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A Robust and Easy Approach for Demand Forecasting in Supply Chains

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Abstract: Demand forecasting plays an important role for supply chains decision making. It also represents a basis step for activity planning in response to customer demand. In this paper, recent advances in times series allow a new robust and easy approach for demand forecasting. Such technique which is based on algebraic methods of estimation seems to be more adequate for such supply chains task. Several computer simulations and comparative studies demonstrate the relevance of the proposed approach. *Copyright 2018 IFAC.*

Keywords: Time series; forecasts; algebraic technics; supply chain control; management systems.

1. INTRODUCTION

Producing the exact amount of a needed product by costumers is the ideal case for a company. Nevertheless, in the real world such a case remains impossible. The efficient solution to this complex problem is to anticipate the customer's demand at sufficient time before its occurrence especially when producing on make on stock. In this frame, supply chain forecasting is the starting step of the production planning. It has been demonstrated that with a simple forecasting method, safety stock can be reduced to half compared to the safety stock calculated without using any forecast (Courtois & al [2003]) which represents a significant decrease in operating costs for the concerned company. Quantitative forecasting methods are techniques that make a formal use of historical data in the form of time series based on identifying, modeling and extrapolating the pattern (trend) found in historical data. Generally, those methods are performed in the following steps (Montgomery & al [2015]):

(1) Problem definition: determine and define the forecasting components.
(2) Data collection.
(3) Data analysis: which consists on visually inspection of the data for pattern recognition (trend, seasonality) and determination of its basic features in order to select the adequate forecasting model.
(4) Model selection and fitting.
(5) Model validation: examine the errors and choose the "best" model.
(6) Forecasting model deployment.
(7) Monitoring forecasting model performance: is an ongoing process activity that ensures that the model still performing satisfactory.

From the modeling view point several approaches based on times series have been developed in order to best describe their behavior such as: regression techniques, smoothing techniques, statistical or neural techniques (Montgomery & al [2015]).
However, as stated in (Lafont & al., [2015]), writing down a "*good*" and a *"precise"* mathematical model is a very difficult task, if not an impossible one, since several phenomena cannot be taking in account [1]. Therefore, the work presented in this paper exploits the new setting of time series and the recent advances in this topic. Such new paradigm permits to bypass this crucial step of modeling like the new concept of ***"model-free control"*** introduced few years ago in control engineering with many successful applications in several areas, ((see, e.g., (Fliess & Join [2008]), (Fliess & Join [2009]), (Fliess & Join [2013])). Notice that this setup is based on the existence of trend in the time series which is estimated via algebraic techniques (Fliess & al [2008]) that are low consuming calculation algorithms. In addition, it is very known that in forecasting domain, selecting the model that provides the best fit to historical data generally does not result in a forecasting method that produces the best forecasts of new data. Concentrating too much on the model that produces the best historical fit often results in overfitting (Montgomery & al [2015]), or including too many parameters or terms in the model just because these additional terms improve the model fit.
On the contrary, the method proposed in this paper which is based on algebraic techniques that is presented here takes an other path and gives a new approach to deal with time series. Such approach, contrary to other methods, does not necessitate large historical data like techniques which are based on big data and without using any:

[1] "All models are wrong. Some are useful", George Box.

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- mathematical model,
- statistic or probabilistic tools,
- global optimization technique.

The proposed new concept on time series has been initially introduced in quantitative finance by (Fliess & Join [2009b]) to forecast economic quantities. The convincing obtained results led the two authors to propose a mathematical definition of volatility, to demonstrate the inanity of a mathematical modeling and probabilistic tools, to give a new definition of causality between time series and to the definition and detection of seasonality and cycles in ((Fliess & al [2011a,b]), (Fliess & Join [2015a,b])) respectively. Besides financial engineering this approach has also been utilized for short-term solar irradiation in meteorological forecasts for the purpose of renewable energy management ((Join & al [2014]), (Voyant & al [2015]) and (Join & al [2016])) and network traffic flow forecasting in (Abouaïssa & al [2016]). To the best knowledge of the authors, the application of this concept seems to be a novel attempt mainly for supply chain.

