





CI2142 - Data Structures & Algorithms

Tree data structure and its application

- ❖ Introduction to the Tree Data Structure
- ❖ **Binary Trees**
- ❖ Traversals in a Binary Tree
- ❖ Java Implementation



Tree based representations

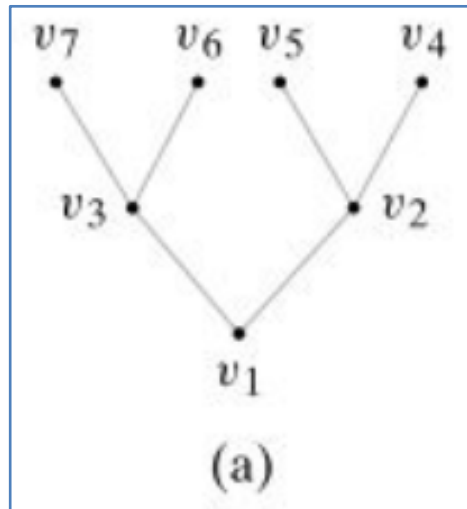
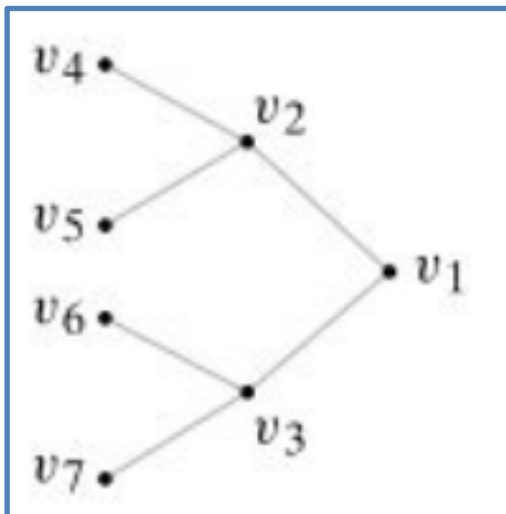
	Monica Seles	6	6	6
	Martina Navratilova	2	7	4

1992 Wimbledon

1		Monica Seles	2	1
2		Steffi Graf	6	6

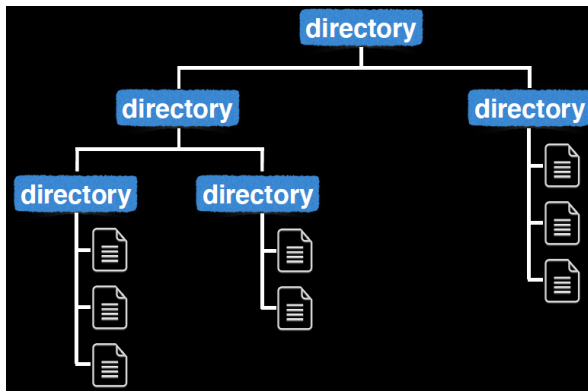
 Steffi Graf

	Gabriela Sabatini	3	3
	Steffi Graf	6	6

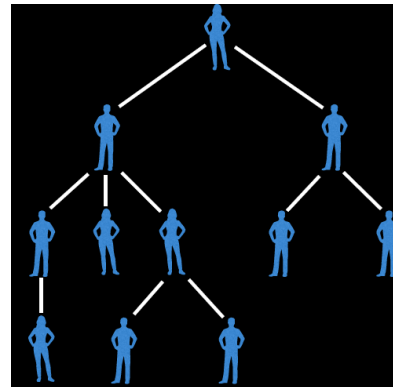


Potential Application of Trees:

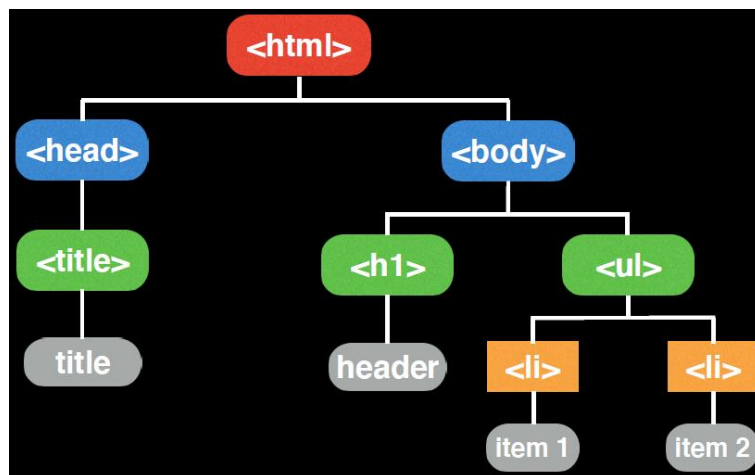
The following applications use a tree data structure as very efficient algorithms exist to process *big data* represented as trees.



