Lecture - 25

Stress: when a deforming force is applied on a body, a restoring force is developed in the body due to the action of interatomic forces.

Since the body is in equilibrium, the restoring force developed is equal in magnitude but opposite in the direction to the applied deforming force.

"The restoring force per unit area is called stress".

If 'F' is the force applied and 'A' is the area of crossection of the body, then

$$\boxed{\text{stress} = \frac{F}{A}}$$

The S.I unit of stress is N/m^2 (or) pascal

Its dimensional formula is $[M L^{-1} T^{-2}]$

On the basis of applied forces on the body, the stress can be classified as

1. longitudinal stress
2. Tangential (or) shearing stress
3. Volume (or) Bulk stress

2. shearing stress:-

The stress which tends to change the shape of a body is called shear stress

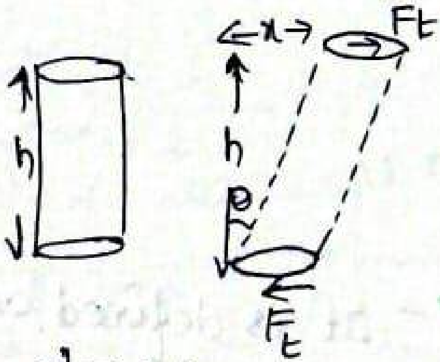
Here, a pair of forces, each of magnitude are applied parallel (or) tangential to the surface of the object.

It is also called tangential stress.

$$\text{shear stress} = \frac{\text{shearing force}}{\text{Area of cross-section}}$$

(or)

$$\text{tangential stress} = \frac{\text{tangential force } (F_t)}{\text{Area } (A)}$$

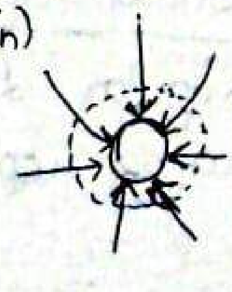


3. Bulk stress :-

The stress which tends to change the volume of a body is called bulk (or) volumetric stress.

Here normal inward forces are applied uniformly over the entire surface of the solid

$$\text{Bulk stress} = \frac{\text{Normal force } (F_n)}{\text{Area } (A)}$$



This type of stress is also called hydraulic stress.

1. Longitudinal stress :-

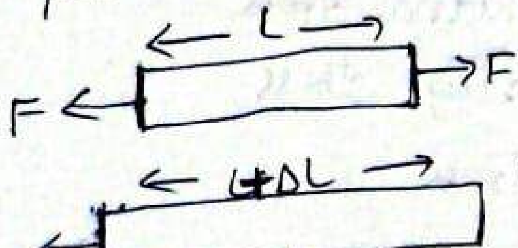
The stress which tends to change the length of a body is called longitudinal stress.

In this type, two forces each of Magnitude F_n are applied to opposite faces of a solid, Each force is normal to the face and is uniformly distributed over the surface.

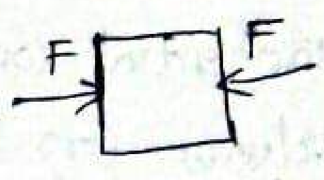
$$\text{Longitudinal stress} = \frac{F_n}{A} = \frac{\text{Normal force}}{\text{Area}}$$

The longitudinal stress is said to be tensile stress (normal stress) ~~if the applied forces~~ If two equal and opposite forces are applied to a rod to increase the length of a rod, then restoring force is equal to the applied force, it is called tensile stress.

When two equal and opposite forces are applied at the end of a rod to decrease its length (or) compress it, then restoring force equal to the applied force, this force per unit area is knows as compressive stress



increased length



Compressive stress

shear strain = $\tan\theta = \frac{x}{h} = \frac{\text{displacement}}{\text{height}}$

If θ is very small, $\tan\theta \approx \theta$

$\therefore \theta = \frac{x}{h}$

$$\boxed{\therefore \text{ shearing strain} = \theta}$$

it can be expressed in radian.

