EEL 6509 Wireless Communications Lecture 36

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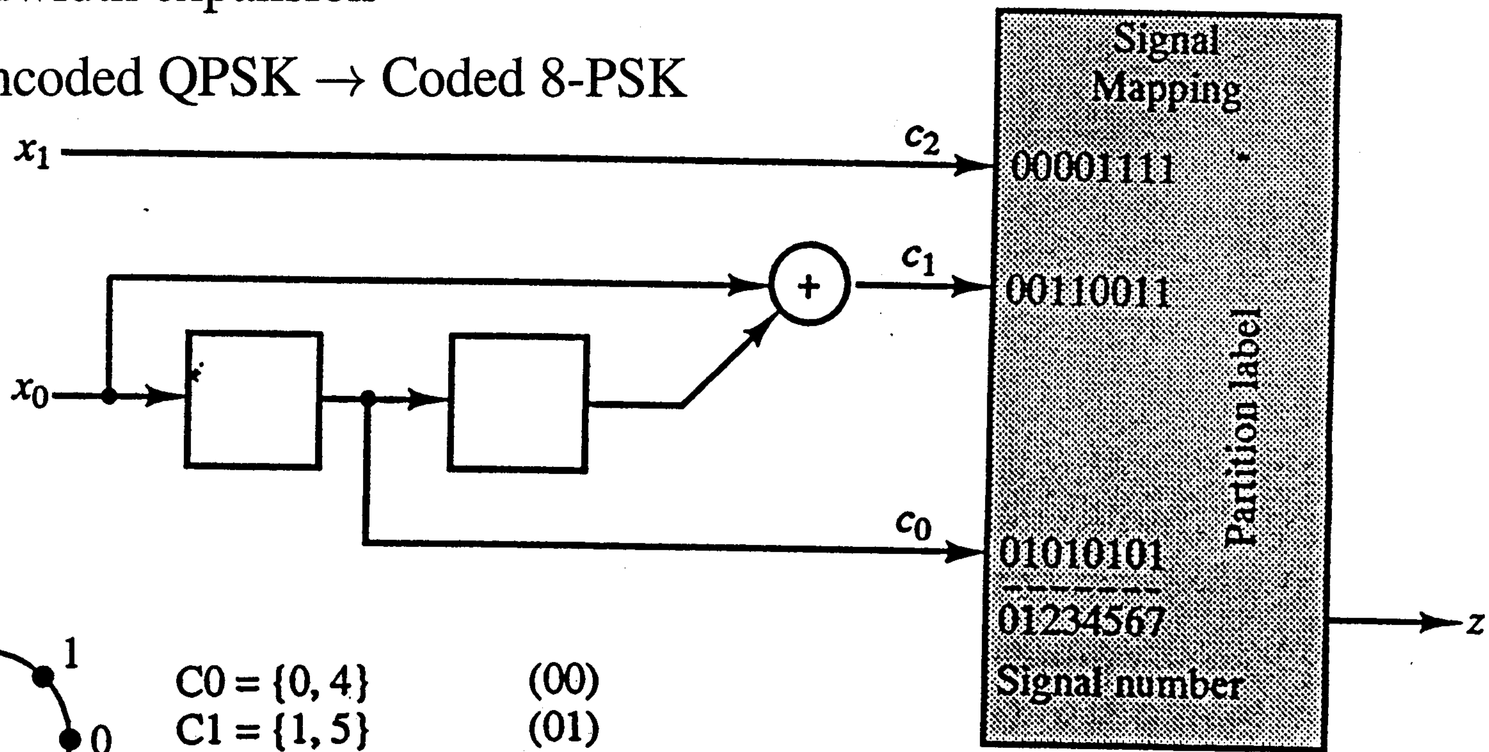
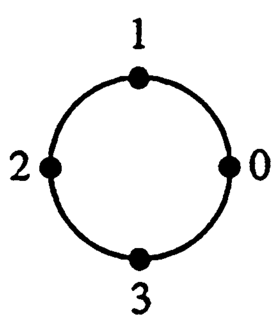
1 Overview

- Advanced Coding Techniques
  - Trellis Codes
  - Interleaving
    - Block
    - Convolutional
    - Pseudorandom
  - Concatenated Codes
  - Turbo Codes

