Activity 3

OBJECTIVE

To verify the conditions of consistency/ inconsistency for a pair of linear equations in two variables by graphical method.

MATERIAL REQUIRED

Graph papers, pencil, eraser, cardboard, glue.

METHOD OF CONSTRUCTION

1. Take a pair of linear equations in two variables of the form

   $a_1x + b_1y + c_1 = 0$ (1)

   $a_2x + b_2y + c_2 = 0$, (2)

   where a_1, b_1, a_2, b_2, c_1 and c_2 are all real numbers; a_1, b_1, a_2 and b_2 are not simultaneously zero.

There may be three cases :

Case I : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Case II: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Case III: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

2. Obtain the ordered pairs satisfying the pair of linear equations (1) and (2) for each of the above cases.
3. Take a cardboard of a convenient size and paste a graph paper on it. Draw two perpendicular lines X′OX and YOY′ on the graph paper (see Fig. 1). Plot the points obtained in Step 2 on different cartesian planes to obtain different graphs [see Fig. 1, Fig. 2 and Fig.3].

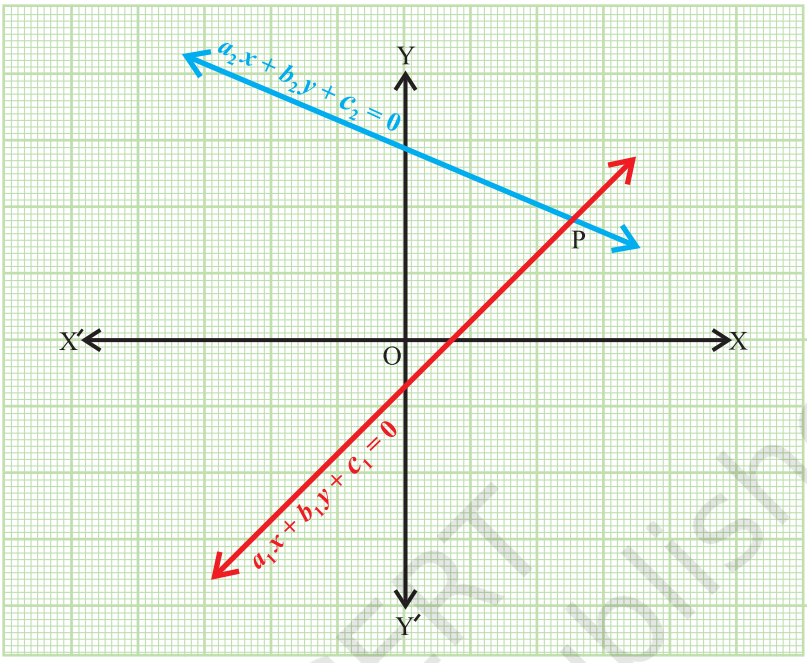

Fig. 1

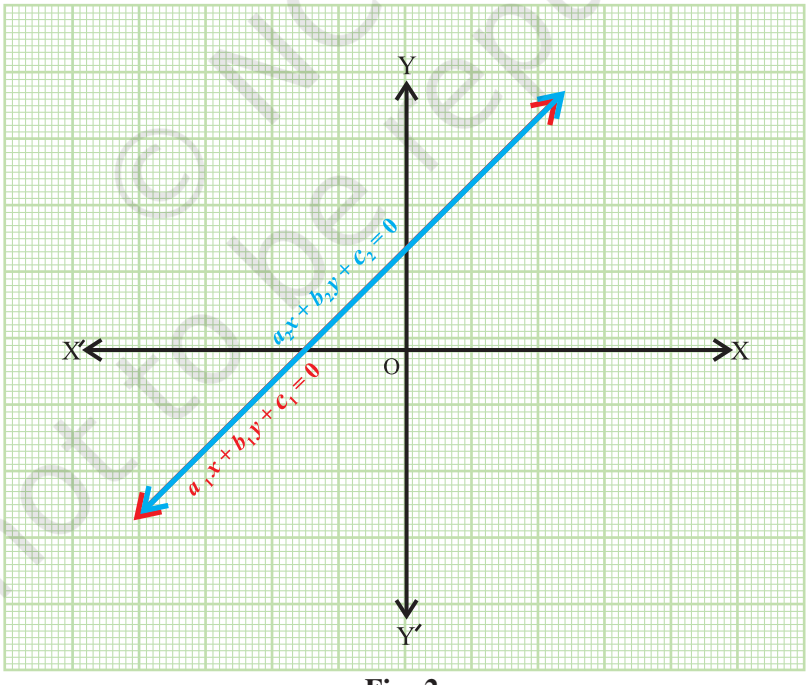

Fig. 2

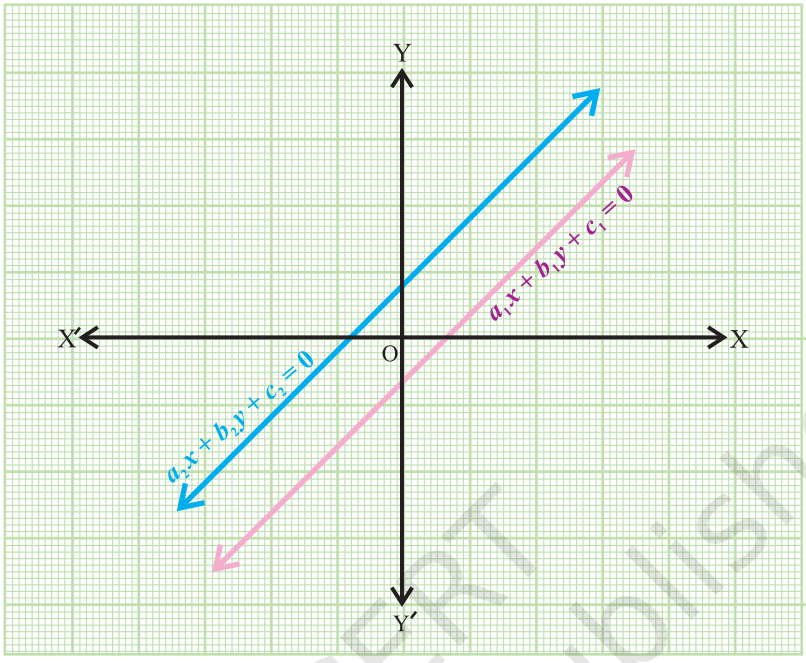

Fig. 3

Demonstration

Case I: We obtain the graph as shown in Fig. 1. The two lines are intersecting at one point P. Co-ordinates of the point P (x,y) give the unique solution for the pair of linear equations (1) and (2).

Therefore, the pair of linear equations with $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ is consistent and has the unique solution.

Case II: We obtain the graph as shown in Fig. 2. The two lines are coincident. Thus, the pair of linear equations has infinitely many solutions.

Therefore, the pair of linear equations with $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ is also consistent as well as dependent.

Case III: We obtain the graph as shown in Fig. 3. The two lines are parallel to each other.

This pair of equations has no solution, i.e., the pair of equations with $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ is inconsistent.

Observation

1. a_1 = _____, a_2 = _____,

   b_1 = _____, b_2 = _____,

   c_1 = _____, c_2 = _____,

So, $\frac{a_1}{a_2}$ = ..., $\frac{b_1}{b_2}$ = ..., $\frac{c_1}{c_2}$ = ...

$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Case I, II or III	Type of lines	Number of solution	Conclusion Consistent/ inconsistent/ dependent

Application

Conditions of consistency help to check whether a pair of linear equations have solution (s) or not.

In case, solutions/solution exist/exists, to find whether the solution is unique or the solutions are infinitely many.