BRAC UNIVERSITY
CSE 330: Numerical Methods (LAB)
LAB 7: Numerical Differentiation

We are familiar with the analytical method of finding the derivative of a function when the functional relation between the dependent variable y and the independent variable x is known. However, in practice, most often functions are defined only by tabulated data or the values of y for specified values of x can be found experimentally. Also in some cases, it is not possible to find the derivative of a function by analytical method. The process of calculating the derivatives of a function by means of a set of given values of that function is called numerical differentiation.

Forward Difference Formula:

All numerical differentiation are done by expansion of Taylor series

$$f(x+h) = f(x) + f'(x)h + f''(x)h^2/2 + f'''(x)h^3/6 + \ldots\ldots\ldots\ldots \quad (1)$$

From (1)

$$f'(x) = (f(x+h) - f(x))/h + O(h) \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Where, O(h) is the truncation error, which consists of terms containing h and higher order terms of h.

Central Difference Formula:

$$f(x+h) = f(x) + f'(x)h + f''(x)h^2/2 + f'''(x)h^3/6 + \ldots\ldots\ldots\ldots \quad (3)$$

$$f(x-h) = f(x) - f'(x)h + f''(x)h^2/2 - f'''(x)h^3/6 + \ldots\ldots\ldots\ldots \quad (4)$$

Using (3) and (4)

$$f'(x) = (f(x+h) - f(x-h))/2h + O(h^2) \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Where, $O(h^2)$ is the truncation error, which consists of terms containing h^2 and higher order terms of h.

Example 1: Given f(x) = e^x, find $f'(1)$ with forward difference formula using h =10^{-1}, 10^{-2}…upto 10^{-10}.

Ans: 2.7183

Solution:

```
1   function [d]= forward_difference(f,x,tol,max)
2 -  h=1;
3 -  D(1)=(f(x+h)-f(x))/(h);
4 -  E(1)=0;
5 -  for n=1:2
6 -  h=h/10;
7 -  D(n+1)=(f(x+h)-f(x))/(h);
8 -  E(n+1)=abs(D(n+1)-D(n));
9 -  end
10
11 -  while((E(n)>E(n+1))& E(n)>tol & n<max)
12 -  h=h/10;
13 -  D(n+2)=(f(x+h)-f(x))/(h);
14 -  E(n+2)=abs(D(n+2)-D(n+1));
15 -  n=n+1;
16 -  end
17 -  d=D(n+1);
```

Command Window

```
>> f=@(x) exp(x);
[d]=forward_difference(f,1,.0001,12)

d =

    2.7183
```

Lab Task 1: : Given f(x) = x^2 + 5, find $f'(2)$ with central difference formula using h =10^{-1}, 10^{-2}…upto 10^{-15}.

Ans: 4.00

Numerical Differentiation with built-in function: "diff **"**

MATLAB software has the ability to determine the derivatives of data based on built-in function **diff.** When it is passed a one-dimensional vector of length *n*, the **diff** function returns a vector of length *n-1* containing the differences between adjacent elements. As described in the following example, these can be employed to determine finite-difference approximations of first derivatives.

Example 2: From the following table of values of x and y, obtain

$$\frac{dy}{dx} \quad for \quad x = 2.0$$

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.953	6.0496	7.3891	9.025

Ans: 7.4385

Solution:

```
1 -  x=1:.2:2.2;
2 -  y=[2.7183 3.3201 4.0552 4.953 6.0496 7.3891 9.025];
3 -  d=diff(y)./diff(x);
4    %The vector d now contains derivative estimates corresponding
5    %to the midpoint between adjacent elements.
6 -  n=length(x);
7 -  xm=(x(1:n-1)+x(2:n))./2; %to create the midpoints
8 -  yd=interp1(xm,d,2)
```

Lab Task 2: In the above example, find

$$\frac{dy}{dx} \quad for \quad x = 1.2$$ Ans: 3.3423

Bonus mark will be given if you can find

$$\frac{d^2y}{dx^2} \quad for \quad x = 1.2$$ Ans: 3.3325

Home Task:

Given f(x) = 0.2 + 25x -200x^2 + 675x^3 – 900x^4 + 400x^5. Find $f'(0)$ with

[1]. Forward difference method
[2]. Central difference method
[3]. Built-in function **diff**

Compare the results and determine which method is more accurate and appropriate with proper explanations.