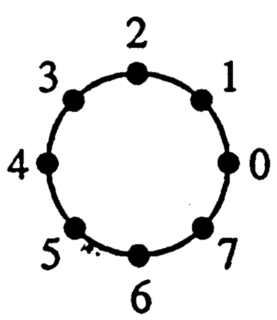
2 Trellis Codes

- trellis coding combines convolutional coding with M-ary modulation
- constellation size is increased (relative to uncoded) to add coding gain without bandwidth expansion
- **Example** Uncoded QPSK → Coded 8-PSK
  [1], p. 364





Uncoded QPSK



Coded 8-PSK

Partitions (v = 2) for 8-PSK	Binary Partition Labels
C0 = {0, 4}	(00)
C1 = {1, 5}	(01)
C2 = {2, 6}	(10)
C3 = {3, 7}	(11)

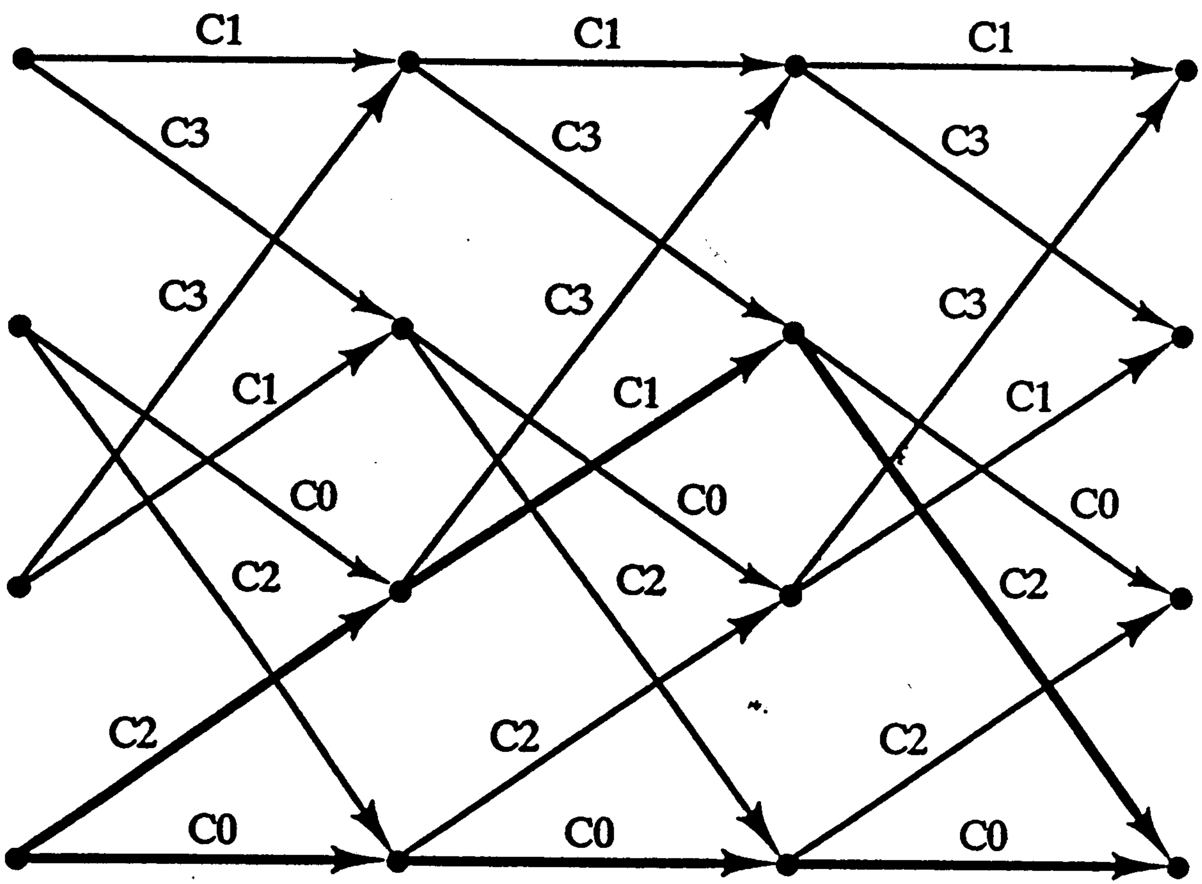
- For uncoded QPSK, any symbol is valid

$$d_{fre} \qquad \sqrt{2E_s}$$

- For the coded 8-PSK shown in the example, there are two types of errors:
  1. One bit is uncoded, so that bit can take on either value for any symbol
     – C=coded bits (Figure was provided on blackboard)

$$d_1 = 2\sqrt{E}$$

2. For coded bits, have to use trellis to find *free distance*, d_{free}
3. d_{free} = min. Euclidean distance between any two paths [1], p. 365



$$d_2 = 2.14\sqrt{E}$$

- The overall free distance for the trellis code is

$$d_{free,coded} = \min\{d_1, d_2\} = 2\sqrt{E}$$

- The coding gain is

$$\frac{d^2_{free,coded}}{d^2_{free,uncoded}} = \frac{4E}{2E} = 2 = 3 \text{ dB}$$

