-2274.
- [8] M.H. Dehkordi, S. Mashhadi. Verl'able secret sharing schemes based on non-homogeneous linear recursions and elliptic curves. Computer Communications, 2008, 31(9): 1777-1754.
- [9] C. Gao, S. Lv, Y. Wei, Z. V. ang, Z. Liu, X. Cheng. M-SSE: An Effective Searchable Symmetric Encryption with Enhanced Security for Mobile Devices. IEEE Access. 6: 388′ 0-38869, 2018.
- [10] L. Harn, C. Lin Strong (n, t, n) verifiable secret sharing scheme, Information Sciences, 2010 180. 3059-3064.
- [11] C.F. Hsu, C. Cheng, X.M. Tang, B. Zeng, An ideal multi-secret sharing scheme bas do MSP, Information Sciences, 2011, 181: 1403-1409.
- [12] Z. Huane, J. Lai, W. Chen, M. Raees-ul-Haq, L. Jiang. Practical public key encrypt on with selective opening security for receivers. Information Sciences, 2019, 478. (5-27).
- [13] Z. Fuang, J. Lai, W. Chen, T. Li, Y. Xiang. Data security against receiver corrotions SOA security for receivers from simulatable DEMs. Information Science, 2018, 471: 201-215.
- [14] M. Ito, A. Saito, T. Nishizeki. Secret sharing scheme realizing general access source realizing. Electronics & Communications in Japan, 1989, 72(9): 56-64.
- [17] I. Jiang, Y. Cheng, L. Yang, J. Li, H. Yan, X. Wang. A Trust-Based Collaboative Filtering Algorithm for E-Commerce Recommendation System. Journal of Ambient Intelligence and Humanized Computing, DOI: 10.1007/s12652-018-0928-7, 2018.

- [16] M. Karchmer, A. Wigderson, On span programs, in: Proceedings of the Eighth Annual Conference on Structure in Complexity. San Diego, CA, 1993: 102-111.
- [17] M. Kumar, C.P. Katti, P.C. Saxena. A Secure Anonymo. E-Voting System Using Identity-Based Blind Signature Scheme. Information Systems Security. ICISS 2017. Lecture Notes in Computer Science, vol 10717. Springer.
- [18] W. Jamroga, M. Knapik, D. Kurpiewski. Moder Checking the SELENE E-Voting Protocol in Multi-agent Logics: Third international Joint Conference, E-Vote-ID 2018, Bregenz, Austria, October 2-5, 2018. Proceedings: Electronic Voting. Springer, 2018.
- [19] K. Yeow, A. Gani, R.W. Ahmad, et al. Decent alized consensus for edgecentric interent of things: a review taxon, my and reserch issues. IEEE Access, 2017, 99: 1-1.
- [20] J. Li, L. Wang, J. Yan, et al. A (k, t, n) rifiable multi-secret sharing scheme based on adversary structure. Ks. Transactions on Internet & Information Systems, 2014, 8(12): 4552-4567.
- [21] H.Y. Lin and Y. Shiung. Dynan we multi-secret sharing scheme. Int. J. Contemp.math.Sciences, 2008, 3(1): 3, 42.
- [22] T. Li, Z. Huang, P. Li, Z. Liu, C. Jia. Outsourced Privacy-Preserving Classification Service over Encrypted Data. Journal of Network and Computer Applications, 2018, 106, 106, 106, 106.
- [23] T. Li, W. Chen, Y. Tan, H. Yan. A Homomorphic Network Coding Signature Scheme for Multiple Sources and its Application in IoT. Security and Communication Networks, 2018, DOI: 10.1155/2018/9641273, 2018.
- [24] T. Li, C. Gao, A. Jiang, W. Pedrycz, J. Shen. Publicly verifiable privacy-preserving agg egation and its application in IoT. Journal of Network and Computer Applications, 2018, 126: 39-44.
- [25] Y.X. Liu, I. Hern, C.N. Yang, Y.Q. Zhang, Efficient (n, t, n) secret sharing schemes, Journal of Systems and Software, 2012, 85: 1325-1332.
- [26] Y. Lyu, V.C.S. Jee, C.Y. Chow, et al. R-Sharing: Rendezvous for Personalized Taxi Shar. 7. LEE Access, 2017, 99: 1-1.
- [27] T.P Peder, en. A threshold cryptosystem without a trusted party, in: Advances in C yptology, Proceedings of the Eurocrypt'91, 8-11 April, Brighton, U.L., Linco, Springer-Verlag, Berlin, 1991, 547: 522-526.
- [28] E.A. Q aglia, B. Smyth. Authentication with Weaker Trust Assumptions for Vering Systems. International Conference on Cryptology in Africa. Springer,
- [29] C.E. Shannon and W. Weaver. The Mathematical Theory of Communication, The University of Illinois Press, Urbana, IL, 1949.

