CSE708 Graph Theory: Assignment 1

ID: 20166051 Name: Shaily Roy
ID: 20366018 Name: Sajjad Ahmed

08/08/2020

1 Question

You will be given a graph $G = (V, E)$. You will need to partition the vertices of V into two independent sets A and B such that $A \cap B = \emptyset$ and $A \cup B = V$. What it means is that A will have no internal edges and B will have no internal edges. So, one needs to partition V into A and B and erase edges inside A and B. Then count the remaining edges. But we are interested in such a partitioning so that the number of remaining edges is at least $\frac{|e|}{2}$.

2 Proof

At first we start with any partition of $V(G)$ into two sets X, Y. Using the edges having one endpoint in each set yields a bipartite subgraph H with partitions X and Y.

If H contains less than $\frac{e(G)}{2}$ then the vertex in H has more vertices(v) in its own class than the other. As our goal of the bipartite graph that there can not be internal edges, we have to decrease the internal edges by moving some of the vertices to the other class.

Thus, we move vertices of G to the other class so that the other class gains and it loses. We move each vertex in this way from X to Y until it terminates. To explain, the algorithm only stops when

$$X(v) >= Y(v) \tag{1}$$

where X(v) means the number of neighbors of X in the opposite set and Y(v) means the number of neighbors in the same set.

$$since, X(v) >= Y(v) \tag{2}$$

$$so, X(v) + X(v) >= Y(v) + X(v) \tag{3}$$

However,here we can see that X(v) + Y(v) is the degree of v in the old graph H and X(v) is the degree of v in the new graph G. Therefore, it can be said that the degree of the new graph should be at least the half of the whole graph. So we can write

$$d_H(v) \geq \frac{d_G(v)}{2} \tag{4}$$

As we know from handshaking lemma:

$$2e_G = \sum_{i=1}^{n} d_G(V_i) \tag{5}$$

Now applying this handshaking lemma on eq.4, we get:

$$2e_H \geq \frac{1}{2} \times 2e_G \tag{6}$$

$$2e_H \geq 2e_G \tag{7}$$

$$e_H \geq \frac{e_G}{2} \tag{8}$$

Here we tried to partition the vertices of V into two independent sets A and B such that $A \cap B = \emptyset$ and $A \cup B = V$ which implies the definition of Bipartition of a graph.

Therefore, from the above discussion, it is proved that if we create a new bipartite sub-graph from a given loopless graph, then there should be at least $\frac{e_G}{2}$ edges.

[In our algorithm, we have assumed that all the vertex will start from 1 and end to V and there is not loop]