Normal Distribution:-

⇒ Also called Gaussian distribution.
⇒ It is bell shaped distribution and unimodel.
⇒ Total Area under the curve is 1(unity)
⇒ It has two Parameters μ and σ^2.
⇒ It is a symmetrical distribution.
⇒ Most important Continuous distribution.
⇒ The probability density function of normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\sigma^2(x-\mu)^2} \qquad -\infty < x < \infty$$

⇒ Examples:-

Physical Measurements in areas, Such as metrological experiments.
⇒ Rainfall Studies
⇒ Measurement of manufactured parts are often explained by normal distribution.

Standard Normal Distribution:-

A normal probability distribution depends on the values of the Parameters μ and σ^2 and the various possible values for these two parameters will result in an unlimited number of different normal distributions. So we transform every normally distributed r.v X into a new normal r.v $Z = \frac{X-\mu}{\sigma}$ that has zero mean and unit variance.

⇒ The normal probability distribution of Z which has zero mean and unit variance is called the standardized normal distribution and is denoted by $N(0,1)$

Example 6.2 (walpole)

Given a standard normal distribution, find the area under the curve that lies

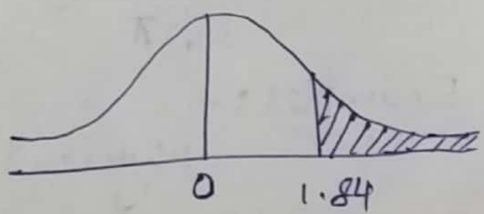
a) To the right of $z = 1.84$ i.e $P(Z > 1.84)$

b) Between $z = -1.97$ and $z = 0.86$
$P(-1.97 < Z < 0.86) = ?$

Solution:–

a) $P(Z > 1.84) = ?$

$$P(Z > 1.84) = 1 - P(Z < 1.84)$$
$$= 1 - 0.9671$$



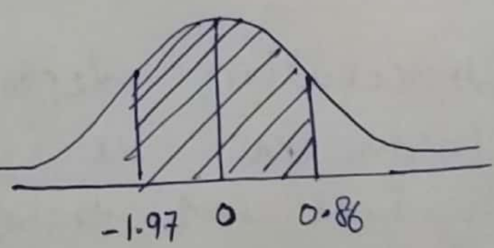
$$\boxed{P(Z > 1.84) = 0.0329}$$

b) $P(-1.97 < Z < 0.86)$

$= P(Z < 0.86) - P(Z < -1.97)$

$= 0.8051 - 0.0244$

$= 0.7807$



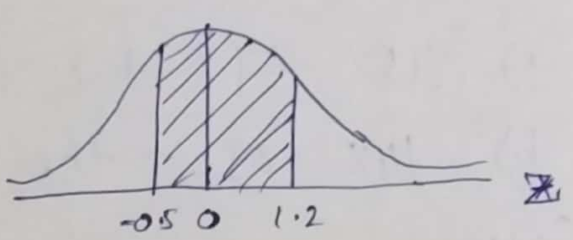
Example 6.4 :-

$\mu = 50$

$\sigma = 10$

$P(45 < x < 62) = ?$

$Z_1 = \dfrac{45 - 50}{10} = -0.5$



$Z_2 = \dfrac{62 - 50}{10} = 1.2$

$P(45 < x < 62) = P(-0.5 < Z < 1.2)$

$P(-0.5 < Z < 1.2) = P(Z < 1.5) - P(Z < -0.5)$

$= 0.8849 - 0.3085$

$= 0.5764$

Example 6.5 :-

$\mu = 300$

$\sigma = 50$

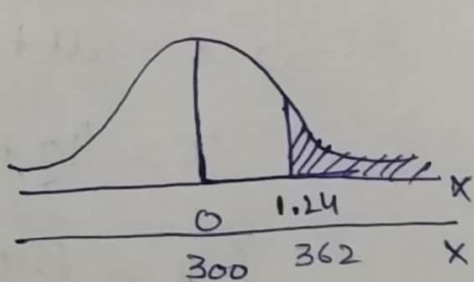
$P(x > 362) = ?$ As $Z = \dfrac{x - \mu}{\sigma} = \dfrac{362 - 300}{50} = 1.24$

