1 Item 1

1.1 Algorithm

Step 1: Let B be the number of bootstrap samples taken. With $n = 15$, do SRSWR from schools 1 to 15.

$$\begin{aligned} B_1^* &= \{(X_{11}^*, Y_{11}^*), (X_{21}^*, Y_{21}^*), \ldots, (X_{n1}^*, Y_{n1}^*)\} \\ B_2^* &= \{(X_{12}^*, Y_{12}^*), (X_{22}^*, Y_{22}^*), \ldots, (X_{n2}^*, Y_{n2}^*)\} \\ \vdots &= \vdots \\ B_B^* &= \{(X_{1B}^*, Y_{1B}^*), (X_{2B}^*, Y_{2B}^*), \ldots, (X_{nB}^*, Y_{nB}^*)\} \end{aligned} \tag{1.1}$$

Step 2: Let $S_{x_b}^*$ and $S_{y_b}^*$ be the standard deviations of the variables, X_b^* and Y_b^*, respectively, where $b = \{1, 2, \ldots, B\}$. Calculate the pearson product coefficient of correlation, r_b^*

$$r_b^* = \frac{\frac{1}{n-1} \sum \left(X_{ib}^* - \bar{X}_b^*\right)\left(Y_{ib}^* - \bar{Y}_b^*\right)}{S_{x_b}^* S_{y_b}^*} \tag{1.2}$$

to yield

$$r = \{r_1^*, r_2^*, \ldots, r_B^*\} \tag{1.3}$$

Step 3: Calculate $\widehat{se}(r)$ using

$$\widehat{se}(r) = \sqrt{\frac{\sum_{b=1}^{B} \left(r_i^* - \bar{r}^*\right)^2}{B-1}} \tag{1.4}$$

1.2 Algorithm implementation

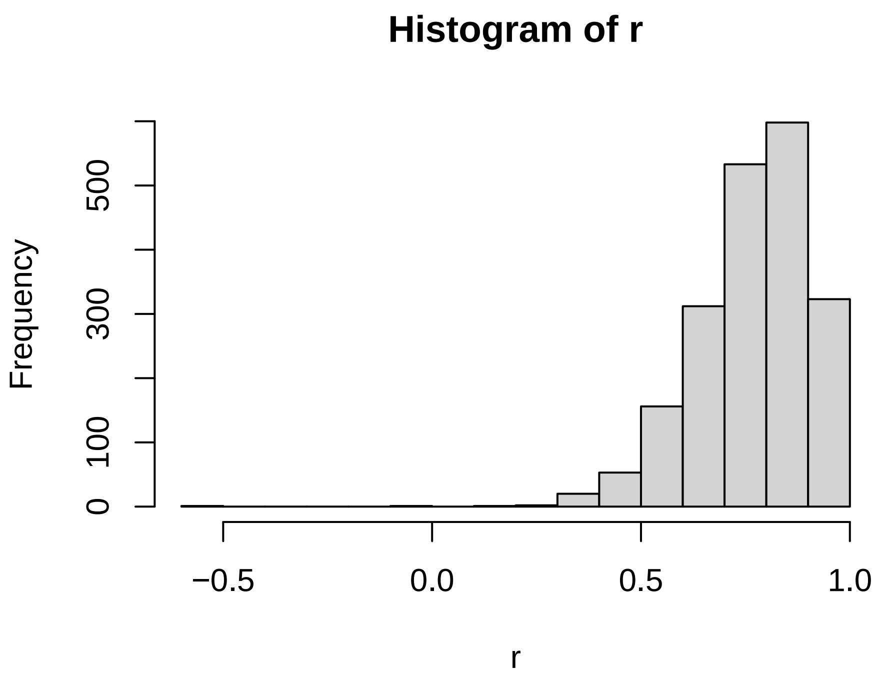
```
# set seed for reproducibility
set.seed(7)

r <-
  # Step 2: take the pearson correlation for each bootstrap sample
  sapply(
    # Step 1: 2000 bootstrap samples (with replacement) of size 15
    lapply(
      1:2000,
      function(x){ law_school_data[sample(law_school_data$School,
                                          15,
                                          replace = TRUE),
                                   c(2,3)] }
    ),
    function(x){cor(x$LSAT,x$GPA, method = "pearson")}
  )

# Step 3: take the bootstrap estimate of the standard error
se_r_boot <- sd(r)

# take the percentile 95% CI for r
ci_r_boot <- c(quantile(r,.025),quantile(r,.975))
```

i. bootstrap estimate of the standard error of r: 0.1369
ii. 95% confidence interval for ρ (the true population correlation): (0.451, 0.9625)
iii. a histogram showing the bootstrap distribution of the correlation r



1.3 Change maximum r_b^*

```
source("../R/s02_i01_bs_sampling.R")

r_max <- max(r)

r_replaced <- replace(r, r==r_max, 100*r_max)
r_replaced_max <- max(r_replaced)
se_r_replaced_boot <- sd(r_replaced)

se_percent_change <- ((se_r_replaced_boot - se_r_boot)/se_r_boot)*100
```

The original maximum of r_b^*'s is 0.9937 while the new one is 99.3656. Calculating the new $\widehat{se}(r)$ gives the value 2.209. This meant an increase of 1514.0138% compared to the original value.

2 Item 2

2.1 Fill in the table

```
    5%    10%    15%    20%    50%    70%    85%    90%    95% 
0.5244 0.5859 0.6224 0.6588 0.7878 0.8517 0.9044 0.9228 0.9472 
[1] 0.9937
[[1]]
   95% 
```

```
0.3477

[[2]]
   90%
0.2159

[[3]]
   85%
0.1461

[[1]]
   95%
0.3477

[[2]]
   90%
0.2159

[[3]]
   85%
0.1461

[1] 99.37
```

2.2 Compute $\tilde{se}_\alpha(r)$

2.3 Change maximum r_b^* and recompute

3 Item 3

References

Fox, J.-P., & Marianti, S. (2017). Person-fit statistics for joint models for accuracy and speed. *Journal of Educational Measurement*.