

# Formula Sheet

(Ungrouped)

→ Mean:

$$\bar{x} = \frac{\sum x}{n}$$

→ Median:

n: odd

$$\left(\frac{n+1}{2}\right)^{\text{th}} \text{ value}$$

n: even

$$\frac{1}{2} \left[ \left(\frac{n}{2}\right)^{\text{th}} \text{ val} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ val} \right]$$

→ Mode:-

Most frequent occurring value

⇒ Quartiles

n: odd

$$Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}} \text{ val}$$

$$Q_3 = \left(\frac{3n+1}{4}\right)^{\text{th}} \text{ val}$$

n: even

$$Q_1 = \frac{1}{2} \left[ \left(\frac{n}{4}\right)^{\text{th}} \text{ val} + \left(\frac{n}{4} + 1\right)^{\text{th}} \text{ val} \right]$$

(Grouped)

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$l + \frac{h}{f} \left( \frac{n}{2} - c \right)$$

$$l + \left[ \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \right] h$$

$$Q_1 = l + \frac{h}{f} \left( \frac{n}{4} - c \right)$$

$$Q_3 = l + \frac{h}{f} \left( \frac{3n}{4} - c \right)$$



$$Q_3 = \frac{1}{2} \left[ \left( \frac{3n}{4} \right)^{\text{th}} \text{val} + \left( \frac{3n}{4} + 1 \right)^{\text{th}} \text{val} \right]$$

⇒ Decile:-

n: odd

$$D_j = \left( \frac{jn}{10} + 1 \right)^{\text{th}} \text{val}$$

$$j = 1, 2, 3, \dots, 9$$

n: even

$$D_j = \frac{1}{2} \left[ \left( \frac{jn}{10} \right)^{\text{th}} \text{val} + \left( \frac{jn}{10} + 1 \right)^{\text{th}} \text{val} \right]$$

$$D_j = L + \frac{h}{f} \left( \frac{jn}{10} - c \right)$$

⇒ Percentile:-

n: odd

$$P_i = \left( \frac{in}{100} + 1 \right)^{\text{th}} \text{val}$$

$$i = 1, 2, \dots, 99$$

n: even

$$P_i = \frac{1}{2} \left[ \left( \frac{in}{100} \right)^{\text{th}} \text{val} + \left( \frac{in}{100} + 1 \right)^{\text{th}} \text{val} \right]$$

$$P_i = L + \frac{h}{f} \left( \frac{in}{100} - c \right)$$

(Population)

(Sample)

$$\text{Variance:- } \sigma^2 = \frac{\sum f(x - \mu)^2}{\sum f}$$

$$\sigma^2 = \frac{\sum x^2}{N} - \left( \frac{\sum x}{N} \right)^2$$

$$\sigma = \sqrt{\frac{\sum f(x - \mu)^2}{\sum f}}$$

Standard Deviation:-

$$\sigma = \sqrt{\frac{\sum x^2}{N} - \left( \frac{\sum x}{N} \right)^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \left( \frac{\sum x}{n} \right)^2$$

$$\sigma^2 = \frac{\sum f(x - \bar{x})^2}{\sum f}$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left( \frac{\sum x}{n} \right)^2}$$

$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$



IQR

$\Rightarrow$  Semi-IQR

$$Q_3 - Q_1$$

$$(Q_3 - Q_1) / 2$$

relative frequency

$\Rightarrow$  Skewness

frequency of a particular class  
total no of observation

Centre : Symmetric

right : positively skewed

left : negatively skewed

$\therefore$  Mean of positively skewed data is greater than median  
Opposite is the case in negatively skewed data.

(Probability)

• Arrangements for  $n$  distinct objects  
 $n!$

• Arrangement for  $n$  non-distinct elements

$$\frac{n!}{h_1! h_2! \dots h_k!}$$

• Circular Arrangement  
 $(n-1)!$

• Permutation (order is important)  
 $nPr = \frac{n!}{(n-r)!}$

• Combination (order is not important)

$$nCr = \frac{n!}{r! (n-r)!}$$



→ Additive law

• Mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

• Non - mutually exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(Conditional Probability)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cup B) = 1 - P(A' \cap B')$$

$$P(A \cap D') = P(A) - P(A \cap D)$$

$$P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C)$$

⇒ Multiplicative law

$$P(A \cap B) = P(A) \cdot P(B) \quad \text{independent}$$

$$P(A \cap B) = P(A) \cdot P(B/A) \quad \text{dependent}$$

⇒ Bayes Theorem

$$P(D) = P(A) P(D/A) + P(B) P(D/B) + \dots$$

↑  
Total probability

$$P(A/D) = \frac{P(A) P(D/A)}{P(D)}$$

⇒ Bayesian Spam Filter

$$P(S/E_i) = \frac{P(S) P(E_i/S)}{P(S) P(E_i/S) + P(S') P(E_i/S')}$$



# Continuous Probability Distribution

$$\int_{-\infty}^{\infty} f(x) = 1$$

$$P(x > 2) = \int_2^{\text{upper limit}} f(x) dx$$

$$P(x < 1) = \int_{\text{lower limit}}^1 f(x) dx$$

CDF :-

$$F(x) = \int_{\text{lower limit}}^x f(x) dx$$

$$P(a < x < b) = F(b) - F(a)$$

$$P(x < b) = F(b)$$

$$P(x \geq b) = 1 - F(b)$$

## $\Rightarrow$ Joint Probability Distribution

Marginal (Discrete)