$P(x > 362) = P(Z > 1.24)$

$= 1 - P(Z < 1.24)$

$= 1 - 0.8925$

$= 0.1075$



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Using the normal curve in Reverse

Example 6.6 :-

If $\mu = 40$ and $\sigma = 6$

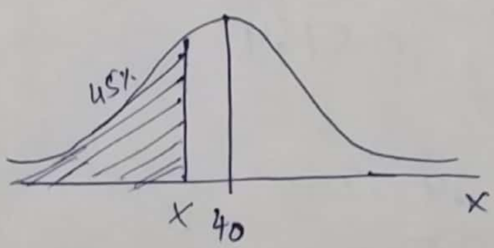
a) 45% of the area to the left.

b) 14% of the area to the right.

Sol:-

Area is given and we need to find value of x.

a) 45% of the area to the left.



As $P(Z < -0.13) = 0.45$

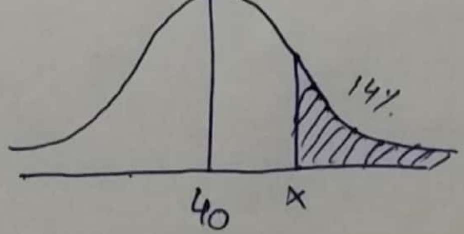
Now $Z = \frac{x - \mu}{\sigma}$

$\Rightarrow x = \mu + \sigma Z$

$x = 40 + 6(-0.13)$

$$\boxed{x = 39.22}$$

b) 14% of area to the right.



we require Z value that leaves 0.14 area to the right and $1-0.14=0.86$ to the left.

$$P(Z<1.08)=0.86$$

$$X=\mu+\sigma Z$$

$$=40+6(1.08)$$

$$\boxed{X=46.48}$$

Ans

Application of Normal distribution:-

Example 6.7

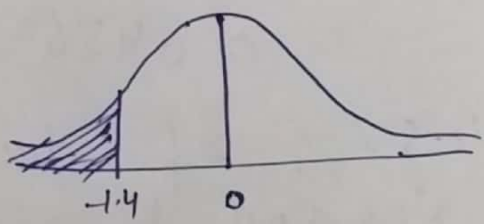
$$\mu=3$$

$$\sigma=0.5$$

$$P(X<2.3)=?$$

$$Z=\frac{2.3-3}{0.5}$$

$$Z=-1.4$$



$$P(X<2.3)=P(Z<-1.4)$$

$$=0.0808$$

Example 6.9 :-

Specification for diameter is 3.0 ± 0.01 cm.

$$\mu=3.0, \quad \sigma=0.005$$

$$x_2=3.0+0.01=3.01, \quad X_1=3.0-0.01=2.99$$

Required Probability is outside these specifications.

Two methods to solve this

1. $P(2.99 < x < 3.01)$

$$Z_1 = \frac{2.99 - 3.0}{0.05} = -2.0$$

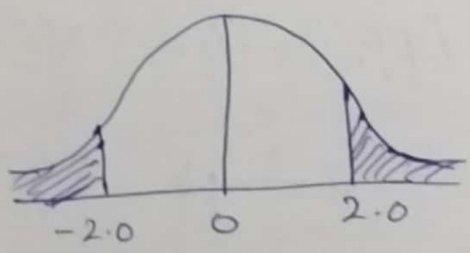
$$Z_2 = \frac{3.01 - 3.0}{0.05} = 2.0$$

$P(2.99 < x < 3.01) = P(-2.0 < Z < 2.0)$

$P(Z < -2.0) = 0.0228$

due to symmetry

$P(Z > 2.0) = 0.0228$



so

$P(Z < -2.0) + P(Z > 2.0)$

$= 2(0.0228)$

$= 0.0456$

4.56% of ball bearing will be scrapped.

or

2) $P(\text{Scrapped}) = 1 - P(2.99 < x < 3.01) = 1 - P(-2.0 < Z < 2.0)$

$= 1 - [P(Z < 2.0) - P(Z < -2.0)]$

$= 1 - [0.9772 - 0.0228]$

$= 1 - 0.9544$

$P(\text{Scrapped}) = 0.0456$

4.56% of ball bearings will be scrapped.