A huge forecasting techniques exist in the literature. The choice of Kalman filter, for comparison purposes, is due to its efficiency in forecasting future demand in many fields. It is used to forecast deformations in large thin-walled to improve compensatory control method deformation in (Wang & al [2016]) and demonstrated that it gives better results than the Autoregressive integrated moving average (ARIMA) modeling. (Takeda & al [2016]) and (Lynch & al [2016]) applied the Kalman filter for electrical load forecasting which is a major issue in power systems operations to garranty a safe and stable electricity. it has also been applied in an important issue for flow traffic forecasting. In Grenoble (France), (Leon & al [2013]) developed a Kalman filter algorithm that allows to forecast multi-step ahead and in USA in (Portugais & Khanal [2014]) in order to use the forecasts in traffic flow ramp metering scheme.

This paper is organized as follows. Section (2) introduces the new view point on times series allowed by Cartier-Perrin (1995) theorem which satisfies weak integrability conditions and demonstrated the existence of a trend in the chronicles (time series) which led to this approach. Section (3) presents Kalman filter based on a state space model which is a learning model for a one-step ahead prediction for comparison purposes. Section(4) illustrates the application of the presented method based on algebraic technics on different types of data that shows a linear trend, seasonality and even a non-linear trend. Finally, section (5), summarizes the main results and lists some perspectives for future researches.

2. NEW SETTING FOR TIME SERIES: A *"MODEL-FREE"* APPROACH

Consider the interval $[0,1] \subset \mathbb{R}$ and let's introduce the infinitesimal sampling as often in *nonstandard analysis*:

$$\Upsilon = \{0 = t_0 < t_1 < ... < t_\nu = 1\}$$

where $t_{i+1} - t_i$ for $0 \leq i \leq \nu$ is *infinitisimal* (very small). A time series $\Omega : \Upsilon \to \mathbb{R}$ is said to be *quickly fluctuating* or *oscillating* if and only if the integral $\int_D \Omega dm$ *infinitisimal* for any *appreciable* interval (which is neither very small nor very large) [2].

The new vision of times series is allowed by the Cartier-Perrin (1995) theorem which states that any time series $X(t)$, if it satisfies weak integrability condition, can be written:

$$X(t) = E(X(t)) + X_{fluctuations}(t) \tag{1}$$

where:

- $E(X(t))$: the trend (mean, expectation) which is quite smooth.
- $X_{fluctuations}(t)$: is quickly fluctuating.

Let's also consider the Taylor formula:

$$f(x) = \sum_{k=o}^{n} \frac{f^{(k)}(t)}{k!}(x-t)^k + R_n(x) \tag{2}$$

where $R_n(x)$ is the approximation error of f.
If we put $x = t + h$ where h is in the neighborhood of t then (2) becomes:

$$f(t+h) = f(t)+\frac{h}{1!}f^{'}(t)+\frac{h^2}{2!}f^{(2)}(t)+...+\frac{h^n}{n!}f^{(n)}(t)+R_n(h) \tag{3}$$

For $n = 1$, (3) becomes:

$$f(t+h) = f(t) + \frac{h}{1!}f^{'}(t) \tag{4}$$

For a forecasting horizon τ of our historical data time series $X(t)$ can be written thanks to (4) and (1) as:

$$X_{forecast}(t+\tau) = E(X)(t) + \left[\frac{dE(X)(t)}{dt}\right]_e \tau \tag{5}$$

We know in control engineering and signal processing that any observed signal contains a useful signal tainted by noise and this corresponds to:

$$Observed\ signal = useful\ signal + noise \tag{6}$$

It is clear from (6) and (5), that extracting the useful information from the observed signal can be done with an efficient denoising method, *i.e.*, forecasting boils down to filtering noisy signals, and determining the expectation derivatives means deriving noisy signals.
Deriving noisy signals is a known issue that gave a huge literature. The solution given in this paper is to use algebraic estimation technique to determine the derivatives *via* integrals [3].
(5) can be written in the form:

$$P(\tau) = a_0(t) + a_1(t)\tau \tag{7}$$

Rewrite (7) thanks to Laplace transform with respect to τ :

$$P(\tau) = \frac{a_0}{s} + \frac{a_1}{s^2} \tag{8}$$

[2] see (Fliess & al [2011b]) for more mathematical details and definitions
[3] The idea of determining derivatives *via* integrals is not new, it goes back to (Lanczos [1956])

