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Describe the model of image degradation restoration process.

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Ans: Model image degradation as an operator that together with an additive noise term operates on input image $f(x, y)$ to produce a degraded image $g(x, y)$.

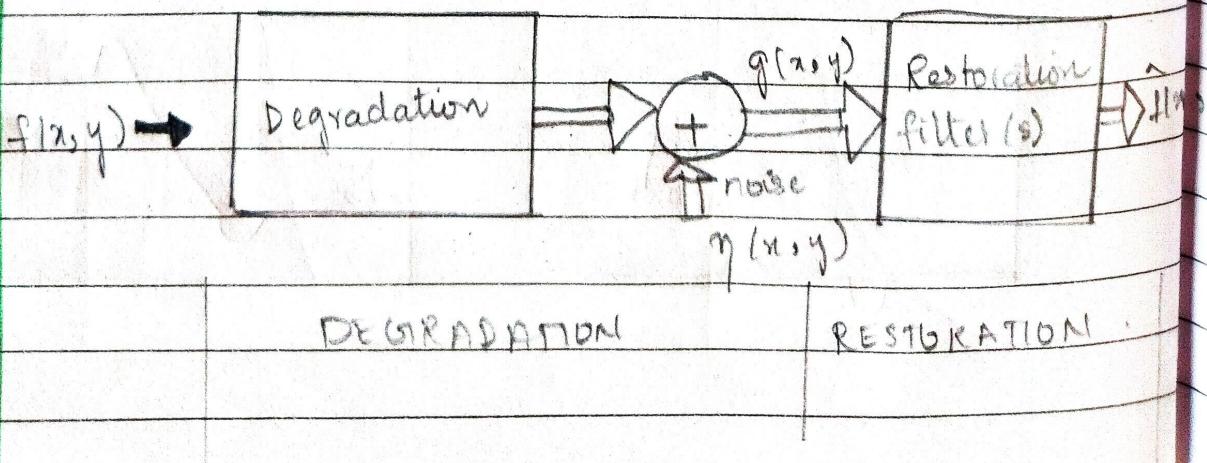
The degraded image is given in the spatial domain by

$$g(x, y) = (h * f)(x, y) + \eta(x, y).$$

where $h(x, y)$ is the spatial representation of the degradation function.
 '*' indicates convolution.

$$G(u, v) = H(u, v)F(u, v) + N(u, v).$$

The capital letters are the Fourier transformations.



Q8 Define noise

Noise module and explain different probability density functions

Ans: the principal source of noise in digital images arise during image acquisition and /or transmission.

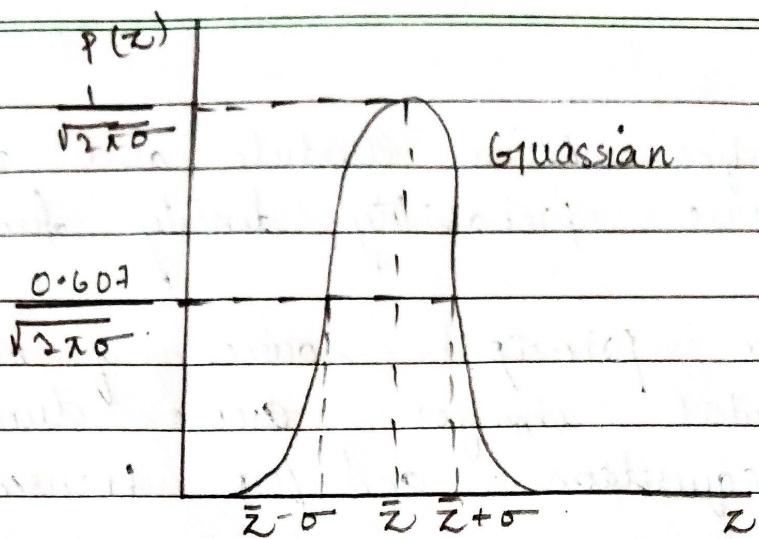
- The performance of imaging sensors is affected by variety of environmental factors during image acquisition.
In acquiring images with CCD camera, light levels & sensor temperature are major factors effecting the amount of noise in the resulting image.

