شهرزاد صمیمی - ۸۱۰۱۰۰۲۷۲

DS

①

الف)

```
if (n%2 == 0){
    for (int i=0 ; i<n ; i++ ){
        for (int j=i ; j>=0 ; j--){
            for(int k=n ; k>=1 ; k/=2){
                cout << 3;
            }
        }
    }
}
```

$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$

$\sum_{j=0}^{i} 1 = \frac{j(j+1)}{2}$

$\sum_{k=1}^{n/2} 1 = \frac{\frac{n}{2}(\frac{n}{2}+1)}{2}$

در بهترین حالت: n فرد است، $T(n) = cn^0 = c$

پاسخ: $\sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{k=0}^{\log n} c2^{-k} = \sum_{i=0}^{n} \sum_{j=0}^{i} c\log n = \sum_{i=0}^{n} ic\log n = c(n^2+n)\log = O(n^2 \log n)$

ب) $\sum_{i=0}^{n} \sum_{j=1}^{\lceil \sqrt{i} \rceil} c = \sum_{i=1}^{n} c\lceil \sqrt{i} \rceil \leq \sum_{i=1}^{n} ci = O(n^2)$

ج) $\sum_{i=0}^{n} \sum_{j=0}^{i} c = \sum_{i=0}^{n} ci = O(n^2)$

②

$(\log n)^2 = O(\log^2 n)$

$\log n! = \log((n)\times(n-1)\times(n-2)\times \dots \times 1) = \log n + \log(n-1) + \log(n-2) + \dots + \log 1 \leq n\log n$

$\rightarrow \log n! = O(n\log n)$

$n^2 = O(n^2)$

$(5/4)^n = O(2^n)$

$(5/4)^n > n^2 > \log n! > \log^2 n$

ب) $(\sqrt{2})^{\log n} > n^{\log\log n} > (\log n)! > \log n!$

۳) الف) با نقض رد می‌کنیم.

$g(n)=2 \qquad f(n)=\tan(n)$

ب) اثبات می‌شود.

$f(n)\le c\,g(n) \xrightarrow{\log} \log_2 f(n)\le \log_2 c+\log_2 g(n)$

$\rightarrow \log_2 f(n)\le c\log_2 g(n) \rightarrow \log_2 f(n)\in O(\log_2 g(n))$

ج) اثبات

$$\left.\begin{array}{l} f(n)\le c\,g(n)\\ g(n)\le c' h(n)\end{array}\right\} \Rightarrow f(n)\le c\,h(n) \rightarrow f(n)=O(h(n))$$

د) اثبات

$\log_k n=\dfrac{\log_2 n}{\underbrace{\log_2 k}_{c_1}}=c_2\log_2 n \rightarrow c_1>\dfrac{1}{\log_2 k}$

$\searrow c_2<\dfrac{1}{\log_2 k}$

$\Rightarrow c_1\log_2 n\le \log_k n\le c_2\log_2 n \rightarrow \log_k n=\Theta(\log_2 n)$

۴)

الف) $T(n)=9T(n/3)+n^2\log^3 n$

$a=9,\ b=3 \quad n^{\log_3 9}=n^2 \quad ,\ f(n)=\Omega(n^2) \quad ,\ n^2\log^{3} n^{\ldots} << n^2\log^3 n$

$\rightarrow T(n)=\Theta(n^2\log^4 n)$

ب) $T(n)=4^n T(n/2)+n^n$

طرفین را در 2^{-2n} ضرب

$\longrightarrow 2^{-2n}T(n)=2^{-n}T(n/2)+2^{-2n}\cdot n^n \xrightarrow{T'(n)=2^{-2n}T(n)} T'(n)=T'(n/2)+(n/4)^n$

$a=1,\ b=2 \longrightarrow g(n)=n^{\log_2 1}=1 \quad ,\ f(n)=(n/4)^n$

$f(n)=\Omega(1)\ ,\ (n/n)^{n/2} << (n/4)^n$ ✓

$\Rightarrow T'(n)=\Theta((n/4)^n) \rightarrow 2^{-2n}T(n)=\Theta((n/4)^n)$

$\longrightarrow T(n)=4^n\,\Theta((n/4)^n)=\boxed{\Theta(n^n)}$

$\log(n!) \xrightarrow{\log} \log\log(n!) < \log(n\log n)$ (I)

$(\log n)! \xrightarrow{\log} \log((\log n)!) = \log(\log n) + \log(\log(n-1)) + \cdots \leq n\log\log n$

$= O(n\log\log n)$ (II)

(I),(II) $\longrightarrow \log(n\log n) < \log n\log\log n < n\log\log n$

$\rightarrow \log(n!) < (\log n)!$

$n^{\log\log n} \xrightarrow{\log} \log n \cdot \log\log n$ (1)' $(\sqrt{2})^{\log^2 n} \xrightarrow{\log} \log^2 n\log\sqrt{2} < \log^2 n$ (2)

(1),(2) $\rightarrow \log n\log\log n < \log n \times \log n \longrightarrow (\sqrt{2})^{\log^2 n} > n^{\log\log n}$

$n > \log n$, $n^{\log\log n} > \log n!$

$\Rightarrow (\sqrt{2})^{\log^2 n} > n^{\log\log n} > (\log n)! > (\log n!)$

ج) $\boxed{\sum_{k=0}^{n}\frac{n^k}{k!} > 2^n > \sum_{i=1}^{n}\sum_{j=1}^{i} i > n^{\frac{1}{\log n}}}$

طبق بسط تیلور $\Rightarrow$ $\sum_{k=0}^{n}\frac{n^k}{k!} < e^n \longrightarrow \sum_{k=0}^{n}\frac{n^k}{k!} = O(e^n)$

