Activity 9

Objective

To establish a formula for the sum of first n terms of an Arithmetic Progression.

Material Required

Cardboard, coloured drawing sheets, white paper, cutter, adhesive.

Method of Construction

1. Take a rectangular cardboard of a convenient size and paste a white paper on it. Draw a rectangle ABCD of length $(2a+9d)$ units and breadth 10 units.
2. Make some rectangular strips of equal length a units and breadth one unit and some strips of length d units and breadth 1 unit, using coloured drawing sheets.
3. Arrange/paste these strips on the rectangle ABCD as shown in Fig. 1.

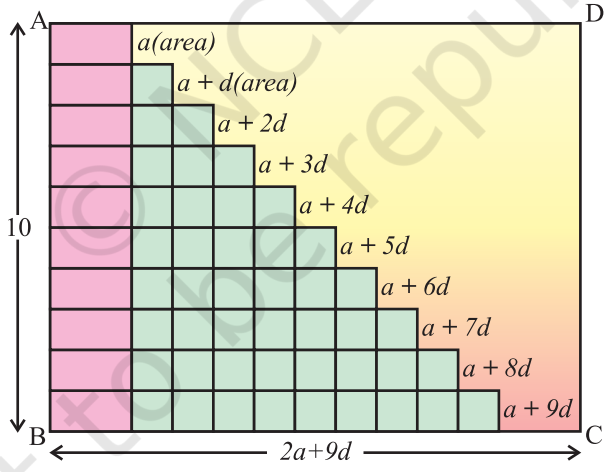

Fig. 1

Demonstration

1. The strips so arranged look like a stair case.
2. The first stair is of length a units, the second stair is of length $a+d$ (units), third of $a+2d$ units and so on and each is of breadth 1 unit. So, the areas (in sq. units) of these strips are $a,\ a+d,\ a+2d,\ \ldots\ldots,\ a+9d,$ respectively.

3. This arrangement of strips gives a pattern a, $a + d$, $a + 2d$, $a + 3d$, ... which is an AP with first term a and the common difference d.
4. The sum of the areas (in square units) of these strips
   $= a + (a + d) + (a + 2d) + ... + (a + 9d) = 10a + 45d$ (1)
5. Area of the designed formed by the stair case = $\frac{1}{2}$ (area of rectangle ABCD)

   $= \frac{1}{2}(10)(2a+9d)$

   $= (10a + 45d)$, which is the same as obtained in (1) above.

   This shows that the sum of first 10 terms of the AP = $\frac{1}{2}(10)(2a+9d)$

   $= \frac{1}{2}(10)\,[2a+(10-1)d]$

   This can be further generalised to find the sum of first n terms of an AP as

   $S_n = \frac{n}{2}\,[2a+(n-1)d\,]$

OBSERVATION

On actual measurement:

a = ---------, d = ----------, n = --------- S_n = ----------

So, $S_n = \frac{n}{2}\,[-+(n-1)-]$.

APPLICATION

This result may be used to find the sum of first n terms of the list of numbers :

1. $1^2, 2^2, 3^2, ...$ 2. $1^3, 2^3, 3^3, ...$

   to be studied in Class XI.