File system in PCs



Social networks

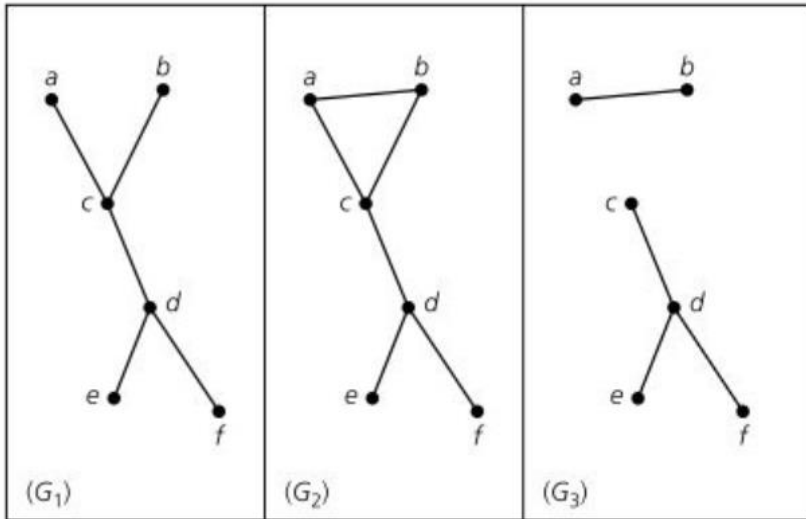


Web pages are basically trees!

Decision Trees, Domain Name Servers (DNS), Databases,...all use trees.

Terminology / Definitions

Let $G = (V, E)$ be a simple undirected graph. The graph G is called a tree if G is connected and contains no cycles.



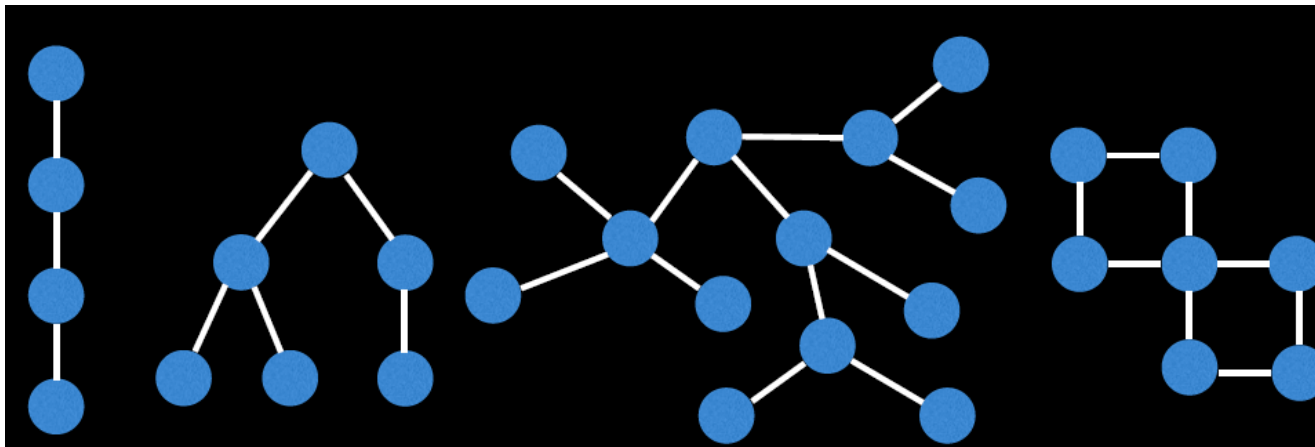
Identify trees?

Is G_1 a tree? Justify?

In G_2 $\{a, b\}$, $\{b, c\}$ and $\{c, a\}$ is called as a _____

G_3 is called as a _____

What is a Forest?



