

1 Elementary Algebra:-

$$1.1: \frac{x^{32}}{x^9 \cdot x^2} \cdot \frac{x^7}{x^2}$$

$$= \frac{x^{39}}{x^{13}} = x^{26}.$$

$$1.6: x \geq e^e \quad [e^e, \infty]$$

$$1.2: 8^2 \cdot 4^x \cdot 2^x = 8^4$$

$$(2^3)^2 \cdot (2^2)^x \cdot 2^x = (2^3)^4$$

$$2^6 \cdot 2^{2x} \cdot 2^x = 2^{12}$$

$$6 + 2x + x = 12.$$

$$3x = 6.$$

$$x = 2.$$

$$1.3: \frac{x}{y} = 3$$

$$\frac{y}{x} = \frac{1}{3}.$$

$$\left(\frac{y}{x}\right)^4 = \left(\frac{1}{3}\right)^4$$

$$\left(\frac{y}{x}\right)^4 = \frac{1}{81}$$

$$1.4: \frac{\sqrt[4]{4^{15}}}{\sqrt[16]{16^7}} = \frac{\sqrt{(2^2)^{15}}}{\sqrt{(2^4)^7}}$$

$$= \frac{2^{30/2}}{2^{28/2}}$$

$$= 2^{15}/2^{14}$$

$$= 2^1.$$

- 1.5:-
- True.
 - True.
 - True.
 - True.

2 Functions of one variable:

$$2.1. \quad 0^\circ\text{C} = 32^\circ\text{F}$$

$$100^\circ\text{C} = 212^\circ\text{F}$$

$$100x = 180$$

$$x = 1.8.$$

$$\text{As } 0^\circ\text{C} = 32^\circ\text{F}$$

$$32 \div 1.8 = -40$$

$$2.5. \quad \pi^{\log(\frac{1}{\pi^5})} = \pi^{-5}$$

$$\log_{\pi}(\frac{1}{\pi^5}) = -5$$

$$2.2. \quad f(x) = 3x - 12$$

$$3x - 12 = 0$$

$$3x = 12.$$

$$x = 4.$$

$$2.3. \quad 9^{x^2 - 6x + 2} = 81$$

$$x^2 - 6x + 2 = 2.$$

$$x^2 - 6x = 0.$$

$$x(x - 6) = 0.$$

$$x = 0 \text{ or } x = 6.$$

$$2.4. \quad (1.03)^x = 3$$

$$-\ln(1.03)^x = \ln(3)$$

$$x \ln(1.03) = \ln(3)$$

$$x = \frac{\ln(3)}{\ln(1.03)}$$

$$x = 37 \text{ years.}$$

3 Calculus:

$$3.1: \sum_{i=0}^{\infty} (1/5^i + 0.3^i)$$

DONE AT END.

$$= \cancel{1/5^0} + 0.3^1$$

$$= \cancel{1/5^0} + \cancel{0} = 0.$$

$$3.5: f(x) = x^9 + 3.$$

$$f'(x) = 9x^8$$

$$f''(x) = 72x^7$$

3.6: 40 is not continuous at $x=0$ as at this point there exists infinite discontinuity.

$$3.2: \lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$$

$$x \rightarrow 4 = \frac{16 - 25}{-1}$$

$$= -9/-1 = 9.$$

$$x \rightarrow 6 = \frac{36 - 25}{1}$$

$$= 11/1 = 11.$$

So $\lim_{x \rightarrow 5} = 10.$

$$3.7: f(x) = 4x^3 - 12x.$$

$$f'(x) = 12x^2 - 12$$

$$12x^2 - 12 = 0.$$

~~$12x^2 = 0$~~

$$12(x^2 - 1) = 0,$$

$$x^2 = 1,$$

$$x = \pm 1.$$

$$3.3: f'(x) = 3x^2$$

when $x = -2.$

$$3(-2)^2 = 12$$

$$3.4: f(x) = \frac{x^5 + 3}{x^2 - 1}$$

$$= \frac{g(x) \cdot df(x) - f(x) \cdot dg(x)}{g(x)^2}$$

$$= \frac{(x^2 - 1)(5x^4) - (x^5 + 3)(2x)}{(x^2 - 1)^2} = \frac{5x^6 - 5x^4 - 2x^6 - 6x}{(x^2 - 1)^2}$$

$$= \frac{3x^6 - 5x^4 - 6x}{(x^2 - 1)^2}$$



3.9. $f(x,y) = \ln(x-3y)$
 $x-3y > 0.$

$x > 3y$, x is +ve
and y is +ve.