Example 6.13

$\mu = 74$

$\sigma = 7$

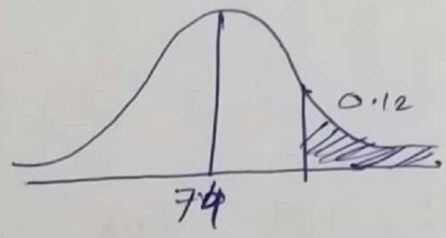
An area of 0.12, corresponding to the fraction of students receiving As so we require a z value that leaves 0.12 of area to the right and hence 0.88 to the left.

$P(Z < 1.18) = 0.88$

$x = \mu + \sigma z$

$= 74 + 7(1.18)$

$= 82.26$



Therefore, the lowest A is 83 and highest B is 82.

Normal approximation to binomial:-

when n is large and p is not so small i.e p is not extremely close to 0.

$\Rightarrow$ Fairly good approximation is when n is small and p is close to $1/2$.

$\Rightarrow$ When np and nq both are equal to or greater than 5.

Example 6.15:-

$n = 100$, $P = 0.4$

$\mu = np$, $\sigma = \sqrt{npq}$

$\mu = 100 \times 0.4$, $\sigma = \sqrt{100 \times 0.4 \times 0.6}$

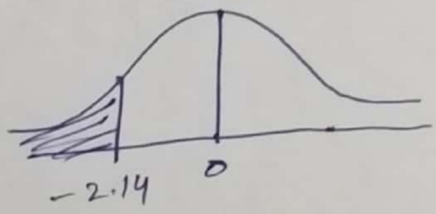
$\mu = 40$, $\sigma = 4.899$

$P(X < 30) = ?$

To obtain desired probability, we adjust the value for continuous and take $x = 29.5$

$$Z = \frac{X - \mu}{\sigma} = \frac{29.5 - 40}{4.899} = -2.14$$

so $P(X < 30) \approx P(Z < -2.14) = 0.0162$



(Pg 376, Shri M. Chaudhary)

Example 9.18

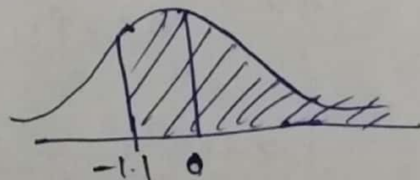
$n = 180$
$p = 1/6$, $q = 5/6$

$\mu = np$
$\mu = 180 \times 1/6$
$\mu = 30$

$\sigma = \sqrt{npq}$
$= \sqrt{180 \times 1/6 \times 5/6}$
$= 5$

i) $P(X \geq 25) = ?$ This interval includes 25 therefore it starts at 24.5 to ∞

$$Z = \frac{24.5 - 30}{5} = -1.1$$

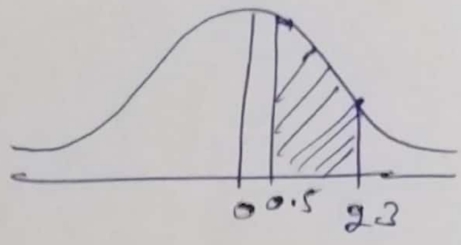


$P(X \geq 25) \approx P(Z \geq 24.5) = 0.8643$

ii) $P(33 \leq X \leq 41) = ?$ This discrete interval will be replaced by $P(32.5 \leq X \leq 41.5)$

$$Z_1 = \frac{32.5 - 30}{5} = 0.5$$

$$Z_2 = \frac{41.5 - 30}{5} = 2.3$$

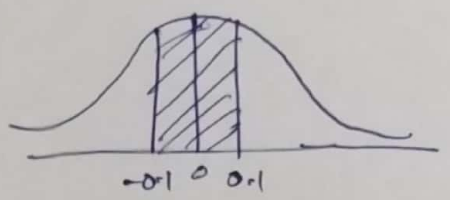


$$P(0.5 \leq Z \leq 2.3) = 0.2978$$

(iii) $P(X = 30)$, it becomes the interval 29.5 to 30.5

$$Z_1 = \frac{29.5 - 30}{5} = -0.1$$

$$Z_2 = \frac{30.5 - 30}{5} = 0.1$$



$$P(-0.1 \leq Z \leq 0.1) = 0.0796$$