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1 Abstract

- To find the antenna currents in a half-wave dipole antenna using:

 (i) Standard Expression. (ii) Magnetic Vector Potential and approximating
- To study the difference between the graphs obtained via estimation and actual values

2 Introduction

We have a long wire carrying a current I(z) in dipole antenna with half length 0f $50 \mathrm{cm}(=1)$ so, wavelength $= 2 \mathrm{m}$. Next, we need to determine the currents in the two wires of the antenna. Next, we have the expressions to calculate the value of currents.

$$I = I_m sin(k(l-z))$$
$$0 < z < l$$

In the next process, we calculate the magnetic vector potential by approximating the integrals (in terms of summation); we next find out P_{ij} and P_B .

$$A_{z,i} = \sum_{j} P_{ij} I_j + P_B I_N = \sum_{j} I_j \left(\frac{\mu_0}{4\pi} \frac{exp(-jkR_{ij})}{R_{ij}} dz_j'\right)$$
$$P_B = \frac{\mu_0}{4\pi} \frac{exp(-jkR_{iN})}{R_{iN} dz_j'}$$

Then, we use the Ampere's circuital law to calculate H_{ϕ} . Again, we get it in terms of some summation involving the matrices Q_{ij} and Q_B .

$$H_{\phi}(r,z_{i}) = \sum_{j} Q_{ij} J_{j} + Q_{Bi} I_{m} = -\sum_{j} P_{ij} \frac{r}{\mu_{0}} (\frac{-jk}{R_{ij}} - \frac{1}{R_{ij}^{2}}) + P_{B} \frac{r}{\mu_{0}} (\frac{-jk}{R_{iN}} - \frac{1}{R_{iN}^{2}})$$

At last we solve the matrix equation to find out the current vector J and then find out I.

$$MJ = QJ + Q_BI_m$$

3 Assignment questions

3.1 Question-1

According to the question, we now need to find vector z and u. And then find the current vectors I (at locations of z) and J (at locations of u) respectively. The following code snippet does the job!

```
# Defining all the given variables
```

```
import pylab
import numpy as np

pi = np.pi

N = 4  # Number of sections in each half section of the antenna
Im = 1.0  # Current injected into the antenna
len = 0.5  # Quarter wavelength
w_no = pi  # Wave number = (2*pi)/(lambda)
dz = len/N  # Spacing of current samples

z = np.linspace(-len,len,2*N+1)  # Points at which we determine the currents.
u = np.delete(z,[0,N,2*N])  #2*(N-1) locations of unknown currents
a = 0.01  # Radius of wire
mu_0 = 4e-7*pi  # Permeability of free space
```

The values obtained after running the code are:

```
z = \begin{bmatrix} -0.5 & -0.38 & -0.25 & -0.12 & 0. & 0.12 & 0.25 & 0.38 & 0.5 \end{bmatrix}

u = \begin{bmatrix} -0.38 & -0.25 & -0.12 & 0.12 & 0.25 & 0.38 \end{bmatrix}
```

3.2 Question-2

According to the question, we now need determine the M vector. I defined a function to determine M vector. The following code snippet does the job!

Defining matrix M

```
M = (np.identity(2*(N-1)))*(1/(2*pi*a))
 return M
M = M()
The values obtained after running the code are:
M: [[15.92 0.
                   0.
                         0.
                                0.
 [ 0.
                           0.
        15.92 0.
                   0.
                                  0.
             15.92 0.
 [ 0.
         0.
                           0.
                                  0.
              0. 15.92 0.
                                  0. ]
 [ 0.
         0.
 Γ0.
                     0.
         0.
               0.
                         15.92 0. ]
 [ 0.
         0.
               0.
                     0.
                           0.
                                 15.92]]
3.3
    Question-3
According to the question, we now need determine the matrices R_z, R_u, R_i n, PandPB.
   The following code snippet does the job!
# Computing vectors R_z, R_u and matrices PB, P
Z = np.meshgrid(z,z)
z_i, z_j = Z[0], Z[1]
#R_z determines distances from source and observer where source is point of wire.
R_z = \text{np.sqrt}((z_i-z_j)**2 + \text{np.ones}([2*N+1,2*N+1],dtype=complex)*(a**2))
U = np.meshgrid(u, u)
u_i, u_j = U[0], U[1]
#R_u determines distances from source and observer where observer is point where we want the
R_u = \text{np.sqrt}((u_i-u_j)**2 + \text{np.ones}([2*N-2,2*N-2],dtype=complex)*(a**2))
# Distances with respect to z = 0
R_in = np.delete(R_z[:][N],[N*2,0,N]) # Removing the three elements (first, middle, last)
# P is the contribution to the vector potential due to unknown currents
P = (mu_0/(4*pi))*(np.cos(w_no*R_u)-(np.sin(w_no*R_u))*1j)*(1/R_u)*(dz)
# PB is the contribution to the vector potential due to current "In"
PB = (mu_0/(4*pi))*(np.cos(w_no*R_in)-(np.sin(w_no*R_in))*1j)*(R_in)*(dz)
The values obtained after running the code are:
Rz =
[[0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j 0.5 +0.j 0.63+0.j 0.75+0.j 0.88+0.j
```

