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by
Nmae

Abstract

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1 Methodology

Let $\theta_1, \theta_2, \dots, \theta_K$ be the true parameter values and $\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_K$ be the estimates obtained.

1.1 Parametric bootstrap

Let $\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_K$ be independent but not identically distributed estimates. For this study, it is assumed that $\hat{\theta}_k \sim N(\theta_k, \sigma_k^2)$, $k = 1, 2, \dots, K$, where σ_k^2 is known. Denote the corresponding ordered values by $\hat{\theta}_{(1)}, \hat{\theta}_{(2)}, \dots, \hat{\theta}_{(K)}$.

Algorithm 1 Computation of Joint Confidence Region via Parametric Bootstrap

1: **for** $b = 1, 2, \dots, B$ **do**

2: Generate $\hat{\theta}_{bk}^* \sim N(\hat{\theta}_k, \sigma_k^2)$, $i = 1, 2, \dots, K$ and let $\hat{\theta}_{b(1)}, \hat{\theta}_{b(2)}, \dots, \hat{\theta}_{b(K)}$ be the corresponding ordered values

	$k = 1$	$k = 2$	\dots	$k = K$
$b = 1$	$\hat{\theta}_{1(1)}^*$	$\hat{\theta}_{1(2)}^*$	\dots	$\hat{\theta}_{1(K)}^*$
$b = 2$	$\hat{\theta}_{2(1)}^*$	$\hat{\theta}_{2(2)}^*$	\dots	$\hat{\theta}_{2(K)}^*$
\vdots	\vdots	\vdots	\dots	\vdots
$b = B$	$\hat{\theta}_{B(1)}^*$	$\hat{\theta}_{B(2)}^*$	\dots	$\hat{\theta}_{B(K)}^*$

3: Compute

$$\hat{\sigma}_{b(k)}^* = \sqrt{\text{kth ordered value among } \{\hat{\theta}_{b1}^{*2} + \sigma_1^2, \hat{\theta}_{b2}^{*2} + \sigma_2^2, \dots, \hat{\theta}_{bK}^{*2} + \sigma_K^2\} - \hat{\theta}_{(k)}^{*2}}$$

4: Compute $t_b^* = \max_{1 \leq k \leq K} \left| \frac{\hat{\theta}_{b(k)}^* - \hat{\theta}_k^*}{\sigma_{b(k)}^*} \right|$

5: **end for**

6: Compute the $(1 - \alpha)$ -sample quantile of $t_1^*, t_2^*, \dots, t_B^*$, call this \hat{t} .

7: The joint confidence region of $\theta_{(1)}, \theta_{(2)}, \dots, \theta_{(K)}$ is given by

$$\mathfrak{R} = [\hat{\theta}_{(1)} \pm \hat{t} \times \hat{\sigma}_{(1)}] \times [\hat{\theta}_{(2)} \pm \hat{t} \times \hat{\sigma}_{(2)}] \times \dots \times [\hat{\theta}_{(K)} \pm \hat{t} \times \hat{\sigma}_{(K)}]$$

where $\hat{\sigma}_{(k)}$ is computed as

$$\hat{\sigma}_{(k)} = \sqrt{\text{kth ordered value among } \{\hat{\theta}_1^2 + \sigma_1^2, \hat{\theta}_2^2 + \sigma_2^2, \dots, \hat{\theta}_K^2 + \sigma_K^2\} - \hat{\theta}_{(k)}^2}$$

Algorithm 2 Computation of Coverage Probability for Parametric Bootstrap

For given values of $\theta_1, \theta_2, \dots, \theta_K$ and thus $\theta_{(1)}, \theta_{(2)}, \dots, \theta_{(K)}$

1: **for** replications = 1, 2, \dots , 5000 **do**

2: Generate $\hat{\theta}_k \sim N(\theta_k, \sigma_k^2)$, for $k = 1, 2, \dots, K$

3: Compute the rectangular confidence region \mathfrak{R} using Algorithm 1.