- 8-PSK and 4-PSK have the same bandwidth, but can achieve 3 dB performance gain by using coded 8-PSK instead of uncoded 4-PSK

3 Interleaving

- We often need to "shuffle" or rearrange code symbols
  - For instance, for a channel with burst errors, rearranging the channel symbols can break up the bursts
- **Definition:** *Interleaving* is the process of reorganizing a group of symbols.
- **Definition:** *Deinterleaving* is the process of reversing an interleaving process.
- 3 Types of interleavers are commonly used in communications:
  1. Block
  2. Convolutional
  3. Pseudorandom

3.1 Block Interleaving

- Uses $M \times N$ arrays for interleaving and deinterleaving
- Interleaving is done by filling array by columns and reading by rows
- **Example**, p. 361 [2]
  Input sequence: 1,2,3,...,24

N = 6 columns

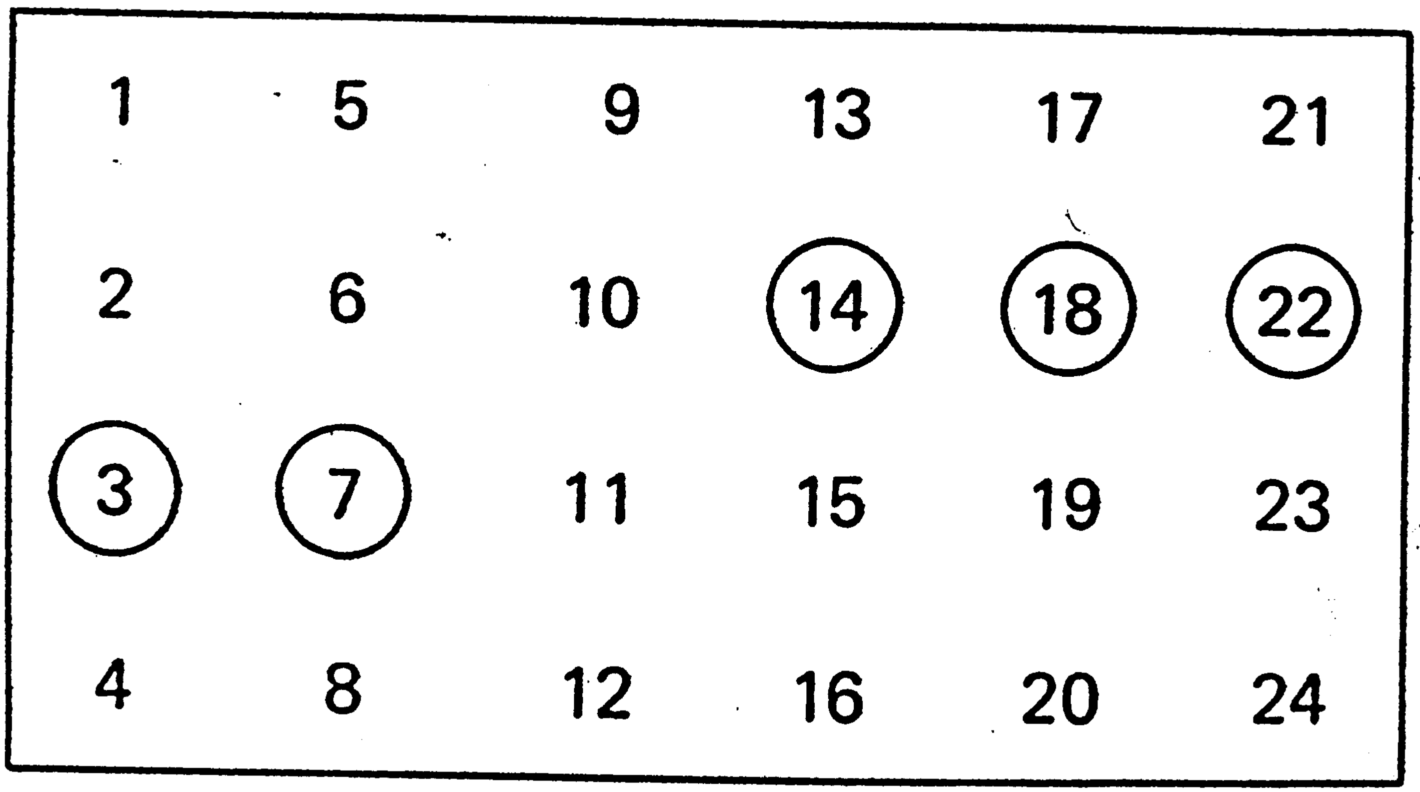
M = 4 rows

1	5	9	13	17	21
2	6	10	14	18	22
3	7	11	15	19	23
4	8	12	16	20	24

Output sequence: 1, 5, 9, 13, 17, 21, 2, 6, ...

- Any burst of length $< N$ channel symbols results in isolated errors at deinterleaver that are separated by $\geq M$ symbols
- **Example**, p. 361 [2] (Figure on next page)

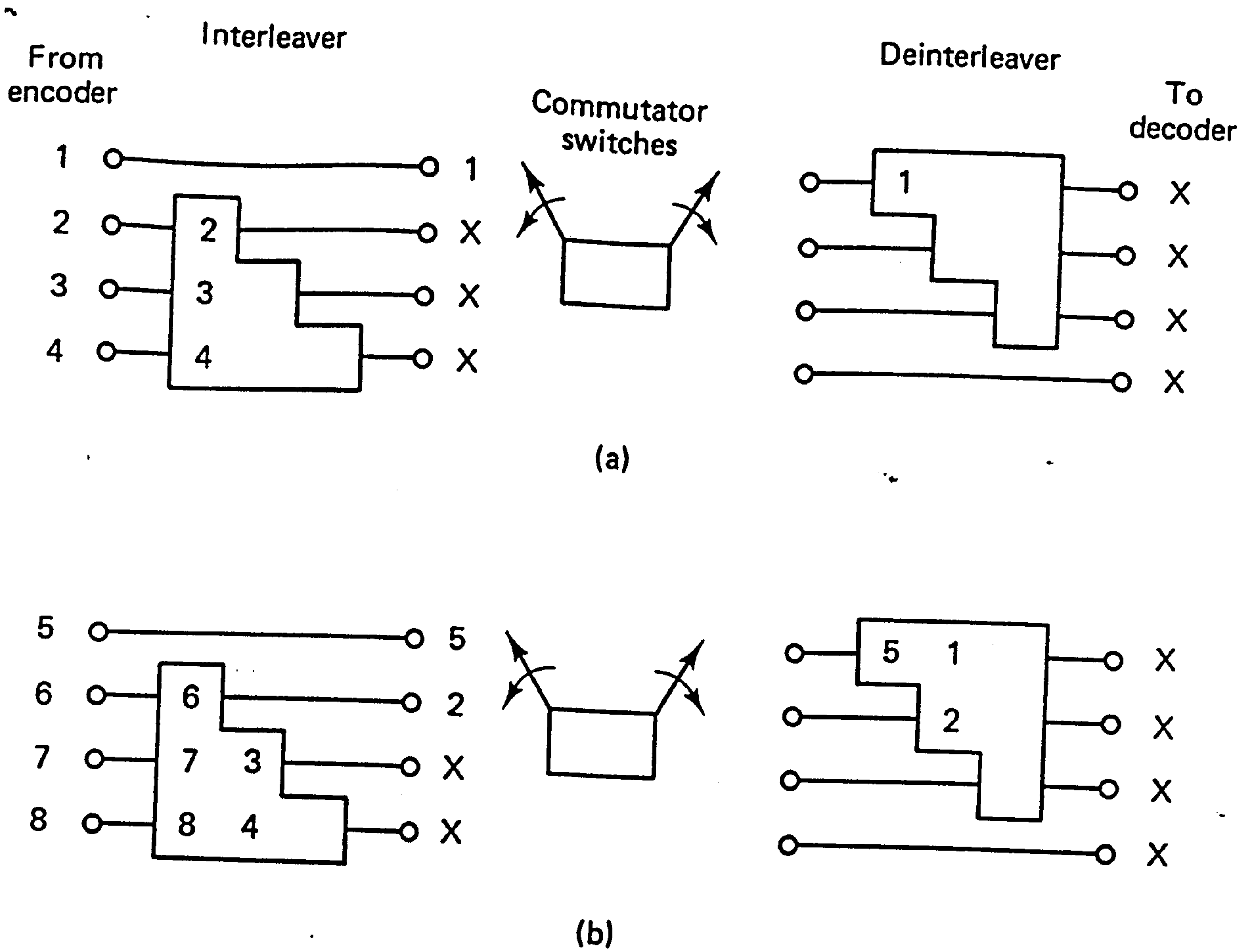
- Interleaver/deinterleaver delay is $\approx 2M$ symbols
- requires minimum M symbol array at Tx and Rx; 2MN symbol array often used to allow one $M \times N$ array to be filled while other is emptied

