

## Analysis of Algorithm

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Here are some equalities that are useful in analyzing algorithms:

### Arithmetic series

$$\sum_{i=1}^n i = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \quad \dots (1)$$

More generally, if  $a_n = a_{n-1} + c$ , where  $c$  is a constant, then

$$\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n = \frac{n(a_n + a_1)}{2} \quad \dots (2)$$

### Geometric series

$$\sum_{i=0}^n 2^i = 1 + 2 + 4 + \dots + 2^n = 2^{n+1} - 1 \quad \dots (3)$$

More generally, if  $a_n = ca_{n-1}$ , where  $c \neq 1$  is a constant, then

$$\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n = a_1 \frac{c^n - 1}{c - 1} \quad \dots (4)$$

If  $0 < c < 1$ , then the sum of the infinite geometric series is

$$\sum_{i=1}^{\infty} a_i = \frac{a_1}{1 - c} \quad \dots (5)$$

### Sum of squares

$$\sum_{i=1}^n i^2 = 1 + 4 + 9 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \dots (6)$$

### Logarithms

$$\log_b a = \frac{1}{\log_a b} \quad \dots (7)$$

$$\log_a x = \frac{\log_b x}{\log_b a} \quad \dots (8)$$