30/01/2025

Experiment-4

Title: Development of Minimum Risk Bayes Classification.

Aim: to Implement a minimum Risk Bayes Classifier that determines the optimal solution based on test Results using Bayesian decision theory to minimize the Overall risk.

Theory:

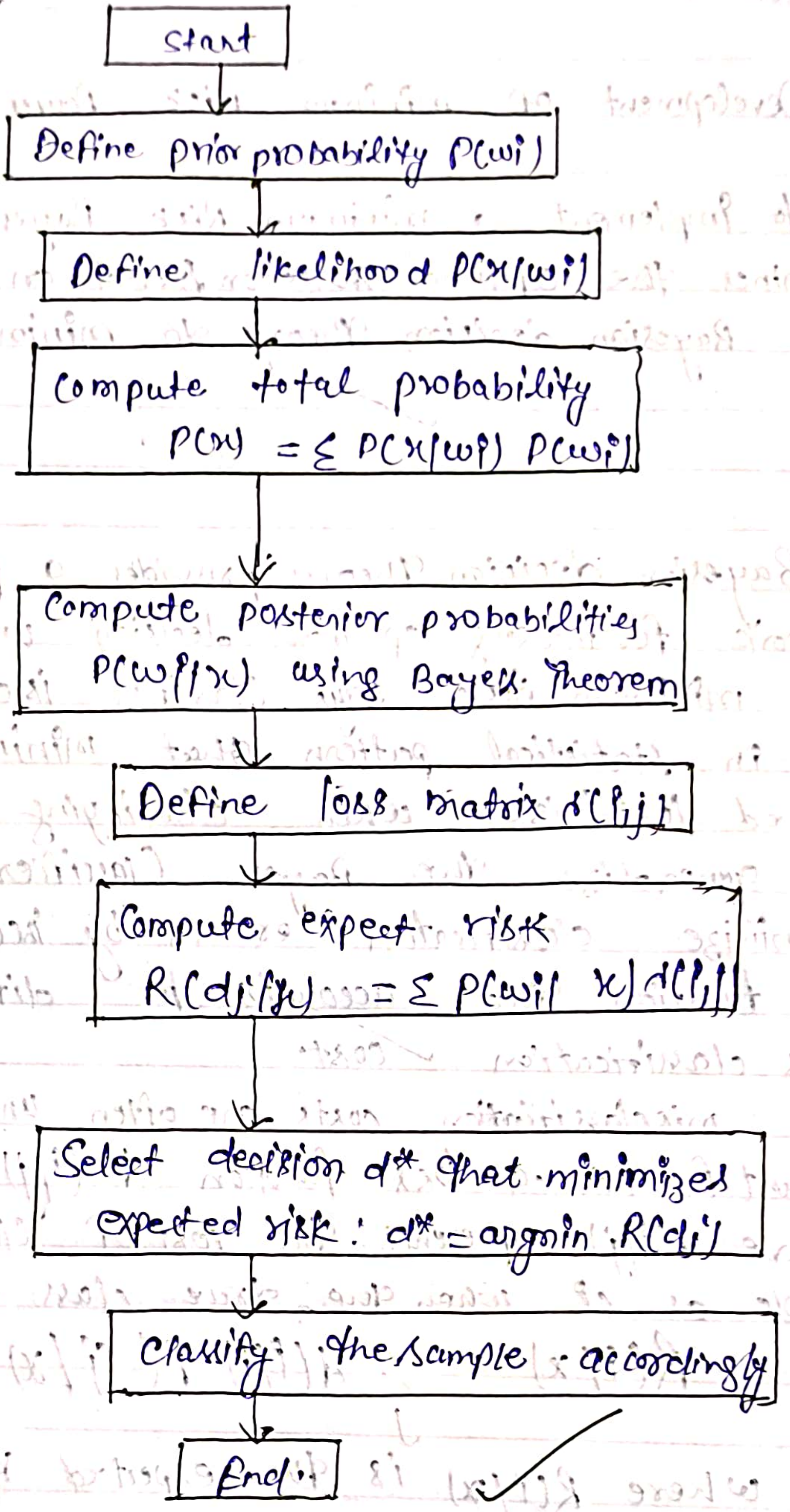
Bayesian decision theory provides a probabilistic framework for making optimal decision under Uncertainty. The minimum Risk Bays Classifier is a decision rule in statistical pattern that minimize the expected loss (risk) when classifying observation. It generalize the Bayes Classifier, which minimize classification errors by incorporating a loss function that accounts for different misclassification cost.

misclassification costs are often unequal. To account for this, a loss function $\lambda(i|j)$ is introduced where $\lambda(i|j)$ represents the cost of classifying a sample as C_i when the true class is C_j

$$R(i/x) = \sum_j \lambda(i|j) \, P(C_j/x)$$

where $R(i/x)$ is the expected risk of Classifying x as class C_i.

Flowchart:


Start
Define prior probability $P(\omega_i)$
Define likelihood $P(x/\omega_i)$
Compute total probability $P(x) = \Sigma P(x/\omega_i) P(\omega_i)$
Compute posterior probabilities $P(\omega_i/x)$ using Bayes Theorem
Define loss matrix $\lambda(i,j)$
Compute expect risk $R(d_j/x) = \Sigma P(\omega_i/x) \lambda(i,j)$
Select decision d^* that minimizes expected risk : $d^* = \text{argmin } R(d_i)$
Classify the sample accordingly
End

Assign x to class c^i if minimize $R(i/x)$

Algorithms

⇒ Start the Classification process.

