Activity 8

OBJECTIVE

To verify the algebraic identity:

$(a - b)^3 = a^3 - b^3 - 3(a - b)ab$

MATERIAL REQUIRED

Acrylic sheet, coloured papers, saw, sketch pens, adhesive, Cello-tape.

METHOD OF CONSTRUCTION

1. Make a cube of side $(a - b)$ units $(a > b)$using acrylic sheet and cellotape/ adhesive [see Fig. 1].
2. Make three cuboids each of dimensions $(a-b) \times a \times b$ and one cube of side b units using acrylic sheet and cellotape [see Fig. 2 and Fig. 3].
3. Arrange the cubes and cuboids as shown in Fig. 4.

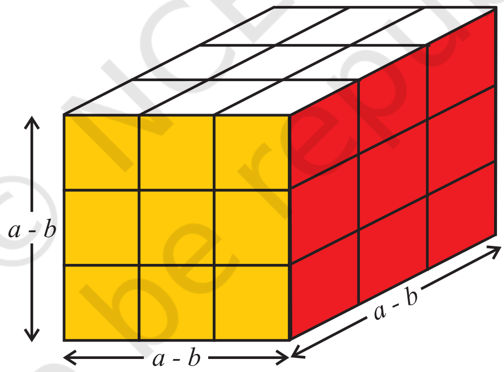

Fig. 1

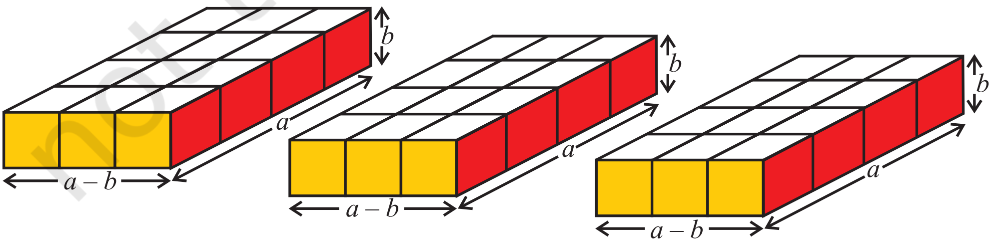

Fig. 2

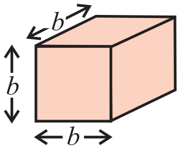

Fig. 3

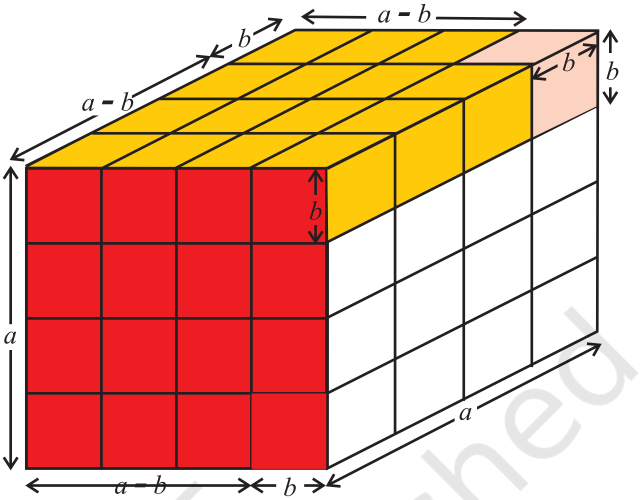

Fig. 4

Demonstration

Volume of the cube of side $(a - b)$ units in Fig. 1 = $(a- b)^3$

Volume of a cuboid in Fig. 2 = $(a-b)\ ab$

Volume of three cuboids in Fig. 2 = $3\ (a-b)\ ab$

Volume of the cube of side b in Fig. 3 = b^3

Volume of the solid in Fig. 4 = $(a-b)^3 + (a-b)\ ab + (a-b)\ ab + (a - b)\ ab + b^3$

$= (a-b)^3 + 3(a-b)\ ab + b^3$ (1)

Also, the solid obtained in Fig. 4 is a cube of side a

Therefore, its volume = a^3 (2)

From (1) and (2),

$(a-b)^3 + 3(a-b)\ ab + b^3 = a^3$

or $(a-b)^3 = a^3 - b^3 - 3ab\ (a-b)$.

Here, volume is in cubic units.

OBSERVATION

On actual measurement:

a = ..., b = ..., $a–b$ = ...,

So, a^3 = ..., ab = ...,

b^3 = ..., $ab(a–b)$ = ...,

$3ab\ (a–b)$ = ..., $(a–b)^3$ = ...,

Therefore, $(a–b)^3 = a^3 – b^3 – 3ab(a–b)$

APPLICATION

The identity may be used for

1. calculating cube of a number expressed as a difference of two convenient numbers
2. simplification and factorisation of algebraic expressions.

NOTE

This identity can also be expressed as :

$(a – b)^3 = a^3 – 3a^2b + 3ab^2 – b^3$.