2D-Transformations

- Translation → t_x, t_y
- Scaling → S_x, S_y < max / min
- Rotation → θ < Clock-wise / Anti-clockwise
- Shearing
- Reflection

Translation
(changing the position)

$P(x, y)$ — original point
$P'(x', y')$ — Point after translation

t_x, t_y

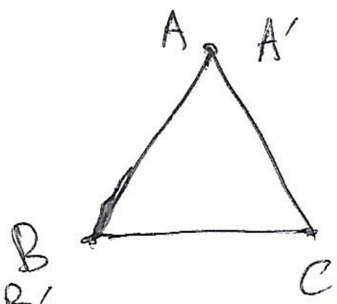
$$\boxed{\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}}$$

$$P' = P + T$$

$$[x' \; y'] = [x \; y] + [t_x \; t_y]$$

or

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$



Scaling (Resize the object)

S_x , S_y

→ If S_x & S_y in between 0 & 1
- Point is closer to origin.
- Size decreases.

→ If S_x & S_y are > 1
- Point is away from the origin.
- Size increases.

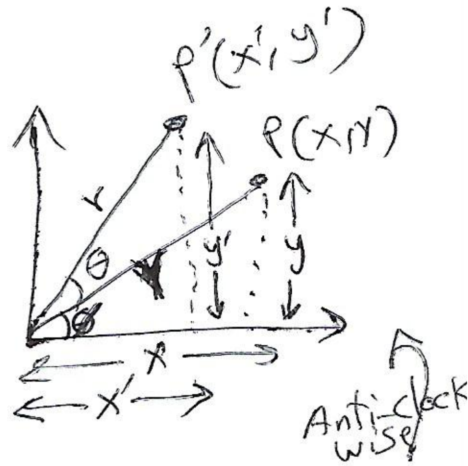
→ If S_x & S_y are equal
- Scaling will be done uniformly

lets consider $P(x, y)$ — before Scaling
$P'(x', y')$ — after scaling

$$\boxed{\begin{aligned} x' &= x \cdot S_x \\ y' &= y \cdot S_y \end{aligned}}$$

$$\boxed{\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}}$$

Rotation



New angle after rotation P to P' $= (\emptyset + \theta)$

$$\cos(\emptyset + \theta) = \frac{x'}{r}$$

$$x' = r\cos(\emptyset + \theta)$$

$$\sin(\emptyset + \theta) = \frac{y'}{r}$$

$$y' = r\sin(\emptyset + \theta)$$

$$\cos\emptyset = \frac{x}{r}$$

$$x = r\cos\emptyset$$

$$\sin\emptyset = \frac{y}{r}$$

$$y = r\sin\emptyset$$

we know that:

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

Therefore: $x' = r\cos\emptyset\cos\theta - r\sin\emptyset\sin\theta$

$$\therefore \boxed{x' = x\cos\theta - y\sin\theta}$$

Similarly $y' = r\sin\emptyset\cos\theta + r\cos\emptyset\sin\theta$

$$\therefore y' = y\cos\theta + x\sin\theta$$

$$\boxed{y' = x\sin\theta + y\cos\theta}$$

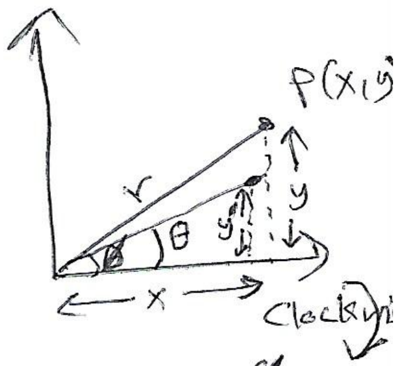
$$\boxed{[x' \; y'] = [x \; y]\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}}$$

Rotation

New angle $= (\phi - \theta)$

$x' = r\cos(\phi - \theta)$

$y' = r\sin(\phi - \theta)$



$x = r\cos\phi$

$y = r\sin\phi$

Then

$x' = \underbrace{r\cos\phi}\cos\theta + \underbrace{r\sin\phi}\sin\theta$

$$\boxed{x' = x\cos\theta + y\sin\theta}$$

$y' = \underbrace{r\sin\phi}\cos\theta - \underbrace{r\cos\phi}\sin\theta$

$$\boxed{y' = y\cos\theta - x\sin\theta}$$

$$\boxed{\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$$

Translation — t_x, t_y

Rotation — θ — Clock / Anti-clock

Scaling — S_y, S_y

$S_x \neq S_y$ — change in shape

$S_x = S_y$ — No change in shape

S_x & $S_y < 1$ — Size decreases

S_x & $S_y > 1$ — Size increases

~~Examples~~

Translation

$t_x, t_y, (x,y), (x', y')$

$x' = x + t_x$

$y' = y + t_y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

Example: Square $(0,0), (2,0), (0,2), (2,2)$

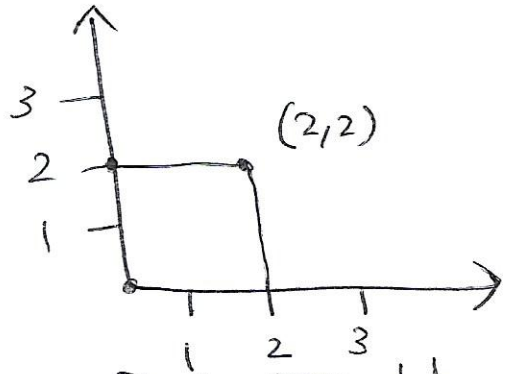
$t_x = 2, t_y = 3$.