3. **Bulk strain :-** It is defined as the ratio of change in volume (ΔV) to the original volume (V) of a body.

$$\text{Bulk strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

It is also called volume strain. It is demensionless and has no unit.

HOOKE'S LAW :-

"Within the elastic limits, the stress is directly proportional to the corresponding strain"

i.e. stress $\propto$ strain

$\therefore$ stress = E × strain

$$\boxed{E = \frac{\text{stress}}{\text{strain}}}$$

here E is a constant for a given material and is called elastic constant or modulus of elasticity.

It's S.I unit is N/m^2 and dimensional formula is $[ML^{-1}T^{-2}]$

Strain :- ~~when a~~
Normal stress on a body causes change in length or volume and tengential stress produces change in shape of the body.

The ratio of the change in dimensions of a body to the original dimensions is called strain.

$$\text{strain} = \frac{\text{change in dimensions}}{\text{original dimensions}}$$

it is a ratio of two like quantities, so it has no unit and dimension.

strain is threes types

1. Longitudinal strain
2. shear strain
3. Volumetric strain

1. longitudinal strain :- It is defined as the ratio of change in length (ΔL) to the original length (L) of a thin rod (or) a wire

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

2. shear strain :- It is defined as the small angular displacement of a reference line on a surface on which a shear stress is acting.

Young's Modulus of Elasticity :- (Y)

"It is defined as the ratio of longitudinal stress to longitudinal strain, within proportionality limit"

Let 'L' be the original length of a wire or a rod of area of cross section A. Its length changed by 'ΔL' when a force F is applied on it.

then Longitudinal stress $= \frac{F}{A}$

Longitudinal strain $= \frac{\Delta L}{L}$

Young's modulus $Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

$$= \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$\boxed{\therefore Y = \frac{FL}{A\Delta L}}$$

2. shear modulus (or) Rigidity modulus (η)

"It is defined as the ratio of shearing stress to the shearing strain of the body, within proportionality limit" is called lateral strain.

$$\therefore \sigma = \frac{-\Delta D/D}{\Delta L/L}$$

poisson's ration, $\sigma = -\frac{L}{D} \cdot \frac{\Delta D}{\Delta L}$

The negative sign indicates that longitudinal and lateral strain are in opposite sense, it has no unit and dimensions.

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{\theta}$$

$$\boxed{\eta = \frac{F}{A\theta}}$$

where, F = tangential force

A = Area of the surface

θ = angular displacement

3. Bulk modulus (K)

"It is defined as the ratio of volume stress to volume strain within proportionality limit"

$$K = \frac{\text{volume stress}}{\text{Volume strain}} = \frac{F/A}{\Delta V/V} = \frac{P}{\Delta V/V}$$

$$\boxed{\therefore K = \frac{P \cdot V}{\Delta V}}$$

where P = Bulk stress = change in pressure

V = original volume

ΔV = change in volume.

Poisson's ration (σ) :-

"It is defined as the ratio of the lateral strain to the longitudinal strain"