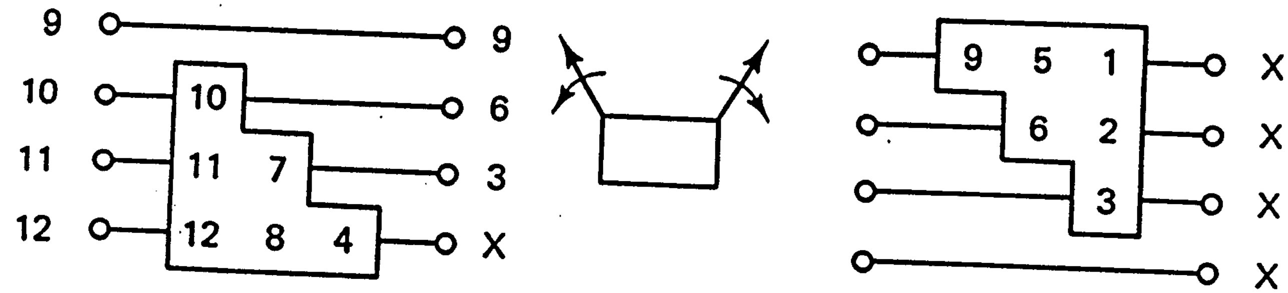


(b)

3.2 Convolutional Interleaver

- Easiest to understand operation through an example
- **Example**, p.364 [2]





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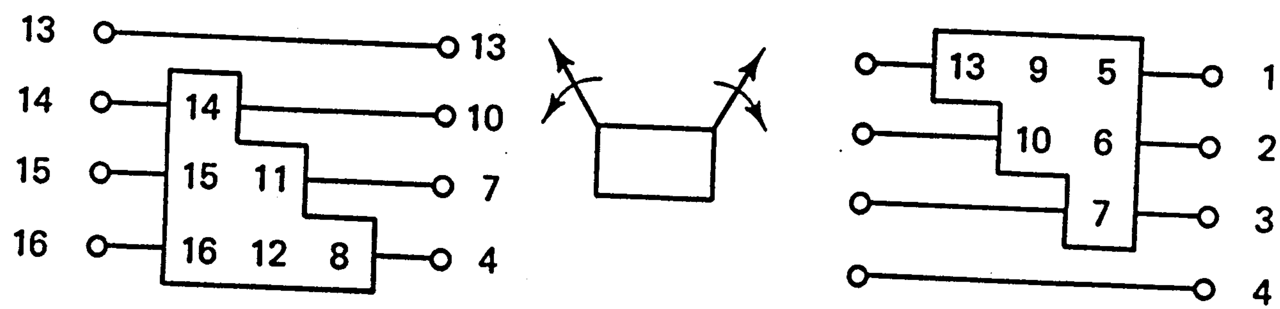

(d)

Figure 6.26 Convolutional interleaver/deinterleaver example.

- similar performance to block interleaver
- end-to-end delay of $M(N-1)$ (vs. $2MN$ for block interleaver)
- memory requirement is $M(N-1)/2$ (vs. MN for block interleaver)

3.3 Pseudorandom Interleaver

- Usually based on block interleaving scheme
- Writes into array by rows or columns; reads out according to specified pseudo-random pattern
- Used with turbo codes to scramble input sequence for different encoders

4 Concatenated Codes

- **Definition:** A *concatenated code* is a code that results from combining two or more codes. The codes that make up the concatenated codes are referred to as the *constituent codes*.
- Will start with discussion of traditional concatenated codes
- In a traditional concatenated coding scheme, 2 levels of coding are used: an *inner code* and an *outer code*
  [2], p.365

