Poisson Distribution

when events occur randomly over a specified interval of time or space or length
e.g the number of telephone calls recieved per hour in an office.

⇒ The number of insurence claim made to a Company in a year.

⇒ The number of typing errors per page in a book. e.t.c

$$P(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} \qquad x = 0, 1, 2 \ldots$$

where

λ : Average number of occurences Per time, distance area

t : Specific time, distance, area or volume.

X : No. of occurences in time t.

$e = 2.71828$

⇒ It has one parameter μ and $\mu = \lambda t$

⇒ Mean = variane = λt

$P(x;\lambda t)$ or $P(x;\mu)$

The poisson dist is also called law of small numbers or the rare events distribution.

Example 5.17 : (Pg 162) walpole

$$\lambda t = 4$$

$$P(x;\lambda t) = \frac{e^{-\lambda t}\,\lambda t^{x}}{x!}$$

$$P(X=6) = \frac{e^{-4}(4)^{6}}{6!}$$

$$\boxed{P(X=6) = 0.1042}$$

Example 5.18

$$\mu = \lambda t = 10$$

$$P(x;\lambda t) = \frac{e^{-\lambda t}\,\lambda t^{x}}{x!}$$

$$P(X>15) = 1 - P(X \leq 15)$$

$$= 1 - \sum_{x=0}^{15} P(x;10)$$

$$= 1 - [P(X=0) + P(X=1) + \dots P(X=15)$$

$$= 1 - 0.9513$$

$$\boxed{P(X>15) = 0.0487}$$

Approximation of binomial dist by Poisson:-

If n is large and p is small then we approximate ~~by~~ Binomial by Poisson.
i.e (when p is 0.05 or less and n is 20 or more)

(Walpole)

Example 5.19 :-

$$P = 0.005$$
$$n = 400$$

As n is large and p is small

$$\mu = np$$
$$= 400 \times 0.005$$
$$\boxed{\mu = 2}$$

$$\mu = 400 \times 00$$

a) $$P(X = x) = \frac{e^{-\mu} \mu^{x}}{x!}$$

$$P(X=1) = \frac{e^{-2} 2^{1}}{1!}$$

$$\boxed{P(X=1) = 0.271}$$

b) $$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{e^{-2} 2^{0}}{0!} + \frac{e^{-2} 2^{1}}{1!} + \frac{e^{-2} 2^{2}}{2!} + \frac{e^{-2} 2^{3}}{3!}$$

$$\boxed{P(X \le 3) = 0.857}$$

1) Probability function :-

Probability function gives the probabilities of possible values of a random variable. It gives the probability that a discrete random variable is exactly equal to some value.

i.e. $P(X = x_i) = f(x_i)$ or $P(x_i)$

2) Distribution function (df) or cummulative distribution function (cdf) :-

This function $F(x)$ gives the probability that event X takes a value less than or equal to a specified value x. i.e $P(X \leq x)$. It is denoted by $F(x)$.

⇒ Also called cummulative distribution function (cdf) as it is cummulative probability function of x from smallest upto specific value of x

$$F(b) - F(a) = P(X \leq b) - P(X \leq a)$$
$$= P(a < X \leq b)$$

⇒ $F(x)$ is non-negative and non-decreasing function of x.

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Example 7.1

S.S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

x	$f(x_i)$	$F(x)$
0	$1/8$	$1/8$
1	$3/8$	$1/8 + 3/8 = 4/8$
2	$3/8$	$4/8 + 3/8 = 7/8$
3	$1/8$	$7/8 + 1/8 = 8/8$
	1	

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/8 & \text{for } 0 \leq x < 1 \\ 4/8 & \text{for } 1 \leq x < 2 \\ 7/8 & \text{for } 2 \leq x < 3 \\ 1 & \text{for } x \geq 3 \end{cases}$$

$$f(2) = F(2) - F(1)$$

$$= \frac{7}{8} - 4/8$$

$$\boxed{f(2) = \frac{3}{8}}$$

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