Multiple Strategies (MS) and Simultaneously Identifying Skills & Misconceptions (SISM)

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Multiple Strategies (MS) part

Suppose that item j has M_j strategies. Each of the strategy involves a unique set of the K attributes. Different items may have different number of strategies. Different strategies correspond to distinct q-vectors. Let $\mathbf{q}_j^m = \{q_{jk}^m\}$ be the q-vector of item j for strategy m.

GMS-CDM defines the likelihood of observing response vector $\mathbf{y}_i$ for individual i with attribute profile $\boldsymbol{\alpha}_c$ as

$$L(\mathbf{y}_i) = \sum_c \pi_c \prod_j^J P_j(\boldsymbol{\alpha}_c)^{y_{ij}} \left[1 - P_j(\boldsymbol{\alpha}_c)\right]^{1-y_{ij}}$$

$P_j(\boldsymbol{\alpha_c})$ is the IRF or the probability of successfully accomplishing item j and is defined by the condensation rule. For GMS-CDM,

$$IRF = P_j(\boldsymbol{\alpha_c}) = \sum_{m=1}^{M_j} P(Y_{ij} = 1 \mid \boldsymbol{\alpha_c}, m) P_j(m \mid \boldsymbol{\alpha}_c)$$

Simultaneously Identifying Skills & Misconceptions (SISM) part

$P(Y_{ij} = 1 \mid \boldsymbol{\alpha_c}, m)$ is the strategy-specific IRF. For the purpose of this study, the IRF for SISM model is used.

$$P(Y_{ij} = 1 \mid \boldsymbol{\alpha_c}, m) = h_j^{\eta_{ij}(1-\gamma_{ij})} \omega_j^{\eta_{ij}\gamma_{ij}} g_j^{(1-\eta_{ij})(1-\gamma_{ij})} \epsilon_j^{(1-\eta_{ij})\gamma_{ij}}$$

where η_{ij} and γ_{ij} are deterministic latent response variables.

$$\eta_{ij} = \prod_{k=1}^{K_s} \alpha_{ik}^{q_{jk}} \text{ and } \gamma_{ij} = 1 - \prod_{k=K_s+1}^{K} (1 - \alpha_{ik})^{q_{jk}}$$

There are 4 parameters for the SISM model:

- h_j: success probability when
  - all required attributes are possessed ($\eta_j = 1$) and
  - no misconception ($\gamma_j = 0$) is possessed.
- ω_j: success probability when
  - all required attributes are possessed ($\eta_j = 1$) but
  - at least one misconception ($\gamma_j = 1$) is possessed.
- g_j: success probability when
  - not all required attributes are possessed ($\eta_j = 0$) and
  - no misconception ($\gamma_j = 0$) is possessed.
- ε_j: success probability when

- not all required attributes are possessed ($\eta_j = 0$) and
- at least one misconception ($\gamma_j = 1$) is possessed.

If no misconceptions are considered in the Q-matrix, number of parameters is reduced to 2 - namely, h_j and g_j - the slip and guess parameters in the DINA model, respectively. Note that the Q-matrix for the SISM model requires the specification of skills (first k columns of the Q-matrix) and the misconceptions (last 1-k columns of the Q-matrix) being measured.

$$P(Y_{ij} = 1 \mid \boldsymbol{\alpha_c}, m) = \begin{cases} h_j, & \eta_j = 1, \gamma_j = 0 \\ \omega_j, & \eta_j = 1, \gamma_j = 1 \\ g_j, & \eta_j = 0, \gamma_j = 0 \\ \varepsilon_j, & \eta_j = 0, \gamma_j = 1 \end{cases}$$

$P_j(m \mid \boldsymbol{\alpha}_c)$ is the probability of an individual with attribute profile $\boldsymbol{\alpha}_c$) choosing strategy m for item j.

$$P_j(m \mid \boldsymbol{\alpha}_c) = \frac{P(Y_j = 1 \mid \boldsymbol{\alpha}_c, m)^s}{\sum_{m=1}^{M_j} P(Y_j = 1 \mid \boldsymbol{\alpha}_c, m)^s}$$

where s is the strategy selection parameter.

- $s = 0$ means that all strategies have the same probability of being selected.
- $s = 1$: probability matching approach
- $s > 1$: overmatching approach
- $s \to +\infty$: maximizing approach

Strategy selection approaches (MS part)

1. Maximizing approach. Choosing the strategy with the highest success probability to maximize the expected item score. Choose strategy A over B if success probabilities are 0.8 and 0.5, respectively.
2. Probability matching approach. Choosing A over B when corresponding attached values are 0.8/(0.8 + 0.5) and 0.5/(0.8 + 0.5), respectively.
3. Overmatching approach. Choosing A when corresponding value > 0.8/(0.8 + 0.5)

Next steps

Hello Sir,

Apologies for my inconsistency when it comes to reporting.

I have not yet thoroughly thought about how I could express the SISM model similar to how the GDINA model is written (in terms of δ parameters). I feel that I have to, since it is mentioned that the estimation procedure is the same as the GDINA model. For the M_j strategies, I assume that the adjustment will be on the coding side. But, to be able to reflect MS (variable M_j) in the SISM IRF, I have to express the equation in terms of δ, as mentioned. This is how MS was done for DINA, DINO, and ACDM.

Please let me know if there are errors or confusions created in the content above and if you have suggestions.

Thank you!

-Shaine