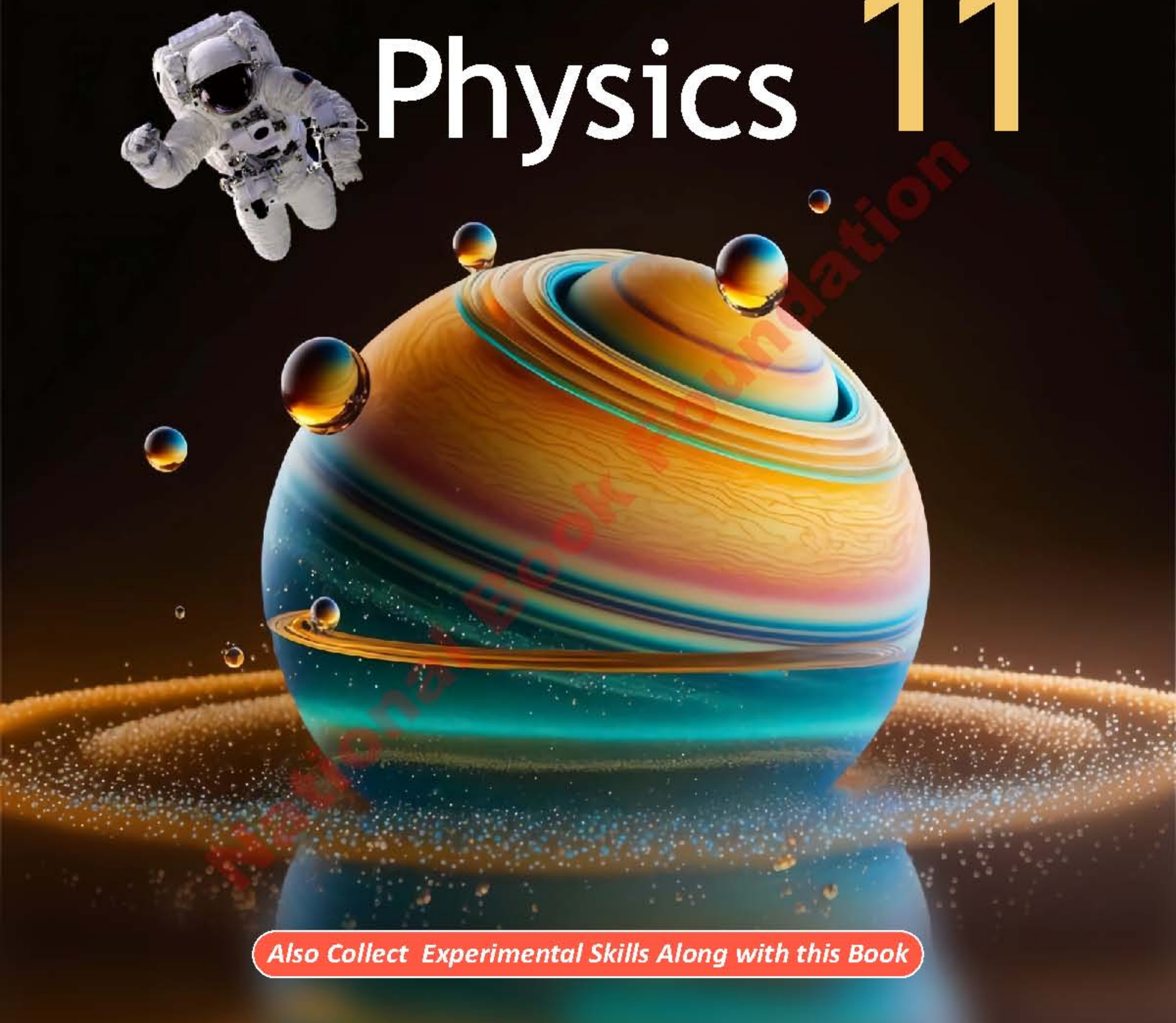
Based on National Curriculum of Pakistan 2022-23

Model Textbook of Physics 11



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Model Textbook of

Physics

Grade

11

National Curriculum Council

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Islamabad

Model Textbook of **Physics**
for Grade 11

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Preface

This Model Textbook for Physics Grade 11 has been developed by NBF according to the National Curriculum of Pakistan 2022-2023. The aim of this textbook is to enhance learning abilities through inculcation of logical thinking in learners, and to develop higher order thinking processes by systematically building the foundation of learning from the previous grades. A key emphasis of the present textbook is creating real life linkage of the concepts and methods introduced. This approach was devised with the intent of enabling students to solve daily life problems as they grow up in the learning curve and also to fully grasp the conceptual basis that will be built in subsequent grades.

