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Abstract

Contents

1 Methodology

1.1 Parametric bootstrap

Algorithm 1 Computation of Joint Confidence Region via Parametric Bootstrap

1: **for** $b = 1, 2, \ldots, B$ **do**

2: Generate $\hat{\theta}^*_{bi} \sim N\left(\hat{\theta}_i, \sigma_i^2\right)$, $i = 1, 2, \ldots, K$ and let $\hat{\theta}_{b(1)}, \hat{\theta}_{b(2)}, \ldots, \hat{\theta}_{b(K)}$ be the corresponding ordered values

	$k=1$	$k=2$	$\ldots$	$k=K$
$b=1$	$\hat{\theta}^*_{1(1)}$	$\hat{\theta}^*_{1(2)}$	$\ldots$	$\hat{\theta}^*_{1(K)}$
$b=2$	$\hat{\theta}^*_{2(1)}$	$\hat{\theta}^*_{2(2)}$	$\ldots$	$\hat{\theta}^*_{2(K)}$
$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$
$b=B$	$\hat{\theta}^*_{B(1)}$	$\hat{\theta}^*_{B(2)}$	$\ldots$	$\hat{\theta}^*_{B(K)}$

3: Compute

$$\hat{\sigma}^*_{b(k)} = \sqrt{\text{kth ordered value among } \left\{\hat{\theta}^{*2}_{b1} + \sigma_1^2, \hat{\theta}^{*2}_{b2} + \sigma_2^2, \ldots, \hat{\theta}^{*2}_{bK} + \sigma_K^2\right\} - \hat{\theta}^{*2}_{(k)}}$$

4: Compute $t^*_b = \max\limits_{1 \le k \le K} \left| \frac{\hat{\theta}^*_{b(k)} - \hat{\theta}^*_k}{\sigma^*_{b(k)}} \right|$

5: **end for**

6: Compute the $(1-\alpha)$-sample quantile of $t^*_1, t^*_2, \ldots, t^*_B$, call this $\hat{t}$.

7: The joint confidence region of $\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}$ is given by

$$\mathfrak{R} = \left[\hat{\theta}_{(1)} \pm \hat{t} \times \hat{\sigma}_{(1)}\right] \times \left[\hat{\theta}_{(2)} \pm \hat{t} \times \hat{\sigma}_{(2)}\right] \times \cdots \times \left[\hat{\theta}_{(K)} \pm \hat{t} \times \hat{\sigma}_{(K)}\right]$$

where $\hat{\sigma}_{(k)}$ is computed as

$$\hat{\sigma}_{(k)} = \sqrt{\text{kth ordered value among } \left\{\hat{\theta}_1^2 + \sigma_1^2, \hat{\theta}_2^2 + \sigma_2^2, \ldots, \hat{\theta}_K^2 + \sigma_K^2\right\} - \hat{\theta}^2_{(k)}}$$

Algorithm 2 Computation of Coverage Probability for Parametric Bootstrap

For given values of $\theta_1, \theta_2, \ldots, \theta_K$ and thus $\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}$

1: **for** replications $= 1, 2, \ldots, 5000$ **do**

2: Generate $\hat{\theta}_i \sim N(\theta_i, \sigma_i^2)$, for $i = 1, 2, \ldots, K$

3: Compute the rectangular confidence region $\mathfrak{R}$ using Algorithm 1.

4: Check if $\left(\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}\right) \in \mathfrak{R}$ and compute $T = \prod_{k=1}^{K} \left|\Lambda_{Ok}\right|$.

5: **end for**

6: Compute the proportion of times that the condition in step 4 is satisfied and the average of T

1.2 Nonrank-based method

The nonrank-based method accounts for potential correlation among items being ranked. For this case, an exchangeable correlation, $\boldsymbol{\rho}$ (See Equation 1.1.), is assumed and used in the calculation of the variance covariance matrix (See Equation 1.2.).

$$\boldsymbol{\rho} = (1 - \rho)\, \mathbf{I}_K + \rho \mathbf{1}_K \mathbf{1}_K' \tag{1.1}$$

$$\boldsymbol{\Sigma} = \boldsymbol{\Delta}^{1/2} \boldsymbol{\rho} \boldsymbol{\Delta}^{1/2} \tag{1.2}$$

where $\boldsymbol{\Delta} = \operatorname{diag}\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2\}$, with known σ_k's and ρ is studied for $0.1, 0.5, 0.9$.

Algorithm 3 Computation of Joint Confidence Region via Nonrank-based Method

Let the data consist of $\hat{\theta}_1, \ldots, \hat{\theta}_K$ and suppose $\boldsymbol{\Sigma}$ is known

1: **for** $b = 1, 2, \ldots, B$ **do**

2: Generate $\hat{\boldsymbol{\theta}}_b^* \sim N_K\left(\hat{\boldsymbol{\theta}}, \boldsymbol{\Sigma}\right)$ and write $\hat{\boldsymbol{\theta}}_b^* = \left(\hat{\theta}_{b1}^*, \hat{\theta}_{b2}^*, \ldots, \hat{\theta}_{bK}^*\right)'$

3: Compute $t_b^* = \max\limits_{1 \le j \le K} \left| \frac{\hat{\theta}_{bj}^* - \hat{\theta}_j^*}{\sigma_j} \right|$

4: **end for**

5: Compute the $(1 - \alpha)$-sample quantile of $t_1^*, t_2^*, \ldots, t_B^*$, call this $\hat{t}$.

6: The joint confidence region of $\theta_1, \theta_2, \ldots, \theta_K$ is given by

$$\mathfrak{R} = \left[\hat{\theta}_1 \pm \hat{t} \times \sigma_1\right] \times \left[\hat{\theta}_2 \pm \hat{t} \times \sigma_2\right] \times \cdots \times \left[\hat{\theta}_K \pm \hat{t} \times \sigma_K\right]$$

Algorithm 4 Computation of Coverage Probability for Nonrank-based Method

For given values of $\theta_1, \theta_2, \ldots, \theta_K$ and $\mathbf{\Sigma}$

1: **for** replications $= 1, 2, \ldots, 5000$ **do**

2: Generate $\hat{\boldsymbol{\theta}} \sim N_K(\boldsymbol{\theta}, \mathbf{\Sigma})$

3: Compute the rectangular confidence region $\mathfrak{R}$ using Algorithm 3.

4: Check if $(\theta_1, \theta_2, \ldots, \theta_K) \in \mathfrak{R}$ and compute $T = \prod_{k=1}^{K} \left|\Lambda_{Ok}\right|$.

5: **end for**

6: Compute the proportion of times that the condition in step 4 is satisfied and the average of T

2 Introduction

2.1 Background of the Study

2.2 Statement of the Problem

2.3 Objective of the Study

2.4 Study Hypothesis

2.5 Significance of the Study

2.6 Scope and Limitation

2.7 Definition of Terms

3 Background

4 `{r, child = "child/rrl.Rmd"} #  #`

THIS IS Rizzo (2008) and Klein et al. (2020)

References

Klein, M., Wright, T., & Wieczorek, J. (2020). *A joint confidence region for an overall ranking of populations.*

Rizzo, M. (2008). *Statistical computing with r.*

Appendices