Joint Confidence Regions for Rankings based on Correlated Estimates

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$$r_k = \sum_{j=1}^{K} I(\theta_j \leq \theta_k) = 1 + \sum_{j:j \neq k} I(\theta_j \leq \theta_k), \qquad \text{for } k = 1, 2, \ldots, K. \tag{1}$$

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Klein's

Suppose that for each $k \in \{1, 2, \ldots, K\}$ there exists values L_k and U_k ST

$$\theta_k \in (L_k, U_k), k = 1, 2, \ldots, K. \tag{2}$$

If the condition in (2) holds, the main result from Klein et al. (2020) gives a range for the value of r_k for each $k \in \{1, 2, \ldots, K\}$ as follows:

$$r_k \in \{|\Lambda_{Lk}| + 1, |\Lambda_{Lk}| + 2, |\Lambda_{Lk}| + 3, \ldots, |\Lambda_{Lk}| + |\Lambda_{Ok}| + 1\} \tag{3}$$

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Suppose that for random quantities L_k and U_k the event defined in (2) satisfies the following probability condition:

$$P\left[\bigcap_{k=1}^{K}\{\theta_k \in (L_k, U_k)\}\right] \geq 1-\alpha, \tag{4}$$

then, by the result of Klein et al. (2020), it also follows that

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Political setting

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Measurement across geographies

- Klein et al. (2020) also noted that states with large unpopulated land areas and relatively few high-density population centers tend to report shorter travel times while longer travel times are typically observed in highly urbanized states with large populations and high population densities.

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Definitions and Assumptions

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Procedure

1. Derive simultaneous confidence intervals for $\theta_1, \theta_2, \ldots, \theta_K$ of the form

$$\mathfrak{R}_1 = [\hat{\theta}_1 \pm t \times \sigma_1] \times [\hat{\theta}_2 \pm t \times \sigma_2] \times \cdots \times [\hat{\theta}_K \pm t \times \sigma_K]. \quad (6)$$

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2. Once the confidence intervals in (6) have been obtained, we can then use the result of Klein et al. (2020) in (5) to get the lower and upper bounds on the ranks $r_k, k = 1, 2, \ldots, K$. That is, we also get a joint confidence region for $r_1, r_2, \ldots, r_K$.

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Proposed methodology to compute the joint confidence region for the unordered parameters: Algorithm 1

Let the data be represented by $\hat{\boldsymbol{\theta}} = \left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_K\right)'$ and suppose that $\boldsymbol{\Sigma}$ is known

1: **for** $b = 1, 2, \ldots, B$ **do**

2: Generate $\hat{\boldsymbol{\theta}}_b^* \sim N_K\left(\hat{\boldsymbol{\theta}}, \boldsymbol{\Sigma}\right)$ and write $\hat{\boldsymbol{\theta}}_b^* = \left(\hat{\theta}_{b1}^*, \hat{\theta}_{b2}^*, \ldots, \hat{\theta}_{bK}^*\right)'$

3: Compute

$$t_b^* = \max_{1 \le k \le K} \left| \frac{\hat{\theta}_{bk}^* - \hat{\theta}_k}{\sigma_k} \right|$$

4: **end for**

5: Compute the $(1-\alpha)$-sample quantile of $t_1^*, t_2^*, \ldots, t_B^*$, call this $\hat{t}$.

6: The joint confidence region for $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_K)'$ is given by

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Algorithm 1: Quantile Calculation

We want the joint confidence region in (6) to satisfy the following probability condition:

$$P\left(\hat{\theta}_k - t \cdot \sigma_k \leq \theta_k \leq \hat{\theta}_k + t \cdot \sigma_k,\ \forall\ k = 1, 2, \ldots, K\right) = 1 - \alpha. \quad (7)$$

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Proposed methodology to compute a joint confidence region for the ordered parameters

for $b = 1, 2, \ldots, B$ **do**

Generate $\hat{\boldsymbol{\theta}}^*_b = \left(\hat{\theta}^*_{b1}, \hat{\theta}^*_{b2}, \ldots, \hat{\theta}^*_{bK}\right)' \sim N_K\left(\hat{\boldsymbol{\theta}}, \boldsymbol{\Sigma}\right)$ and let $\hat{\theta}^*_{b(1)}, \hat{\theta}^*_{b(2)}, \ldots, \hat{\theta}^*_{b(K)}$ be the corresponding ordered values