After amalgamation of the efforts of experts and experienced authors, this book was reviewed and finalized after extensive reviews by professional educationists. Efforts were made to make the contents student friendly and to develop the concepts in interesting ways.

The National Book Foundation is always striving for improvement in the quality of its textbooks. The present textbook features an improved design, better illustration and interesting activities relating to real life to make it attractive for young learners. However, there is always room for improvement, the suggestions and feedback of students, teachers and the community are most welcome for further enriching the subsequent editions of this textbook.

May Allah guide and help us (Ameen).

Dr. Raja Mazhar Hameed
Managing Director

Contents

UNIT
1

MEASUREMENT

Student Learning Outcomes (SLOs)

The students will

- Make reasonable estimates of physical quantities [of those quantities that are discussed in the topics of this grade].
- Express derived units as products or quotients of the SI base units.
- Analyze the homogeneity of physical equations [Through dimensional analysis].
- Derive formulae in simple cases [Through using dimensional analysis].
- Analyze and critique the accuracy and precision of data collected by measuring instruments.
- Assess the uncertainty in a derived quantity [By simple addition of absolute, fractional or percentage uncertainties].
- Justify why all measurements contain some uncertainty.

Physics is based on experimental observations. Observations may be qualitative or quantitative. Qualitative observations have no associated numbers. It deals with facts that can be observed with our five senses: sight, smell, taste, touch and hearing. For instance, colors, shapes and textures of objects are examples of qualitative observations. Observations, like 'water keeps its level' is also an example of qualitative observation. A quantitative observation includes numbers, and is also called a measurement. We can measure mass, time, distance, speed, pressure, force, torque, momentum, and energy. Quantitative observations are more useful to a scientist.

1.1 ESTIMATION OF PHYSICAL QUANTITIES

In our daily life, we may face some situations like: What will be the height of this building? Will the piece of equipment fit in the back of our car or do we need to rent a truck? How long will this download take? About how large a current will be there in this circuit? How many houses could a proposed power plant actually power if it is built? Usually we solve such problems by estimations. In many circumstances, scientists and engineers also need to make estimates of some specific physical quantity with the help of little or no actual data.

Estimation does not mean guessing a formula or a number at random. Let us understand the estimation with the help of a simple example:

Estimation of length: To estimate the height of building, we first count the number of floors it has. Then, estimate the height of a single floor by imagining how many people would have to stand on each other's shoulders to reach the ceiling. In the last, we estimate the height of a person. These estimates give you the height of the building.

An estimation is a rough educated guess to the value of a physical quantity by using prior experience and sound physical reasoning.

An estimation usually includes the identification of correct physical principles and a good guess about the relevant variables. Estimation is very useful in developing a physical sense.

Some of the following kind of strategies may help to improve our skill of estimation:

- **Breaking big things into smaller things or aggregating smaller things into a bigger thing:** When estimating lengths, remember that anything can be a ruler. Thus, imagine breaking a big thing into smaller things, estimate the length of one of the smaller things, and multiply to obtain the length of the big thing. For example, we have estimated the height of a building in the previous paragraphs. Sometimes it also helps to do this in reverse, i.e., to estimate the length of a small thing, imagine a bunch of them making up a bigger thing. For example, to estimate the thickness of a sheet of paper, estimate the thickness of a stack of paper and then divide by the number of pages in the stack. These same strategies of breaking big things into smaller things or aggregating smaller things into a bigger thing can sometimes be used to estimate other physical quantities, such as mass and time. In such situations some of the length, mass and time scales, as shown in table 1.1 may be helpful.

Table 1.1: The estimation of some physical quantities.		
Length (m)	**Mass (kg)**	**Time (s)**
Diameter of proton = 10^{-15}	Mass of electron = 10^{-30}	Mean lifetime of unstable nucleus = 10^{-22}
Diameter of large nucleus = 10^{-14}	Mass of proton = 10^{-27}	Time for single floating-point operating in a supercomputer = 10^{-17}
Diameter of H-tom = 10^{-10}	Mass of bacterium = 10^{-15}	Time period of visible light = 10^{-15}
Diameter of typical virus = 10^{-7}	Mass of mosquito = 10^{-5}	Time period of an atom in solid = 10^{-13}
width of pinky fingernail = 10^{-2}	Mass of hummingbird = 10^{-2}	Time period of nerve impulse = 10^{-3}
Height of 4 year old child = 10^{0}	Mass of 1 liter water = 10^{0}	Time for 1 heartbeat = 10^{0}
length of football ground = 10^{2}	Mass of a Motorcycle= 10^{2}	One day = 10^{5}
Diameter of Earth = 10^{7}	Mass of atmosphere = 10^{19}	One year = 10^{7}
Diameter of solar system = 10^{13}	Mass of Moon = = 10^{22}	Human lifetime = 10^{9}
1 light-year = 10^{16}	Mass of Earth = = 10^{25}	Recorded human history = 10^{11}
Diameter of Milky-Way = 10^{21}	Mass of Sun = = 10^{30}	Age of Earth = 10^{17}
Distance b/w edges of observable universe = 10^{26}	Mass of known universe = 10^{53}	Age of universe = 10^{18}

