Number System Assignment 1

1)

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010

2) Decimal to Binary conversions

a) 12

2	12
	6 – 0
	3 – 0
	1 – 1

12 → 1100

b) 20

2	20
	10 – 0
	5 – 0
	2 – 1
	1 – 0

20 → 10100

c) 45

2	45
	22 − 1
	11 − 0
	5 − 1
	2 − 1
	1 − 0

45 ⟶ 101101

d) 77

2	77
	38 − 1
	19 − 0
	9 − 1
	4 − 1
	2 − 0
	1 − 0

77 ⟶ 1001101

e) 103

2	103
	51 − 1
	25 − 1
	12 − 1
	6 − 0
	3 − 0
	1 − 1
	1 − 0

103 ⟶ ~~10100111~~
1100111

```
38
2)77
  6
  17
  16
   1

  19
2)38
   2
   18
   18
    0

  51
2)103
   10
    03

   25
2)51
   4
   11
```

3) Octal equivalent of $(9910)_{10}$

8	9910
	1238 − 6
	154 − 6
	19 − 2
	2 − 3

$(9910)_{10} \xrightarrow{\text{octal}} 23266$

```
      1238
8) 9910
   -8
    19
    16
     31
    -24
      70
      64
       6

      154
8) 1238
   8
    43
    40
     38
     32
      6

      19
8) 154
   8
    74
    72
     2

      2
8) 19
   16
    3
```

4) Binary to Decimal

a) 1101

$2^3\ 2^2\ 2^1\ 2^0$

$8+4+1 = 13$

$1101 \rightarrow 13$

b) 1110

$2^3\ 2^2\ 2^1\ 2^0$

$8+4+2 = 14$

$1110 \rightarrow 14$

c) 1111 0101

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

~~64+32+16+8+ 4 +1 = 125~~

128+64+32+16+ 4 +1

1111 0101 = ~~125~~ 245

d) 0101 0101

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

~~32+8+4+1 = 45~~

64+16+4+1 = 85

0101 0101 → 85

```
2) 85
   42 − 1
```

e) 1000 1111

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

128+ 8+4+2+1 = 143

1000 1111 → 143

5) $(1101)_2 \longrightarrow (15)_8$

$\searrow (13)_{10} \nearrow$

1101

$2^3\ 2^2\ 2^1\ 2^0$

$8+4+1 = \underline{\underline{13}}$

8	13
	1 — 5

$= \underline{\underline{15}}$

~~octal~~ Verification

1 5

$8^1\ 8^0$

$8+5 = \underline{\underline{13}}$

2	13
	6 — 1
	3 — 0
	1 — 1

$\underline{\underline{1101}}$

∴ Verified

b) $(1111\ 1111\ 1110)_2 \longrightarrow (4094)_{10}$

1 1 1 1 1 1 1 1 1 1 1 0

$2^{11}\ 2^{10}\ 2^9\ 2^8\ 2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$2048+1024+512+256+128+64+32+16+8+4+2 = \underline{\underline{4094}}$

2	4094
	2047 — 0
	1023 — 1
	511 — 1
	255 — 1
	127 — 1
	63 — 1
	31 — 1
	15 — 1
	7 — 1
	3 — 1
	1 — 1

∴ Verified

c) $(221201)_3 \longrightarrow (694)_{10}$

2 2 1 2 0 1

$3^5\ 3^4\ 3^3\ 3^2\ 3^1\ 3^0$

$486+162+27+18+1 = \underline{\underline{694}}$

~~Base 3 doesn't exist~~

d) $(76)_8 \longrightarrow \cancel{(10)_8}\ (\ )_{10}$

7 6

$8^1\ 8^0$

$56+6 = \underline{\underline{62}}$

8	62
	7 — 6

$\begin{array}{r} 7 \\ 8\overline{)62} \\ 56 \\ \hline 6 \end{array}$

e) $(231)_8 \rightarrow (\ )_2$

$\rightarrow (153)_{10} \rightarrow (\ )_2$

2 3 1

8^2 8^1 8^0

$128 + 24 + 1 = 153$

2	153
	76 – 1
	38 – 0
	19 – 0
	9 – 1
	4 – 1
	2 – 0
	1 – 0

$(10011001)_2$

cross verification

1 0 0 1 1 0 0 1

2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

$128 + 3 \cdot 16 + 8 + 1 = 153$

8	153
	19 – 1
	2 – 3

231

∴ verified

f) $(0xF00)_{16} \rightarrow (\ )_8$

$\rightarrow (\ )_{10}$

$x = 56$

0x only denotes that the number is a hex.