Ans: $(0,0)$

$x' = 0 + 2 = 2$

$y' = 0 + 3 = 3$

$\boxed{(0,0) \rightarrow (2,3)}$



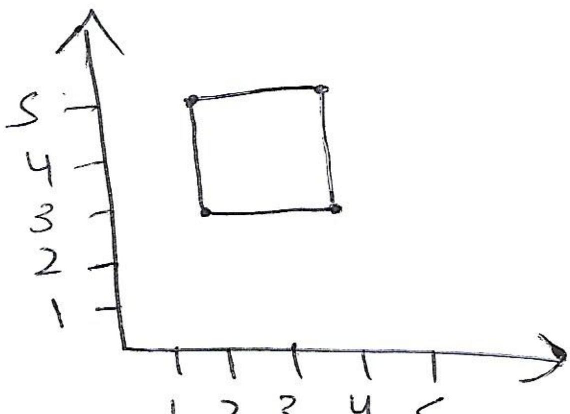
Before Traslation

$(2,0)$

$x' = 2 + 2 = 4$

$y' = 0 + 3 = 3$

$\boxed{(2,0) \rightarrow (4,3)}$



After Translation

$(0,2)$

$x' = 0 + 2 = 2$

$y' = 2 + 3 = 5$

$\boxed{(0,2) \rightarrow (2,5)}$

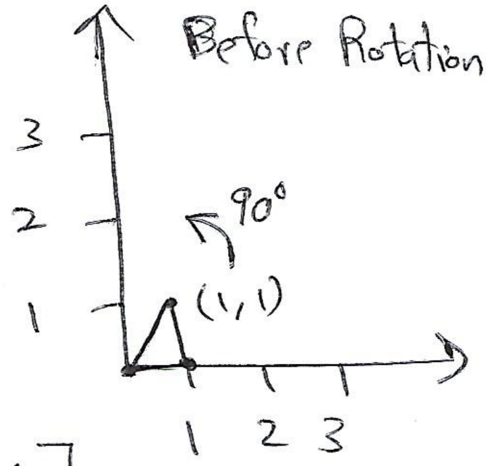
$(2,2)$

$x' = 2 + 2 = 4$

$y' = 2 + 3 = 5$

$\boxed{(2,2) \rightarrow (4,5)}$

Example on Rotation:



Traingle

(0,0) (1,0) (1,1)

$\theta = 90^\circ$ (Anti-Clock)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{\begin{matrix} \sin 90^\circ = 1 \\ \cos 90^\circ = 0 \end{matrix}}$$

<u>(0,0)</u>

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boxed{(0,0) \longrightarrow (0,0)}$$

<u>(1,0)</u>

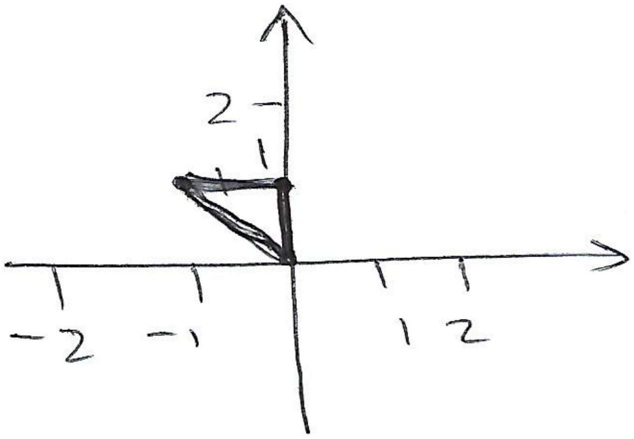
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boxed{(1,0) \longrightarrow (0,1)}$$

<u>(1,1)</u>

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\boxed{(1,1) \longrightarrow (-1,1)}$$



the rotation

what about if we are doing ~~[scribble]~~ clock wise?

Scaling Example:

$S_x, S_y, (x, y), (x', y')$

$x' = x \cdot S_x$

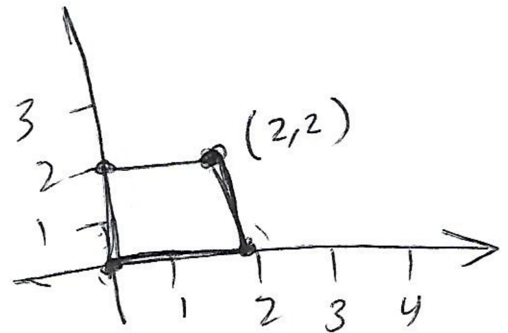
$y' = y \cdot S_y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Q: Square

(0,0) (2,0) (0,2) (2,2)

$S_x = 2$, $S_y = 3$. Compute the Scaling and draw it in the 2D spatial domain.



$\underline{(0,0)}$

$x' = 0 \times 2 = 0$

$y' = 0 \times 3 = 0$

$\boxed{(0,0) \longrightarrow (0,0)}$

$\underline{(2,0)}$

$x' = 2 \times 2 = 4$

$y' = 0 \times 3 = 0$

$\boxed{(2,0) \longrightarrow (4,0)}$

$\underline{(0,2)}$

$x' = 0 \times 2 = 0$

$y' = 2 \times 3 = 6$

$\boxed{(0,2) \longrightarrow (0,6)}$

$\underline{(2,2)}$

$x' = 2 \times 2 = 4$

$y' = 2 \times 3 = 6$

$\boxed{(2,2) \longrightarrow (4,6)}$