- **Estimate Areas and Volumes from Lengths:** When dealing with an area or a volume of a complex object, introduce a simple model of the object, such as a sphere or a box. Then, estimate the linear dimensions (such as the radius of the sphere or the length, width, and height of the box) first, and use the estimates to find the volume or area from standard geometric formulas. If you happen to have an estimate of an object's area or volume, you can also do the reverse; that is, use standard geometric formulas to get an estimate of its linear dimensions.
- **Estimate Mass from Volume and Density:** When estimating the mass of an object, it can help first to estimate its volume and then to estimate its mass from estimate of its average density. (recall, density has dimension of mass/length^3, so mass = density × volume). For this, it helps to remember that the density of air is about 1 kg/m^3, the density of water is 10^3 kg/m^3, and the densest everyday solids max out at around 10^4 kg/m^3. Asking yourself whether an object floats or sinks in either air or water gets you a rough estimate of its density. You can also do the reverse: if you have an estimate of an object's mass and its density, you can use them to get an estimate of its volume.

Example 1.1: Estimate the energy required for an adult man to walk up through stairs from ground floor to 1st floor?

Solution:

As, the energy required = $m\,g\,h$

We have to take the following estimations:

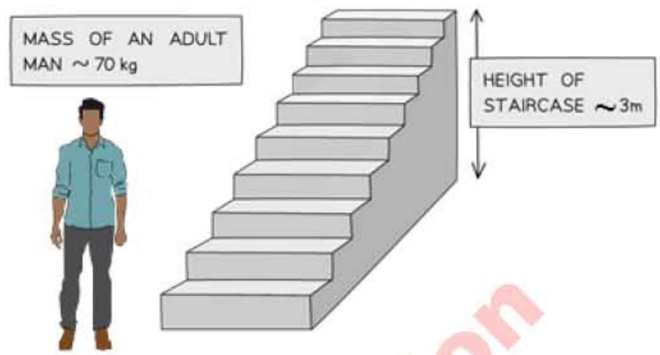
Mass of an adult ≈ 70 kg

Distance between 2 floors ≈ 3 m

So,

Energy required ≈ $70 \times 10 \times 3$

= 2100 J



Assignment 1.1

Estimate that how many floating-point operations can a supercomputer do in 1 day?

1.2 DERIVED UNITS IN TERMS OF BASE UNITS

In Grade 9, we have studied about base and derived physical quantities and also their units. We know that derived units can be expressed in terms of base units and are obtained by multiplying or dividing base units with each other. Here we will express some more derived units as products or quotients of the SI base units. Let us first we take force: as,

$$Force = mass \times acceleration$$

$$Force = mass \times \frac{velocity}{time}$$

$$Force = mass \times \frac{displacement}{time^2}$$

Now, we put SI unit for each physical quantity i.e., N for force, kg for mass, m for displacement and s for time, so we get:

$$N = kg \times m/s^2$$

For work we proceed as:

$$Work = Force \times displacement$$

$$Work = mass \times acceleration \times displacement$$

$$Work = mass \times \frac{velocity}{time} \times displacement$$

$$Work = mass \times \frac{displacement}{time^2} \times displacement$$

$$Work = mass \times \frac{(displacement)^2}{(time)^2}$$

Now, we put SI unit for each physical quantity i.e., J for work, kg for mass, m for displacement and s for time, so we get:

$$J = kg \times \frac{m^2}{s^2}$$

Similarly, we can express other derived units as products or quotients of the SI base units. Some examples are shown in the table 1.2.

TABLE 1.2

Name of Derived Quantity	SI Unit	Symbol	In terms of base units
Power	watt	W	$J/s = kg\ m^2/s^3$
Pressure	pascal	Pa	$N\ m^{-2} = kg\ m^{-1}s^{-2}$
Electric Charge	coulomb	C	A s

1.3 DIMENSIONS OF PHYSICAL QUANTITIES

Dimension denotes the qualitative nature of a physical quantity. For example, length, width, height, distance, displacement, radius etc. all are measured in meter because of having same nature so have the same dimensions.

Dimension of a physical quantity is often represented by capital letter enclosed in square brackets [ ]. Dimensions for base quantities are given in the table 1.3.