(15)

F 0 0

~~8^2 8^1 8^0~~

16^2 16^1 16^0

$256 = 3840$

8	3840
	480 – 0
	60 – 0
	7 – 4

$(7400)_8$

verification

7 4 0 0

8^3 8^2 8^1 8^0

$3584 + 256 = 3840$

16	3840
	240 – 0
	15 – 0
	F00

F00

∴ verified

g) (0x D

D = 13

A = 10

C = 1

E =

532

12 |

2

12

138

414

h)

cation

0 1
2^1 2^0
+1 = 153

19
8) 153
8
73
72
1

240
6) 3840
32
64
64
00

g) $(0x DACE)_{16} \longrightarrow (\ )_{12}$

~~12 base doesnt exist~~

D = 13
A = 10
C = 12
E = 14

13 10 12 14
16^3 16^2 16^1 16^0

53248 + 2560 + 192 + 14 = 56014

12 | 56014
4667 - 10
388 - 11
32 - 4
2 - 8

B A
2 8 4 11 10
12^4 12^3 12^2 12^1 12^0

13824 + 576 + 132 + 10
4147 2 +
= 56014

32
12) 388
36
28
24
4

2
12) 32
24
8

4667
12) 56014
-48
80
72
81
72
94
84
10

388
12) 4667
36
106
96
107
-96
11

46677
12) 560014
-48
80
72
81
72
91
84
74
72
2

h) $(0x2B)_{16} \longrightarrow (53)_8$

$\searrow (43)_{10} \nearrow$

2 11
16^1 16^0
32 + 11 = 43

16 | 43
2 - 11

2
16) 43
32
11

8 | 43
5 - 3

5
8) 43
40
3

5 3
8^1 8^0
40 + 3 = 43

6) Convert to base 10 (Decimal)

a) $(3312)_8$

3 3 1 2

8^3 8^2 8^1 8^0

$1536 + 192 + 8 + 2 = \underline{\underline{1738}}$

b) $(167)_8$

1 6 7

8^2 8^1 8^0

$64 + 48 + 7 = 119$

c) $(202103)_9$

2 0 2 1 0 3

9^5 9^4 9^3 9^2 9^1 9^0

$\cancel{+729} + 81 \quad + 3 = 119640$

$118098 + 1458$

d) $(3132334)_{16}$

3 1 3 2 3 3 4

16^6 16^5 16^4 16^3 16^2 16^1 16^0

$1048576 + 196608 + 8192 + 768 + 43 + 4$

$\cancel{16777216 +}$

$50331648 = 51585839$

e) $(0x F2)_{16}$

1 5 2

16^1 16^0

$240 + 2 = 242$

7) Base 10 to given base

a) $(5610)_{10} \rightarrow (\ )_2$

2	5610
	2805 – 0
	1402 – 1
	701 – 0
	350 – 1
	175 – 0
	87 – 1
	43 – 1
	21 – 1
	10 – 1
	5 – 0
	2 – 1
	1 – 0