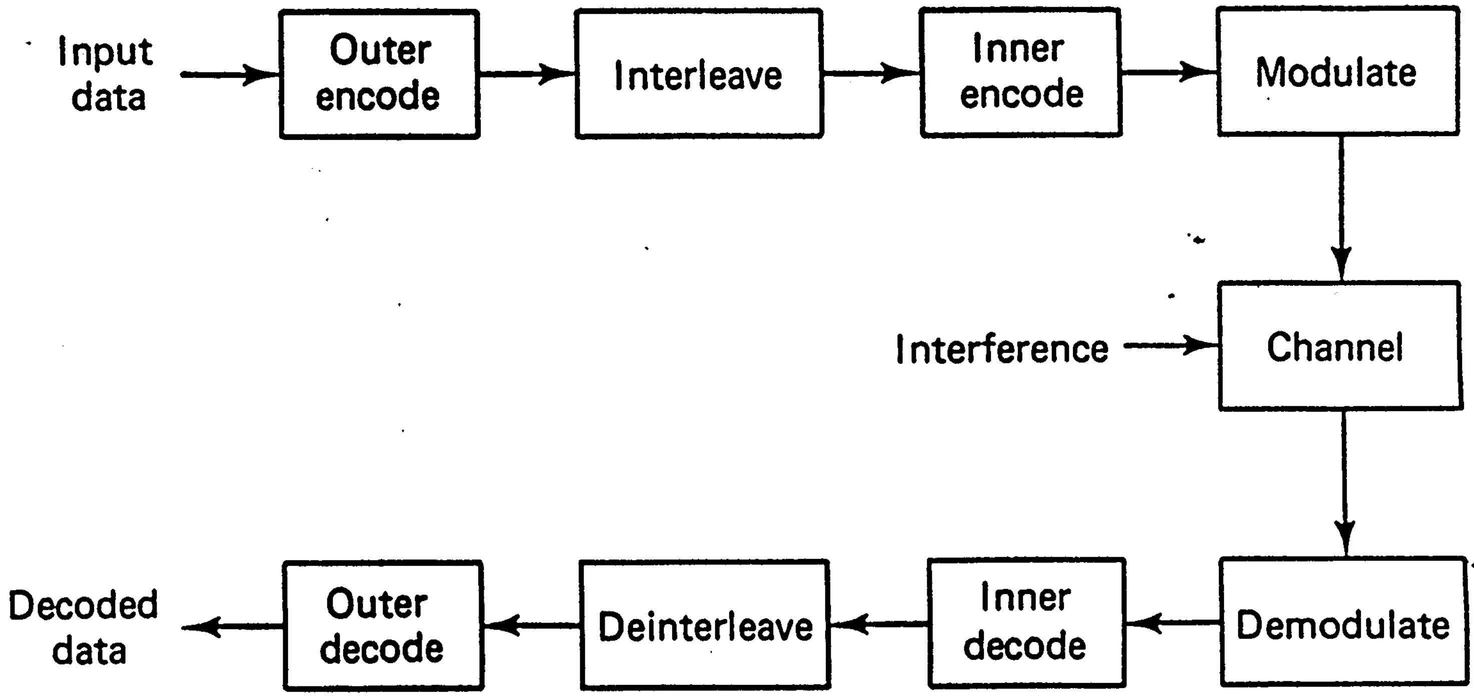

Figure 6.27 Block diagram of a concatenated coding system.

- Concatenated coding achieves low error rates with a lower complexity than would be required with a single code
- Provides power of block codes with that of soft-decision decoding when inner code is convolutional and outer code is a block code
- Popularly used with inner convolutional and outer Reed-Solomon codes
- Interleaver is used between inner and outer codes to break up error bursts at output of inner decoder
- Used in Voyager and Galileo deep-space explorers
- Standardized by CCSDS for spacecraft telemetry
- Concatenated Reed-Solomon codes are used for CD and DVD

5 Turbo Codes

- Also known as *parallel concatenated convolutional codes (PCCCs)*
  - *concatenated convolutional codes* $\Rightarrow$ more than 1 conv. encoder is employed
  - *parallel* $\Rightarrow$ the information bits are fed into each encoder, but in different orders