TABLE 1.3

Sr. No	Physical Quantity	Dimensions
1	mass	[M]
2	length	[L]
3	time	[T]
4	electric current	[I]
5	temperature	[θ]
6	intensity of light	[J]
7	amount of substance	[N]

Dimensions of derived quantities are obtained by multiplication or division of the dimensions of base quantities i.e., from which these quantities are derived. For example, the dimension for area, volume, velocity and acceleration are $[L^2]$, $[L^3]$, $[LT^{-1}]$ and $[LT^{-2}]$ respectively.

Thus, dimensions give the relation of a given physical quantity with base quantities i.e. mass, length, time etc. There are following essential terms which are used in dimensional analysis:

Dimensional Variables: Those physical quantities that have dimensions and have variable magnitude are called dimensional variables. Some dimensional variables are length, velocity, acceleration, force, energy and acceleration etc.

Dimensional Constant: Those physical quantities that have dimensions and constant in magnitude are called dimensional constant. Some examples of dimensional constants are

Planck's constant, gravitational constant, speed of light in vacuum and ideal gas constant etc.

Dimensionless Variables: Those physical quantities that have no dimensions and have variable magnitude are called dimensionless variables. Some examples of dimensionless variables are plane angle, solid angle, strain and coefficient of friction etc.

Dimensionless Constant: Those physical quantities that have no dimensions and constant in magnitude are called dimensionless constant. A pure number (1, 2, 3, ...), the exponential constant (e = 2.718) and π are some examples of dimensionless constant.

1.3.1 Advantages of Dimensions

Using the method of dimensions (called dimensional analysis) we can check the homogeneity of an equation, to derive a possible formula and its units. Dimensional analysis makes use of the fact that dimensions can be treated as algebraic quantities. That is, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions.

(i) The Homogeneity of an Equation

In order to check the correctness of an equation, we have to show that both sides of the equation have the same dimensions, otherwise the equation cannot be considered as physically correct equation. This is called the principle of homogeneity of dimensions.

Let us check whether the equation $v_f = v_i + at$ is dimensionally correct.

Dimensions of L.H.S. = $[LT^{-1}]$

Dimensions of R.H.S. = $[LT^{-1}] + [LT^{-2}][T]$

$= [LT^{-1}] + [LT^{-1}]$

$= 2[LT^{-1}]$

As, 2 is dimensionless constant, so

Dimensions of L.H.S = Dimensions of R.H.S

Hence, the equation is dimensionally correct.

(ii) To Derive a Possible Formula

Deriving a relation for a physical quantity depends on the correct guessing of various factors on which the physical quantity depends. Let us derive the formula for wavelength of matter waves using dimensional analysis.

As wavelength (λ) of matter waves may depend upon Plank's constant (h), velocity (v) and mass (m) of the particle.

So, the relation for the wavelength (λ) will be of the form:

$$\lambda \propto h^a m^b v^c$$

$$\lambda = (\text{constant})\, h^a m^b v^c \text{ ----------------- } (1)$$

We have to find the values of powers i.e. a, b and c:

Using the dimension on both sides, we get:

$$[L] = \text{constant } [ML^2T^{-1}]^a [M]^b [LT^{-1}]^c$$

OR $$[M^0 L^1 T^0] = \text{constant } [M]^{a+b} [L]^{2a+c} [T]^{-a-c} \text{ ----------------- } (2)$$

Equating the powers of M on both sides of equation (2), we get:

$$a + b = 0 \text{ ----------------- } (3)$$

Equating the powers of L on both sides of equation (2), we get:

$2a + c = 1$ ----------------- (4)

Equating the powers of T on both sides of equation (2), we get:

$-a - c = 0$ ----------------- (5)

On solving (3), (4) and (5), we get:

$a = 1, b = -1$ and $c = -1$

Put the values of a, b, and c in (1), we get:

$\lambda = (\text{constant})\ h^1\ m^{-1}\ v^{-1}$

OR $\lambda = (\text{constant}) \times \dfrac{h}{mv}$

1.3.2 Limitations of Dimensional Analysis

Some limitations of dimensional analysis are:

1) Dimensional analysis does not distinguish between the physical quantities having same dimensions. For example, if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be work or energy or torque.
2) Dimensional analysis cannot be used to derive a formula containing trigonometric function, exponential functions, logarithmic function, etc.
3) Dimensional analysis cannot determine the dimensionless constant when deriving a possible formula.
4) Dimensional analysis doesn't always prove that a relation is physically correct although relation is dimensionally correct. However, a dimensionally wrong equation is always wrong.

Example 1.2: Derive formula for the time period of simple pendulum using dimensional analysis.