Compute $\hat{\sigma}^*_{b(k)}$ using:

- asymptotic variance definition
- second-level bootstrap

Compute

$$t^*_b = \max_{1 \le k \le K} \left| \frac{\hat{\theta}^*_{b(k)} - \hat{\theta}^*_k}{\hat{\sigma}^*_{b(k)}} \right|$$

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Compute the $(1 - \alpha)$-sample quantile of $t^*_1, \ldots, t^*_B$, call this $\hat{t}$.

The joint confidence region of $\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}$ is given by

$$\mathfrak{R}_2 = \left[\hat{\theta}_{(1)} \pm \hat{t} \times \hat{\sigma}_{(1)}\right] \times \left[\hat{\theta}_{(2)} \pm \hat{t} \times \hat{\sigma}_{(2)}\right] \times \cdots \times \left[\hat{\theta}_{(K)} \pm \hat{t} \times \hat{\sigma}_{(K)}\right]$$

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Algorithm 2: Asymptotic Definition of Variance

$$\hat{\sigma}^*_{b(k)} = \sqrt{\left[\text{kth ordered value among } \left\{\hat{\theta}^{*2}_{b1} + \sigma^2_1, \ldots, \hat{\theta}^{*2}_{bK} + \sigma^2_K\right\}\right] - \hat{\theta}^{*2}_{(k)}}$$

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Algorithm 3: Variance from Second-Level Bootstrap

for $c = 1, 2, \ldots, C$ **do**

  Generate $\hat{\boldsymbol{\theta}}^{**}_{bc} = \left(\hat{\theta}^{**}_{bc1}, \hat{\theta}^{**}_{bc2}, \ldots, \hat{\theta}^{**}_{bcK}\right) \sim N_K\left(\hat{\boldsymbol{\theta}}^{*}_{b}, \boldsymbol{\Sigma}\right)$ and let $\hat{\theta}^{**}_{bc(1)}, \hat{\theta}^{**}_{bc(2)}, \ldots, \hat{\theta}^{**}_{bc(K)}$ be the corresponding ordered values of $\hat{\theta}^{**}_{bc1}, \hat{\theta}^{**}_{bc2}, \ldots, \hat{\theta}^{**}_{bcK}$

  Compute $\hat{\sigma}^{*}_{b(k)} = \dfrac{\sum_{c=1}^{C}\left(\hat{\theta}^{**}_{bc(k)} - \bar{\hat{\theta}}^{**}_{b\cdot(k)}\right)^2}{C-1}, \quad \bar{\hat{\theta}}^{**}_{b\cdot(k)} = \dfrac{1}{C}\sum_{c=1}^{C}\hat{\theta}^{**}_{bc(k)}$

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Evaluation Algorithm

For given values of $\boldsymbol{\Sigma}$ and $\theta_1, \theta_2, \ldots, \theta_K$ (with corresponding $\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}$ for rank-based methods)

1: **for** replications $= 1, 2, \ldots, 5000$ **do**
2: Generate $\hat{\boldsymbol{\theta}} \sim N_K(\boldsymbol{\theta}, \boldsymbol{\Sigma})$
3: Compute the confidence region $\mathfrak{R}_1$ for the unordered parameters using Algorithm 1 and the confidence region for the ordered parameters $\mathfrak{R}_2$ using Algorithms 2 and 3.
4: For the unordered parameters, check if $(\theta_1, \theta_2, \ldots, \theta_K) \in \mathfrak{R}_1$ and compute T_1, T_2, and T_3. For the ordered parameters, check if $\left(\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}\right) \in \mathfrak{R}_2$
5: **end for**
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Covariance Matrix $\boldsymbol{\Sigma}$

- The covariance matrix $\boldsymbol{\Sigma}$ need not be a diagonal matrix.
- We assume that $V(\hat{\boldsymbol{\theta}}) = \boldsymbol{\Sigma}$ is known and express $\boldsymbol{\Sigma}$ as in (9), where $\mathbf{R}$ is the population correlation matrix.