- [30] S. Tamura, H.A. Haddad, N. Islam, et al. An Incoercible T-Voting Scheme Based on Revised Simplified Verifiable Re-encryption of x-ix-necs. Computer Science, 2015.
- [31] Q. Xia, E.B. Sifah, K.O. Asamoah, et al. MeDShare: Trust Tess Medical Data Sharing Among Cloud Service Providers via Bloc' chain. IEEE Access, 2017, 5: 14757-14767.
- [32] J. Zhang, Z. Zhang. Secure and efficient data-sharing in clouds. Concurrency & Computation Practice & Experience, 2015, 27(3): 2125-2143.
- [33] Y. Zhang, D Zheng, R.H. Deng, Security and p. ivacy in smart health: Efficient policy-hiding attribute-based access control, IEE1 Internet of Things Journal, 2018, 5(3): 2130-2145.

# Appendix A. Toy example

We now give an example to show the process of the e-voting system.

• Multi-target access structure:

Let 
$$\mathcal{P} = \{P_1, P_2, P_3\}$$
 and  $\Omega = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_1, P_2\}, \{P_1, P_3\}, \{P_2, P_3\}\}$ ,  $\{P_1, P_2, P_3\}\}$ . It can be seen that  $n = 3, m = 7, \varphi : \{1, \dots, 7\} \to \Omega$  and there are sever master secrets to be shared in such a 7-tuple  $\vec{\Gamma} = \{\Gamma_1, \dots, \Gamma_7\}$  of access structures as follows:  $(\Gamma_1)_{min} = \{\{P_1\}\}, (\Gamma_2)_{min} = \{\{P_2\}\}, (\Gamma_3)_{min} = \{\{P_3\}\}, (\Gamma_4)_{min} = \{\{P_1, P_2\}\}, (\Gamma_7)_{min} = \{\{P_1, P_3\}\}, (\Gamma_6)_{min} = \{\{P_2, P_3\}\}, (\Gamma_7)_{min} = \{\{P_1, P_2, P_3\}\}$ 

• Target-vector generation:

Suppose that p = 6045133 and  $\overline{V} = F_p^3$  with a basis  $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ , where  $\vec{e}_1 = (1, 2, 5)$ ,  $\vec{e}_2 = (3, 7, 10)$  and  $\vec{e}_3 = (4, 1, 9)$ . Let  $\vec{u}_1 = \mathbf{v}(1) = (8, 10, 2^4)$  associated with  $P_1$ ,  $\vec{u}_2 = \mathbf{v}(3) = (46, 32, 116)$  associated with  $P_2$  and  $\vec{v}_3 = \mathbf{v}(4) = (77, 46, 189)$  associated with  $P_3$ . Let seven target vectors be as follows:  $\vec{v}_1 = \mathbf{v}(1) = (8, 10, 24)$ ,  $\vec{v}_2 = 2\mathbf{v}(3) = (92, 64, 232)$ ,  $\vec{v}_1 = 2 \cdot (4) = (231, 138, 567)$ ,  $\vec{v}_4 = 2\mathbf{v}(1) + \mathbf{v}(3) = (62, 52, 164)$ ,  $\vec{v}_5 = \mathbf{v}(1) - 2\mathbf{v}(4) = (162, 102, 402)$ ,  $\vec{v}_6 = 3\mathbf{v}(3) + 2\mathbf{v}(4) = (292, 188, 726)$ ,  $\vec{v}_7 = 4\mathbf{v}(1) + 2\mathbf{v}(3) + 3\mathbf{v}(4) = (355, 242, 895)$ , where the coefficients in the expressions above are randomly chosen from field  $F_p$ .