→ Some important noise probability density functions:

(i) Gaussian Noise

$$P(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}$$

where z represent intensity
 \bar{z} is mean (average of z)
 σ standard deviation



(iii)

(ii) Rayleigh Noise

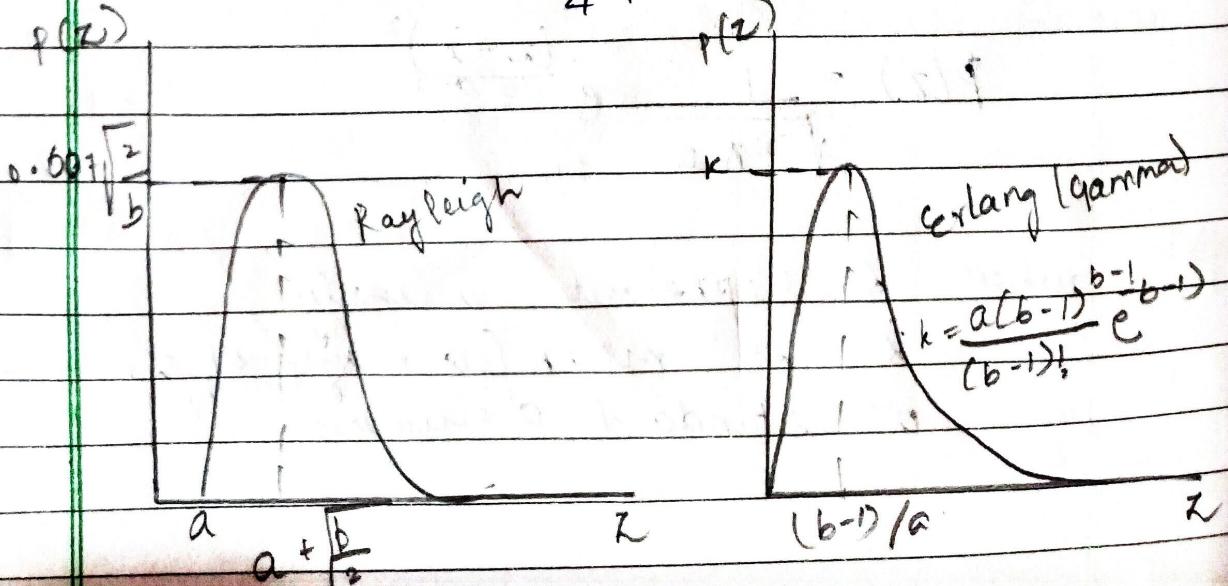
$$p(z) = \begin{cases} \frac{2}{b} (z-a) e^{-(z-a^2)/b} & z \geq a \\ 0 & z < a \end{cases}$$

The mean & variance of Z when this random variable is characterized by.

$$\bar{z} = a + \sqrt{\pi b/4}$$

σ^2

$$\sigma^2 = \frac{b(4-\pi)}{4}$$



(iv) Ex

m

(iii) Erlang (Gamma) Noise

$$P(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

• It indicates factorial.

mean & variance are.

$$\bar{z} = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$

LB

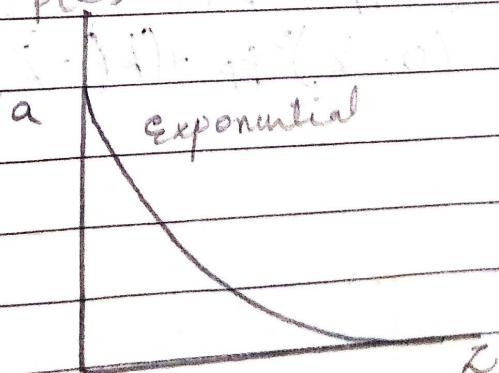
This (iv) Exponential Noise :-

$$P(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

mean & variance $P(z)$.

$$\bar{z} = \frac{1}{a}$$

$$\sigma^2 = \frac{1}{a^2}$$



gamma

$\frac{b-1}{e}$

z

(v) Uniform Noise

$$P(z) = \begin{cases} \frac{1}{b-a} & a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

mean's Variance

$$\bar{z} = \frac{a+b}{2}$$

$$\sigma^2 = \frac{(b-a)^2}{12}$$

(vi)