$(1010111101010)_2$

b) $(5610)_{10} \rightarrow (\ )_3$

3	5610
	1870 – 0
	623 – 1
	207 – 2
	69 – 0
	23 – 0
	7 – 2
	2 – 1

$(2120021 0)_3$

```
     623
3 ) 1870
    18 ↓
    ---
     07
      6
     ---
     10
      9
     ---
      1

     207
3 ) 623
    6
    ---
    023
     21
    ---
      2
```

c) $(5610) \rightarrow (\ )_8$

8	5610
	701 – 2
	87 – 5
	10 – 7
	1 – 2

$(12752)_8$

$$\begin{array}{r} 701 \\ 8\overline{)5610} \\ 56 \\ \hline 010 \\ 8 \\ \hline 2 \end{array}$$

d) $(5610)_{10} \rightarrow (\ )_{12}$

12	5610
	467 – 6
	38 – 11
	3 – 2

$(32B6)_{12}$

e) $(5610)_{10} \rightarrow (\ )_{16}$

16	5610	
	350 – 10	A
	21 – 14	E
	1 – 5	

15EA

$$\begin{array}{r} 350 \\ 16\overline{)5610} \\ 48\downarrow \\ \hline 81 \\ 80 \\ \hline 10 \end{array}$$

~~f) $(22110)_{10} \rightarrow (\ )_2$~~

operand 1

a ⊙ b

operator

$$\begin{array}{r} 12 \\ +13 \\ \hline \end{array}$$

$$\begin{array}{r} 35 \\ +17 \\ \hline 52 \end{array}$$

Assignment 1 Cont.

f) $(22110)_{10} \rightarrow (\ )_2$

2	22110
	11055 - 0
	5527 - 1
	2763 - 1
	1381 - 1
	690 - 1
	345 - 0
	172 - 1
	86 - 0
	43 - 0
	21 - 1
	10 - 1
	5 - 0
	2 - 1
	1 - 0

$(1010110010111 0)_2$

g) $(22110)_{10} \rightarrow (\ )_3$

3	22110
	7370 - 0
	2456 - 2
	818 - 2
	272 - 2
	90 - 2
	30 - 0
	10 - 0
	3 - 1
	1 - 0

$(1010022220)_3$

h) $(22110)_{10} \rightarrow (\ )_8$

8	22110
	2763 – 6
	345 – 3
	43 – 1
	5 – 3

$(53136)_8$

i) $(22110)_{10} \rightarrow (\ )_{12}$

12	22110
	1842 – 6
	153 – 6
	12 – 9
	1 – 0

$(10966)_{12}$

j) $(22110)_{10} \rightarrow (\ )_{16}$

16	22110
	1381 – 14 E
	86 – 5
	5 – 6

$(565E)_{16}$

8) 9 + 12

$$\begin{array}{r} 12 \rightarrow 1100 \\ +\ 9 \rightarrow 1001 \\ \hline 21 \quad 10101 \end{array}$$

b) 40 + 31

```
  40 → 101000
+ 31 → 11111
  71 → 1000111
```

c)

```
  1110 → 14
+ 0101 → 5
  1001   19
```

d)

```
  11110101 → 245
+ 01111100 → 124
  10111 0001 → 369
```

e)

```
  11000011 → 195
+ 01011110 → + 94
  100100001    289
```

a) 8 → 0x0000

```
- 3 → 11
  5 → 101
```

b) 17 → 0x0001

```
-11 → 1011
  6 → 0110
```

c) 15 → 0001

```
- 7 → 111
  8 → 1000
```

d) 86 → [illegible]
−31 → − 11111
55 → 0110111

e) 110[illegible]0001 → 110 204
−0100001x1 → − 71
10001010 138

10) a) 12 → 1100
× 3 → 11
36
11100
1100x
1001000 → 36

b) 20 → 10100
× 5 → × 101
100
10100
00000x
101000xx
1001000 → 100

c) 01100111 → 103
× 101 → × 5
515
01100111
00000000x
01100111xx
100000011

d) 10101010 → 170
× 0101 → × 5
850
10101010
00000000x
10101010xx
000000000xxx
01101010010 → 850