Multiplying both sides by s and derive by ds to get rid a_0. (8) becomes:

$$s\frac{dP}{ds} + P = -\frac{a_1}{s^2} \tag{9}$$

Noisy signal numerical derivation is a problem as stated previously. To get rid of the positive power, represented here by the derivative s, we multiply by s^N, $N = -2$, in our case. Then we get:

$$s^{-2}P + s^{-1}\frac{dP}{ds} = -s^{-4}a_1 \tag{10}$$

As we can see in (10), only integrals appear in the equation. then using the correspondance to time domaine :

$$\begin{cases} \dfrac{1}{s^n} \to \dfrac{t^{n-1}}{(n-1)!} \\ \dfrac{d}{ds} \to -t \end{cases} \tag{11}$$

(10) becomes:

$$\int_{t_0}^{t}\int_{t_0}^{\tau} x(\kappa)d\kappa d\tau - \int_{t_0}^{t} \tau x(\tau)d\tau = -\frac{t^3}{6}a_1 \tag{12}$$

Multiple integrals transformation to simple integrals is obtained thanks to Cauchy formula:

$$\int \cdots \int_n f(\tau_1, \ldots, \tau_k)\, d\tau_1 \ldots d\tau_k = \int_{t_0}^{t} \frac{(t-\tau)^{(n-1)}}{(n-1)!} f(\tau)\, d\tau \tag{13}$$

The final expression of a_1 estimate is:

$$a_1 = \frac{6(\int_{t_0}^{t} \tau x(\tau)d\tau - \int_{t_0}^{t}(t-\tau)x(\tau)d\tau)}{t^3} \tag{14}$$

Using the same methodology for a_0, we get :

$$a_0 = \frac{2(\int_{t_0}^{t} 2(t-\tau)x(\tau)d\tau - \int_{t_0}^{t} \tau x(\tau)d\tau)}{t^2} \tag{15}$$

Note that denoising is made by integrals, that are considered as low pass filters which allow us to obtain the smoothed trend of the time series.

3. KALMAN FILTER

Consider the following state representation for discrete time k [4]:

$$\begin{cases} x_k = G_k x_{k-1} + \omega_k \\ y_k = F_k x_k + \nu_k \end{cases} \tag{16}$$

where:

- $x \in \mathbb{R}^n$ is the state variable,
- $G_k \in \mathbb{R}^{n\times n}$ and $F_k \in \mathbb{R}^{m\times n}$ are known matrices.
- $y \in \mathbb{R}^m$ is the measurement state variable,
- ω_k and ν_k are two independent gaussian noises such ($\omega_k \backsim \mathcal{N}(0, Q_k)$ and $\nu_k \backsim \mathcal{N}(0, R_k)$).

Kalman filter is a two-stage filter:

- time update equations: predicts the forward (in time) current state and error covariance to obtain the *a priori* estimate,
- measurement update equations: incorporates into the *a priori* to obtain an improved *a posteriori* estimate.

prediction stage:

Update algorithm equations are given by:

a priori state estimation:

$$\widehat{x}_{k|k-1} = G_k \widehat{x}_{k-1|k-1} \tag{17}$$

a priori error covariance estimate:

$$P_{k|k-1} = G_k P_{k-1|k-1} G_k^T + Q_k \tag{18}$$

Measurement update:

The residual is defined as the difference between the observation (measurement) and its prediction using the information available when the new measurement is taken. The measurement update equations are given by:

$$\tilde{y}_k = y_k - F_k \widehat{x}_{k|k-1} \tag{19}$$

Residual covariance:

$$S_k = F_k P_{k|k-1} F_k^T + R_k \tag{20}$$

The weight correction (Kalman gain):

$$K_k = P_{k|k-1} F_k^T S_k^{-1} \tag{21}$$

state estimation a posteriori

$$\widehat{x}_{k|k} = \widehat{x}_{k|k-1} + K_k \tilde{y}_k \tag{22}$$

error covariance estimate (a posteriori):

$$P_{k|k} = P_{k|k-1} - K_k F_k P_{k|k-1} \tag{23}$$

Numerical implementation of (23) generates numerical errors. It is replaced by the following formula suggested by Joseph (2005):

$$P_{k|k} = (I - K_k F_k) P_{k|k-1} (I - K_k F_k)^T + K_k R_k K_k^T \tag{24}$$

4. NUMERICAL SIMULATION

The simulation study is conducted using Matlab software on three types of noisy data:

- Without a trend and without seasonality
- With a linear trend
- With a linear trend and seasonality

Data in the time series are tainted by a gaussian white noise that follows a normal distribution ($\mathcal{N}(0, 150)$) which is daily collected.

[4] The control variable u_k does not apprear in forecasting state model because we do not have feedback.

4.1 Comparison criterion

To have an idea on the efficiency of a forecast method we use here the mean squared error. We first calculate the mean on the collected data by the following formula:

$$M = \frac{1}{n}\sum_{i=0}^{n} X(i) \tag{25}$$

Then the mean is calculated on the forecasted data:

$$M_f = \frac{1}{n}\sum_{i=0}^{n} X_{forecast}(i) \tag{26}$$

it yields the standard deviation on the forecasted data as follow:

$$\sigma = \sqrt{M_f} \tag{27}$$

Finally, the error in percentage is given by:

$$E_p = \frac{\sigma}{M} \tag{28}$$

4.2 Simulation resultats

First, we compare the method based on algebraic methods developed here to the Kalman filter for a one step ahead forecasting. Simulation results are given in the following figures where the blue curve represents the real collected data :

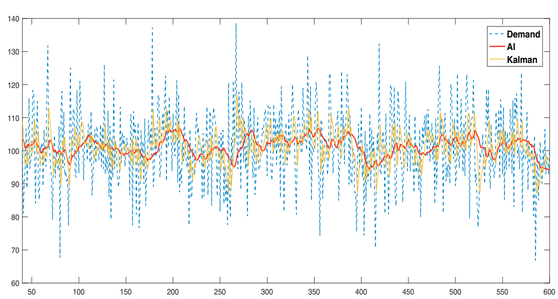

Fig. 1. One day forecast for noisy data (No Trend, No Seasonality) (NT & NS).

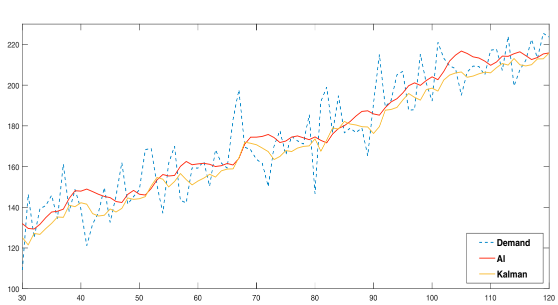

Fig. 2. One day forecast for noisy data with a linear trend (LT).

Table 1: Forecast error comparison between algebraic technics (Al) and Kalman filter for 1 step ahead prediction.

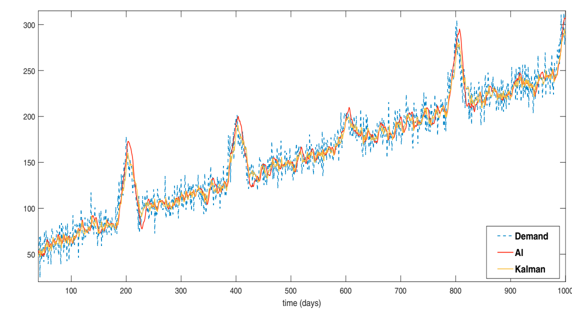

Fig. 3. One day forecast for noisy data with a linear trend and seasonality (LT & S).

The two forecasting techniques provide satisfactory results for a one step ahead prediction. But results obtained by the algebraic technic presented here are slightly better than the ones obtained by the Kalman filter as it is show in Table 1.