def M(): # Function to determine and return the matrix M.

```
1. +0.j]
 [0.13+0.j 0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j 0.5 +0.j 0.63+0.j 0.75+0.j
 0.88+0.i]
 [0.25+0.j 0.13+0.j 0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j 0.5 +0.j 0.63+0.j
 0.75+0.i
 [0.38+0.j 0.25+0.j 0.13+0.j 0.01+0.j 0.13+0.j 0.25+0.j 0.38+0.j 0.5 +0.j
 0.63+0.j
 [0.5 +0.j \ 0.38+0.j \ 0.25+0.j \ 0.13+0.j \ 0.01+0.j \ 0.13+0.j \ 0.25+0.j \ 0.38+0.j
 0.5 + 0.i
 [0.63+0.j \ 0.5 +0.j \ 0.38+0.j \ 0.25+0.j \ 0.13+0.j \ 0.01+0.j \ 0.13+0.j \ 0.25+0.j
 0.38+0.j]
 [0.75+0.j 0.63+0.j 0.5 +0.j 0.38+0.j 0.25+0.j 0.13+0.j 0.01+0.j 0.13+0.j
 0.25+0.j]
 [0.88+0.j 0.75+0.j 0.63+0.j 0.5 +0.j 0.38+0.j 0.25+0.j 0.13+0.j 0.01+0.j
 0.13+0.j]
 [1. +0.j 0.88+0.j 0.75+0.j 0.63+0.j 0.5 +0.j 0.38+0.j 0.25+0.j 0.13+0.j
 0.01+0.j
R_u =
[[0.01+0.j \ 0.13+0.j \ 0.25+0.j \ 0.5 \ +0.j \ 0.63+0.j \ 0.75+0.j]
 [0.13+0.j \ 0.01+0.j \ 0.13+0.j \ 0.38+0.j \ 0.5 +0.j \ 0.63+0.j]
 [0.25+0.j \ 0.13+0.j \ 0.01+0.j \ 0.25+0.j \ 0.38+0.j \ 0.5 +0.j]
 [0.5 +0.j 0.38+0.j 0.25+0.j 0.01+0.j 0.13+0.j 0.25+0.j]
 [0.63+0.j 0.5 +0.j 0.38+0.j 0.13+0.j 0.01+0.j 0.13+0.j]
 [0.75+0.j \ 0.63+0.j \ 0.5 +0.j \ 0.25+0.j \ 0.13+0.j \ 0.01+0.j]]
R_{in} =
[0.38+0.j 0.25+0.j 0.13+0.j 0.13+0.j 0.25+0.j 0.38+0.j]
P*1e8
[[124.94-3.93] 9.2 -3.83[3.53-3.53] -0. -2.5[-0.77-1.85]
  -1.18-1.18j]
 [ 9.2 -3.83j 124.94-3.93j 9.2 -3.83j
                                            1.27-3.08j -0. -2.5j
  -0.77-1.85j]
 [ 3.53-3.53j
                9.2 -3.83j 124.94-3.93j
                                            3.53-3.53j
                                                         1.27-3.08j
  -0. -2.5j]
 [-0. -2.5j]
                 1.27-3.08j
                              3.53-3.53j 124.94-3.93j
    3.53-3.53j
 [-0.77-1.85j -0. -2.5j
                            1.27-3.08j
                                            9.2 -3.83j 124.94-3.93j
   9.2 - 3.83j
 [ -1.18-1.18j -0.77-1.85j -0. -2.5j
                                            3.53-3.53j 9.2 -3.83j
 124.94-3.93j]]
PB*1e8 =
[0.18-0.43j 0.22-0.22j 0.14-0.06j 0.14-0.06j 0.22-0.22j 0.18-0.43j]
```