11) a) $\frac{15}{2}$

```
      111  -> 7
10 ) 1111
     -10
      11
     -10
      11
     -10
       1 -> 1
```

```
      7
2 ) 15
    14
     1
```

b) $\frac{45}{5}$ → 101101 → 101

```
        100101 -> 9
101 ) 101101
      -101
      000101
         101
           0
```

```
      9
5 ) 45
    45
     0
```

c) $\frac{121}{14}$

```
        1000 -> 8
1110 ) 1111001
       1110
        1001 - 9
```

```
      8
14 ) 121
     -112
        9
```

d)

```
        101010
101 ) 1101100
      -101
       00110
        101
       00110
      -101
       00010
```

```
      42
5 ) 212
    20
     12
     10
      2
```

e) 7 < 0111

e)

```
        10000 -> 24
0111 ) 10101010
     - 0111
       00011010
     -  0111
        00001
        0111
        000010 -> 2
```

```
      24
7 ) 170
    - 14
      30
    - 28
       2
```

Assignment 1

12) a) $(34.34)_{10}$

$$\begin{array}{r|l} 2 & 34 \\ \hline & 17 - 0 \\ \hline & 8 - 1 \\ \hline & 4 - 0 \\ \hline & 2 - 0 \\ \hline & 1 - 0 \end{array}$$

$34 + 0.34 \times 2 = \underline{0}.68 \quad 0$

$0.68 \times 2 = \underline{1}.36 \quad 1$

$0.36 \times 2 = \underline{0}.72 \quad 0$

$0.72 \times 2 = \underline{1}.44 \quad 1$

$0.44 \times 2 = \underline{0}.88 \quad 0$

$0.88 \times 2 = \underline{1}.76 \quad 1$

$0.76 \times 2 = \underline{0}.5776 \quad 0$

$(100010.0101\cancel{010})_2$

100010

~~1.36~~

~~b) (125.12~~

Base - 3

$$\begin{array}{r|l} 3 & 34 \\ \hline & 11 - 1 \\ \hline & 3 - 2 \\ \hline & 1 - 0 \end{array}$$

