



# BRAC UNIVERSITY

## CSE 330: Numerical Methods (LAB)

### LAB 7: Numerical Differentiation

We are familiar with the analytical method of finding the derivative of a function when the functional relation between the dependent variable  $y$  and the independent variable  $x$  is known. However, in practice, most often functions are defined only by tabulated data or the values of  $y$  for specified values of  $x$  can be found experimentally. Also in some cases, it is not possible to find the derivative of a function by analytical method. The process of calculating the derivatives of a function by means of a set of given values of that function is called numerical differentiation.

#### Forward Difference Formula:

All numerical differentiation are done by expansion of Taylor series

$$f(x+h) = f(x) + f'(x)h + f''(x)h^2/2 + f'''(x)h^3/6 + \dots \quad (1)$$

From (1)

$$f'(x) = (f(x+h) - f(x)) / h + O(h) \quad (2)$$

Where,  $O(h)$  is the truncation error, which consists of terms containing  $h$  and higher order terms of  $h$ .

#### Central Difference Formula:

$$f(x+h) = f(x) + f'(x)h + f''(x)h^2/2 + f'''(x)h^3/6 + \dots \quad (3)$$

$$f(x-h) = f(x) - f'(x)h + f''(x)h^2/2 - f'''(x)h^3/6 + \dots \quad (4)$$

Using (3) and (4)

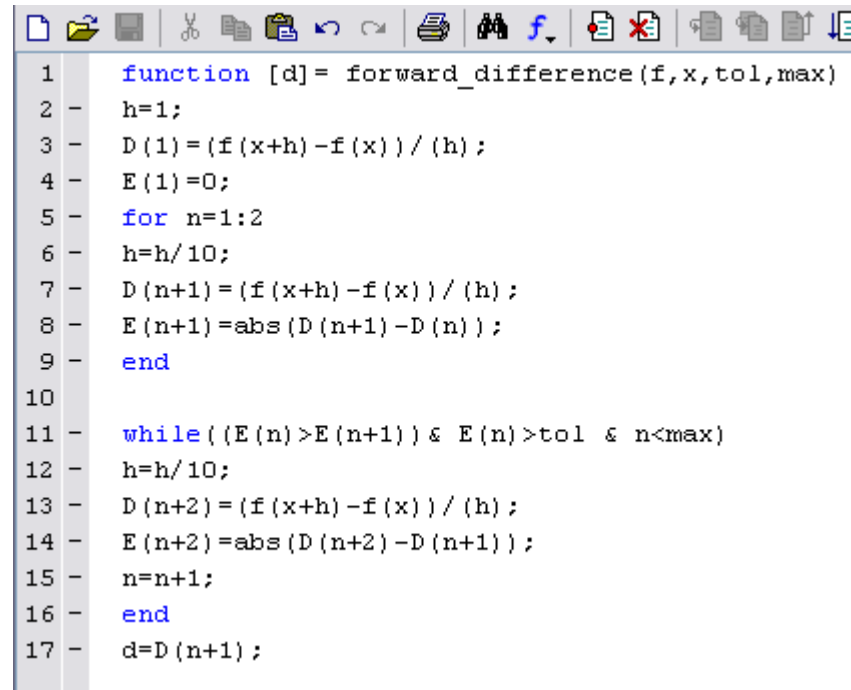
$$f'(x) = (f(x+h) - f(x-h)) / 2h + O(h^2) \quad (5)$$

Where,  $O(h^2)$  is the truncation error, which consists of terms containing  $h^2$  and higher order terms of  $h$ .

**Example 1:** Given  $f(x) = e^x$ , find  $f'(1)$  with forward difference formula using  $h = 10^{-1}$ ,  $10^{-2}$ ...upto  $10^{-10}$ .

