Ex 1 (4 pts). Logistic Regression

$k=2$; $w = \begin{bmatrix} 0 & 0 \end{bmatrix}$; $b = 0$

$\dfrac{y_i \, GPA_i}{1 + e^{y_i w x_i}}$: for $x = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $\dfrac{(1)(1)}{1 + e^0} = 0.5$

for $x = \begin{bmatrix} 0.9 & 1 \end{bmatrix}$, $\dfrac{(1)(0.9)}{1 + e^0} = 0.45$

for $x = \begin{bmatrix} 0.9 & 0.875 \end{bmatrix}$, $\dfrac{(1)(0.9)}{1 + e^0} = 0.45$

for $x = \begin{bmatrix} 0.7 & 0.75 \end{bmatrix}$, $\dfrac{(-1)(0.7)}{1 + e^0} = -0.35$

for $x = \begin{bmatrix} 0.6 & 0.875 \end{bmatrix}$, $\dfrac{(-1)(0.6)}{1 + e^0} = -0.3$

$\dfrac{y_i \, GRE_i}{1 + e^{y_i w x_i}}$: for $x = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $\dfrac{(1)(1)}{1 + e^0} = 0.5$

for $x = \begin{bmatrix} 0.9 & 1 \end{bmatrix}$, $\dfrac{(1)(1)}{1 + e^0} = 0.5$

for $x = \begin{bmatrix} 0.9 & 0.875 \end{bmatrix}$, $\dfrac{(1)(0.875)}{1 + e^0} = 0.4375$

for $x = \begin{bmatrix} 0.7 & 0.75 \end{bmatrix}$, $\dfrac{(-1)(0.75)}{1 + e^0} = -0.375$

for $x = \begin{bmatrix} 0.6 & 0.875 \end{bmatrix}$, $\dfrac{(-1)(0.875)}{1 + e^0} = -0.4375$

$E_i = \ln\left(1 + e^{-y_i w x_i}\right):$

for $x = [1 \quad 1]$, $E = \ln\left(1 + e^{(-1)(0.1)}\right) = 0.693$

$x = [0.9 \quad 1]$, $E = 0.693$

$x = [0.9 \quad 0.875]$, $E = 0.693$

$x = [0.7 \quad 0.75]$, $E = 0.693$

$x = [0.6 \quad 0.875]$, $E = 0.693$

$\dfrac{y_i \, Dummy_i}{1 + e^{y_i w x_i}}$: for $y=1, x = [1 \quad 1]$, $\dfrac{(1)(1)}{1+1} = 0.5$

$y=1; x[0.9 \quad 1]$, $= 0.5$

$y=1; x[0.9 \quad 0.875]$, $= 0.5$

$y=-1; x = [0.7 \quad 0.75]$, $= -0.5$

$y=-1; x=[0.6 \quad 0.875]$, $= -0.5$

$w_1 = \text{mean}\left(\dfrac{y\,GPA}{1+e^{ywx}}\right) = \dfrac{0.5 + 0.45 + 0.45 - 0.35 - 0.3}{5} = \boxed{0.15}$

$w_2 = \text{mean}\left(\dfrac{y\,GRE}{1+e^{ywx}}\right) = \dfrac{0.5 + 0.5 + 0.4375 - 0.375 - 0.4375}{5} = \boxed{0.125}$

$b = \text{mean}\left(\dfrac{y\,Dummy}{1+e^{ywx}}\right) = \dfrac{0.5 + 0.5 + 0.5 - 0.5 - 0.5}{5} = \boxed{0.1}$

	$\dfrac{y_i\, GPA_i}{1+e^{y_i w x_i}}$	$\dfrac{y_i\, GRE_i}{1+e^{y_i w x_i}}$	$\dfrac{y_i\, Dummy_i}{1+e^{y_i w x_i}}$	$E = \ln\left(1+e^{-y_i w x_i}\right)$
$x_i = \begin{bmatrix} 1 & 1 \end{bmatrix}$	0.5	0.5	0.5	0.693
$x_i = \begin{bmatrix} 0.9 & 1 \end{bmatrix}$	0.45	0.5	0.5	0.693
$x_i = \begin{bmatrix} 0.9 & 0.875 \end{bmatrix}$	0.45	0.4375	0.5	0.693
$x_i = \begin{bmatrix} 0.7 & 0.75 \end{bmatrix}$	-0.375	-0.375	-0.5	0.693
$x_i = \begin{bmatrix} 0.6 & 0.875 \end{bmatrix}$	-0.3	-0.4375	-0.5	0.693

$$W^i_u = W^i + W^i_{j}\, k$$

$$\Rightarrow W^1_u = 0 + (0.15)(2) = 0.3$$

$$W^2_u = 0 + (0.125)(2) = 0.25$$

$$b_u = 0 + (0.1)(2) = 0.2$$

after 1st iteration:

$$W = \begin{bmatrix} 0.3 & 0.25 \end{bmatrix}$$

$$b = 0.2$$