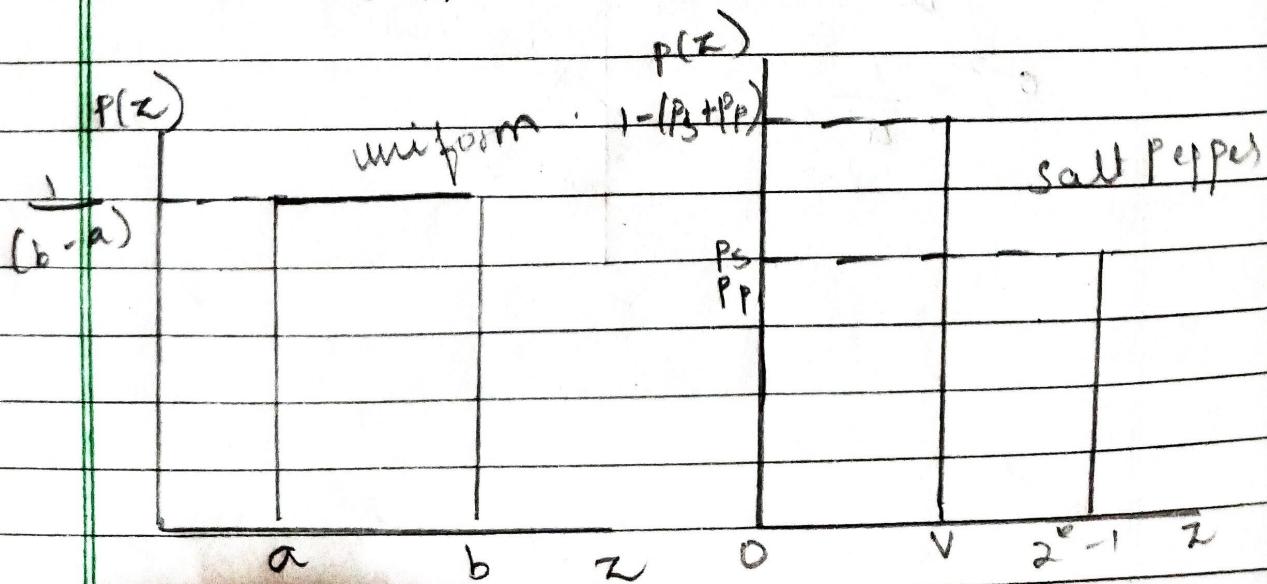
Salt & pepper Noise

$$P(z) = \begin{cases} P_s & \text{for } z = 2^k - 1 \\ P_p & \text{for } z = 0 \\ 1 - (P_s + P_p) & \text{for } z = v \end{cases}$$

mean & variance =

$$\bar{z} = (0)P_p + k(1 - P_s - P_p) + (2^k - 1)P_s$$

$$\sigma^2 = (0 - \bar{z})^2 P_p + (k - \bar{z})^2 [1 - P_s - P_p] + (2^k - 1)^2 P_s$$



Q6. Describe restoration in the presence of noise only.

Ans: When an image is degraded only by additive noise, it becomes:

$$g(x, y) = f(x, y) + n(x, y)$$

$$G(u, v) = F(u, v) + N(u, v).$$

$n(u, v)$ can be subtracted from $G(u, v)$ to obtain an estimate of original image.

Spatial filtering is the method of choice for estimating $f(x, y)$ in situation when only additive random noise is present.

Q7. Describe mean filters:-

→ Arithmetic Mean filter:-

It is the simplest of the mean filters. Let S represent the set of coordinates in a rectangular subimage window of size $m \times n$. center point (x, y) .

$$f(x, y) = \frac{1}{mn} \sum_{(r, c) \in S_{xy}} g(r, c)$$

r & c are row and column coordinates
of the pixels contained in the
neighbourhood say

(ii) \rightarrow Geometric Mean filter :-

$$\hat{f}(x, y) = \left[\prod_{(r, c) \in S_{xy}} g(r, c) \right]^{\frac{1}{mn}}$$

where \prod indicates multiplication.

(iii) \rightarrow Harmonic Mean filter :-

$$f(x, y) = \frac{mn}{\sum_{(r, c) \in S_{xy}} \frac{1}{g(r, c)}}$$

This mean filter works well for salt noise but fails for pepper noise.

(iv) \rightarrow Contraharmonic Noise Mean Filter :-

$$f(x, y) = \frac{\sum_{(r, c) \in S_{xy}} g(r, c)^{q+1}}{\sum_{(r, c) \in S_{xy}} g(r, c)^q}$$

q is order of filter.

