Activity 6

Objective

To find the sum of first n natural numbers.

Material Required

Cardboard, coloured papers, white paper, cutter, adhesive.

Method of Construction

1. Take a rectangular cardboard of a convenient size and paste a coloured paper on it. Draw a rectangle ABCD of length 11 units and breadth 10 units.
2. Divide this rectangle into unit squares as shown in Fig. 1.
3. Starting from upper left-most corner, colour one square, 2 squares and so on as shown in the figure.

Demonstration

1. The pink colour region looks like a stair case.
2. Length of 1st stair is 1 unit, length of 2nd stair is 2 units, length of 3rd stair 3 units, and so on, length of 10th stair is 10 units.

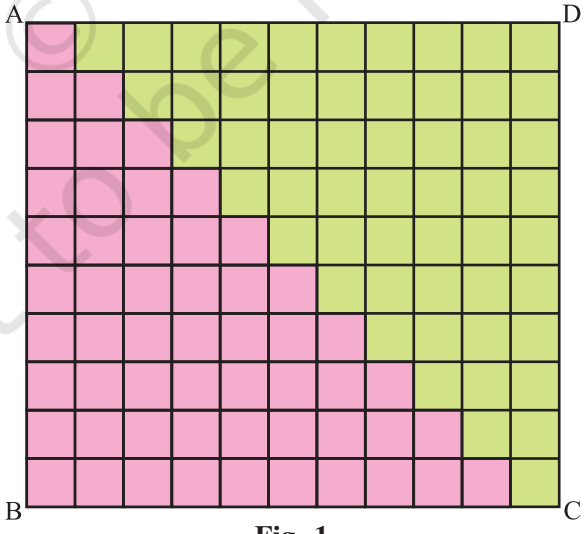

Fig. 1

3. These lengths give a pattern

   1, 2, 3, 4, ..., 10,

   which is an AP with first term 1 and common difference 1.

4. Sum of first ten terms

   $= 1 + 2 + 3 + ... + 10 = 55$ (1)

   Area of the shaded region $= \frac{1}{2}$ (area of rectangle ABCD)

   $= \frac{1}{2} \times 10 \times 11$, which is same as obtained in (1) above. This shows that the sum of the first 10 natural numbers is $\frac{1}{2} \times 10 \times 11 = \frac{1}{2} \times 10(10+1)$.

   This can be generalised to find the sum of first n natural numbers as

   $$S_n = \frac{1}{2} n(n+1) \quad (2)$$

OBSERVATION

For $n = 4$, S_n = ...

For $n = 12$, S_n = ...

For $n = 50$, S_n = ...

For $n = 100$, S_n = ...

APPLICATION

Result (2) may be used to find the sum of first n terms of the list of numbers:

1. $1^2, 2^2, 3^2, ...$
2. $1^3, 2^3, 3^3, ...$

   to be studied in Class XI.