Solution: The time period of the simple pendulum is possibly depending on mass of the bob (m), length of the pendulum (l), angle which the string makes with vertical (θ) and acceleration due to gravity (g). So, the relation for the time period T will be of the form:

$T \propto m^a\ l^b\ \theta^c\ g^d$

$T = (\text{constant})\ m^a\ l^b\ \theta^c\ g^d$ ----------------- (1)

We have to find the values of powers i.e. a, b, c and d:

Using the dimension on both sides, we get:

$[M^0\ L^0\ T] = \text{constant}\ [M]^a\ [L]^b\ [LL^{-1}]^c\ [LT^{-2}]^d$

$[M^0\ L^0\ T] = \text{constant}\ [M]^a\ [L]^{b+d}\ [T]^{-2d}$ ----------------- (2)

Equating the powers of M on both sides of equation (2), we get:

$a = 0$ ----------------- (3)

Equating the powers of L on both sides of equation (2), we get:

$b + d = 0$ ----------------- (4)

Equating the powers of T on both sides of equation (2), we get:

$-2d = 1$

Or $d = -1/2$ ----------------- (5)

Put $d = -1/2$, in (4), we get:

$b = 1/2$

Put the values of a, b, c and d in (1)

$$T = (\text{constant})\ m^{o}\ l^{1/2}\ g^{-1/2}$$

$$T = (\text{constant}) \times \sqrt{\frac{l}{g}}$$

Where the constant can be found by experiment, which is 2π.

Assignment 1.2

Which of the following relationships is dimensionally consistent with an expression yielding a value for acceleration? In these equations, x is a distance, t is time, and v is velocity.

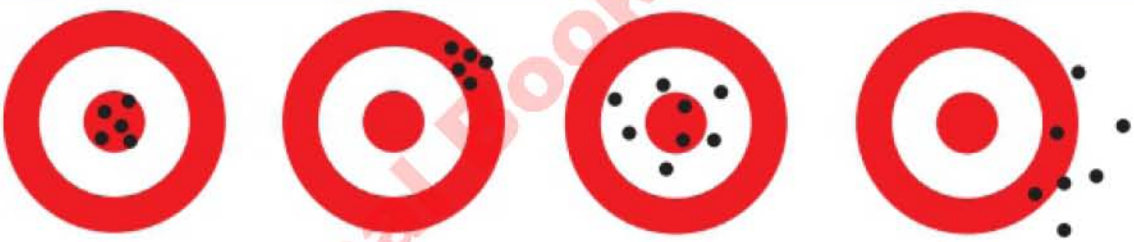
(a) v/t^2 (b) v/x^2 (c) v^2/t (d) v^2/x

1.4 PRECISION AND ACCURACY

Science is built on observation and experiment, i.e., on measurements. Precision is measurements of the same physical quantity agree with each other. Accuracy is measurements of the same physical quantity agree with standard or true value. Accuracy describes how well we eliminate systematic error. Hence:

Precision refers to the closeness of measured values to each other, while accuracy refers to the closeness of a measured value to a standard or true value.

For Your Information



(a) precise and accurate (b) precise but not accurate (c) not precise but accurate (d) neither precise nor accurate

Several independent trials of shooting at a bullseye target illustrate the difference between being accurate and being precise. Accuracy is how close an arrow gets to the bull's-eye center. Precision is how close a second arrow is to the first one (regardless of closeness to the target).

To understand the concept of precision and accuracy, consider that a person weighs exactly 160.0 pounds and he weight himself three times on three different scales. Results of the scales are:

Scale A: 170.1, 169.9 and 170.0 pounds.

Scale B: 161, 162 and 158 pounds.

Scale C: 159.9. 160.0 and 160.1 pounds.

In this case, weight measured by scale A is very precise, but not accurate. Weight measured by scale B is fairly accurate but not precise. Weight measured by scale C is both precise and accurate.

The precision of a measurement is associated with least count of the measuring instruments. Smaller the least count of the measuring instrument greater will be its precision.
Precision is indicated by absolute uncertainty in measurement. Accuracy is indicated by the fractional or percentage uncertainty or error in measurement. Smaller the magnitude of fractional or percentage uncertainty or error, greater will be its accuracy.

1.5 UNCERTAINTIES

In Grade 9, we have studied about the sources of human error, systematic error and random error in experiments. We studied that the difference between the true value and observed value of a measurement is called error. i.e.,

Error = observed value − true value

In a measurement the error may occur due to

- Negligence or inexperience of a person.
- Using a faulty apparatus.
- Inappropriate method or technique.

Error may be divided into the following three types:

- Personal Error
- Systematic Error
- Random Error

Here we will study about uncertainty.