Q99 Describe Order static filters?

Ans is median filter :- It is the best known order static filter in image processing. As the name implies it replaces the value of pixel by the median of intensity level.

$$f(x, y) = \text{median}_{(r, c) \in S_{xy}} \{g(r, c)\}.$$

S_{xy} = subimage

median filters are quite popular because of types of random noise.

(ii) \rightarrow Max and min filters

$$f(x, y) = \max_{(r, c) \in S_{xy}} \{g(r, c)\}$$

This filter is useful for finding the brightest points in an image or for eroding dark regions adjacent to bright areas.

The 0th percentile filter is the min filter

$$f(x, y) = \min_{(r, c) \in S_{xy}} \{g(r, c)\}.$$

It is used to find the darkest point in an image

iii → Midpoint filter :-

It computes the midpoint between the maximum & minimum values in the area compressed.

$$\hat{f}(x, y) = \frac{1}{2} \left[\max_{(r, c) \in S_{xy}} \{g(r, c)\} + \min_{(r, c) \in S_{xy}} \{g(r, c)\} \right]$$

iv → Alpha Trimmed Mean filter

Suppose we delete the $d/2$ lowest & $d/2$ highest intensity values of $g(r, c)$ in neighbourhood S_{xy} . Let $g_R(r, c)$ represent the remaining $m-n-d$ pixels in S_{xy} .

- A filter formed by averaging these remaining pixels is known as alpha trimmed mean filter.

$$\hat{f}(x, y) = \frac{1}{m-n-d} \sum_{(r, c) \in S_{xy}} g_R(r, c).$$

where d changes from 0 to $mn-1$
if $d=0$ then Alpha trimmed reduces
to arithmetic mean filter $d=mn-1$
then becomes median filter

Ques. Discuss adaptive local noise reduction filters and Adaptive median filter?

Ans. The mean gives a measure of average intensity in the region over the appearance of an image which the mean is computed & variance gives a measure of image contrast in that region.

Our filters i.e. do operate on a neighbourhood say, centered on coordinates (x, y) . It contains the following quantities:

$g(x, y)$, value of the noisy image at (x, y) , σ_n^2 the variance of noise \bar{I}_{avg} local average intensity of pixel in say., σ_{avg}^2 local variance of intensities of pixel in say.

1. If σ_n^2 is zero the filter should return simply the value of g at (x, y) . This is the trivial, zero noise case in which g is equal to f at (x, y) .

2. If the local variable variance σ_{avg}^2 is high relative to σ_n^2 , the filter

to return a value close to $g(x, y)$.
 A high local variance typically is associated with edges & there should be preserved.

3. if the two variance are equal as want the filter to return the arithmetic mean value of pixel in say. This condition occurs when the local area has the same properties as the overall image, & local noise is to be reduced by averaging.

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_{xy}^2}{\sigma_{say}^2} [g(x, y) - \bar{z}_{say}]$$

→ Adaptive Mean Filter . . .

An additional benefit of the adaptive median filtering can handle noise with probabilities larger than 50%. It seeks to preserve detail while simultaneously smoothing non-uniform noise. It also works on rectangular neighbourhood say.

we use following notations.

Z_{\min} = minimum intensity value in S_{xy}

Z_{\max} = maximum intensity value in S_{xy}

Z_{med} = median of intensity values in S_{xy}

Z_{xy} = intensity at coordinates (x, y)

S_{\max} = maximum allowed size of S_{xy} .

The adaptive median filtering uses two levels: denoted by level A & level B.

level A : if $Z_{\min} < Z_{\text{med}} < Z_{\max}$, go to level B
else, increase size of S_{xy} .

if $S_{xy} \leq S_{\max}$, repeat level A
else, output Z_{med} .

level B : if $Z_{\min} < Z_{xy} < Z_{\max}$, output Z_{xy} .
else output Z_{med} .

S_{xy} & S_{\max} are odd positive integers
greater than 1. Another option in
the last step of level A is to
output Z_{xy} instead of Z_{med} . This
produce slightly less blurred result.
but can fail to detect salt
noise.

This algorithm has 3 principal objectives.

- To remove salt-pepper noise
- To produce smoothing of other noise than may not be unimpressive
- To reduce distortion such as excessive thinning or thickness of object boundaries