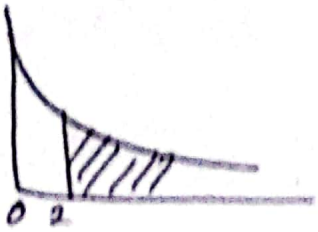
Assignment-9

Answer to the question no 1

a

We know,

$$f(x) = \frac{1}{\theta} e^{-x/\theta}$$



So, $P(x \geq 2) = 1 - P(x < 2)$

$$= 1 - \int_0^2 \frac{1}{0.37} e^{-x/0.37} dx$$

$$= 1 - \frac{1}{0.37} \left[e^{-x/0.37} \right]_0^2$$

$$= 1 - (-1)\left(e^{-\frac{2}{0.37}} - e^{0}\right)$$

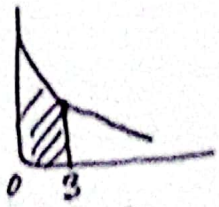
$$= 1 - 0.996$$

$$= 0.0095$$

Here, $\theta = 0.37$

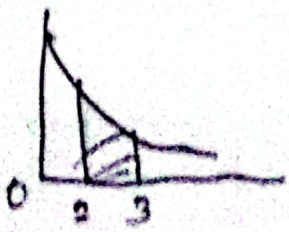
b

So, $P(x \leq 3) = 1 - e^{-3/0.37} = 0.9996$



So, $P(2 < x < 3) = P(x < 3) - P(x < 2) = \left(1 - e^{-\frac{3}{0.37}}\right) - \left(1 - e^{-\frac{2}{0.37}}\right)$

$$= 0.0001$$



b

$P(x<a) = 0.05$

$\int_0^a \frac{1}{0.37} e^{-x/\theta} dx = 0.05$

$\Rightarrow 1 - e^{-a/0.37} = 0.05$

$\Rightarrow e^{-a/0.37} = 0.95$

$\Rightarrow -a/0.37 = \ln(0.95)$

$\Rightarrow a = -0.37 \ln(0.95)$

$\Rightarrow a = 0.019\ h^{-1}$

Here,
$P(x<a) = 0.05$
$\theta = 0.37$

Answer to the question no 2

a

$P(x > 20{,}000) = 1 - P(x < 20000)$

$= 1 - \{1 - e^{-20000/0.00004}\}$

$= 1 - 1 = 0$

Here,
$\theta = 0.00004$

b

$P(x<a) = 0.01$

$a = -4 \times 10^{-5} \ln(0.99) = 4.02 \times 10^{-7}$

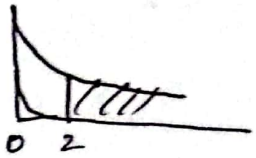
Answered to the question no 3

a

$$P(x>2) = 1 - P(x<2)$$

$$= 1 - (1 - e^{-2/\theta})$$

$$= 0.6 \times 10^{-5}$$

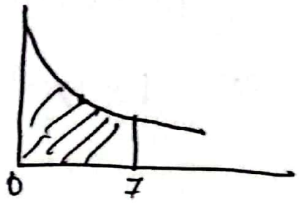


here, $\theta = 1/6$

b

$$P(x<7) = 1 - e^{-7/\theta}$$

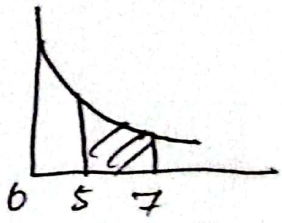
$$= 1$$



$$P(5<x<7) = P(x<7) - P(x<5)$$

$$= \left(1 - e^{-7/\theta}\right) - \left(1 - e^{-5/\theta}\right)$$

$$= 0$$



b

$$P(x<a) = 0.02$$

$$a = 3.37 \times 10^{-3}$$

Answer to the question no 4

a

$$P(T_w > 2) = e^{-2(3-2)}$$

$$= 0.135$$

here,
$\theta = 3$
$\lambda = 2$
$t = 2$

b

$$L_q = \frac{\lambda^2}{\theta(\theta-\lambda)}$$

$$= \frac{2^2}{3-2} = 2$$

c

$$W = \frac{1}{\theta - \lambda} = \frac{1}{3-2} = 1 \text{ hour}$$

Answer to the question no 5

a

$\lambda = 10\ h^{-1}$ $\theta = 13.33\ h^{-1}$

a

$$W_q = \frac{\lambda}{\theta(\theta-\lambda)} = \frac{10}{13.33(13.33-10)} = 0.23$$

Answer to the question no 6

$$TPM, P = \begin{vmatrix} 1 & 0 & 0 \\ \frac{2}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 1 \end{vmatrix}$$

As we need data for 2 days after, so,

$$P^2 = \begin{vmatrix} 1 & 0 & 0 \\ 0.78 & 0.28 & 0.194 \\ 0 & 0 & 1 \end{vmatrix}$$

So, $P_{02}^{\sim} = 0$

Answer to the question no 7

$\theta = 2\ h^{-1}$ $\lambda = 0.5\ h^{-1}$

$1 - \pi_0 = \frac{\lambda}{\theta} = \frac{0.5}{2} = 0.25$ or 25% or 15 ~~min~~ hour

Answer to the question no 8

a

Metrix, P =

Previous day \ Next day	0	1
0	0.4	0.6
1	0.5	0.5

Here,
clean day = 0
rainy " = 1
P_{RC} or P_{10} = 0.5
P_{CR} or P_{01} = 0.6

b

Steady-state probability of rainy day

$$\pi_1 = 0.6\pi_0 + 0.5\pi_1$$

$$\pi_0 = 1 - \pi_1$$

So,

$$\pi_1 = 0.6 - 0.6\pi_1 + 0.5\pi_1$$

$$\pi_1 + 0.1\pi_1 = 0.6$$

$$\pi_1 = \frac{0.6}{1.1} = 0.55$$

c

P^2

$$P^2 = \begin{vmatrix} 0.96 & 0.54 \\ 0.45 & 0.55 \end{vmatrix}$$

$P^2_{00} = 0.46$

Here,
$P^2 = P \times P$

P^2_{00} or $P^2_{\text{clean to clean}}$

$= P^2_{00}$

Answer to the question no 9

Given,

$$P = \begin{vmatrix} 0.3 & 0.1 & 0.6 \\ 0.2 & 0.7 & 0.1 \\ 0.5 & 0.3 & 0.2 \end{vmatrix}$$

$$P^3 = \begin{vmatrix} 0.334 & 0.33 & 0.336 \\ 0.288 & 0.466 & 0.246 \\ 0.342 & 0.366 & 0.292 \end{vmatrix}$$

$P_{02} = 0.336$

Answer to the question no 10

$\lambda = 5 \ h^{-1}$ $\qquad \theta = 7.5 \ h^{-1}$

a

$$P_0 = 1 - \frac{\lambda}{\theta} = 1 - \frac{5}{7.5} = 0.33$$

b

$$1 - P_0 = \frac{\lambda}{\theta} = 0.67$$

c

$$L = \frac{\lambda}{\theta - \lambda} = \frac{5}{7.5 - 5} = 2$$

d

$$W_q = \frac{\lambda}{\theta(\theta - \lambda)} = \frac{5}{7.5(7.5 - 5)} = 0.267$$