Example 7.3

$$f(x) = \begin{cases} Kx & 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

a) Find K

b) $P(x > 1)$

c) $F(x) = ?$

Solution:-

The function $f(x)$ will be a density function, if

$$\int_{-\infty}^{\infty} f(x).dx = 1$$

$$\int_{0}^{2} Kx\,dx = 1$$

$$K\left|\frac{x^2}{2}\right|_0^2 = 1$$

$$K\left[\frac{2^2}{2} - \frac{0^2}{2}\right] = 1$$

$$2K = 1$$

$$\boxed{K = \frac{1}{2}}$$

$$\text{Hence } f(x) = \begin{cases} \frac{1}{2}x & 0 \le x \le 2 \\ 0 & \text{else when} \end{cases}$$

b) $$P(x>1) = \int_1^2 f(x)\,dx$$

$$= \int_1^2 \frac{x}{2}\,dx$$

$$= \frac{1}{2}\int_1^2 x\,dx$$

$$= \frac{1}{2}\left|\frac{x^2}{2}\right|_1^2$$

$$= \frac{1}{2}\left[\frac{4}{2} - \frac{1}{2}\right]$$

$$= \frac{1}{2}\left(\frac{3}{2}\right)$$

$$\boxed{P(x>1) = \frac{3}{4}}$$

c) $$F(x) = P(x \le x) = \int_{-\infty}^{x} f(x)\,dx$$

$$F(x) = \int_0^x \frac{x}{2}\,dx$$

$$= \frac{1}{2}\left|\frac{x^2}{2}\right|_0^x$$

$$= \frac{1}{2}\left[\frac{x^2}{2} - 0\right]$$

$$\boxed{F(x) = \frac{x^2}{4}}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{4} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

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Example 7.4

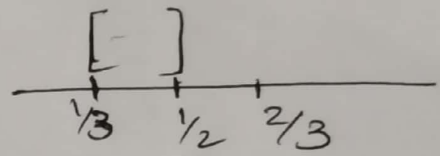
(v) $P\left[x \le \frac{1}{2} / \frac{1}{3} \le x \le 2/3\right] = ?$

As $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

So

$$P\left[x \le \frac{1}{2} / 1/3 \le x \le 2/3\right] = \frac{P\left[(x \le 1/2) \cap (1/3 \le x \le \frac{2}{3})\right]}{P(1/3 \le x \le 2/3)}$$

$$= \frac{P(1/3 \le x \le 1/2)}{P(1/3 \le x \le \frac{2}{3})}$$



$$= \frac{\int_{1/3}^{1/2} 2x\,dx}{\int_{1/3}^{2/3} 2x\,dx}$$

$$= \left[x^2\right]_{1/3}^{1/2} \div \left[x^2\right]_{1/3}^{2/3}$$

$$= \frac{5}{36} \times \frac{9}{3}$$

$$= \frac{5}{12}$$

Example 7.5

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 2x^2/5 & \text{for } 0 < x \leq 1 \\ -\frac{3}{5} + \frac{2}{5}\left(3x - \frac{x^2}{2}\right) & 1 < x \leq 2 \\ 1 & x > 2 \end{cases}$$

Find p.d.f and $P(|X| < 1.5)$.

Solution

$$f(x) = \frac{d}{dx} F(x)$$

So

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 4x/5 & \text{for } 0 < x \leq 1 \\ \frac{2}{5}(3 - x) & 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Now $P(|x| < 1.5) = P(-1.5 < x < 1.5)$

$$= \int_{-1.5}^{0} 0\,dx + \int_{0}^{1} \frac{4x}{5}\,dx + \int_{1}^{1.5} \frac{2}{5}(3 - x)\,dx$$

$$= \frac{\cancel{4}^{2}}{5}\left|\frac{x^2}{\cancel{2}}\right|_0^1 + \left|\frac{2}{5}\left(3x - \frac{x^2}{2}\right)\right|_1^{1.5}$$

$$= \frac{2}{5}(1) + \frac{2}{5}\left[\left((3 \times 1.5) - \frac{(1.5)^2}{2}\right) - \left(3 - \frac{1}{2}\right)\right]$$

$$= 0.40 + 0.35 = 0.75$$

Example 3.5 Gartia

$$P[X > x] = e^{-\lambda x} \qquad x > 0$$

1) Find CDF of X.
2) Find $P[T < X \leq 2T]$ where $T = \frac{1}{\lambda}$

Solution:-

$$F(x) = P(X \leq x)$$

$$F(x) = 1 - P(X > x)$$

$$F(x) = 1 - e^{-\lambda x}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

As $P(a < X \leq b) = F(b) - F(a)$

so

$$P(T < X \leq 2T) = 1 - e^{-\lambda(2T)} - (1 - e^{-\lambda(T)})$$

As $T = 1/\lambda$

so

$$= 1 - e^{-\cancel{\lambda}(2\frac{1}{\cancel{\lambda}})} - [1 - e^{-\cancel{\lambda}(1/\cancel{\lambda})}]$$

$$= \cancel{1} - e^{-2} - \cancel{1} - e^{-1}$$

$$= e^{-2} - e^{-1}$$

$$= 0.233$$

Ans