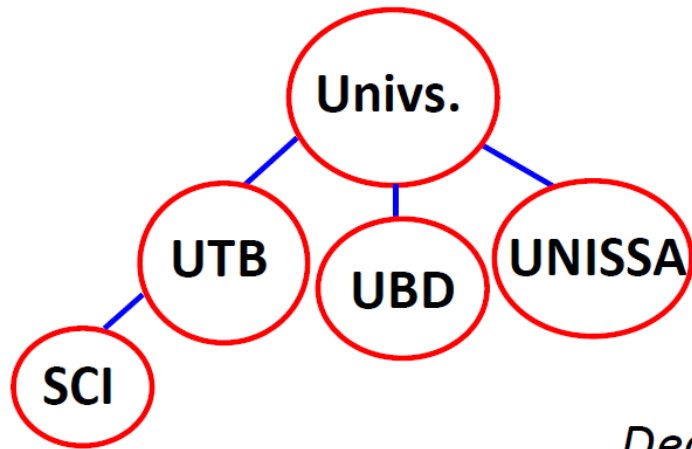
G_4

G_5

G_6

G_7

Terminology / Definitions



A tree is a _____ data structure that has a finite set of _____ and _____

The number of subtrees of a node is called the _____ of the node.

Degree of the node "Univs." is _____

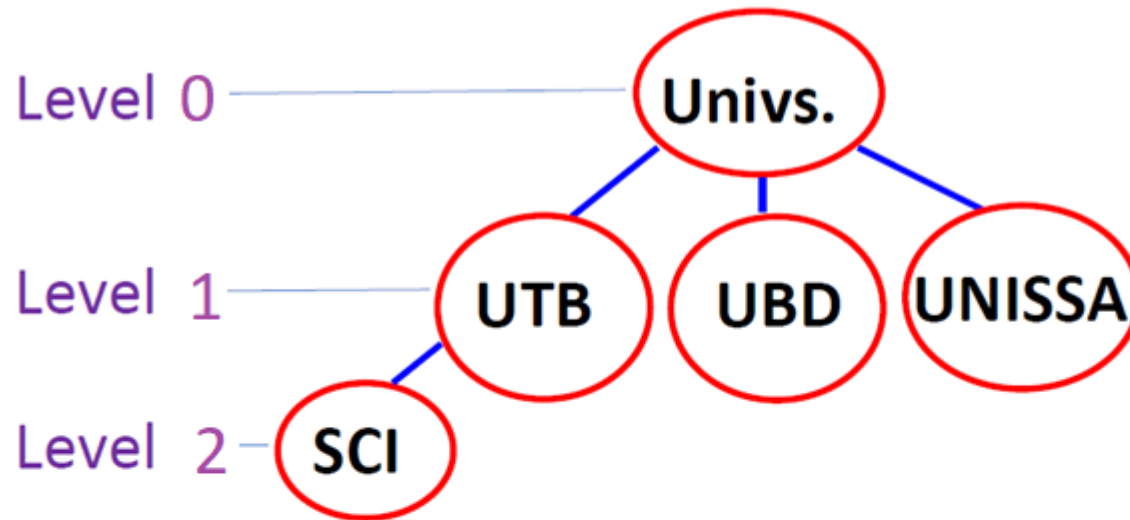
A node of degree zero is called a *terminal node* or _____ node.

Is the node "UTB" a leaf? What are leaf nodes in the eg.?

The *degree of a tree* is the maximum degree of a node in the tree.

What is the degree of the eg. tree?

Terminology / Definitions



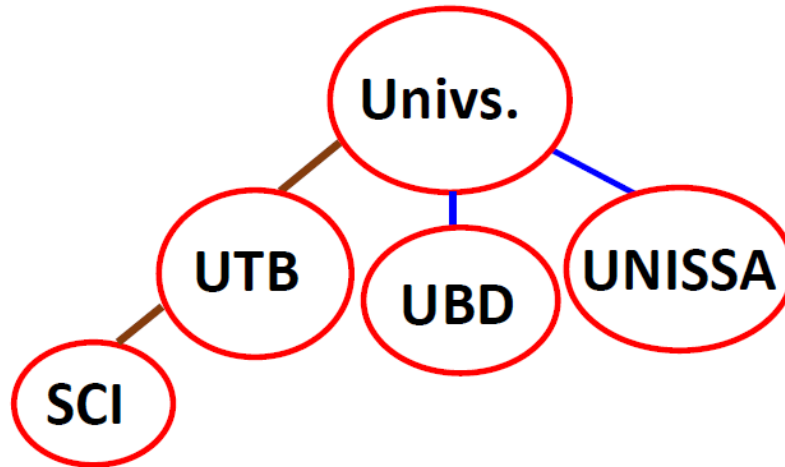
The _____ of a node is a measure of its distance from the root.