	NT & NS		LT		LT & S	
Error	Al	Kal-man	Al	kal-man	Al	Kal-man
1 day forecast	12.28 %	12.37 %	2.68 %	3.05 %	9.18 %	9.16 %

To forecasting for inventory control in supply chain framework, we should properly define the components of the entire problem such as the time needed to get row materials and the production lead time to choose the appropriate forecasting time horizon τ. τ should be bigger than the sum of these delays [5] . For example, in our application case of a semi conductor supply chain:

- Sort/fabrication actor has a 3 days lead time,
- test/assembly actor has a 7 days lead time.

Therefore, the forecasting horizon should be bigger than those lead times.
Contrary to the Kalman filter and other forecasting algorithms where we should develop another method to be able to forecast on a longer horizon, the algebraic methods based technique can do it without, additionally to its adaptation to different type of data (trend, seasonality,...), changing the algorithm.

Table 2: Forecast error using algebraic technics (Al) for multi-step ahead prediction.

	NT & NS	LT	LT & S	NL
Error	Al	Al	Al	Al
3 days forecast	6.89 %	3.09 %	4.39%	1.82%
7 days forecast	25.1 %	6.97 %	12.36%	2.08%
10 days forecast	26.57 %	7.43 %	14.53%	5.24%

The following figures (4), (5) and (6) show that the obtained results for the different types of data cited previously for 3, 7 and 10 days ahead prediction are satisfying based on the mean squared error in table 2. Except in the case (NT & NS) where the mean squared error reaches 25.1% and 26.57% for 7 and 10 days respectively that should be more investigated.
We added to the previous data, a time series that shows

[5] For simplicity sake, transport times are supposed to be integrated in the production lead times.

a nonlinear trend to demonstrate the efficiency of the algebraic techniques where we do not have to determine any mathematical model to adapt to the non linearities. The forecast as it is shown in figure (7) gives convincing results due the small error obtained for different forecasting horizons (see Table 2).

5. CONCLUSION

The work presented in this paper shows a new setting of chronicles using algebraic techniques concept in the frame of supply chain management and gave a problem formulation that can be used by engineers. The proposed forecasting method does not need a model or information about the pattern to predict like Kalman filters, ARIMA models or smoothing techniques which makes the implementation easy. The suggested forecasting algorithm is based on replacing the whole complex mathematical model by a local approximation using the Taylor formula. This was allowed by Cartier-Perrin (1995) for time series that satisfies the weak integrability condition. The concept requires derivative estimation of noisy signal which is possible thanks to Laplace transform that allows us to estimate derivatives *via* integrals and attenuated the noise. This technique provides a viable and effective alternative to classical forecasting strategies in the field of supply chain management in our point of view. The simplest and elegant formulation of the proposed algorithm can be effortlessly implemented for real-time supply chain forecasting for more complex dynamical data. The authors will further explore the relevance of the method combined with an inventory control technic and stock optimization.

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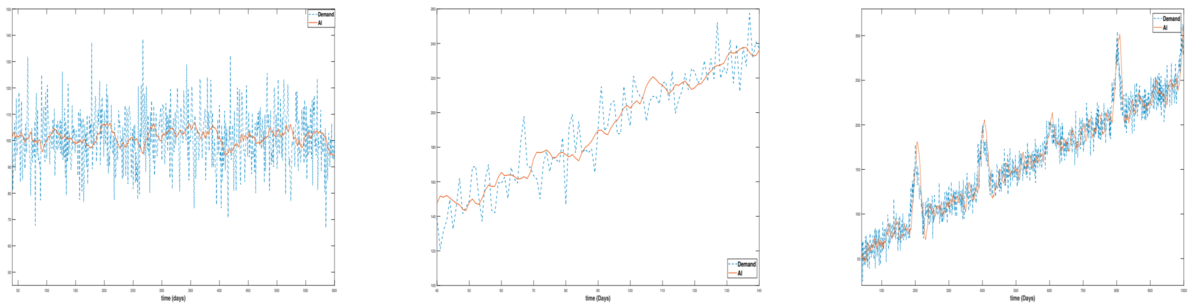
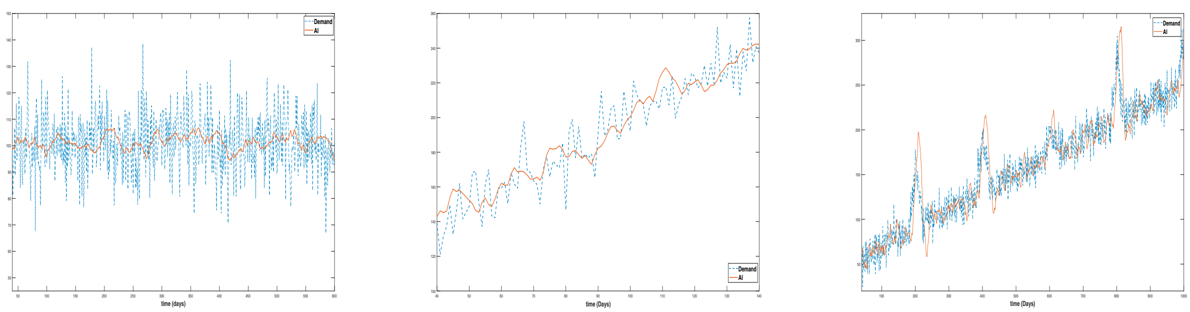