We will give the continued example with respect to  $\Gamma_5$  and  $\Gamma_6$ .

# • Master secret generation:

 $P_1$ ,  $P_2$  and  $P_3$  selects  $\vec{r}_1 = (7,4,13)$ ,  $\vec{r}_2 = (11,3,6)$  and  $\vec{r}_3 = (1,19,5)$ , respectively. Then participant  $P_i$  computes  $S_{i,l} = \vec{r}_1 \cdot \vec{v}_l$   $(i=1,\ldots,3,l=5,6)$  as  $S_{1,5} = 6768$ ,  $S_{1,6} = 12234$ ;  $S_{2,5} = 4500$ ,  $S_{2,6} = 8132$ ;  $S_{3,5} = 4110$ ,  $S_{3,6} = 7494$ . Then master see ets are  $S_5 = 15378$  and  $S_6 = 27860$ .

# • Master share generation:

Participant  $P_i$  computes sub-share  $s_{ij} = \vec{c} \cdot \vec{u}_j$  (i = 1, ..., 3, j = 1, ..., 3) that  $s_{11} = 408$ ,  $s_{12} = 1958$  and  $s_{13} = 3180$ ;  $s_{21} = 262$ ,  $s_{22} = 1298$  and  $s_{23} = 2119$ ;  $s_{31} = 318$ ,  $s_{32} = 1234$  and  $s_{33} = 1896$ . Then master shares are  $s_1 = 988$ ,  $s_2 = 490$  and  $s_3 = 7195$ .

#### • Master secret reconstruction:

For master secret  $S_6$  with rapped to  $\Gamma_6$  is determined as  $S_6 = 3s_2 + 2s_3 = 3 \cdot 4490 + 2 \cdot 7195 = 2.7863$  where the coefficients "3" and "2" are obtained from the example of target vector  $\vec{v}_6$ , while  $s_2$  and  $s_3$  are the master shares of participants  $P_2$  and  $P_3$ , respectively.

Note that the above or eratic is in MSSS are finished over field  $F_p$ .

#### • Public parameter for e-ting:

Let the set of candidates be  $C_5 = C_6 = \{C_1, C_2, C_3, C_4\}$ . The central authority CA choose  $v_{yes} = 13$ ,  $v_{no} = 4$ ,  $v_0 = 1$  and then selects  $p_1 = 41$ ,  $p_2 = 45$ ,  $p_3 = 47$ ,  $p_4 = 53$  that can satisfy the conditions of  $n \cdot v_{yes} < r_A$  and  $\prod_{\lambda=1}^4 p_{\lambda} < p$ .

#### • Ballot Cene. tion:

Each vote P generates his votes  $b_{i,l,\lambda} \in \{1,4,13\}$  for  $C_{\lambda}$   $(l = 5,6, \lambda = 1, ..., 4)$ , where

$$b_{1,5,1} = 4$$
,  $b_{1,5,2} = 13$ ,  $b_{1,5,3} = 13$ ,  $b_{1,5,4} = 1$ ;  
 $b_{2,5,1} = 13$ ,  $b_{2,5,2} = 4$ ,  $b_{2,5,3} = 13$ ,  $b_{2,5,4} = 1$ ;  
 $b_{3,5,1} = 1$ ,  $b_{3,5,2} = 13$ ,  $b_{3,5,3} = 13$ ,  $b_{3,5,4} = 4$ ;  
 $b_{1,6,1} = 1$ ,  $b_{1,6,2} = 13$ ,  $b_{1,6,3} = 13$ ,  $b_{1,6,4} = 1$ ;

$$b_{2,6,1} = 13$$
,  $b_{2,6,2} = 13$ ,  $b_{2,6,3} = 13$ ,  $b_{2,6,4} = 1$ ;  $b_{3,6,1} = 13$ ,  $b_{3,6,2} = 13$ ,  $b_{3,6,3} = 4$ , i.  $a_{4} = 1$ .