Recursively,

- if node n is the root of the tree T , its _____ is 0
- if not then the level is $1 + \text{_____}$ of its parent

The alternative term for _____ is depth

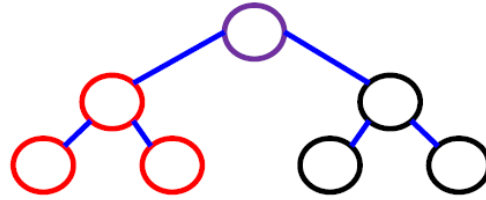
Terminology / Definitions



The height of a tree is the number of nodes in the longest path from the root node to a leaf node. (Height of the Eg. Tree is 3)

- if T is empty, its height is zero
- if not then the height is the max. depth of its nodes.

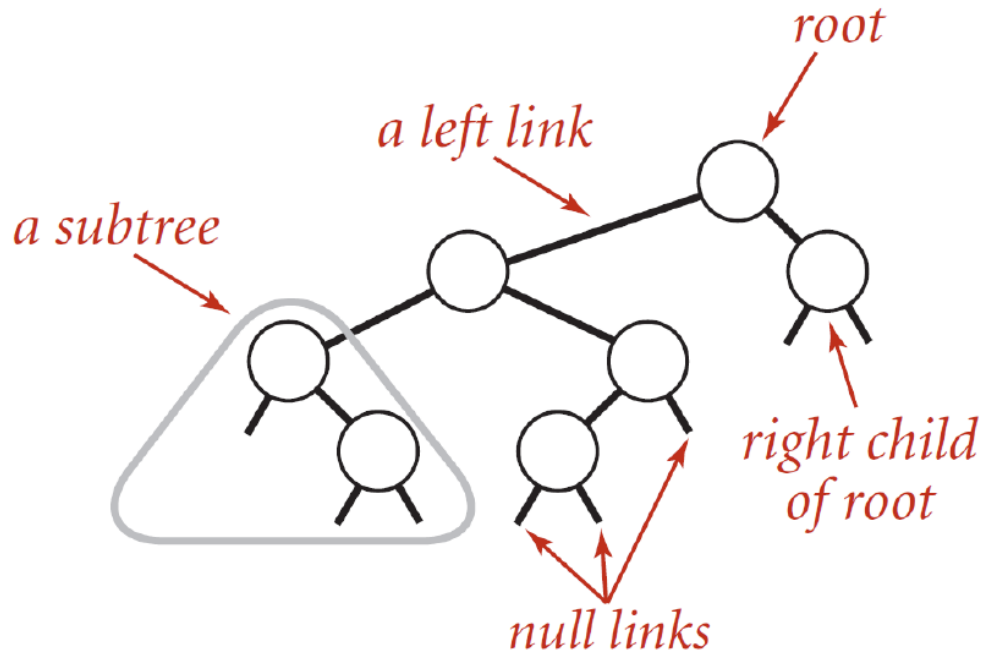
Binary Tree



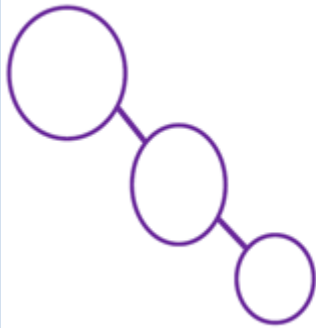
A set of nodes T is a binary tree if either of the following is true

- T is empty*
- If not then its root has two subtrees T_L and T_R*

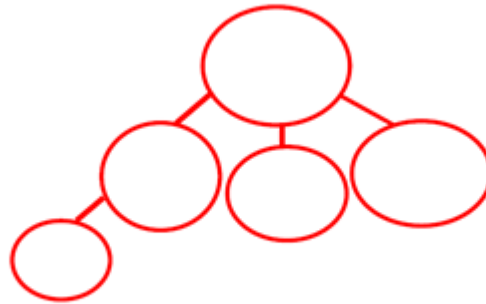
Characteristics of a Binary Trees



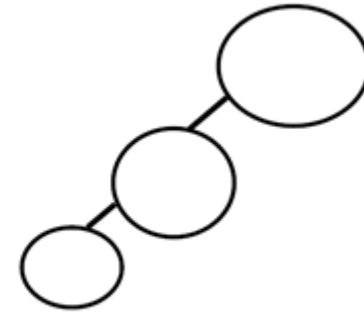
Which of the following represent binary trees?



DEGREE?