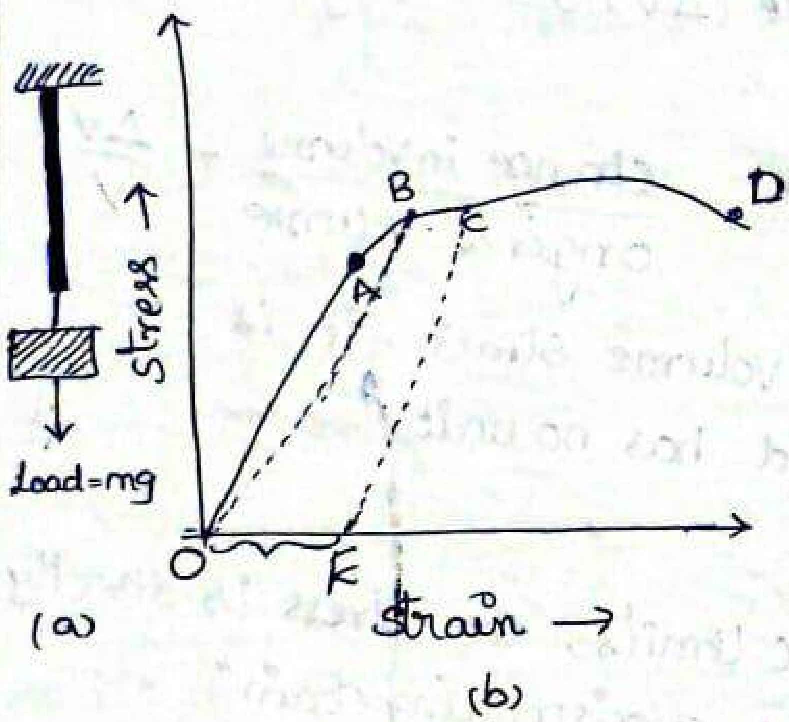
When a wire is stretched by tensile stress, there will be a contraction or decrease in the thickness of the wire. The ratio of the change in the thickness or diameter to the original thickness or diameter (D)

Lecture - 27

Stress - Strain curve :- To study the behaviour of a metal wire under increasing load, a metal wire is suspended from a rigid support and loaded at the other end.

The load is increased gradually until it breaks.

A graph is plotted between the stress on the Y-axis and strain on the X-axis.



A - proportional limit
B - elastic limit
CD - plastic region
D - Breaking point
OE - permanent set

i) Between O and A, the curve is linear and hence stress is proportional to strain, which obey's Hooke's law.

• "The value of stress upto which stress and strain are proportional to each other is called proportional limit". Here A is known as proportional limit.

ii) When stress is increased beyond A, then for small stress, there is a large strain in the wire upto point B.

"The minimum value of stress at which permanent deformation occurs is called the elastic limit"

here "B" is known as point of elastic limit or yield point.

iii) If the stress or load increases beyond point B, the strain further increases. Now, If the load is removed, the wire does not regain its original length.

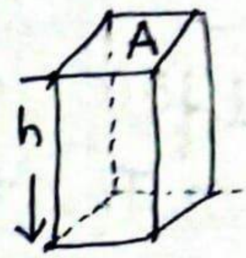
- "The permanent strain produced in the wire when the stress is removed is called permanent set"
- "OE" represents permanent set.

iv) when the stress is increased beyond the yield point the strain increases more rapidly and breaks at point D. "D" is called the fracture point. the corresponding stress is called "breaking strength"

The material is said to exhibit plastic behaviour in the region b/n elastic limit and the breaking point.

The pressure from the weight of a column of liquid of area 'A' and height 'h' is

$$\text{pressure} = \frac{\text{weight}}{\text{area}}$$



$W = mg = \rho V g \quad [\because \rho = \frac{m}{V}]$

here V = volume = hA

volume = hA

weight = mg

$\therefore W = \rho h A g$

$$\therefore \text{pressure } (P) = \frac{W}{A} = \frac{\rho h \cancel{A} g}{\cancel{A}} = \rho h g$$

$$\therefore \boxed{P = \rho g h}$$

Static fluid pressure does not depend on the shape, total mass or surface Area of the liquid. It depends on the depth 'h' within the fluid.

Lecture - 28

Fluid statics

pressure :- The pressure may be defined as the normal force exerted on a unit area around that point.