Uncertainty is the range of possible values within which the true value of the measurement lies.

Tip for Solving Numerical Problems Symbolic Solutions!

When solving problems, it is very useful to perform the solution completely in algebraic form and wait until the very end to enter numerical values into the final symbolic expression. This method will save many calculator keystrokes, especially if some quantities cancel so that you never have to enter their values into your calculator! In addition, you will only need to round once, on the final result.

For example, a measurement of 3.06 mm ± 0.02 mm means that the experimenter is confident that the actual value for the quantity being measured lies between 3.04 mm and 3.08 mm. "Uncertainty is a quantitative measurement of variability in the data".
All measurements have a degree of uncertainty. This is caused by two factors, the limitation of the measuring instrument (systematic error) and the skills of the experimenter making the measurements.
Absolute uncertainty is equal to the least count of a measuring instrument, for example the length of a glass slab measured with meter rod is 37.5 cm. The least count of meter rod is 1 mm = 0.1 cm, so the absolute uncertainty in measured value will be ± 0.1 cm i.e., ±0.05 cm uncertainty develops at each end. For example, if one end of the slab coincides with 20.5 cm mark and the other coincides with 58.0 cm mark of meter rule, the length of the slab along with uncertainty is given by

$$(58.0 \pm 0.05)\text{ cm} - (20.5 \pm 0.05)\text{ cm} = (37.5 \pm 0.1)\text{ cm}$$

It means that the length of slab is between 37.4 cm and 37.6 cm.

Remember!

If $x + \Delta x = (2.0 \pm 0.1)$ mm, then Actual/Absolute uncertainty is
$\Delta x = \pm 0.1$ mm
Fractional uncertainty is
$\frac{\Delta x}{x} = 0.05$
Percentage uncertainty is
$\frac{\Delta x}{x} \times 100\% = 5\ \%$

In the above measurement precision is ±0.1 cm, which is equal to the magnitude of absolute uncertainty.
The accuracy in the measurement is the magnitude of fractional error. Here

$$\text{Fractional uncertainty} = \frac{\text{absolute uncertainty}}{\text{measured value}}$$

$$= \frac{\pm 0.1}{37.5} = \pm 0.003$$

$$Percentage\ uncertainty = fractional\ uncertainty \times 100\,\%$$

Smaller the magnitude of fractional (relative) uncertainty or error greater will be the accuracy in measurement.

1.5.1 Rules for Calculating Uncertainties in Final Result

There are some rules for calculating uncertainties in different cases but we need to be very careful whether we use the absolute or percentage uncertainty in each case.

Let x and y are two different physical quantities with uncertainties Δx and Δy respectively. If z is a physical quantity which is obtained by operating x and y then the propagated uncertainty Δz in the result can be calculated by using the following rules.

a) Rule for Sum and Difference

If two or more than two measured quantities are added or subtracted, then their absolute uncertainties are added to get uncertainty in the result.

If	$z = x + y$	or	$z = x - y$
then	$\Delta z = \pm (\Delta x + \Delta y)$		

For example, if $x \pm \Delta x = (24.0 \pm 0.1)$ cm

and $y \pm \Delta y = (30.0 \pm 0.1)$ cm

then $\Delta z = \pm 0.2$ cm

b) Rule for Multiplication and Division

If two or more than two quantities are multiplied or divided, then their percentage uncertainties are added to get uncertainty in the result.

If	$z = xy$	or	$z = x/y$,
Then	% uncertainty in z = % uncertainty in x + % uncertainty in y		

c) Rule for Power of a Quantity

The total uncertainty in power of a quantity is equal to the percentage uncertainty multiplied with that power.

If	$z = x^3$,
then	percentage uncertainties in z = ± 3 (percentage uncertainty in x)

d) Uncertainties in average values of many measurements

The uncertainty in the average value is calculated by adopting the following steps.

i) Find the average of measured values.
ii) Find the deviation of each value from the average.
iii) The mean deviation is the uncertainty in the average.

For example, three readings are recorded for the radius of a small cylinder as

$r_1 = 1.50$ cm, $r_2 = 1.51$ cm and $r_3 = 1.52$ cm

The uncertainty in the average radius is calculated as

Finding average $\bar{r} = \dfrac{r_1 + r_2 + r_3}{3}$

$$= \frac{1.50\ cm + 1.51\ cm + 1.52\ cm}{3} = 1.51\ cm$$

Finding deviation $\Delta r_1 = \bar{r} - r_1 = 1.51 - 1.50 = 0.01$ cm

$\Delta r_2 = \bar{r} - r_2 = 1.51 - 1.51 = 0$ cm

$$\Delta r_3 = \bar{r} - r_3 = 1.51 - 1.52 = 0.01 \text{ cm}$$

Finding mean deviation $\quad < \Delta r > = \dfrac{\Delta r_1 + \Delta r_2 + \Delta r_3}{3}$

$$= \frac{0.01\,cm + 0\,cm \quad 0.01\,cm}{3}$$

$$= 0.0067 \text{ cm} = 0.007 \text{ cm}$$

e) Uncertainty in Timing Experiment

The time period T of a vibrating body can be found by dividing time of multiple vibrations to the number of vibrations.