$$\boldsymbol{\Sigma} = \boldsymbol{\Delta}^{1/2}\mathbf{R}\boldsymbol{\Delta}^{1/2}; \quad \boldsymbol{\Delta} = \operatorname{diag}\left\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2\right\}. \tag{9}$$

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- This assumes that the k variables are equally correlated, i.e., that $\rho_{jk} = \rho$ where $\rho \in [-1, 1]$ for $j \neq k \in \{1, \ldots, K\}$.

$$\mathbf{R}_{\text{eq}} = (1-\rho)\,\mathbf{I}_K + \rho \mathbf{1}_K \mathbf{1}_K' = \begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{bmatrix}_{K \times K} \tag{10}$$

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- The full block correlation matrix can be expressed as in (11).

$$\mathbf{R}_{\text{block}} = \begin{bmatrix} \mathbf{R}_{eq,1} & \mathbf{C}_{12} & \cdots & \mathbf{C}_{1G} \\ \mathbf{C}_{21} & \mathbf{R}_{eq,2} & \cdots & \mathbf{C}_{2G} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{G1} & \mathbf{C}_{G2} & \cdots & \mathbf{R}_{eq,G} \end{bmatrix}_{K \times K} \tag{11}$$

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$$\rho_{\text{matern}} = \frac{2^{1-\nu}}{\Gamma(\nu)}(\kappa \parallel \mathbf{s}_i - \mathbf{s}_j \parallel)^{\nu} K_{\nu}(\kappa \parallel \mathbf{s}_i - \mathbf{s}_j \parallel)$$

where $\parallel \cdot \parallel$ denotes the Euclidean distance and K_{ν} is the second kind of the modified Bessel function. It has a scale parameter $\kappa > 0$ and a smoothness parameter $\nu > 0$. ρ_{matern} reduces to the exponential correlation when $\nu = 0.5$ and to Gaussian correlation function when $\nu = \infty$.

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Code blocks

Alert block

$$E = mc^2$$

Examples

Example blocks are automatically green in color

Blue block

- happens with level 2, 3 headings
- this is only true for ‘Madrid’ theme in R Markdown!!

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This works, incremental bullets

- Bullet 1

This works, incremental bullets

- Bullet 1

This works, incremental bullets

- Bullet 1
- Bullet 2

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Joint Confidence Regions for Rankings
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- Bullet 2

This nests, but does not increment

- Bullet 1
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This nests, but does not increment

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Algo Font Size Adjustments sample

for $b = 1, 2, \ldots, B$ **do**

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Compute $\hat{\sigma}_{b(k)}^* = \hat{\sigma}_{b(k)}^* = \sqrt{\left[\text{kth ordered value among } \left\{\hat{\theta}_{b1}^{*2} + \sigma_1^2, \ldots, \hat{\theta}_{bK}^{*2} + \sigma_K^2\right\}\right] - \hat{\theta}_{(k)}^{*2}}$

Compute

$$t_b^* = \max_{1 \le k \le K} \left| \frac{\hat{\theta}_{b(k)}^* - \hat{\theta}_k^*}{\hat{\sigma}_{b(k)}^*} \right|$$

end for

Compute the $(1-\alpha)$-sample quantile of $t_1^*, \ldots, t_B^*$, call this $\hat{t}$.

The joint confidence region of $\theta_{(1)}, \theta_{(2)}, \ldots, \theta_{(K)}$ is given by

$$\mathfrak{R}_2 = \left[\hat{\theta}_{(1)} \pm \hat{t} \times \hat{\sigma}_{(1)}\right] \times \left[\hat{\theta}_{(2)} \pm \hat{t} \times \hat{\sigma}_{(2)}\right] \times \cdots \times \left[\hat{\theta}_{(K)} \pm \hat{t} \times \hat{\sigma}_{(K)}\right]$$

where

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Algo Font Size Adjustments sample

Text

Today is going to be a great day

Today is. . .

A new beginning.

Your Reference

- slideshare
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