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Abstract

Contents

1 Introduction

2 Background

3 Methodology

3.1 Parametric bootstrap

Algorithm 1 Computation of Joint Confidence Region via Parametric Bootstrap

for $b = 1, 2, \dots, B$ **do**

Generate $\hat{\theta}^*_{bi} \sim N\left(\hat{\theta}_i, \sigma_i^2\right)$, $i = 1, 2, \dots, K$ and let $\hat{\theta}_{b(1)}, \hat{\theta}_{b(2)}, \dots, \hat{\theta}_{b(K)}$ be the corresponding ordered values

	$k = 1$	$k = 2$	$\dots$	$k = K$
$b = 1$	$\hat{\theta}^*_{1(1)}$	$\hat{\theta}^*_{1(2)}$	$\dots$	$\hat{\theta}^*_{1(K)}$
$b = 2$	$\hat{\theta}^*_{2(1)}$	$\hat{\theta}^*_{2(2)}$	$\dots$	$\hat{\theta}^*_{2(K)}$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
$b = B$	$\hat{\theta}^*_{B(1)}$	$\hat{\theta}^*_{B(2)}$	$\dots$	$\hat{\theta}^*_{B(K)}$

Compute

$$\hat{\sigma}^*_{b(k)} = \sqrt{\text{kth ordered value among } \left\{\hat{\theta}^{*2}_{b1} + \sigma_1^2, \hat{\theta}^{*2}_{b2} + \sigma_2^2, \dots, \hat{\theta}^{*2}_{bK} + \sigma_K^2\right\} - \hat{\theta}^{*2}_{(k)}}$$

Compute $t^*_b = \max_{1 \le k \le K} \left| \frac{\hat{\theta}^*_{b(k)} - \hat{\theta}^*_k}{\sigma^*_{b(k)}} \right|$

end for

Compute the $(1-\alpha)$-sample quantile of $t^*_1, t^*_2, \dots, t^*_B$, call this $\hat{t}$.

The joint confidence region of $\theta_{(1)}, \theta_{(2)}, \dots, \theta_{(K)}$ is given by

$$\mathfrak{R} = \left[\hat{\theta}_{(1)} \pm \hat{t} \times \hat{\sigma}_{(1)}\right] \times \left[\hat{\theta}_{(2)} \pm \hat{t} \times \hat{\sigma}_{(2)}\right] \times \dots \times \left[\hat{\theta}_{(K)} \pm \hat{t} \times \hat{\sigma}_{(K)}\right]$$

where $\hat{\sigma}_{(k)}$ is computed as

$$\hat{\sigma}_{(k)} = \sqrt{\text{kth ordered value among } \left\{\hat{\theta}^2_1 + \sigma_1^2, \hat{\theta}^2_2 + \sigma_2^2, \dots, \hat{\theta}^2_K + \sigma_K^2\right\} - \hat{\theta}^2_{(k)}}$$

Algorithm 2 Computation of Coverage Probability for Parametric Bootstrap

For given values of $\theta_1, \theta_2, \ldots, \theta_K$ and thus $\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}$

1: **for** replications $= 1, 2, \ldots, 5000$ **do**
2: Generate $\hat{\theta}_i \sim N(\theta_i, \sigma_i^2)$, for $i = 1, 2, \ldots, K$
3: Compute the rectangular confidence region $\mathfrak{R}$ using Algorithm 1.
4: Check if $\left(\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}\right) \in \mathfrak{R}$ and compute $T = \prod_{k=1}^{K} \left|\Lambda_{Ok}\right|$.
5: **end for**
6: Compute the proportion of times that the condition in step 4 is satisfied and the average of T

3.2 Nonrank-based method

The nonrank-based method accounts for potential correlation among items being ranked. For this case, an exchangeable correlation, $\boldsymbol{\rho}$ (See Equation 3.1.), is assumed and used in the calculation of the variance covariance matrix (See Equation 3.2.)

$$\boldsymbol{\rho} = (1 - \rho)\, \mathbf{I}_K + \rho \mathbf{1}_K \mathbf{1}_K' \tag{3.1}$$

$$\boldsymbol{\Sigma} = \boldsymbol{\Delta}^{1/2} \boldsymbol{\rho} \boldsymbol{\Delta}^{1/2} \tag{3.2}$$

where $\boldsymbol{\Delta} = \text{diag}\left\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2\right\}$, with known σ_k's.

Algorithm 3 Computation of Joint Confidence Region via Nonrank-based Method

Let the data consist of $\hat{\theta}_1, \ldots, \hat{\theta}_K$ and suppose $\boldsymbol{\Sigma}$ is known

1: **for** $b = 1, 2, \ldots, B$ **do**
2: Generate $\hat{\boldsymbol{\theta}}_b^* \sim N_K\left(\hat{\boldsymbol{\theta}}, \boldsymbol{\Sigma}\right)$ and write $\hat{\boldsymbol{\theta}}_b^* = \left(\hat{\theta}_{b1}^*, \hat{\theta}_{b2}^*, \ldots, \hat{\theta}_{bK}^*\right)'$
3: Compute $t_b^* = \max\limits_{1 \le j \le K} \left|\frac{\hat{\theta}_{bj}^* - \hat{\theta}_j^*}{\sigma_j}\right|$
4: **end for**
5: Compute the $(1 - \alpha)$-sample quantile of $t_1^*, t_2^*, \ldots, t_B^*$, call this $\hat{t}$.
6: The joint confidence region of $\theta_1, \theta_2, \ldots, \theta_K$ is given by

$$\mathfrak{R} = \left[\hat{\theta}_1 \pm \hat{t} \times \sigma_1\right] \times \left[\hat{\theta}_2 \pm \hat{t} \times \sigma_2\right] \times \cdots \times \left[\hat{\theta}_K \pm \hat{t} \times \sigma_K\right]$$

Algorithm 4 Computation of Coverage Probability for Nonrank-based Method

For given values of $\theta_1, \theta_2, \ldots, \theta_K$ and $\boldsymbol{\Sigma}$

1: **for** replications $= 1, 2, \ldots, 5000$ **do**

2: Generate $\hat{\boldsymbol{\theta}} \sim N_K(\boldsymbol{\theta}, \boldsymbol{\Sigma})$

3: Compute the rectangular confidence region $\mathfrak{R}$ using Algorithm 3.

4: Check if $(\theta_1, \theta_2, \ldots, \theta_K) \in \mathfrak{R}$ and compute $T = \prod_{k=1}^{K} \left|\Lambda_{Ok}\right|$.

5: **end for**

6: Compute the proportion of times that the condition in step 4 is satisfied and the average of T

THIS IS Rizzo (2008) and Klein et al. (2020)

References

Klein, M., Wright, T., & Wieczorek, J. (2020). *A joint confidence region for an overall ranking of populations.*

Rizzo, M. (2008). *Statistical computing with r.*

Appendices