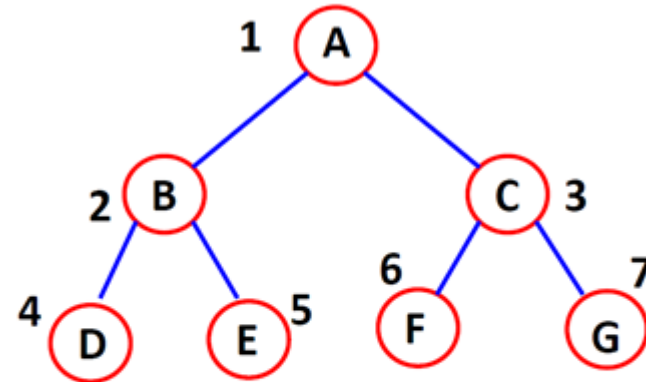
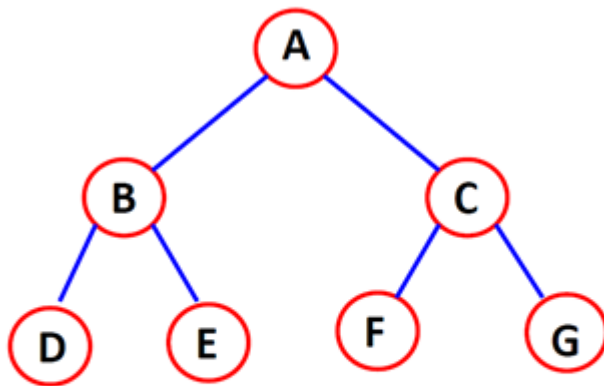


DEGREE?



DEGREE?

How to represent binary trees in a programming language?



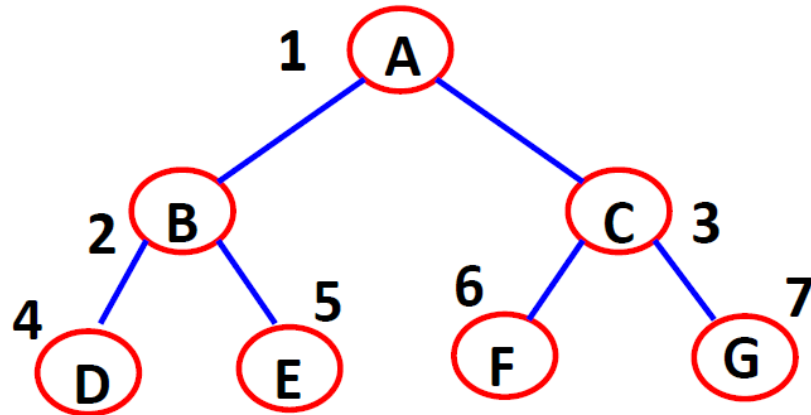
1	2	3	4	5	6	7
A	B	C	D	E	F	G

Store the nodes
according to
their ordering

Sequentially number the nodes from root (left to right) until the lowest level

Note: The starting index for some programming languages (Eg. MATLAB) is one

How to represent binary trees in a programming language?