$\sum_{i=1}^{n}\sum_{j=1}^{i} i = \sum_{i=1}^{n} i^2 = O(n^3)$

$n^{\frac{1}{\log n}} \xrightarrow{\log} \log n^{\frac{1}{\log n}} = \frac{1}{\log n}\log n = 1 \rightarrow n^{\frac{1}{\log n}} = O(1)$

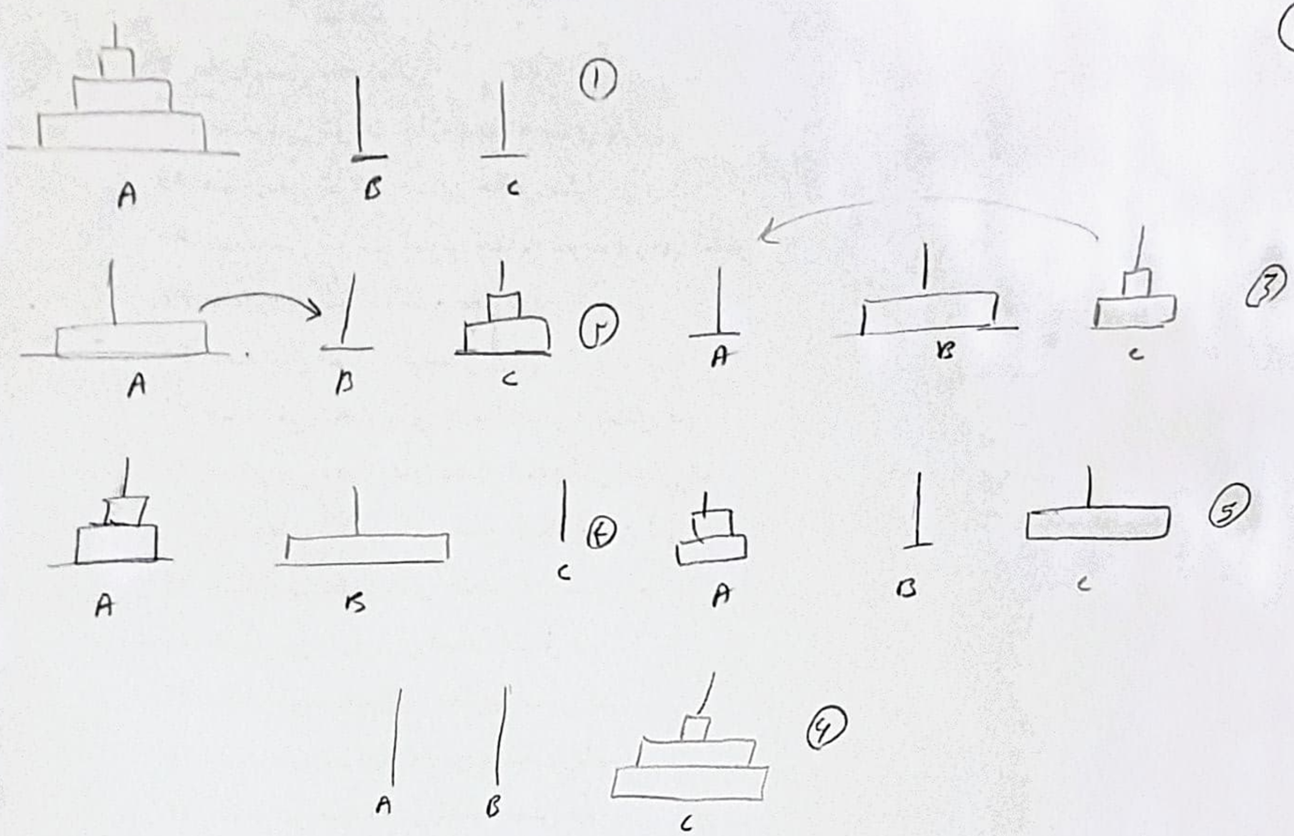
$2^n = O(2^n)$

۲) $T(n) = \sqrt{n}\, T(\sqrt{n}) + n \xrightarrow{\text{تقسیم بر } n} \frac{T(n)}{n} = \frac{T(\sqrt{n})}{\sqrt{n}} + 1$

$\xrightarrow{S(n) = \frac{T(n)}{n}} S(n) = S(\sqrt{n}) + 1 \xrightarrow{n = 2^m} S(2^m) = S(2^{m/2}) + 1$

$S(2^m) = F(m) \xrightarrow[F(m) = \lg m]{} F(m) = F(m/2) + 1 \longrightarrow F(m)$

۵



$1 \to 2 : T(n-1)$ $\quad 2 \to 3 : 1$ $\quad 3 \to 4 : T(n-1)$ $\quad 4 \to 5 : 1$

$5 \to 6 : T(n-1)$

$$\begin{aligned}
T(n) &= 3T(n-1) + 2 = \\
&= 9(n-2) + 3\times 2 + 2 \\
&= 27(n-3) + 9\times 2 + 3\times 2 + 2 \\
&\vdots \\
&= 3^{k-1} T(1) + 3^{k-2} \times 2 + 3^{k-3} \times 2 + \dots \\
&= O(3^n)
\end{aligned}$$

①

```
devider = input()
divisor = input()
int, number_of_digit_devider
int R → باقیمانده
int Q → خارج قسمت
int H=0, d=0

    for (int i=n-1 ; i<=0 ; i++){

        H = H + devider [i]
        d = H%/divisor
        R = H/ divisor
        if (Q=0){
          H *=10}
        else{ Q. push(d) & H =0}

print Q: ; print R
```

تعداد مقایسه‌ها از مرتبه O(n) است