

Activity 10

OBJECTIVE

To verify the algebraic identity :

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

MATERIAL REQUIRED

Acrylic sheet, sketch pen, glazed papers, scissors, adhesive, cello-tape, coloured papers, cutter.

METHOD OF CONSTRUCTION

1. Make a cuboid of dimensions $(a-b) \times a \times a$ ($b < a$), using acrylic sheet and cello-tape/adhesive as shown in Fig. 1.
2. Make another cuboid of dimensions $(a-b) \times a \times b$, using acrylic sheet and cello-tape/adhesive as shown in Fig. 2.
3. Make one more cuboid of dimensions $(a-b) \times b \times b$ as shown in Fig. 3.
4. Make a cube of dimensions $b \times b \times b$ using acrylic sheet as shown in Fig. 4.

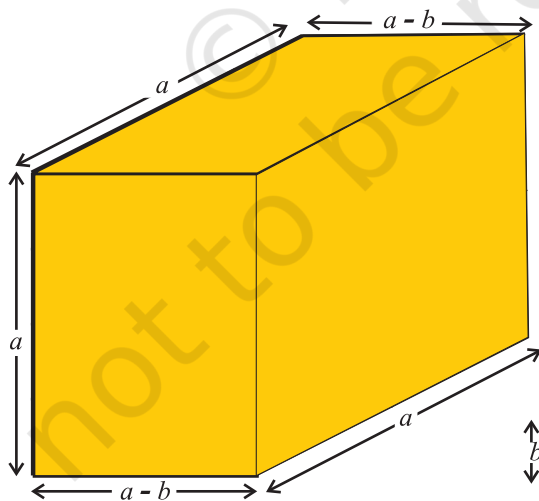


Fig. 1

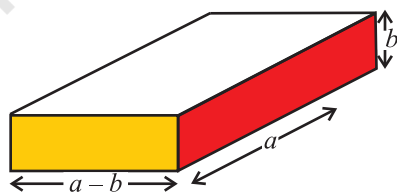


Fig. 2

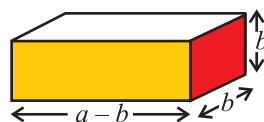


Fig. 3

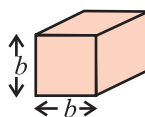


Fig. 4

5. Arrange the cubes and cuboids made above in Steps (1), (2), (3) and (4) to obtain a solid as shown in Fig. 5, which is a cube of volume a^3 cubic units.

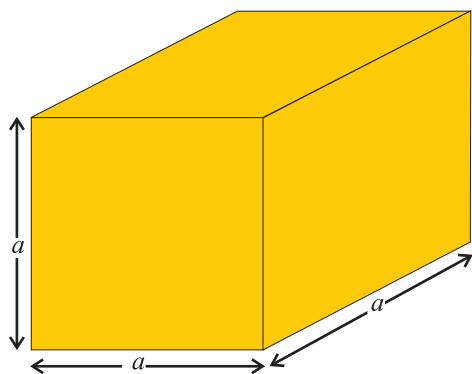


Fig. 5

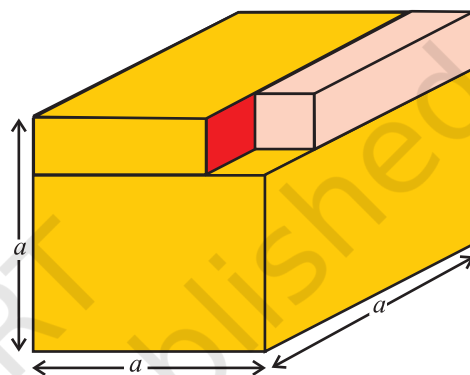


Fig. 6

DEMONSTRATION

Volume of cuboid in Fig. 1 = $(a-b) \times a \times a$ cubic units.

Volume of cuboid in Fig. 2 = $(a-b) \times a \times b$ cubic units.

Volume of cuboid in Fig. 3 = $(a-b) \times b \times b$ cubic units.

Volume of cube in Fig. 4 = b^3 cubic units.

Volume of solid in Fig. 5 = a^3 cubic units.

Removing a cube of size b^3 cubic units from the solid in Fig. 5, we obtain a solid as shown in Fig. 6.

$$\begin{aligned}\text{Volume of solid in Fig. 6} &= (a-b) a^2 + (a-b) ab + (a-b) b^2 \\ &= (a-b) (a^2 + ab + b^2)\end{aligned}$$

$$\text{Therefore, } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

OBSERVATION

On actual measurement:

$$a = \dots\dots\dots, \quad b = \dots\dots\dots,$$

$$\text{So, } a^3 = \dots\dots\dots, \quad b^3 = \dots\dots\dots, \quad (a-b) = \dots\dots\dots, \quad ab = \dots\dots\dots,$$

$$a^2 = \dots\dots\dots, \quad b^2 = \dots\dots\dots,$$

$$\text{Therefore, } a^3 - b^3 = (a - b) (a^2 + ab + b^2).$$

APPLICATION

The identity may be used in simplification/factorisation of algebraic expressions.