$$P_C(z) = \arg\min \frac{1}{2\gamma}||x - z||^2$$

$$C = x \mid a^T x \le c \longrightarrow g(x) = a^T x$$

$$L(\lambda, z) = \frac{1}{2\gamma}||x - z||^2 + \lambda(a^T x - c)$$

if $(a^T x < c)$ } Strictly Inside

↳ Normal Gradient Descent $\longrightarrow \lambda = 0$

↳ minimize $f = \frac{1}{2\gamma}||x - z||^2$

$$f_{t+1} = f_t - k\nabla f_t$$

$$= f_t - \frac{k(x - z)}{\gamma}$$ } One iteration

If Optimal point z is on boundary

i.e $\boxed{a^T x = c}$ & $\nabla f = -\lambda \nabla g(x)$

$$\Downarrow$$

$$\frac{x - z}{\gamma} = -\lambda a$$

$$x - z = -\lambda a \gamma$$

$\therefore$ If Strictly inside $P(c) = 0 \longrightarrow x = z$

if on boundary $\rightarrow x = z - \lambda a \gamma$

$$P(c) = \frac{\lambda^2 a^2 \gamma}{2}$$