$$g(x) = \sum_y f(x, y)$$

$$h(y) = \sum_x f(x, y)$$

(Continuous)

$$\int_{\text{Y limits}} \int_{\text{X limits}} f(x, y) dx dy = 1$$

(Marginal)

$$g(x) = \int_{\text{Y limits}} f(x, y) dy$$

$$h(y) = \int_{\text{X limits}} f(x, y) dx$$



→ (Independent)

$$f(x, y) = g(x) h(y) \quad \text{Independent}$$

$$f(0, 1) = g(0) h(1) \quad \text{for all pairs of } x \text{ and } y$$

⇒ Mathematical Expectation / Mean

$$\mu = E(x) = \sum x P(x) \quad (\text{Discrete})$$

$$\mu = E(x) = \int_{-\infty}^{\infty} x P(x) \quad (\text{Continuous})$$

⇒ Variance

$$\text{Var}(x) = E(x^2) - [E(x)]^2 \quad (\text{Discrete})$$

$$\text{Var}(x) = E(x^2) - [E(x)]^2 \quad (\text{Continuous})$$
$$\int_{-\infty}^{\infty} x^2 P(x)$$

⇒ Expectation of a function

$$E(g(x)) = \sum g(x) P(x) \quad (\text{Discrete})$$

$$E(g(x)) = \int_{-\infty}^{\infty} g(x) P(x) \quad (\text{Continuous})$$

⇒ Variance of a function

$$\text{Var}(g(x)) = E(g(x)^2) - [E(g(x))]^2 \quad (\text{Both})$$



## Binomial Distribution

- Trials are independent

$$P(X=x) = \binom{n}{x} p^x q^{n-x}$$

$$\mu = np$$

$$\sigma^2 = npq$$

$$P(X=n) = \sum_0^n - \sum_0^{n-1}$$

$$P(X \leq n) = \sum_0^n$$

$$P(n \leq X \leq 2) = \sum_0^2 - \sum_0^{n-1}$$

$$P(X \geq n) = 1 - \sum_0^{n-1}$$

## ⇒ Multinomial Distribution

$$f(x_1, x_2, \dots, x_k, n, p_1, p_2, \dots, p_k) = \frac{n!}{x_1! x_2! \dots x_k!} (p_1)^{x_1} (p_2)^{x_2} \dots (p_k)^{x_k}$$

$$\mu = np_i$$

$$i = 1, 2, \dots, k$$

$$\sigma^2 = np_i q_i$$

## ⇒ Hypergeometric Distribution

$$f(x, N, n, k) = \frac{\binom{k}{x} \binom{N-k}{n-x}}{\binom{N}{n}}$$

$$\mu = \frac{nk}{N}$$

$$\sigma^2 = \frac{N-n}{N-1} \cdot n \cdot \frac{k}{N} \left(1 - \frac{k}{N}\right)$$

(approximation)

$$p = k/N$$

$$n/N < 0.05$$

we hypergeometric

## ⇒ Poisson Distribution

$$P(x, \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}$$



(approximation)

$$P \rightarrow 0$$

$$n \rightarrow \infty$$

$$p < 0.05$$

$$\lambda = np$$

⇒ Normal Distribution (Use table)

$$Z = \frac{x - \mu}{\sigma}$$

⇒ Point Estimation

$$\text{Mean} = \frac{\sum x}{n}$$

$$\text{STDError} = \sigma / \sqrt{n}$$

$$\text{Std deviation} = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\text{Var} = \frac{\sigma^2}{n}$$

⇒ Hypothesis Testing

(Z-test)

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$Z > Z_{\alpha}$$

$$Z < -Z_{\alpha}$$

$$Z < -Z_{\alpha/2} \quad \text{or} \quad Z > Z_{\alpha/2}$$

(Confidence Interval)

(T-test)

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$v = n - 1$$

$$t > t_{\alpha, v}$$

$$t < -t_{\alpha, v}$$

$$t < -t_{\alpha/2, v} \quad \text{or} \quad t > t_{\alpha/2, v}$$



$$\bar{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

⇒ Regression

$$Y = \alpha + \beta x + \epsilon$$

$$\hat{Y} = a + bx$$

$$a = \bar{Y} - b \bar{x}$$

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$\bar{Y} = \frac{\sum Y}{n}$$

$$\bar{x} = \frac{\sum x}{n}$$

⇒ Correlation

range = +1 → -1

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{[n \sum x^2 - (\sum x)^2] \cdot [n \sum y^2 - (\sum y)^2]}}$$

⇒ Coefficient of determination

$$r^2 = \frac{\text{Explained Variation}}{\text{total variation}}$$

OR

$$r^2 = 1 - \frac{\text{Unexplained}}{\text{total}}$$



$$\text{Total Variation} = \sum Y^2 - \frac{(\sum Y)^2}{n}$$

$$\text{Unexplained} = \sum Y^2 - a \sum Y - b \sum XY$$

$$\text{explained} = \text{Total} - \text{unexplained.}$$