3.10.

$$\begin{aligned} & \frac{\partial}{\partial x} \left(x^5 y^7 + \frac{x^2}{y^3} \right) \\ &= \frac{\partial}{\partial x} 5x^4 y^7 + \frac{2x}{y^3} \\ &= 5x^4 y^7 + 2x/y^3. \end{aligned}$$

3.11. $f(x,y) = \sqrt{xy} - x - y$

$$f_{xx} = \frac{y}{2\sqrt{xy}} - 1, \quad f_{yy} = \frac{x}{2\sqrt{xy}} - 1.$$

$$f_{xx} = -\frac{y^2}{4(yx)^{3/2}}, \quad f_{yy} = -\frac{x^2}{4(xy)^{3/2}}$$

As both the second differentials are < 0 , this equation has a local maxima.

3.1. $\sum_{i=0}^{\infty} \left(\underbrace{\frac{1}{5^i}}_{1st} + \underbrace{\frac{0.3^i}{2^{nd}}} \right)$

$$\Rightarrow \frac{a}{1-r}, \quad \frac{1/5^0}{1-1/5} = \frac{1}{4/5} = 1.25. \quad (\text{for 1st})$$

$$\Rightarrow \frac{0.3^0}{1-3/10} = 1.43, \quad \text{So } \sum = 1.25 + 1.43 = 2.68$$

(for 2nd)

4 Linear Algebra :-

4.12- $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix}.$

~~$$B \cdot A = \begin{bmatrix} (1 \times 2 + 9 \times 5) \\ (1 \times 2 + 9 \times 5) \\ (1 \times 2 + 9 \times 5) \end{bmatrix}$$~~

$$B \cdot A = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix},$$

$$= \begin{bmatrix} (1 \times 2 + 1 \times 7) & (1 \times 5 + 1 \times 6) \\ (9 \times 2 + 1 \times 2 + 5 \times 7) & (9 \times 5) + 1 \times 1 + 5 \times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 11 \\ 55 & 76 \end{bmatrix}.$$

4.22- $A \cdot B = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix}$

~~$$(5 \times 8 + 0 \times 4 + 1 \times 0)$$~~

$$= \begin{bmatrix} (5 \times 8 + 3 \times 2) & (5 \times 4 + 3 \times 1) & (5 \times 0 + 3 \times 2) \\ (0 \times 8 + 1 \times 2) & (0 \times 4 + 1 \times 1) & (0 \times 0 + 1 \times 2) \\ (1 \times 8 + 2 \times 2) & (1 \times 4 + 2 \times 1) & (1 \times 0 + 2 \times 2) \end{bmatrix}$$

$$= \begin{bmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 4 \end{bmatrix}$$

4.3₂- Transpose:

$$= \begin{bmatrix} 2 & 2 & 4 \\ 93 & 6.1 & \pi \\ 4.7 & 4.22 & 0 \end{bmatrix}.$$

$$\begin{aligned} 4.4_2 \quad \det &= ad - bc \\ &= 2 \times 8 - 6 \times 2 \\ &= 16 - 12 \\ &= 4. \end{aligned}$$

5 Probability Theory:

5.1. Sample Space:

$$\{(1,1), (1,2), (1,3), \dots, (1,6) \\ (2,1), (2,2), \dots, (2,6) \\ \vdots \\ (6,1), (6,2), \dots, (6,6)\}$$

$$5.2_2- P(\text{Drug User}) = 0.001.$$

$$P(\text{PR}) = 0.98$$

$$P(\text{NIR}) = 0.997.$$

$$P(\text{PR} | \text{Drug User}) = \frac{P(\text{PR} \cap \text{Drug User})}{P(\text{Drug User})}$$

$$= \frac{0.98 \times 0.001}{0.001}$$

$$= 98\%.$$



$$5.3. \quad P(5) = 1/6.$$

$$RV = 1/6 \times 20,$$

$$= 3.3.$$

Or share of each number on dice

$$= 20/6 = 3.3.$$