18 Describe fundamentals of Spatial filtering

Ans: Spatial filtering is used in a broad spectrum of image processing applications

- The name filter is borrowed from frequency domain processing. it refers to passing, modifying or rejecting specified frequency components of an image eg:- filter that pass low frequencies is called as lowpass filter.

- Spatial filtering modifies an image by replacing the value of each pixel by function of value of the pixel and its neighbours

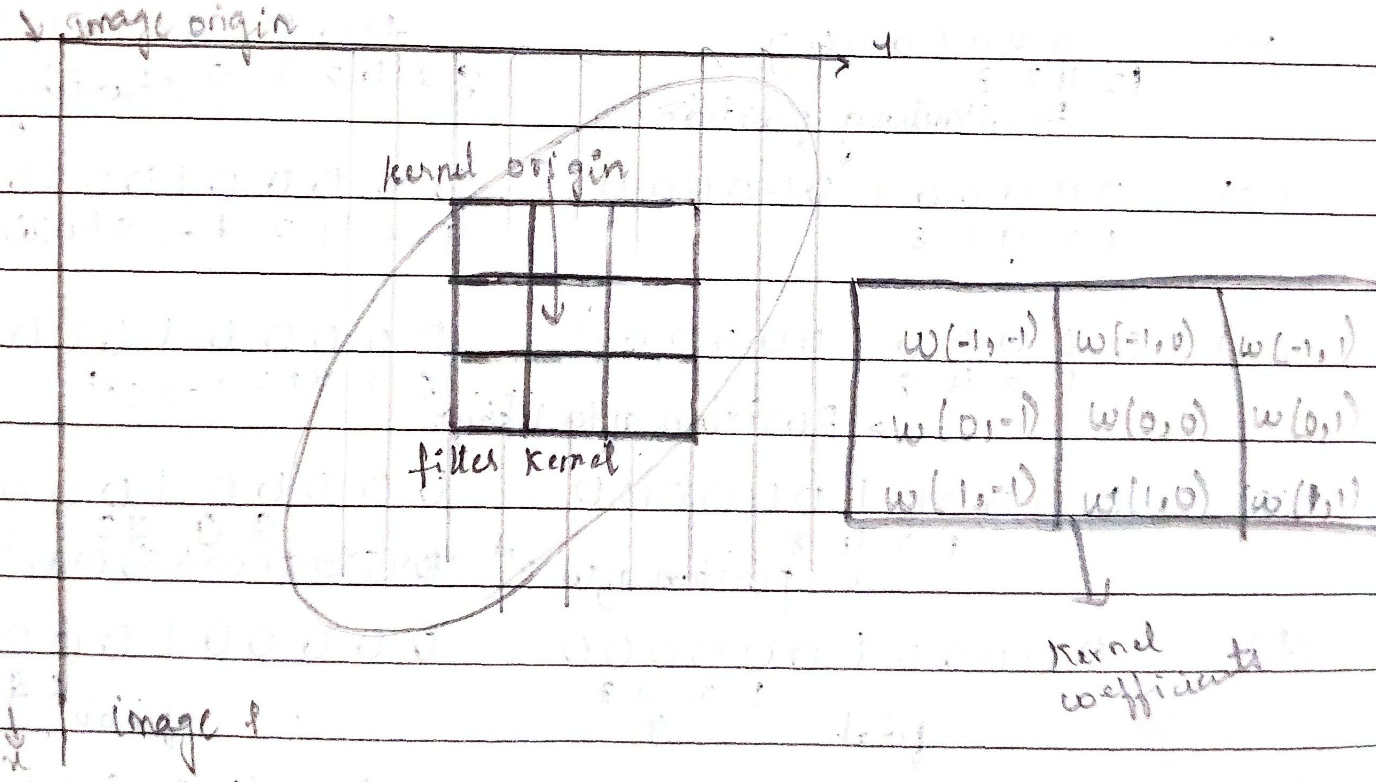
- if operation performed on the image pixel is linear then filter is called as linear spatial filter or else that filter is non linear spatial filter

→ linear Spatial filtering

A linear spatial filter performs a sum of product operations between an image f and filter Kernel w

- The Kernel is an array whose size defines the neighbourhood of operation & Coefficient defines the nature of filter

- Kernel is also known as mask, template and window



$$g(x,y) = w(-1,-1)\,f(x-1,y-1) + w(-1,0)\,f(x-1,y) + \dots + w(0,0)\,f(x,y) + \dots + w(1,1)\,f(x+1,y+1)$$

2Q Illustrate spatial Correlation & Convolution

Ans: correlation consists of moving the centre of a kernel over an image & computing the sum of product at each location