$$T = \frac{\textit{Time of multiple vibrations}}{\textit{No. of vibrations}}$$

The uncertainty in time period ΔT is found by dividing least count (L.C) of the time recording device to the number of vibrations.

$$\Delta T = \frac{L.C}{\textit{No. of vibrations}}$$

For example, the time recorded for 20 vibrations of a pendulum is t = 35.2 s. Let the least count of stop watch used is 0.1 s (1/10 s). So, the uncertainty in measured time is (35.2 s ± 0.1 s).

Then the time period of the pendulum is obtained as:

$$T = 35.2/20 = 1.76 \text{ s}$$

Uncertainty in time period = ΔT = 0.1/20 = 0.005 s

So, $\quad T + \Delta T = (1.76 \pm 0.005)$ s

Example 1.3: If voltage measured across a conductor is V ± ΔV = (7.3 ± 0.1) volts and current is I ± ΔI = (2.73 ± 0.051) A. Find the resistance and uncertainty in it.

Given: $V \pm \Delta V = (7.3 \pm 0.1)$ volts $\qquad I \pm \Delta I = (2.73 \pm 0.051)$ A

To Find: $R \pm \Delta R = ?$

Solution: According to ohm's law, R is calculated as

$$R = V/I = 7.3/2.73 = 2.7\ \Omega$$

Percentage uncertainty in *V* is

$$= \frac{\Delta V}{V} \times 100\% = \frac{0.1}{7.3} \times 100\ \% = 1.37\ \% = 1\ \%$$

Percentage uncertainty in *I* is

$$= \frac{\Delta I}{I} \times 100\% = \frac{0.05}{2.73} \times 100\ \% = 1.83\ \% = 2\ \%$$

Thus, the total uncertainty in R is

$$= 1\ \% + 2\ \% = 3\ \%$$

So $\quad R \pm \Delta R = 2.7 \pm 3\ \%$

$$= 2.7\ \Omega \pm \left(\frac{3}{100} \times 2.7\right)\ \Omega$$

$$= (2.7 \pm 0.08)\ \Omega$$

Example 1.4: If radius of a circular disc is measured as 2.25 cm with uncertainty ± 0.01 cm. Find its surface area with uncertainty in it.

Given: $\quad r = 2.25$ cm, $\qquad \Delta r = \pm 0.01$ cm

To Find: $A \pm \Delta A = ?$

Solution: As, $A = \pi r^2 = 3.14 \times 2.25^2 = 15.90\ cm^2$

Percentage uncertainty in r $= \frac{\Delta r}{r} \times 100\ \% = \frac{0.01}{2.25} \times 100\ \% = 0.4\ \%$

Percentage uncertainty in area is $= 2 \times 0.4\ \% = 0.8\ \%$

So, $\Delta A = 0.8\ \% \times 15.90\ cm^2 = 0.13\ cm^2$

Thus $A \pm \Delta A = (15.90 \pm 0.13)\ cm^2$

Assignment 1.3

The radius of a circle is measured to be (10.5 ± 0.2) m. Calculate (a) the area and (b) the circumference of the circle, also give the uncertainty in each value.

SUMMARY

- **Estimation** does not mean guessing a formula or a number at random. An estimation is a rough educated guess to the value of a physical quantity by using prior experience and sound physical reasoning.
- **Derived units** can be expressed in terms of base units and are obtained by multiplying or dividing base units with each other.
- **Dimension** denotes the qualitative nature of a physical quantity.
- In order to check the correctness of an equation, we have to show that both sides of an equation have the same dimensions, otherwise the equation cannot be physically correct. This is called the **principle of homogeneity of dimensions.**
- **Uncertainty** is the range of possible values within which the true value of the measurement lies.
- **Absolute uncertainty** is equal to the least count of a measuring instrument.
- **Precision** refers to the closeness of measured values to each other.
- **Accuracy** refers to the closeness of a measured value to a standard or true value.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) The mean diameter of a wire is found to be (0.50±0.02) mm. The percentage uncertainty in the diameter is:

A. 2% B. 4% C. 6% D. 8%

2) A reaction takes place that is expected to yield 171.9 g of product, but it only yields 154.8 g. What is the percent error for this experiment?