3.4 Question-4

According to the question, we now need determine the matrices Q and QB. The following code snippet does the job!

```
# Computing Q and QB.
# Matrix corresponding to unknown currents
Q = (a/mu_0)*(P)*((1j)*(w_no)+(1/R_u))*(1/R_u)
# Matrix corresponding to the boundary current
QB = (a/mu_0)*(PB)*((1j)*(w_no)+(1/R_in))*(1/R_in)
```

The values obtained after running the code are:

```
 \begin{array}{l} Q = \\ & [[9.952\,e+01-0.j \quad 5.000\,e-02-0.j \quad 1.000\,e-02-0.j \quad 0.000\,e+00-0.j \quad 0.000\,e+00-0.j \\ & 0.000\,e+00-0.j] \\ & [5.000\,e-02-0.j \quad 9.952\,e+01-0.j \quad 5.000\,e-02-0.j \quad 0.000\,e+00-0.j \quad 0.000\,e+00-0.j \\ & 0.000\,e+00-0.j] \\ & [1.000\,e-02-0.j \quad 5.000\,e-02-0.j \quad 9.952\,e+01-0.j \quad 1.000\,e-02-0.j \quad 0.000\,e+00-0.j \\ & 0.000\,e+00-0.j] \\ & [0.000\,e+00-0.j \quad 0.000\,e+00-0.j \quad 1.000\,e-02-0.j \quad 9.952\,e+01-0.j \quad 5.000\,e-02-0.j \\ & 1.000\,e-02-0.j] \\ & [0.000\,e+00-0.j \quad 0.000\,e+00-0.j \quad 0.000\,e+00-0.j \quad 5.000\,e-02-0.j \quad 9.952\,e+01-0.j \\ & 5.000\,e-02-0.j] \\ & [0.000\,e+00-0.j \quad 0.000\,e+00-0.j \quad 0.000\,e+00-0.j \quad 1.000\,e-02-0.j \quad 5.000\,e-02-0.j \\ & 9.952\,e+01-0.j \] \end{array}
```

$$QB = \begin{bmatrix} 0. - 0.j & 0. - 0.j \end{bmatrix}$$

3.5 Question-5

According to the question, we now need determine the current density vector J and currents $I_e sm$, $I_a sm$.I defined a function to determine M vector The following code snippet does the job!

Here we are computing Estimated currents and Assumed currents.

```
J = np.linalg.inv(M-Q)@QB*Im
#estimated currents
I_esm = np.concatenate(([0],J[:N-1],[Im],J[N-1:],[0]))
#assumed currents
I_asm = Im*np.sin(w_no*(len-abs(z)))
```

The values obtained after running the code are:

$$\begin{split} &I_{-}esm \,=\, [\,0\,.\quad 0\,.\quad 0\,.\quad 0\,.\quad 1\,.\quad 0\,.\quad 0\,.\quad 0\,.\,) \\ &I_{-}asm \,=\, [\,0\,.\quad \quad 0\,.38\ \ 0.71\ \ 0.92\ \ 1\,.\quad \quad 0\,.92\ \ 0.71\ \ 0.38\ \ 0\,.\quad] \end{split}$$

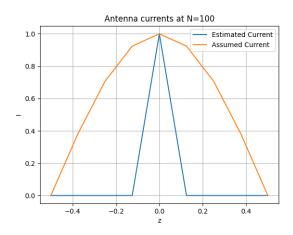


Figure 1: Plot of assumed current Vs estimated current

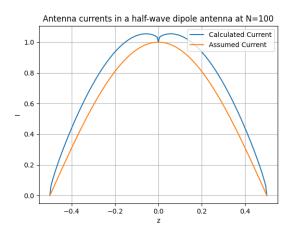


Figure 2: Plot of assumed current Vs estimated current

4 CONCLUSION

- On increasing the value of N, the both graph will merge each other.
- On increasing N,the magnitude of point which are away from the centre are increasing.