Session 3 Presentation
Statistical Methods in Research
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Overview

1. Hypothesis tests
   - μ
   - p
   - σ^2
2. Assumptions and Violations
   - Tests of Normality
3. Pooled t test
4. Paired t test

Hypothesis testing

1: Specify H_0, H_1, and an acceptable level of α.

2: Define a sample-based test statistic and the rejection region for the specified H_0.

3: Collect the sample data and calculate the test statistic

4: Make a decision to either reject or fail to reject H_0. The decision will normally result in a recommendation for action.

5: Interpret the results in the language of the problem. It is imperative that the results be usable by the practitioner. since H_1 is of primary interest, this conclusion should be stated in terms of whether there was or was not evidence of the alternative hypothesis.

Estimation

- A **confidence interval** consists of a range of values together with a percentage that specifies how confident we are that the parameters lies in the interval.
- The **maximum error of estimation**, also called the margin of error is an indicator of the precision of an estimate and is defined as one-half the width of a confidence interval.
- A hypothesis test for $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ will be rejected at a significance level of α if μ_0 is not in the $(1 - \alpha)$ confidence interval for μ.
- Conversely - Any value of μ inside the $(1 - \alpha)$ confidence interval will not be rejected by α-level significance test.

Hypothesis test on μ

- To test the hypothesis

$$H_0 : \mu = \mu_0 \quad \textit{versus} \quad H_1 : \mu \neq \mu_0 \tag{1}$$

- Compute the test statistic

$$t = \frac{(\bar{y} - \mu_0)}{\sqrt{(s^2/n)}} = \frac{(\bar{y} - \mu_0)}{s/\sqrt{n}} \tag{2}$$

- H_0 is rejected if the calculated value of t is in the rejection region, as defined by a specified α, found in the table of the t distribution, or if the calculated p value is smaller than a specified value of α.

Estimation of μ

- The general formula of the $(1-\alpha)$ confidence interval for μ is

$$\bar{y} \pm t_{\alpha/2}\sqrt{\frac{s^2}{n}} \tag{3}$$

- where $t_{\alpha/2}$ has $(n-1)$ degrees of freedom.

Hypothesis test on p

- The hypothesis are

$$H_0 : p = p_0 \quad \text{versus} \quad H_1 : p \neq p_0 \tag{4}$$

- Compute the test statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \tag{5}$$

Estimation of p

- A $(1-\alpha)$ confidence interval on p based on a sample size of n with y successes is

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \tag{6}$$

Hypothesis test on σ^2

- The hypothesis are

$$H_0 : \sigma^2 = \sigma_0^2 \quad \textit{versus} \quad H_1 : \sigma^2 \neq \sigma_0^2 \tag{7}$$

- Compute the test statistic

$$X^2 = SS/\sigma_0^2 \tag{8}$$

- where $SS = \sum (y - \bar{y})^2$
- If the null hypothesis is true, this statistic has the χ^2 distribution with $(n - 1)$ degrees of freedom.

Estimation of σ^2

- A confidence interval can be constructed for the value for the parameter σ^2 using the χ^2 distribution.
- Since the distribution is not symmetric, the confidence interval is not symmetric about s^2 and, we need two individual values from the χ^2 distribution to calculate the confidence interval.
- The lower limit of the confidence interval is

$$L = SS/\chi^2_{\alpha/2} \tag{9}$$

- and the upper limit is

$$U = SS/\chi^2_{(1-\alpha)/2} \tag{10}$$

Assumptions

- If the data are deficient, the results may be less reliable than indicated.
- The 2 major causes for data to be deficient are
  - sloppy data gathering and recording
  - failure of the distribution of the variable(s) to conform to the assumptions underlying the statistical inference procedure.

Assumptions

- Two major assumptions are needed to assure correctness for statistical inferences
  - randomness of the sample observations, and
  - the distribution of the variable(s) being studied.

Detection of Violations

- A routine part of an analysis could be to produce a box plot and see that there is no obvious problem with the normality assumption.
- This gives us confidence that the conclusions based on the t test and the χ^2 test are valid.
- The use of **normal probability plot** allows a slightly more rigorous test of the normality assumption.
- A special plot called a Q-Q plot (quantile - quantile) plot shows the observed value on one axis (usually the horizontal axis) and the value that is expected if the data are sample from the normal probability distribution on the other axis.
- The points should cluster around a straight line for a normally distributed variable.

Tests for Normality

- Formally called as **goodness-of-fit tests**.
  - χ^2 test
  - Kolmogoroff-Smirnoff test

Pooled t Test

- To test the hypothesis

$$H_0 : \mu_1 - \mu_2 = \delta_0 \tag{11}$$

$$H_1 : \mu_1 - \mu_2 \neq \delta_0 \tag{12}$$

- We use the test statistic

$$t = \frac{\bar{y_1} - \bar{y_2} - \delta_0}{\sqrt{(s_P^2/n_1) + (s_P^2/n_2)}} \tag{13}$$

- or equivalently

$$t = \frac{\bar{y_1} - \bar{y_2} - \delta_0}{\sqrt{s_P^2[(1/n_1) + (1/n_2)]}} \tag{14}$$

Pooled t Test

- This statistic is often called the **pooled t statistic** since it uses pooled variance estimate.
- The pooled t statistic has t distribution and $(n_1 + n_2 - 2)$ degrees of freedom.
- Similarly, the confidence interval on $\mu_1 - \mu_2$ is

$$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2}\sqrt{s_P^2[(1/n_1) + (1/n_2)]} \tag{15}$$

Paired t Test

- Inferences on the difference in means of two populations based on paired samples use the sample differences between the paired values.
- For example, in a diet study the observed value for each individual is obtained by subtracting the after weight from the before weight.
- The result becomes a single sample of differences. Thus the basic statistic is,

$$t = \frac{\bar{d} - \delta_0}{\sqrt{s_D^2/n}} \tag{16}$$