A.17.1% B. 90.1% C. 111.0% D. 9.9%

3) Three different people weigh a standard mass of 2.00 g on the same balance. Each person obtains a reading of exactly 7.32 g for the mass of the standard. These results imply that the balance that was used is

A. both accurate and precise B. neither accurate nor precise

C. accurate but not precise D. precise but not accurate.

4) Dimension of universal gravitational constant is

A. $[M^{-2}L^3T^{-2}]$ B. $[M^3L^{-1}T^{-2}]$ C. $[M^{-1}L^3T^{-2}]$ D. $[M^{-3}L^3T^{-2}]$

5) A measurement which on repetition gives same or nearly same result is called

A. Accurate B. average C. precise D. estimated

6) A student is measuring the time of an event by using stopwatch. He takes 5 measurements as: 3.0 s, 3.2 s, 3.4 s, 2.8 s, 3.1 s. What is the uncertainty in the results?

A. ±0.3 s B. ±0.6 s C. ±3.1 s D. ±7.75 s

7) Which of the following quantity has different dimension?

A. Force B. weight C. Modulus of elasticity D. Tension

8) If the dimensions of a physical quantity are given by $[L^a M^b T^{-c}]$, then the physical quantity will be

A. force, if a = -1, b = 0, c = -2 B. pressure, if a = -1, b = 1, c = -2

C. velocity, if a = 1, b = 0, c = 1 D. acceleration, if a = 1, b = 1, c = -2

9) Order of magnitude of $(10^6 + 10^3)$ is

A. 10^{18} B. 10^9 C. 10^6 D. 10^3

10) Which of the following may be used as a valid formula to calculate speed of ocean waves? [v = speed, g = acceleration due to gravity, λ = wavelength, ρ=density, h= depth].

A. $v = \sqrt{\lambda g}$ C. v = ρgh C. v = gh/λ D. v = λgh

Short Questions

1) Create a table to show reasonable estimate of some physical quantities.

2) Express the units of the following derived quantities in term of base units. (a) Force (b) Work (c) Power (d) Pressure (e) Electric charge.

3) Why is it important to use an instrument of smallest resolution?

4) What is the importance of increasing the number of readings in an experiment?

5) What is the difference between precision and accuracy?

6) What is the principle of homogeneity of dimensions?

7) A ball is thrown in the air and 5 different students are individually measuring the time it takes to fall back down using stopwatches. The times obtained by each student are the following: 6.2 s, 6.0 s, 6.4 s, 6.1 s, 5.8 s. (i) What is the uncertainty of the results? (ii) How should the resulting time be expressed?

8) The energy of a photon is given by $E = hf$, find the dimensions of Plank's constant h, where f is frequency.

Comprehensive Questions

1) Define and explain the term uncertainty.
2) Discuss the rules for calculating uncertainty propagation in the final results in different cases.
3) What does the dimension of a physical quantity mean? What are its advantages, explain with the help of examples?
4) What is meant by estimation of a physical quantity? Explain with the help of an example.

Numerical Problems

(1) Estimate number of heartbeats in a lifetime?

(2) Determine the dimensions of each of the following quantities.

a) $\frac{v^2}{a\,x}$ b) $\frac{a\,t^2}{2}$

(Ans: No, [L])

(3) If A = $\frac{X^2}{Y^2\,Z}$ then find the percentage uncertainty in A. The percentage uncertainties in X, Y and Z are 1 %, 1 % and 2 % respectively.

(Ans: 6 %)

(4) A spherical ball of radius r experiences a resistive force F due to the air as it moves through the air at speed v. The resistive force F is given by the expression

$$F = c\,r\,v$$

Where c is constant. By using dimensions, derive the SI base unit of the constant c.

(Ans: kg $m^{-1}s^{-1}$)

(5) The pressure (P) at a depth (h) in an incompressible fluid of density (ρ) is given by

$$P = \rho\,g\,h$$

Where g is acceleration due to gravity. Check the homogeneity of this equation.

(6) Estimate that how many protons are there in a bacterium? **(Ans: 10^{12} protons)**

(7) Estimate that how many hydrogen atoms does it take to stretch across the diameter of the Sun? **(Ans: 10^{19} hydrogen atoms)**

(8) The current passing through a resistor R = (13 ± 0.5) Ω is I = (3 ± 0.1) A.

a) Calculate the power consumed (correct to one significant figure).

b) Find the percentage uncertainty of the current passing through the resistor.

d) Find the percentage uncertainty of the resistance.

c) Find the absolute uncertainty of the electrical power.

(Ans: 117 W, 3 %, 3.84 %, 11.7 W)