If the force F acts normally over a flat area A, then the pressure is

$$\boxed{P = \frac{F}{A}} \quad N/m^2 \text{ (or) pascal}$$

$$1 \text{ pascal} = 10 \text{ dyne}/cm^2$$

another common unit of pressure is atmosphere (atm).

$$1\,atm = 1.013 \times 10^5 \text{ pa}$$

It is a scalar quantity.

pressure due to a fluid column :-

The pressure exerted by a static fluid depends only upon the depth of the fluid, the density of the fluid, and the acceleration of gravity

$$P_{\text{static fluid}} = \rho g h$$

where $\rho = \frac{m}{V}$ = fluid density

g - acceleration of gravity

h - depth of fluid

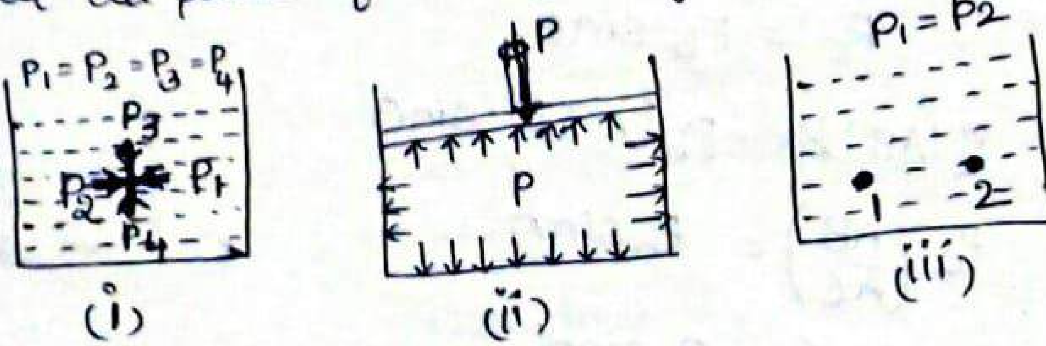
Lecture - 29

pascal's law:- This law tells about the transmission of pressure in a liquid. It can be stated in the following equivalent ways

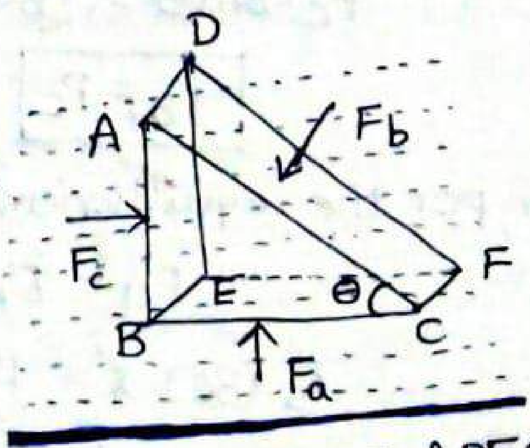
i) The pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.

ii) A change in pressure applied to an enclosed incompressible liquid is transmitted undiminished to every point of the liquid and the walls of the container

iii) The pressure in a liquid at rest is same at all points if we ignore gravity.



proof of pascal's law:-



Consider a small element ABC-DEF in the form of a right angled prism inside a liquid at rest.

Suppose the exerts pressure P_a, P_b and P_c on the faces BEFC, ADFC ADEB respectively of the element.