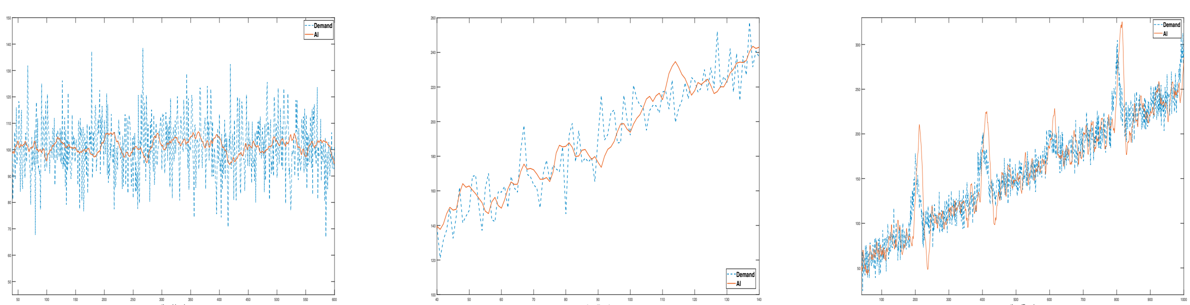
(a) NT & NS (b) LT (c) LT & S

Fig. 4. 3 steps ahead prediction (3 days forecast).



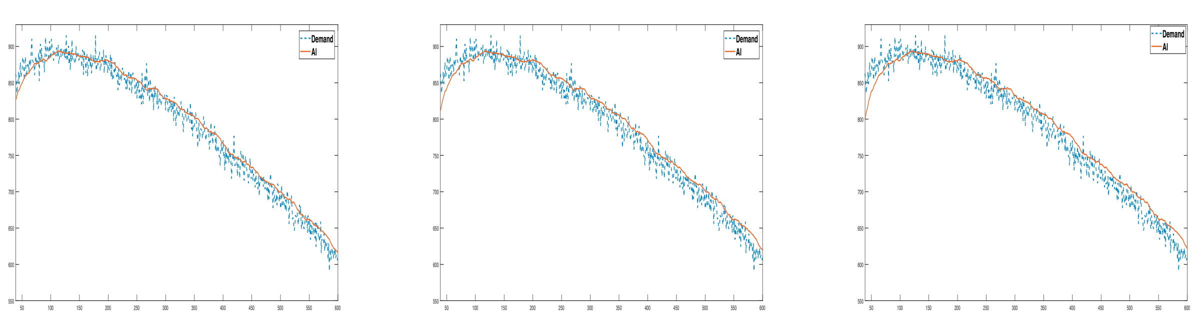
(a) NT & NS (b) LT (c) LT & S

Fig. 5. 7 steps ahead prediction (7 days forecast).



(a) NT & NS (b) LT (c) LT & S

Fig. 6. 10 steps ahead prediction (10 days forecast).



(a) 3 days forecast (b) 7 days forecast (c) 10 days forecast

Fig. 7. Forecasting for data showing a non linear trend (NL).