Assignment 4 (105 marks)

Due Monday Nov. 18, by 11:59pm.

- Assignments in MS Word or PDF format should be handed in via D2L.
- Your source code for Question 6 must be submitted with your Word or PDF document, and must also be submitted separately via D2L.

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1. (10 marks) The number of different ways to parenthesize an expression with $n+1$ variables is the nth Catalan number, $C(n)$, which can be calculated according to the following recurrence:

$$C(n) = \begin{cases} 1 & \text{if } n = 0 \\ \sum_{i=1}^{n} C(i-1)C(n-i) & \text{otherwise} \end{cases}$$

   Give an iterative dynamic programming algorithm to find $C(n)$ for a given positive integer n.

2. (15 marks) Given a set S of n positive integers and a positive integer g, design an efficient dynamic programming algorithm to find the smallest subset of S that sums to exactly g, or indicate such a subset does not exist. Notice that your algorithm returns the subset instead of its size. The size of a set is the number of its elements.

3. (15 marks) Given a one dimensional array that may contain both positive and negative integers, find the sum of contiguous subarray of numbers which has the largest sum. For example, if the given array is {-2, -5, 6, -2, -3, 1, 5, -6}, then the maximum subarray sum is 7. Design a dynamic programming algorithm to solve this problem, and analyze the time complexity of your algorithm.

4. (15 marks) Number Solitaire is a game played with a linear sequence of numbers. You start with no points. In each round, you remove two numbers, multiply them together, and add the product to your total. Only numbers at the two ends of the sequence (the rightmost and leftmost of the remaining numbers) are accessible; note that once you remove the first number in a round, the number next to it will be accessible and thus removable as the second number of the round. The sequence can include both positive and negative numbers, and its length is always even.

Design a dynamic programming algorithm that will compute the largest amount of points obtainable for the number sequence. The algorithm will be given the sequence length n and the sequence $L =< L_1, L_2, \cdots, Ln >$.

5. (15 marks) Given a sequence of distinct integers $a_1, a_2, \cdots, a_n$, design a dynamic programming algorithm to find the length of the longest decreasing subsequence. More specifically, if $1 \leq i_1 < i_2, \cdots, < i_k \leq n$ is a sequence of integers such that $a_{i_1} > a_{i_2}, \cdots, > a_{i_k}$, we say $a_{i_1}, a_{i_2}, \cdots, a_{i_k}$ is an decreasing subsequence. Notice that the required algorithm gives only the length of the longest decreasing subsequence, not the subsequence itself.

6. Given a string S and a string T, count the number of distinct subsequences of T in S.

   A subsequence of a string is a new string which is formed from the original string by deleting some (can be none) of the characters without disturbing the relative positions of the remaining characters. (ie, "ACE" is a subsequence of "ABCDE" while "AEC" is not).

   Here is an example:
   $S =$ "rabbbit", $T =$ "rabbit"
   Return 3.

   (a) (15 marks) Write a dynamic programming algorithm solving the number of distinct subsequence problem, and analyze its running time.

   (b) (20 marks) Implement it in Java. Your implementation should take two sequences of letters as input, and output the number of distinct subsequences.