If F_a, F_b, F_c are the corresponding forces on these faces, then

$F_a = P_a (BC) l$

$F_b = P_b (AC) l$

$F_c = P_c (AB) l$

$P = \frac{F}{A}$

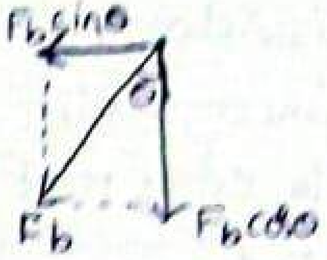
$F = PA$

$= P\, l b$

$F = P\, b \times l$

As the element is at rest, so net force on it must be zero. we can write

~~for the equilibrium in horizontal direction~~



Along horizontal direction

$$F_c = F_b \sin\theta$$

Along vertical direction

$$F_a = F_b \cos\theta$$

i, for the equilibrium horizontal direction

$$F_c = F_b \sin\theta$$

$$P_c (AB) \cancel{l} = P_b (AC) \cancel{l} \sin\theta$$

$$P_c \left(\frac{AB}{AC}\right) = P_b \sin\theta$$

$$P_c \cancel{\sin\theta} = P_b \cancel{\sin\theta}$$

$$\boxed{P_b = P_c}$$

ii, for the equilibrium Vertical direction

$$F_a = F_b \cos\theta$$

$$P_a (BC) \cancel{l} = P_b (AC) \cancel{l} \cos\theta$$

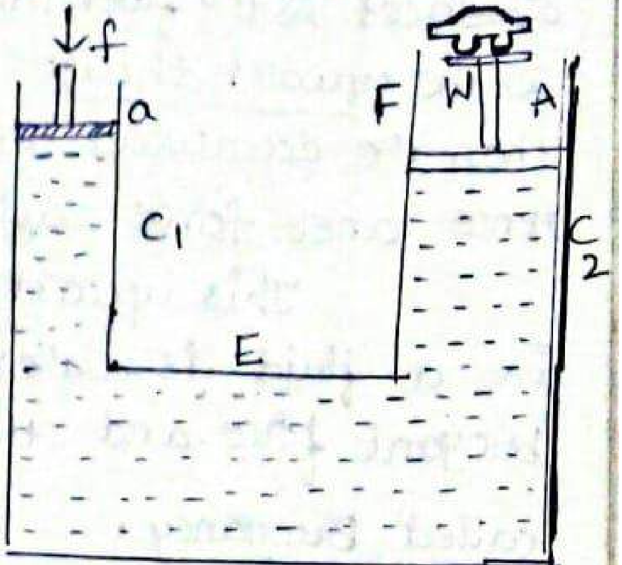
$$P_a \left(\frac{BC}{AC}\right) = P_b \cos\theta$$

$$P_a \cancel{\cos\theta} = P_b \cancel{\cos\theta}$$

$$\boxed{P_a = P_b}$$

Hence, the pressure exerted by the fluid at rest on a body in the fluid is same in all directions This proves pascal's law.

Application of pascal's law [Hydraulic Lift]



It is used to lift heavy loads (cars, trucks) for small height.

A piston of small cross-sectional area 'a' is used to exert a small effort "f" on a liquid such as oil.

The pressure $p = \frac{f}{a}$.

This pressure is transmitted to a larger cylinder (C_2) equipped with a larger piston of area 'A" through a pipe (E).

According to pascal's law

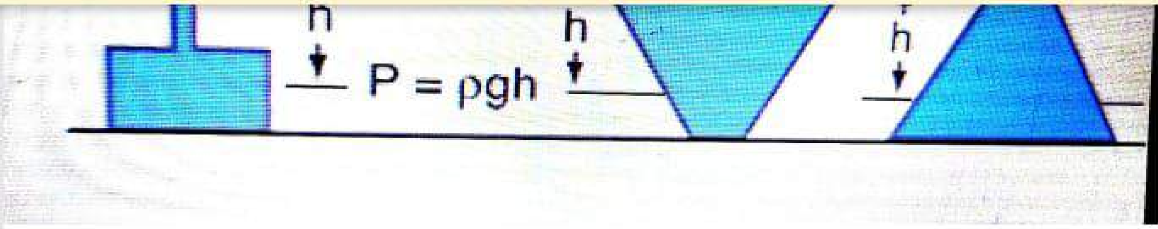
pressure at smaller piston = pressure at larger piston

$$\frac{f}{a} = \frac{W}{A}$$

$$W = f\left(\frac{A}{a}\right)$$

as $A > a$, $\therefore$ $W > f$

Hence by making $\left(\frac{A}{a}\right)$ larger, heavy loads can be lifted by applying small effort.