Ans: 2.7183

**Solution:**



```

1  function [d]= forward_difference(f,x,tol,max)
2  - h=1;
3  - D(1)=(f(x+h)-f(x))/(h);
4  - E(1)=0;
5  - for n=1:2
6  - h=h/10;
7  - D(n+1)=(f(x+h)-f(x))/(h);
8  - E(n+1)=abs(D(n+1)-D(n));
9  - end
10
11 - while (E(n)>E(n+1)) & E(n)>tol & n<max
12 - h=h/10;
13 - D(n+2)=(f(x+h)-f(x))/(h);
14 - E(n+2)=abs(D(n+2)-D(n+1));
15 - n=n+1;
16 - end
17 - d=D(n+1);

```

**Command Window**

```

>> f=@(x) exp(x);
[d]=forward_difference(f,1,.0001,12)

d =

    2.7183

```

**Lab Task 1:** : Given  $f(x) = x^2 + 5$ , find  $f'(2)$  with central difference formula using  $h = 10^{-1}$ ,  $10^{-2}$ ...upto  $10^{-15}$ .

Ans: 4.00

**Numerical Differentiation with built-in function: “diff”**

MATLAB software has the ability to determine the derivatives of data based on built-in function **diff**. When it is passed a one-dimensional vector of length  $n$ , the **diff** function returns a vector of length  $n-1$  containing the differences between adjacent elements. As described in the following example, these can be employed to determine finite-difference approximations of first derivatives.

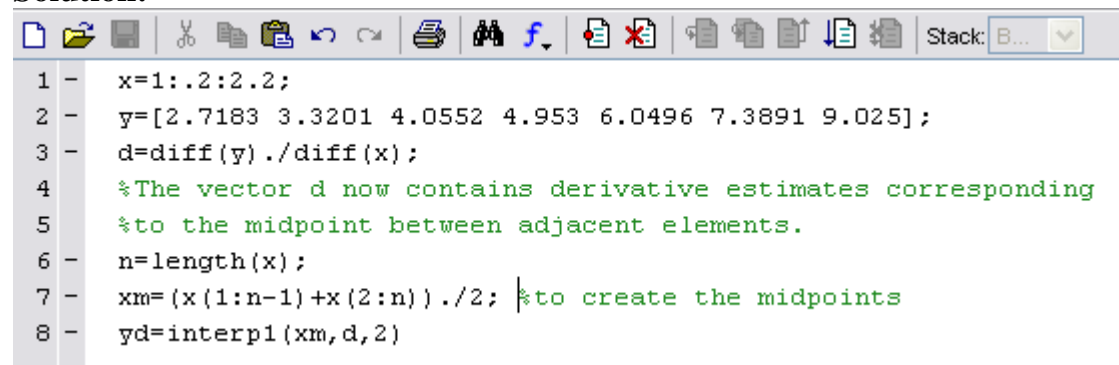
**Example 2:** From the following table of values of  $x$  and  $y$ , obtain

$$\frac{dy}{dx} \quad \text{for } x = 2.0$$

$x$	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$y$	2.7183	3.3201	4.0552	4.953	6.0496	7.3891	9.025

Ans: 7.4385

**Solution:**



```

1 - x=1:.2:2.2;
2 - y=[2.7183 3.3201 4.0552 4.953 6.0496 7.3891 9.025];
3 - d=diff(y)./diff(x);
4 - %The vector d now contains derivative estimates corresponding
5 - %to the midpoint between adjacent elements.
6 - n=length(x);
7 - xm=(x(1:n-1)+x(2:n))./2; %to create the midpoints
8 - yd=interp1(xm,d,2)

```

**Lab Task 2:** In the above example, find

$$\frac{dy}{dx} \quad \text{for } x = 1.2$$

Ans: 3.3423

Bonus mark will be given if you can find

$$\frac{d^2y}{dx^2} \quad \text{for } x = 1.2$$

Ans: 3.3325

**Home Task:**

Given  $f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$ . Find  $f'(0)$  with

- [1]. Forward difference method
- [2]. Central difference method
- [3]. Built-in function **diff**

Compare the results and determine which method is more accurate and appropriate with proper explanations.