For any i th node

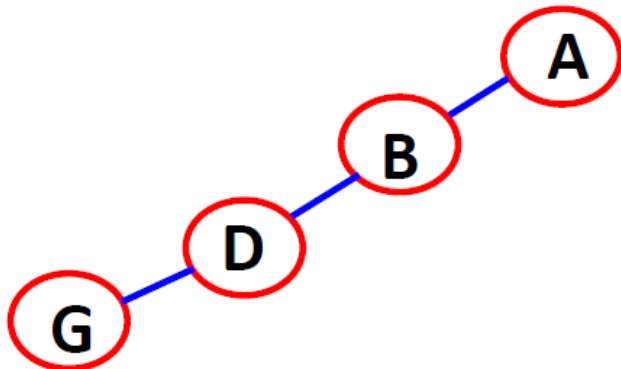
Parent(i) $\Rightarrow i / 2$

Left Child(i) $\Rightarrow 2i$

Right Child(i) $\Rightarrow 2i+1$

1	2	3	4	5	6	7
A	B	C	D	E	F	G

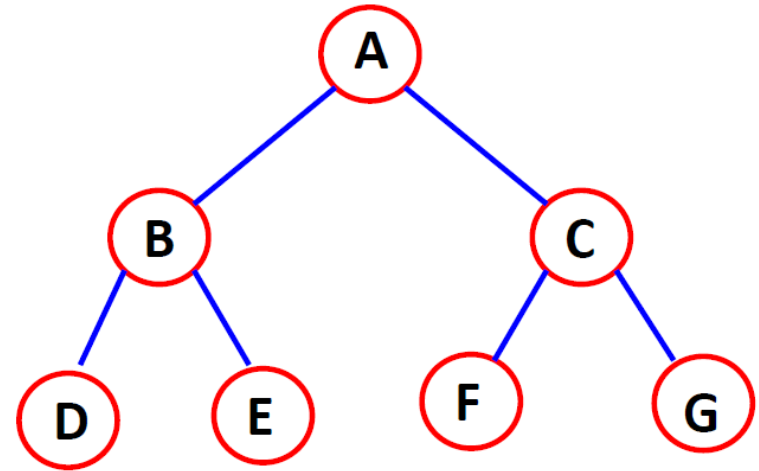
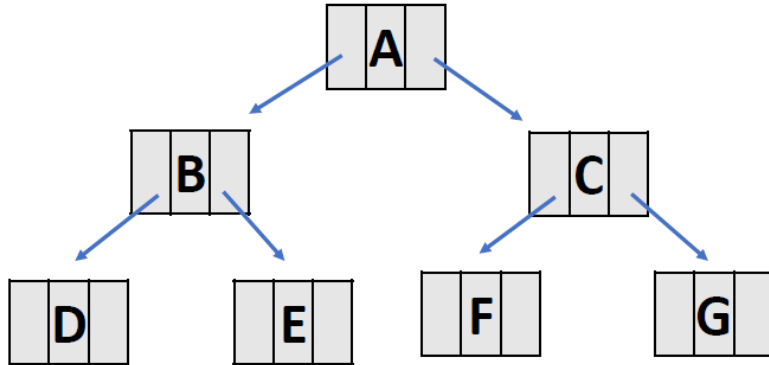
Represent the following
binary tree (skewed BT) in an array



Linked list representation of binary trees

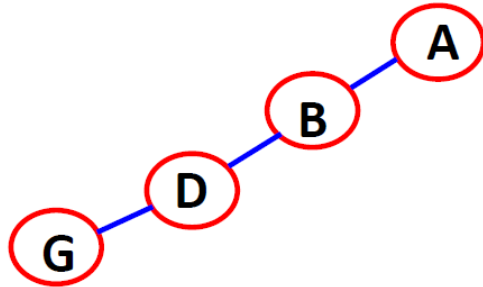
Left Child	Data	Right Child
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Left child and Right child are pointers



While traversing a binary tree, each node is **visited** exactly once.