EXAMPLE |1| Pressure Exerted by Human Body

The two thigh bones (femurs), each of cross-sectional area 10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs. [NCERT]

Sol. Given, $A = 20 \times 10^{-4} m^2$

Weight of body acting vertically downwards

Force on bones, $F = 40$ kg - wt $= 400$ N $\quad [\because g = 10 \ m/s^2]$

$$p_{av} = \frac{F}{A} = \frac{400}{20 \times 10^{-4}}$$

$$= 2 \times 10^5 \ N/m^2$$

Pascal's law

Pascal's law states that "A change in pressure exerted at any part of

$$\Rightarrow \quad \Delta L = \frac{FL}{AY} = \frac{3\times10^{3}\times4}{2\times110\times10^{9}} = 0.0545\times10^{-6}\ \text{m}$$

$$\Delta L = 54.5\times10^{-3}\ \text{mm}$$

EXAMPLE |3| Finding Young's Modulus

The ball of 200 g is attached to the end of a string of an elastic material (say rubber) and having length and cross-sectional area of 51 cm and 22 mm², respectively. Find the Young's modulus of this material if string is whirled round, horizontally at a uniform speed of 50 rpm in a circle of diameter 104 cm.

Sol. Mass of the ball, $M = 200\ \text{g} = 0.2\ \text{kg}$

Area of cross-section, $A = 22\,\text{mm}^2 = 22\times10^{-6}\ \text{m}^2$

Radius of the circle, $r = \frac{D}{2} = \frac{104}{2} = 52\ \text{cm} = 0.52\ \text{m}$

Length of the string, $l = 51\ \text{cm} = 0.51\ \text{m}$

Revolution per second, $= 50\times60\ \text{rps} = 3000\ \text{rps}$

Certain petal force, $F = mr\omega^2 = 0.2\times0.52\times(2\pi\times N)^2$

$$F = 36.95\times10^{6}\ \text{N}$$

The change in length Δl

$$\Delta l = \text{radius of the circle} - \text{length of the string}$$
$$= 0.52 - 0.51$$
$$\Delta l = 0.01\ \text{m}$$

Young's modulus of the material

$$Y = \frac{F}{A}\frac{l}{\Delta l} = \frac{36.95\times10^{6}}{22\times10^{-6}}\times\frac{0.51}{0.01} = 85.67\times10^{12}\ \text{Nm}^{-2}$$

Stress - Strain curve :-

Stress and strain curve

Vo)) LTE 1 4:06 p.m.

8. **Find the increase in pressure required to decrease the volume of a water sample by 0.05%. Bulk modulus of water = 2.1 × 10⁹ Pa**

Solution

$$\frac{dV}{V} = -0.05\% = \frac{-0.05}{100}$$

$$B = -\frac{dp}{\left(\frac{dV}{V}\right)} \Rightarrow dp = 2.1\times10^9 \times \frac{0.05}{100} = 1.05 \times 10^6 \text{ Pa.}$$

Hence, to decrease the volume of water by 0.05% pressure should be increased by 1.05×10^6 Pa.

9. **Compute the bulk modulus of water from the following data:**

Initial volume = 100.5 litre

Pressure increase = 100.0 atm

Final volume = 100.0 litre

Solution

$\Delta V = -0.5$ litre $= -0.5 \times 10^{-3}$ m³

$\Delta P = 100.0$ at $m = 100 \times 1.013 \times 10^5$ Pa

$V = 100.5$ litre $= 100.5 \times 10^{-3}$ m³

$$B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{1.013\times10^7 \times 100.5\times10^{-3}}{0.5\times10^{-3}} = 2.04 \times 10^9 \text{ N m}^{-2}$$

13. **Calculate the work done in stretching a 2 m long wire uniformly by 0.5 cm. Given : Young's modulus of the material of the wire is 8 × 10¹⁰ N m⁻². Radius of the wire is 0.89 mm.**