⇒ Define priors probabilities $P(\omega_i)$ based on statistical data.

⇒ Define the Conditional probabilites $P(x/\omega_i)$, which describe the probability of obtaining a certain test result x given that the patient belong to class ω_i

⇒ Use the law of the total probability to compute the probability of each test result

$$P(x) = \sum_i P(x/\omega_i) P(\omega_i)$$

⇒ Compute the posterior (Bay's Theorem)

$$P(\omega_i/x) = \frac{P(x/\omega_i) P(\omega_i)}{P(x)}$$

⇒ Define the loss matrix $\lambda(i,j)$ where $\lambda(i,j)$ represent the cost of classifying a sample as ω_j when it actually belongs to ω_i.

⇒ Calcute the expected risk

$$R(d_i/x) = \sum_i P(\omega_i/x) \lambda(j,i)$$

⇒ Select the decision d_j that minimize the expected risk $d^* = \arg\min_j R(d_j/x)$

⇒ assign the sample to the class to the class with minimum risk.

Result:

(Problem-1)

-- Minimum Risk Bayes Classifier ---
Prior Probability of Cancer Patients (P(w1)): 0.20
Prior Probability of Non-Cancer Patients (P(w2)): 0.80
Risk for Chemotherapy (a1): 8.00
Risk for Medication (a2): 4.00
Optimal Decision: Choose Medication (a2)

(Problem-2)

-- Minimum Risk Bayes Classifier ---

Prior Probabilities
P(Cancer) = 0.20
P(Non-Cancer) = 0.80

Likelihood Probabilities (Test Outcome Given Class)
P(Negative | Cancer) = 0.10
P(Negative | Non-Cancer) = 0.90
P(Positive | Cancer) = 0.90
P(Positive | Non-Cancer) = 0.10

Marginal Probabilities of Test Outcomes
P(Negative Test) = 0.74
P(Positive Test) = 0.26

Posterior Probabilities (Class Given Test Outcome)
P(Cancer | Negative) = 0.03
P(Non-Cancer | Negative) = 0.97
P(Cancer | Positive) = 0.69
P(Non-Cancer | Positive) = 0.31

Expected Risks for Each Decision
Risk (Chemo | Negative Test) = 9.73
Risk (Medication | Negative Test) = 0.54
Risk (Chemo | Positive Test) = 3.08
Risk (Medication | Positive Test) = 13.85

Final Decision Based on Minimum Risk
For a Negative test result, the decision is: Medication
For a Positive test result, the decision is: Chemotherapy

Problem 1 $P(\text{Cancer}) = 0.2$ $P(\text{Non-cancer}) = 0.8$

Action	Cancer patient (20%)	Non Cancer (80%)
chemo therapy (α_1)	0	10
medication (α_2)	20	0

$R(\text{Chemo}/\cancel{\text{cancer}}) = 0 \times 0.2 + 10 \times 0.8$
$= 8$

$R(\text{medication}) = 20 \times 0.2 + 0 \times 80 = 4$

$R(\text{medication}) < R(\text{Chemotherapy})$

Problem-2

$P(\text{cancer}) = 0.2$ $P(\text{non cancer}) = 0.8$

Test	Cancer	non-cancer
Negative	0.10	0.90
Positive	0.90	0.10

Action	Cancer	Non cancer
Chemotherapy	0	10
Medication	20	0

$$R(\alpha_i/x) = \sum_{j} \lambda(i,j)\, P(w_i/x)$$

$$P(w_i/x) = \frac{P(x/w_i)\, P(w_i)}{P(x)}$$

$$P(x) = \sum_{i} P\left(\frac{x}{w_i}\right) P(w_i)$$

Discussion:

1) The loss matrix reflects the consequences of misclassification. like wrong treating a cancer patient with medication is riskier than giving chemotherapy to a non cancer patient.

2) $R(a_1) = 8$ & $R(a_2) = 4$, the optimal decision is to choosen Medication (a_2) This suggest that medication is preffered based on the given probabilities and loss values.

3). patient with cancer may recieve medication instead of chemotherapy which could lead to incorrect treatment

4) The model considers the lower overall risk in the given population. Significantly

5. Adjusting these loss values could shift the decision boundary.

for the second problem:

→ In the experiment the Risk of Chemotherapy is higher than the medication for the negative test so It choose the medication for the negative test result.

→ for the positive test result, the risk of the chemotherapy is less than the risk of the medication so choose the chemo therapy for the positive test results.

→ Chemo risk: low risk for actual cancer patient but high risk for non cancer patients.

→ medication risk: High risk for cancer patient due to lack of treatment but low risk for non cancer patients

Conclusion:

Use minimum Risk Bayes classifier when some error are more costly than other. On the other hand Bayes classifier where all error consider equall costly. So it choose Minimize classification error by choosing the class with the highest posterior probability.

So the minimum risk classifier minimize expected risk by consider bot misclassification probabilities and loss values.

02.25