Represent the following binary tree (skewed BT) in an array



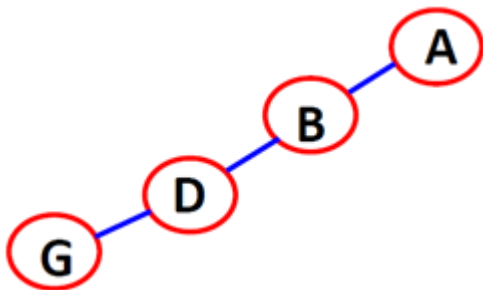
For any i th node

$\text{Parent}(i) \Rightarrow i / 2$

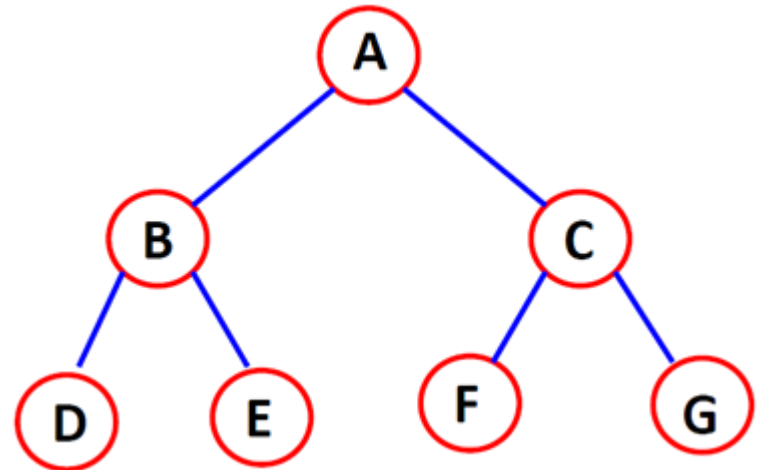
$\text{Left Child}(i) \Rightarrow 2i$

$\text{Right Child}(i) \Rightarrow 2i+1$

1	2	3	4	5	6	7	8
A	B		D				G



Imbalanced binary tree



Balanced binary tree

Traversing in a Binary tree

Pre-Order Traversal: Root, Left ,Right

-	x	+	A					
---	---	---	---	--	--	--	--	--

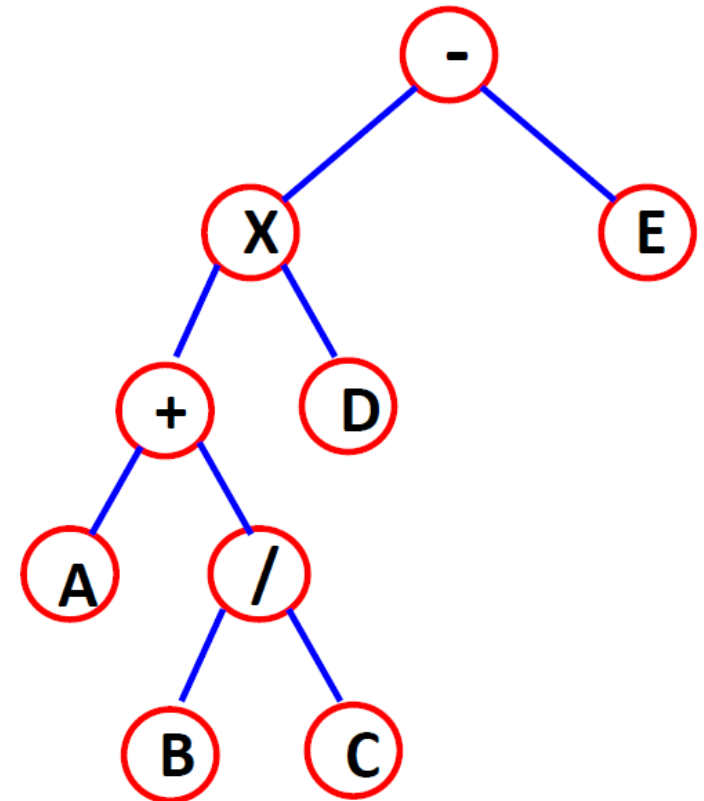
Since there are no left or right Subtrees after A we climb up to + and move down right

-	x	+	A	/	B	C		
---	---	---	---	---	---	---	--	--

Move down to the right of X

-	x	+	A	/	B	C	D	E
---	---	---	---	---	---	---	---	---

Do in-order and post-order yourself !



In-Order Traversal: Left Most, Root, Right

Post-Order Traversal: Left , Right, Root

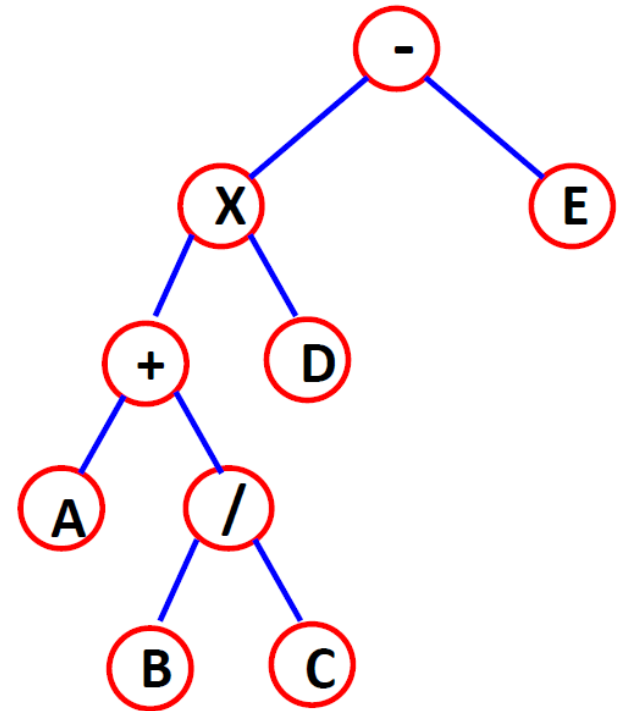
Traversing in a Binary tree

Level-Order Traversal:

The root is visited first and then all
Nodes at depth 1 (going from left to right),
Then all nodes at depth 2 (going from left to right)
and so on...