Solution

$$\text{Work done} = \frac{1}{2} \times Stress \times Strain \times Volume$$

$$= \frac{1}{2} \times Y \times Strain \times Strain \times \pi r^2 l$$

$$= \frac{1}{2} \times Y \times Strain^2 \times \pi r^2 l$$

$$= \frac{1}{2} \times \left(\frac{e}{l}\right)^2 \times \pi r^2 l \times Y$$

$$= \frac{1}{2} \times \frac{e^2 \times \pi r^2}{l} \times Y$$

$$= \frac{1}{2} \times \frac{(0.5\times10^{-2})^2 \times \pi \times (0.89\times10^{-3})^2}{2} \times 8\times10^{10}$$

$= 1.24$ J

EXAMPLE |1| Stress in a Wire

EXAMPLE |1| Stress in a Wire

Calculate the value of stress in a wire of steel having radius of 2 mm of 10 kN of force is applied on it.

Sol. Force, $F = 10\text{ kN} = 1 \times 10^4\text{ N}$

Radius, $r = 2\text{ mm} = 2 \times 10^{-3}\text{ m}$

Area, $A = \pi r^2 = \pi \times (2 \times 10^{-3})^2$

$$= 12.56 \times 10^{-6}\text{ m}^2$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{1 \times 10^4\text{ N}}{12.56 \times 10^{-6}\text{ m}^2}$$

$$= 0.0796 \times 10^{10}$$

$$= 7.96 \times 10^8\text{ N/m}^2$$

EXAMPLE |1| An Elongated Wire

If a wire of length 4 m and cross-sectional area of 2m^2 is stretched by a force of 3 kN, then determine the change in length due to this force. Given Young's modulus of material of wire is $110 \times 10^9\text{ N/m}^2$.

Sol. Given, area of cross-section, $A = 2\text{ m}^2$

Force, $F = 3\text{ kN} = 3 \times 10^3\text{ N}$

Length, $L = 4\text{ m}$

Young's modulus, $Y = 110 \times 10^9\text{ N/m}^2$

Change in length, $\Delta L = ?$

Apply, $Y = \dfrac{FL}{A\Delta L}$

$$\Rightarrow \quad \Delta L = \frac{FL}{AY} = \frac{3 \times 10^3 \times 4}{2 \times 110 \times 10^9} = 0.0545 \times 10^{-6}\text{ m}$$

$$\Delta L = 54.5 \times 10^{-3}\text{ mm}$$

EXAMPLE |3| Finding Young's Modulus

The ball of 200 g is attached to the end of a string of an elastic material (say rubber) and having length and cross-sectional area of 51 cm and 22 mm^2 respectively

$T = mg = 10 \times 9.8 = 98$ N (as both masses matter)

$$\text{Young's modulus}\,(Y) = \frac{Stress}{Strain} = \frac{\left(\frac{F}{\pi r^2}\right)}{\left(\frac{e}{l}\right)} = \frac{Fl}{\pi r^2 e}$$

$$\therefore e = \frac{Fl}{\pi r^2 Y} = \frac{98 \times 1.5}{\pi (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.5 \times 10^{-4}\ \text{m} = 0.15\ \text{mm}$$

(b) Elongation in brass

$$\therefore e = \frac{Fl}{\pi r^2 Y} = \frac{6 \times 98 \times 1.0}{\pi (0.125 \times 10^{-2})^2 \times 1.3 \times 10^{11}} = 9.21 \times 10^{-5}\ \text{m} = 92.1\ \mu\text{m}.$$

4. A copper wire of cross-sectional area 0.001 cm² is under a tension of 15 N. Find the decrease in the cross-sectional area. Young's modulus of copper = 1.2×10^{11} N m^{-2} and Poisson's ratio = 0.31.