- For best performance,
  - Constituent codes are systematic codes, and information bits are only transmitted once
  - Constituent codes are recursive codes
  - Pseudorandom interleaver is used to avoid creating error patterns that reduce code performance
- Probability of error is proportional to inverse of interleaver size; this is referred to as the *interleaver gain*
- Error probabilities very close to theoretical limits can be achieved for large interleavers
- No computationally efficient ML decoder is known for turbo codes
  - decoding is usually done by iterating between two MAP decoders for the constituent codes
  - on each iteration, the probabilities for the information bits are updated

D. Divsalar and F. Pollara, "On the design of turbo codes", JPL TDA - Progress Report 42-123 Nov. 1995

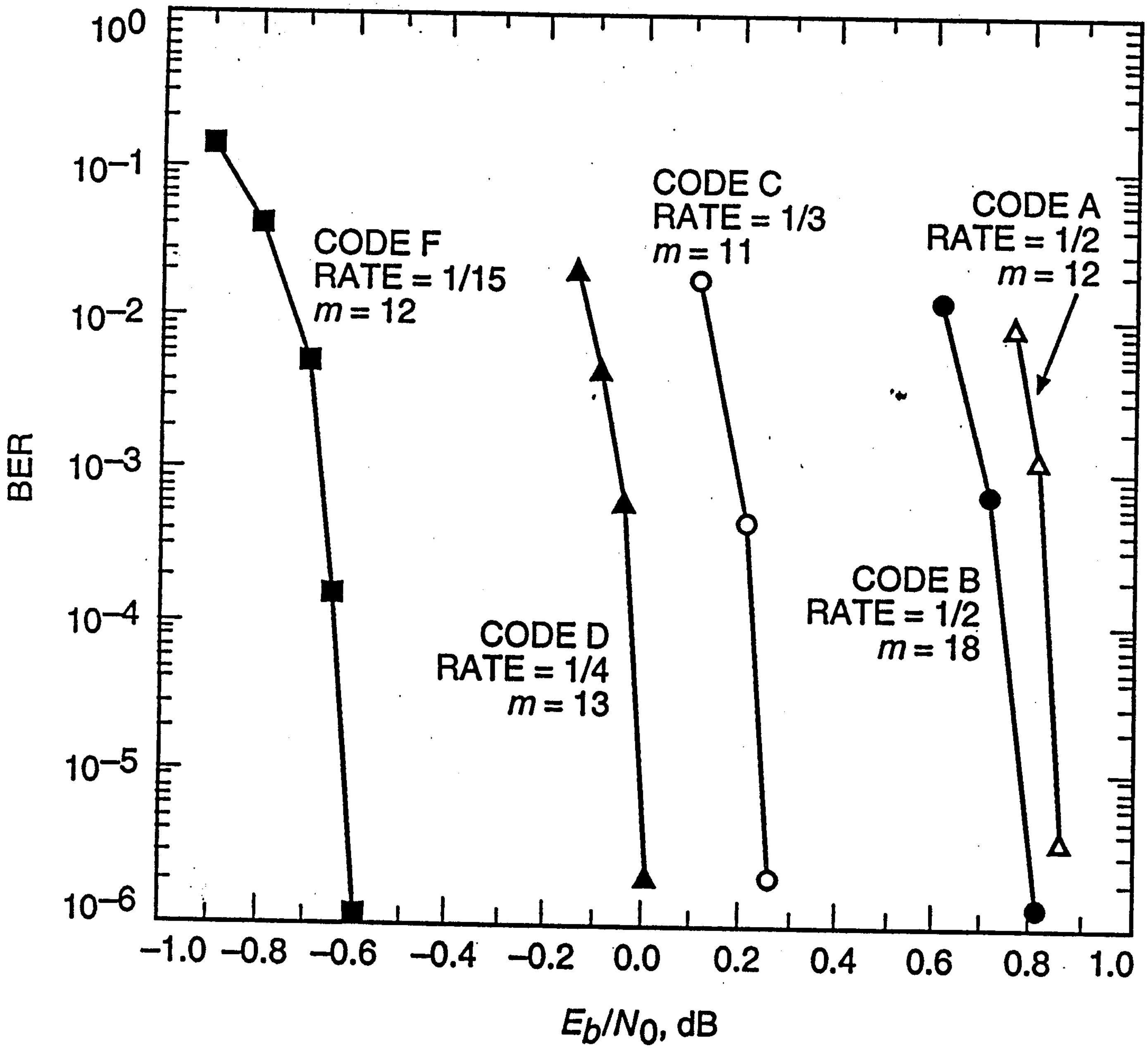

Fig. 5. Performance of turbo codes.

