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by
Nmae

Abstract

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1 Introduction

1.1 Background of the Study

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1.7 Definition of Terms

2 Background

3 Methodology

3.1 Nonrank-based method

Algorithm 1 Computation of Joint Confidence Region via Nonrank-based Method

Let the data consist of $\hat{\theta}_1, \dots, \hat{\theta}_K$ and suppose Σ is known

- 1: **for** $b = 1, 2, \dots, B$ **do**
 - 2: Generate $\hat{\theta}_b^* \sim N_K(\hat{\theta}, \Sigma)$ and write $\hat{\theta}_b^* = (\hat{\theta}_{b1}^*, \hat{\theta}_{b2}^*, \dots, \hat{\theta}_{bK}^*)'$
 - 3: Compute $t_b^* = \max_{1 \leq j \leq K} \left| \frac{\hat{\theta}_{bj}^* - \hat{\theta}_j}{\sigma_j} \right|$
 - 4: **end for**
 - 5: Compute the $(1 - \alpha)$ -sample quantile of $t_1^*, t_2^*, \dots, t_B^*$, call this \hat{t} .
 - 6: The joint confidence region of $\theta_1, \theta_2, \dots, \theta_K$ is given by
$$\mathfrak{R} = [\hat{\theta}_1 \pm \hat{t} \times \sigma_1] \times [\hat{\theta}_2 \pm \hat{t} \times \sigma_2] \times \dots \times [\hat{\theta}_K \pm \hat{t} \times \sigma_K]$$
-

As shown in Algorithm 1, the nonrank-based method...

Algorithm 2 Computation of Coverage Probability for Nonrank-based Method

For given values of $\theta_1, \theta_2, \dots, \theta_K$ and Σ

- 1: **for** replications = 1, 2, ..., 5000 **do**
 - 2: Generate $\hat{\theta} \sim N_K(\theta, \Sigma)$
 - 3: Compute the rectangular confidence region \mathfrak{R} using Algorithm 1.
 - 4: Check if $(\theta_1, \theta_2, \dots, \theta_K) \in \mathfrak{R}$ and compute $T = \prod_{k=1}^K |\Lambda_{Ok}|$.
 - 5: **end for**
 - 6: Compute the proportion of times that the condition in step 4 is satisfied and the average of T
-

Algorithm 3 Computation of Joint Confidence Region via Parametric Bootstrap

- 1: **for** $b = 1, 2, \dots, B$ **do**
- 2: Generate $\hat{\theta}_{bi}^* \sim N(\hat{\theta}_i, \sigma_i^2)$, $i = 1, 2, \dots, K$ and let $\hat{\theta}_{b(1)}, \hat{\theta}_{b(2)}, \dots, \hat{\theta}_{b(K)}$ be the corresponding ordered values

	$k = 1$	$k = 2$	\dots	$k = K$
$b = 1$	$\hat{\theta}_{1(1)}^*$	$\hat{\theta}_{1(2)}^*$	\dots	$\hat{\theta}_{1(K)}^*$
$b = 2$	$\hat{\theta}_{2(1)}^*$	$\hat{\theta}_{2(2)}^*$	\dots	$\hat{\theta}_{2(K)}^*$
\vdots	\vdots	\vdots	\dots	\vdots
$b = B$	$\hat{\theta}_{B(1)}^*$	$\hat{\theta}_{B(2)}^*$	\dots	$\hat{\theta}_{B(K)}^*$

- 3: Compute
$$\hat{\sigma}_{b(k)}^* = \sqrt{\text{kth ordered value among } \{\hat{\theta}_{b1}^{*2} + \sigma_1^2, \hat{\theta}_{b2}^{*2} + \sigma_2^2, \dots, \hat{\theta}_{bK}^{*2} + \sigma_K^2\} - \hat{\theta}_{b(k)}^{*2}}$$
- 4: Compute $t_b^* = \max_{1 \leq k \leq K} \left| \frac{\hat{\theta}_{b(k)}^* - \hat{\theta}_k^*}{\sigma_{b(k)}^*} \right|$
- 5: **end for**
- 6: Compute the $(1 - \alpha)$ -sample quantile of $t_1^*, t_2^*, \dots, t_B^*$, call this \hat{t} .
- 7: The joint confidence region of $\theta_{(1)}, \theta_{(2)}, \dots, \theta_{(K)}$ is given by
$$\mathfrak{R} = [\hat{\theta}_{(1)} \pm \hat{t} \times \hat{\sigma}_{(1)}] \times [\hat{\theta}_{(2)} \pm \hat{t} \times \hat{\sigma}_{(2)}] \times \dots \times [\hat{\theta}_{(K)} \pm \hat{t} \times \hat{\sigma}_{(K)}]$$

where $\hat{\sigma}_{(k)}$ is computed as

$$\hat{\sigma}_{(k)} = \sqrt{\text{kth ordered value among } \{\hat{\theta}_1^2 + \sigma_1^2, \hat{\theta}_2^2 + \sigma_2^2, \dots, \hat{\theta}_K^2 + \sigma_K^2\} - \hat{\theta}_{(k)}^2}$$

Algorithm 4 Computation of Coverage Probability for Parametric Bootstrap

For given values of $\theta_1, \theta_2, \dots, \theta_K$ and thus $\theta_{(1)}, \theta_{(2)}, \dots, \theta_{(K)}$

- 1: **for** replications = 1, 2, ..., 5000 **do**
 - 2: Generate $\hat{\theta}_i \sim N(\theta_i, \sigma_i^2)$, for $i = 1, 2, \dots, K$
 - 3: Compute the rectangular confidence region \mathfrak{R} using Algorithm 3.
 - 4: Check if $(\theta_{(1)}, \theta_{(2)}, \dots, \theta_{(K)}) \in \mathfrak{R}$ and compute $T = \prod_{k=1}^K |\Lambda_{Ok}|$.
 - 5: **end for**
 - 6: Compute the proportion of times that the condition in step 4 is satisfied and the average of T
-

3.2 Parametric bootstrap

THIS IS Rizzo (2008) and Klein et al. (2020)

References

Klein, M., Wright, T., & Wieczorek, J. (2020). *A joint confidence region for an overall ranking of populations.*

Rizzo, M. (2008). *Statistical computing with r.*

Appendices