Solution

$T = 15$ N, $A = 1 \times 10^{-7}$ m², $Y = 1.2 \times 10^{11}$ N m^{-2},

$\sigma = 0.31$.

$$Y = \frac{Stress}{Longitudinal\ strain\ (\alpha)} = \frac{\frac{T}{A}}{\alpha}$$

$$\alpha = \frac{T}{AY}$$

$$\sigma = \frac{Lateral\ strain\ (\beta)}{Longitudinal\ strain (\alpha)}$$

$$\beta = \sigma\alpha = \frac{\sigma T}{AY}$$

$$\frac{dr}{r} = \frac{\sigma T}{AY}$$ where dr = decrease in radius and r = original radius

$$\frac{dr}{r} = \frac{0.31 \times 15}{1 \times 10^{-7} \times 1.2 \times 10^{11}}$$

$$= 3.875 \times 10^{-4}$$

$$A = \pi r^2$$

$$\therefore dA = 2\pi r dr$$

$$\frac{dA}{A} = \frac{2\pi r dr}{\pi r^2} = 2\frac{dr}{r} = 2 \times 3.875 \times 10^{-4}$$

Decrease in area $= 7.75 \times 10^{-4} \times 10^{-7} = 7.75 \times 10^{-11}$ m²

$\therefore e = 12 \times 10^{-?} \times 3 = 36 \times 10^{-?}$ m.

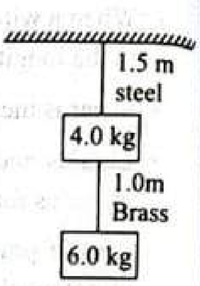
2. Calculate the longest length of steel wire that can hang vertically without breaking. Breaking stress for steel = 7.982×10^{8} N m^{-2} and density of steel = 8.1×10^{3} kg m^{-3}.

Solution

$$Stress = \frac{Force}{Area} = \frac{mg}{A} = \frac{\rho(Al)g}{A}$$

$\therefore$ Breaking stress = $\rho(l_{max})g$.

$$\Rightarrow l_{max} = \frac{7.982 \times 10^{8}}{8.1 \times 10^{3} \times 9.8} = 1.01 \times 10^{4}\,\text{m} = 10.1\,\text{km}$$



3. Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and brass wires.

Young's modulus of steel = 2.0×10^{11} N m^{-2}

Young's modulus of brass = 1.3×10^{11} N m^{-2}

Solution

(a) Elongation in steel

$T = mg = 10 \times 9.8 = 98$ N (as both masses matter)

$$Young's\ modulus\ (Y) = \frac{Stress}{Strain} = \frac{\left(\frac{F}{\pi r^2}\right)}{\left(\frac{e}{l}\right)} = \frac{Fl}{\pi r^2 e}$$

$$\therefore e = \frac{Fl}{\pi r^2 Y} = \frac{98 \times 1.5}{\pi (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.5 \times 10^{-4}\,\text{m} = 0.15\,\text{mm}$$

(b) Elongation in brass

$$\therefore e = \frac{Fl}{\pi r^2 Y} = \frac{6 \times 98 \times 1.0}{\pi (0.125 \times 10^{-2})^2 \times 1.3 \times 10^{11}} = 9.21 \times 10^{-5}\,\text{m} = 92.1\,\mu\text{m}.$$

4. A copper wire of cross-sectional area 0.001 cm^2 is under a tension of 15 N. Find the decrease in the cross-sectional area. Young's modulus of copper = 1.2×10^{11} N m^{-2} and Poisson's ratio = 0.31.

Solution

$T = 15$ N, $A = 1 \times 10^{-7}$ m^2, $Y = 1.2 \times 10^{11}$ N m^{-2},

$\sigma = 0.31$.

$$Y = \frac{Stress}{Longitudinal\ strain\ (\alpha)} = \frac{\frac{T}{A}}{\alpha}$$

$$\alpha = \frac{T}{AY}$$

$$\sigma = \frac{Lateral\ strain\ (\beta)}{Longitudinal\ strain (\alpha)}$$