4: Check if $(\theta_{(1)}, \theta_{(2)}, \dots, \theta_{(K)}) \in \mathfrak{R}$ and compute

$$\begin{aligned} T_1 &= \frac{1}{K} \sum_{k=1}^K |\Lambda_{Ok}| \\ T_2 &= \prod_{k=1}^K |\Lambda_{Ok}| \\ T_3 &= 1 - \frac{K + \sum_{k=1}^K |\Lambda_{Ok}|}{K^2} \end{aligned}$$

5: **end for**

6: Compute the proportion of times that the condition in step 4 is satisfied and the average of T_1, T_2 , and T_3 .

1.2 Nonrank-based method

The nonrank-based method assumes that $\hat{\boldsymbol{\theta}} = (\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_K) \sim N(\boldsymbol{\theta}, \boldsymbol{\Sigma})$. It accounts for potential correlation among items being ranked. For this case, an exchangeable correlation, $\boldsymbol{\rho}$ (See Equation 1.1.), is assumed and used in the calculation of the variance covariance matrix (See Equation 1.2.).

$$\boldsymbol{\rho} = (1 - \rho) \mathbf{I}_K + \rho \mathbf{1}_K \mathbf{1}_K' \quad (1.1)$$

$$\boldsymbol{\Sigma} = \boldsymbol{\Delta}^{1/2} \boldsymbol{\rho} \boldsymbol{\Delta}^{1/2} \quad (1.2)$$

where $\boldsymbol{\Delta} = \text{diag}\{\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2\}$, with known σ_k 's and ρ is studied for 0.1, 0.5, 0.9.

Algorithm 3 Computation of Joint Confidence Region via Nonrank-based Method

Let the data consist of $\hat{\theta}_1, \dots, \hat{\theta}_K$ and suppose Σ is known

- 1: **for** $b = 1, 2, \dots, B$ **do**
- 2: Generate $\hat{\theta}_b^* \sim N_K(\hat{\theta}, \Sigma)$ and write $\hat{\theta}_b^* = (\hat{\theta}_{b1}^*, \hat{\theta}_{b2}^*, \dots, \hat{\theta}_{bK}^*)'$
- 3: Compute $t_b^* = \max_{1 \leq j \leq K} \left| \frac{\hat{\theta}_{bj}^* - \hat{\theta}_j}{\sigma_j} \right|$
- 4: **end for**
- 5: Compute the $(1 - \alpha)$ -sample quantile of $t_1^*, t_2^*, \dots, t_B^*$, call this \hat{t} .
- 6: The joint confidence region of $\theta_1, \theta_2, \dots, \theta_K$ is given by

$$\mathfrak{R} = [\hat{\theta}_1 \pm \hat{t} \times \sigma_1] \times [\hat{\theta}_2 \pm \hat{t} \times \sigma_2] \times \dots \times [\hat{\theta}_K \pm \hat{t} \times \sigma_K]$$

Algorithm 4 Computation of Coverage Probability for Nonrank-based Method

For given values of $\theta_1, \theta_2, \dots, \theta_K$ and Σ

- 1: **for** replications = 1, 2, \dots , 5000 **do**
 - 2: Generate $\hat{\theta} \sim N_K(\theta, \Sigma)$
 - 3: Compute the rectangular confidence region \mathfrak{R} using Algorithm 3.
 - 4: Check if $(\theta_1, \theta_2, \dots, \theta_K) \in \mathfrak{R}$ and compute T_1, T_2 , and T_3 .
 - 5: **end for**
 - 6: Compute the proportion of times that the condition in step 4 is satisfied and the average of T_1, T_2 , and T_3 .
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2 Introduction

2.1 Background of the Study

2.2 Statement of the Problem

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2.5 Significance of the Study

2.6 Scope and Limitation

2.7 Definition of Terms

3 Background

4 {r, child = "child/rrl.Rmd"} #

THIS IS Rizzo (2008) and Klein et al. (2020)

References

Klein, M., Wright, T., & Wieczorek, J. (2020). *A joint confidence region for an overall ranking of populations.*

Rizzo, M. (2008). *Statistical computing with r.*

Appendices