And then each voter uses Chinese Remaind r Theorem to solve the corresponding equations. Hence,  $B_{1,5} = 1225697 \mod \hat{p}$ ,  $B_{2,5} = 799718 \mod \hat{p}$ ,  $B_{3,5} = 1986656 \mod \hat{p}$ ,  $B_{1,6} = 5780^{\circ}9$  n od  $\hat{p}$ ,  $B_{2,6} = 3148731 \mod \hat{p}$ ,  $B_{3,6} = 2868414 \mod \hat{p}$ , here  $\hat{p} = \prod_{\lambda=1}^{4} \lambda_{\lambda} = 4391633$ . Then  $P_1$  publishes  $T_{1,5}, T_{1,6}, P_2$  publishes  $T_{2,5}, T_{2,6}$ , and  $P_3$  publishes  $T_{3,5}, T_{3,6}$ , where  $T_{i,l} = S_{i,l} + B_{i,l}$  for i = 1, 2, 3, l = 5.6

# • Vote Counting:

The cloud server collects the needed shares from voters in  $\Gamma_l$  (l=5,6) and recovers blinding factors  $\Sigma_c$  and  $S_6$  by master secret algorithm. Then  $B_5 = 4007071 \mod \hat{p}$ ,  $C_6 = 203531 \mod \hat{p}$ , and solve equations

$$x_{l,\lambda} \cdot v_{yes} + y_{l,\lambda} \cdot v_{no} + z_{l,\lambda} \cdot v_0 \equiv B_l \pmod{p_\lambda}$$

for  $l=5,6,\,\lambda=1,\ldots$  4, where  $3^2v_0<3v_{no}< v_{yes}$  and  $x_{l,\lambda}+y_{l,\lambda}+z_{l,\lambda}=3$ . Take the vote counting for candidate  $C_1$  as an example: Cloud computes

$$\hat{x}_{5,1} = 18, \quad x_{l,\lambda} = 1;$$
 $\hat{y}_{5,1} = 5, \quad y_{5,1} = 1;$ 
 $\hat{z}_{5,1} = 1, \quad z_{l,\lambda} = 1.$ 

by the desc. stion in vote counting. That is,  $C_1$  gets 2+1+0=3 points. And of ner results can be obtained by the same method. Then cloud broade of sprints 3, 5, 6, 1 for candidates  $C_1, C_2, C_3, C_4$  related to  $\Gamma_5$ ; and points 4, 6, 5, 0 for candidates  $C_1, C_2, C_3, C_4$  related to  $\Gamma_6$ . It can be seen to at  $C_3$  and  $C_2$  are winners for two roles, respectively.

# **Author Biography**

Jing Li received the B.S. degree from I are Tongol Normal University in 2010, the M.S. degree from Shananxi Normal University in 2013 and PhD degree in Beijing University of Posts and Telecommunications. Currently, she works at Guangzhou University. Her research interests include cloud computing, applied cryptography and privacy-preserving, etc.

Xianmin Wang race ved his BS degree from Suzhou

University, Jiangsu, China, in 2006, and his MS degree in computer science from Jiangxi University of Scie. ce and Technology, Jiang Xi, China, in 2013. He received the Phi degree in computer science in 2017 from Beihang University. Currently, he is working in the institution of School of computer science in Guar zhou University. His research interests include deep learning, image recessing and understanding.

Departn ent o Mathematics, Sun Yat-sen University in 2009 and 2011, respectively, and his Ph.D. degree from Department of Computer Science and Engineering, Shanghai Jiao Tong University in 2015. He served as a security engineer in Huawei Technologies Co. Ltd. from 2015 to 2016. Currently, he is a PostDoc in Guangzhou University. His research interests include public-key cryptography and information security.

Licheng Wang received the B.S. degree from Northwest Normal University in 1995, the M.S degree from Nanjing University in 2001 and the PhD degree from Shanghai Jiao Tong University in 2007. His current research interests include modern cryptog aphy, network security, trust management, etc. He is an associate professor in Beijing University of Posts and Telecommunications.

Yang Xiang received the Ph.D. degree in computer science from Deakin University, Australia. He is currently a Dean at the Digital Research & Innovation Capability Platform, Swinburne University of Technology. He is the director of the Network Security and Computing Lab (NSCLab). His research interests include network and system security, distributed systems, and letworking. He is the Chief Investigator of several projects funded by the Australian Research Council. He serves as an Associate Editor of the IEEE Transactions on Computers, the IEEE Transactions on Parallel and Distributed Systems, Security and Communication Networks, and the Editor of the Journal of Network and Computer Applications.