	Correlation	Convolution
	origin f w	origin f w rotated 180°
(a)	0 0 0 1 0 0 0 0 1 2 4 2 8	0 0 0 1 0 0 0 0 8 2 4 2 1
(b)	0 0 0 1 0 0 0 0 1 2 4 2 8 starting position	0 0 0 1 0 0 0 0 8 2 4 2 1 → starting
(c)	0 0 0 0 0 1 0 0 0 0 0 0 1 2 4 2 8	0 0 0 0 0 1 0 0 0 0 0 0 8 2 4 2 1 → starting
(d)	0 0 0 0 0 1 0 0 0 0 0 0 1 2 4 8 → position after 1st shift	0 0 0 0 0 1 0 0 0 0 0 0 8 2 4 2 1 → 1st
(e)	0 0 0 0 0 1 0 0 0 0 0 0 1 2 4 8 → position after 3rd	0 0 0 0 0 1 0 0 0 0 0 0 8 4 2 1 → 3th
(f)	0 0 0 0 0 1 0 0 0 0 0 0 1 2 4 8 final	0 0 0 0 0 1 0 0 0 0 0 0 final 8 4 2 2 1
(g)	0 8 2 4 2 1 0 0 Correlation	0 1 2 4 2 8 0 0 Convolution
(h)	0 0 0 8 2 4 2 1 0 0 0 0 Extended (full) Correlation	0 0 0 1 2 4 2 8 0 0 0 0 Extended Convolution

Spatial correlation is illustrated graphically and described mathematically by

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s,\, y+t) \quad -(1)$$

The mechanism of spatial convolution are same except that the corelation kernel is rotated by 180°. So when the values of kernel are symmetric about its center correlation & convolution yeild the same result.

for 1D

$$g(x) = \sum_{s=-a}^{a} w(s)\, f(x+s) \quad -(2)$$

Origin

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad \begin{array}{c} w \\ \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix} \end{array}$$

padded f.

$$\begin{matrix} 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0 \end{matrix}$$

Correlation result.

$$\begin{matrix} 0&0&0&0&0 \\ 0&9&8&7&0 \\ 0&6&5&4&0 \\ 0&3&2&1&0 \\ 0&0&0&0&0 \end{matrix}$$

39.

Ans:

Convolution result.

$$\begin{matrix} 0&0&0&0&0 \\ 0&1&2&3&0 \\ 0&4&5&6&0 \\ 0&7&8&9&0 \\ 0&0&0&0&0 \end{matrix}$$

Convolution of kernel w of size m×n with an image f(x,y) denoted by $(w * f)$, (x,y) is defined as.

$$(w * f)(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x-s, y-t).$$

where minus sign align the coordinates of f and w when one function is rotated 180°. because Convolution is Commutative

39. Explain different Smoothing Spatial filters

Ans: Smoothing is also known as averaging Spatial filter are used to reduce Sharp transitions in intensity. Because random noise consists of Sharp transitions in intensity. Eg of Smoothing is noise reduction.

- Smoothing is used to reduce irrelevant details in an image. Irrelevant refers to pixel region that are Small compare to the size of the filter kernel.

- Smoothing filters are used in Combination with other techniques for image enhancement, histogram processing, unsharp masking.. etc.

- Linear Smoothing filter consists of Convolving an image with a filter kernel. It blurs the image with the degree of blurring being determined by the size of kernel & values of coefficient.

4Q Explain Sharpening Spatial filters.

Ans: Smoothing is often referred to as lowpass filtering a term borrowed from frequency domain processing.

- Sharpening are based on 1st order & 2nd order derivatives

- We required that any definition we use for 1st derivative.

1. Must be zero in areas of constant intensity
2. Must be nonzero at the onset of an intensity step or ramp.
3. Must be nonzero along intensity ramp.

for 2nd derivative.

1. Must be zero in areas of constant intensity
2. Must be nonzero at the onset and end of an intensity step or ramp.
3. Must be zero along intensity ramp.

So the first order derivative of a 1D function $f(x)$ is

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- The values denoted by small squares are the intensity values along horizonal intensity profile
- The circle indicate the onset or end of intensity transition.
- The first and second order derivatives computed using two preceding transition.
- When computed the first derivative at a location x we subtract the value of the function at that location from the next point. So this is look ahead operation.
- Similarily to compute second derivation at x we use the previous as the next points in the computation.
- The 1^{st} derivative is nonzero & the 2^{nd} is zero along the ramp.
- Edges in digital image are ramp like transition in intensity in the 1^{st} derivative of the image would be thick edge because the derivative is non zero along ramp.

- 2nd derivative would be double edge one pixel thick seperated by zeroes.

→ Using the second derivative for image sharpening :- THE LAPLACIAN.

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \quad -(1)$$

The laplacian is a linear operation. to express this in discreate form we have. for x direction

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \quad -(2)$$

for y direction

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \quad -(3)$$

from the preceding 3 eqn we get.

$$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y).$$

0	-1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

The basic way in which we use the laplacian for image sharpening is

$$g(x,y) = f(x,y) + c\,[\nabla^2 f(x,y)]$$

where $f(x,y)$ & $g(x,y)$ are the input and sharpened image

$c = -1$ if laplacian kernel in 3.45 a or b
$c = 1$ if either or the two other kernel is used.

→ Unsharp Masking and highboost filtering

Subtracting an unsharp version of an image from original image is called as unsharp masking.
It contains the following steps:

1. Blur the original image

2. Subtract the blurred image from the original

3. Add the mask to the original.

$\bar{f}(x,y)$ denote the blurred image the mask eq^n is.

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

Then add weighted portion of the mask to original image.

$$g(x,y) = f(x,y) + k\,g_{mask}(x,y).$$

if $k = 1$ we have unsharp masking.
$k > 1$ highboost filtering
$k < 1$ reduce the contribution of unsharp mask.