$34 + 0.34 \times 3 = \underline{1}.02$

$0.02 \times 3 = \underline{0}.06$

$0.06 \times 3 = \underline{0}.18$

$0.18 \times 3 = \underline{0}.54$

$(1021 \cancel{00010}\ 1000\ldots)_3$

octal

$$\begin{array}{r|l} 8 & 34 \\ \hline & 4 - 2 \end{array}$$

$34 + 0.34 \times 8 = \underline{2}.72$

$0.72 \times 8 = \underline{5}.76$

$0.76 \times 8 = \underline{6}.08$

$0.08 \times 8 = \underline{0}.64$

$(42506\ldots)_8$

Hexadecimal

$$\begin{array}{r|l} 16 & 34 \\ \hline & 2 - 2 \end{array}$$

$34 + 0.34 \times 16 = \underline{5}.44$

$0.44 \times 16 = \underline{7}.04$

$0.04 \times 16 = \underline{0}.64$

$0.64 \times 16 = \underline{10}.24$

$(22570A\ldots)_{16}$

b) $(125.125)_{10}$

Binary

$125 + 0.125 \times 2 = 0.250$

$0.250 \times 2 = 0.500$

$0.500 \times 2 = 1.000$

$(1111101.001)_2$

	125
2	62 – 1
	31 – 0
	15 – 1
	7 – 1
	3 – 1
	1 – 1

1111101

Base-3

$125 + 0.125 \times 3 = 0.375$

$0.375 \times 3 = 1.125$

$0.125 \times 3 = 0.375$

$0.375 \times 3 = 1.125$

$(11221010\ldots)_3$

	125
3	41 – 2
	13 – 2
	4 – 1
	1 – 1

1122

Octal

$125 + 0.125 \times 8 = 1.000$

$(1751)_8$

	125
8	15 – 5
	1 – 7

175

Hexadecimal

$125 + 0.125 \times 16 = 2.000$

$(7D2)_{16}$

	125
16	7 – 13 D

7D

c) $(10.16)_{10}$

Binary

$10 + 0.16 \times 2 = \underline{0}.32$

$0.32 \times 2 = \underline{0}.64$

$0.64 \times 2 = \underline{1}.28$

$0.28 \times 2 = \underline{0}.56$

$(1010\ 0010\ldots)_2$

2	10
	5 - 0
	2 - 1
	1 - 0

1010

Base-3

$10 + 0.16 \times 3 = \underline{0}.48$

$0.48 \times 3 = \underline{1}.44$

$0.44 \times 3 = \underline{1}.32$

$0.32 \times 3 = \underline{0}.96$

$(101\ 0110\ldots)_3$

3	10
	3 - 1
	1 - 0

101

Octal

$10 + 0.16 \times 8 = \underline{1}.28$

$0.28 \times 8 = \underline{2}.24$

$0.24 \times 8 = \underline{1}.92$

$0.92 \times 8 = \underline{7}.36$

$(12\ 1217\ldots)_8$

8	10
	1 - 2

12

Hexadecimal

$10 + 0.16 \times 16 = \underline{2}.56$

$0.56 \times 16 = \underline{8}.96$

$0.96 \times 16 = \underline{F}15.36$

$0.36 \times 16 = \underline{5}.76$

$(A28F5\ldots)_{16}$

16	10 A
	0

13) In 12-bits, the largest positive number must ~~ha~~ not have leading 1, as it would have to be negative, therfore, the largest positive number we can represent is 0111 1111 1111 in ~~decim~~ binary. 2047 in decimal.

14) C O D E / T H S 2 0 2 2
67 79 68 69 47 84 72 83 32 50 48 50 50

15) 1111 1

16)

	B	F	C
2	11	15	12
+ 5	4	10	7
7	15	25	19
7	F	19	13

1	1	1	
1	B	F	C
2	11	15	12
+ 5	4	10	7

$(8\ 0\ A\ 3)_{16}$

16	19
	1 - 3

~~16 | 25~~
~~1 - 9~~

16	26
	1 - 0A

16	16
	1 - 0

17) ~~10 11 12 7~~

A	B	C	7
10	11	12	7
16^3	16^2	16^1	16^0

$40960 + 2816 + 192 + 7 = (43975)_{10}$

2	43975
	21987 - 1
	10993 - 1
	5496 - 1
	2748 - 0
	1374 - 0
	687 - 1
	343 - 1
	171 - 1
	85 - 1
	42 - 1

$(1010\ 1011\ 1100\ 111)_2$

18) $\overset{(10)}{A}\ \overset{(12)}{C}\ \overset{5}{7}\ \overset{4}{4}$

$-\ \ \overset{(11)}{B}\ 3\ \overset{(15)}{F}$

$(A\ 1\ 3\ 5)_{16}$

19) a) 0.1001

$2^{-1}\ 2^{-4}$

$\frac{1}{2} + \frac{1}{16} = \frac{9}{16}$

$= 0.5625$

b) 1.0011

$2^0\ 2^{-3}\ 2^{-4}$

$1 + \frac{1}{8} + \frac{1}{16} = \cancel{\frac{14}{16}} = \cancel{\frac{7}{8}}\ \frac{19}{16}$

c) 1.1111

2^0

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} =$

$\frac{3}{4} + \frac{3}{16} = 1 + \frac{15}{16} = \frac{31}{16}$

20) $0.647 \times 2 = \underline{1}.294$

$0.294 \times 2 = \underline{0}.498$

$0.498 \times 2 = \underline{0}.996$

$0.996 \times 2 = \underline{1}.992$

$0.992 \times 2 = \underline{1}.984$

$0.984 \times 2 = \underline{1}.986$

$0.986 \times 2 = \underline{1}.972$

11/17 in binary $= (1001111\ldots)_2$

21) $0.2727 \times 2 = \underline{0}.5454$

$0.5454 \times 2 = \underline{1}.0908$

$0.0908 \times 2 = \underline{0}.1816$

$0.1816 \times 2 = \underline{0}.3632$

$0.3632 \times 2 = \underline{0}.7264$

$0.7264 \times 2 = \underline{1}.4528$

$0.\cancel{405}$

$4528 \times 2 = \underline{0}.9056$

3/11

$\downarrow$

$(01000010\ldots)_2$

Only fractions with a denominator which is a power of two can be finitely represented in a binary form. In order to store them as a IEEE-754 floating point, they have to be rounded to the number of available bits for mantissa. 10 bits for half precision, 23 bits for single precision or 52 bits for double precision.