-	X	E	+	D	A	/	B	C
---	---	---	---	---	---	---	---	---

Application of BTs: Develop Compilers,
Interpreters, Virtual Machines (Java),
DBMS, OS and many more



Traversing in a Binary tree

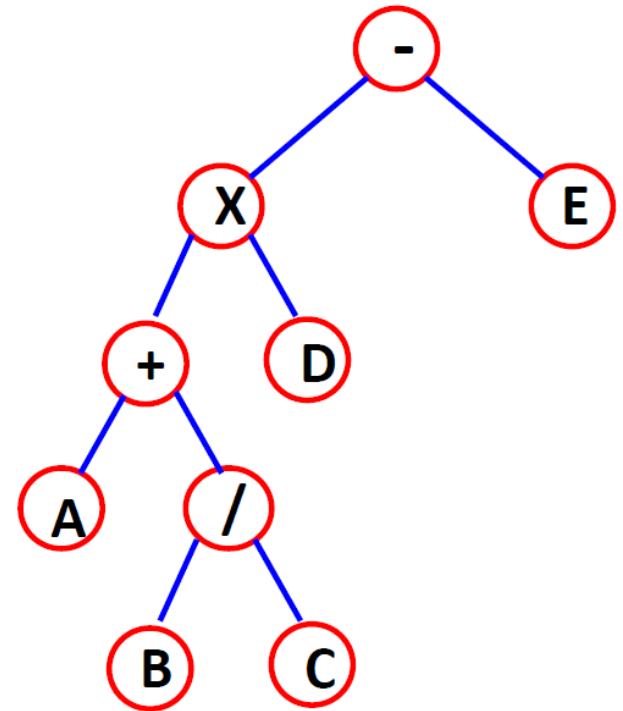
In-Order Traversal: Left Most, Root, Right

A	+	B	/	C	X	D	-	E
---	---	---	---	---	---	---	---	---

Post-Order Traversal: Left , Right, Root

A	B	C	/	+	D	X	E	-
---	---	---	---	---	---	---	---	---

Level – Order Traversal: Can you guess?



```

1  /* Defines an ADT for a Binary tree */
2  public abstract class AbstractBinaryTree {
3      protected Node rootNode;
4      public AbstractBinaryTree(Node rootNode) {
5          this.rootNode = rootNode;
6      }
7      /* Represents a node in binary tree. */
8      protected static class Node{
9          private int value; private Node left; private Node right;
10         public Node(int value, Node left, Node right) {
11             this.value = value; this.left = left; this.right = right;
12         }
13         public int getValue(){ return value;}
14         public Node getLeft() {return left; }
15         public Node getRight() { return right;}
16     } //end of class Node
17
18     /* Find the height of the binary tree given the root node.
19     input parameter root; Returns height of the longest path. */
20     protected int computeHeight(Node root) {
21         if(null == root){ return 0;}
22         int leftHeight = computeHeight(root.getLeft());
23         int rightHeight = computeHeight(root.getRight());
24         return leftHeight > rightHeight ? leftHeight + 1 : rightHeight + 1;
25     }
26 } //end of class AbstractBinaryTree

```

Note: Refer to the Terminology/Definitions and linked-list representation (early slides); We implement those concepts what we have learned.


```

1 public class BinaryTreeTraversals extends AbstractBinaryTree{
2     public BinaryTreeTraversals(Node rootNode) {
3         super(rootNode);
4     }
5     public static void main(String[] args) {
6         /* Construct tree */
7         BinaryTreeTraversals.Node r4 = new BinaryTreeTraversals.Node(4, null, null);
8         BinaryTreeTraversals.Node r5 = new BinaryTreeTraversals.Node(5, null, null);
9         BinaryTreeTraversals.Node r3 = new BinaryTreeTraversals.Node(3, null, null);
10        BinaryTreeTraversals.Node r2 = new BinaryTreeTraversals.Node(2,r4,r5);
11        BinaryTreeTraversals.Node r1 = new BinaryTreeTraversals.Node(1,r2,r3);
12        BinaryTreeTraversals tree = new BinaryTreeTraversals(r1);
13        System.out.println("InOrder traversal");
14        tree.traverseInOrder(r1);
15    }
16    public void traverseInOrder(Node node){
17        if(null == node){
18            return;
19        }
20        /* Recur left */
21        traverseInOrder(node.getLeft());
22        /* Visit root */
23        System.out.print(node.getValue()+" ");
24        /* Rcur right */
25        traverseInOrder(node.getRight());
26    }
27 }

```

```
D:\IBM\JavaWork>javac AbstractBinaryTree.java
```

```
D:\IBM\JavaWork>javac BinaryTreeTraversals.java
```

```
D:\IBM\JavaWork>java BinaryTreeTraversals
InOrder traversal
4 2 5 1 3
```

Lab Task

Write methods for pre-order, post-order and level-order.

Workout all the traversals by hand (pen and paper) for few binary tree examples.

Develop clients to verify the outputs with your hand-worked examples.