Perez, Seghers, Costello, IT 42-6, 11/96
p. 698

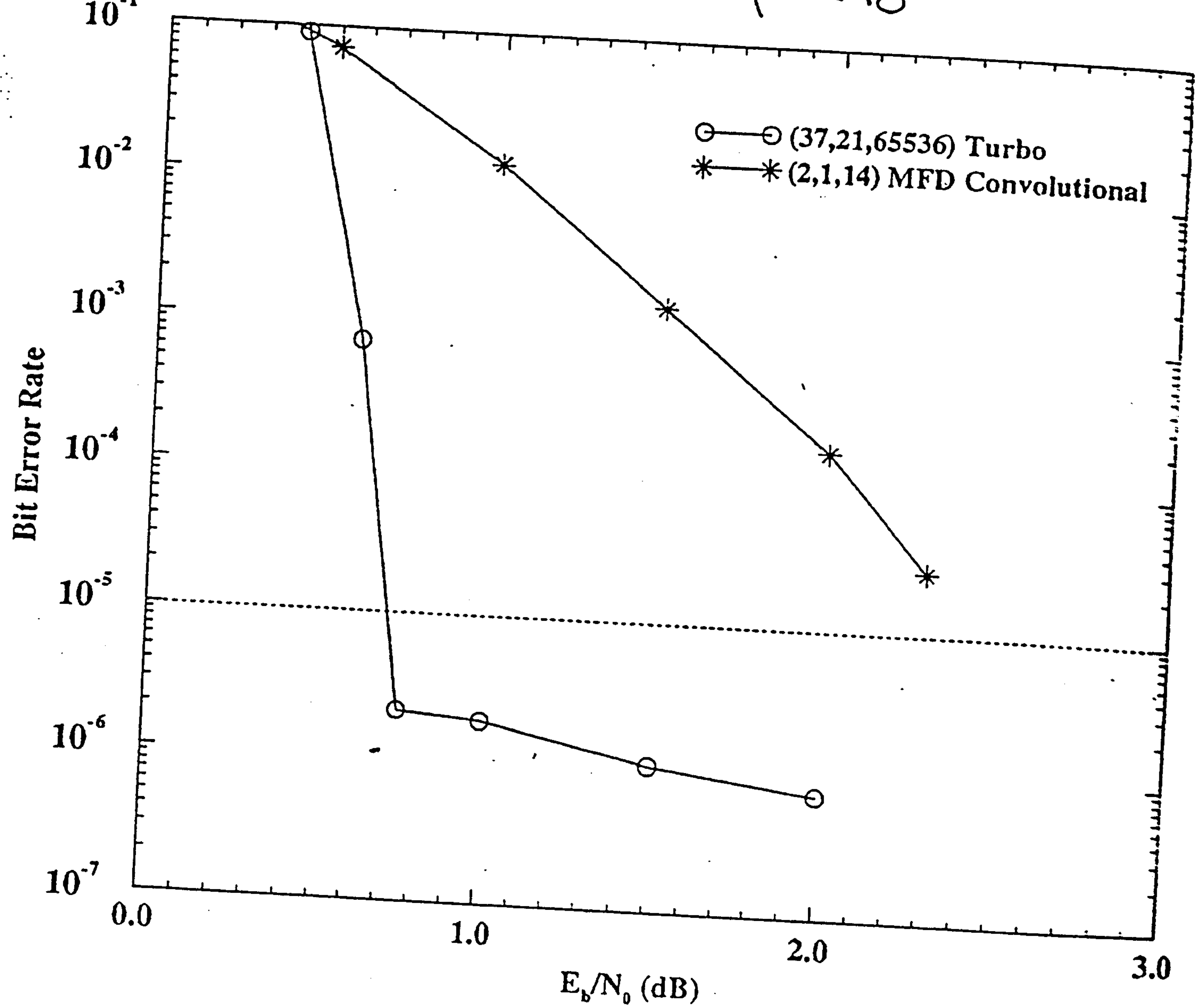

Fig. 1. Simulation results for a(37, 21, 65536) Turbo code and a (2, 1, 14) MFD convolutional code.

References

[1] S. B. Wicker, *Error Control Systems for Digital Communication and Storage*. Upper Saddle River, New Jersey: Prentice Hall, 1995.

[2] B. Sklar, *Digital Communications: Fundamentals and Applications*. Englewood Cliffs, New Jersey: Prentice Hall, 1988.