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# Recitation 1

### Part 1: Probability Theory and Inequalities

1. **The union bound:** Let be k different events (might be dependent) then we have:
2. **Markov’s inequality:** For every **non-negative** random variable , and let it holds that:

Proof:

1. **­­Chebyshev’s inequality:** Let be a random variable with finite expected and finite variance  
   . It holds that for any real number :

Proof:

1. **Chernoff Bound (Hoeffding inequality):** Let be independent and identical distributed (i.i.d) random variables bounded by the interval and   
   let to be their empirical mean. Then for every :

Proof: First denote: so .

### Part 2: ERM – Empirical risk minimization

Recall that in the supervised learning settings, we are given a training set and would like to learn a “good” hypothesis (function) , from the input space to the output space.  
we shall consider a binary classification in with . Let be a training set of samples drawn i.i.d from some (unknown) distribution .

Given a hypothesis , we define its empirical risk (or the training error) as the empirical error over the training set :

The generalization error (true error) of is defined as

We define the hypothesis class used by a learning algorithm as the set of all possible hypotheses (binary classifiers in this case) considered by it.

Consider a finite hypothesis class consisting of hypotheses.   
**Empirical Risk Minimization** is a learning algorithm that returns a hypothesis that minimize the empirical risk on . I.e., empirical risk minimization return .

We would like to give guarantees on the generalization error of . Let be some hypothesis. Now, consider a Bernoulli random variable whose distribution is defined as follows, we are going to sample then we set to indicate whether is misclassified a sample .  
The training error can be written: . Thus is exactly the mean of the m random variables that are drawn i.i.d from Bernoulli distribution with mean .

Hence, we can apply the Chernoff inequality to obtain:

This shows that for out particular , the training error will be close to generalization error with high probability, assuming is large. We can prove that this will be true simultaneously for all . To do so, let denote the event that when .

We have already shown that for any particular , it holds that , thus using the union bound we have that: (

And the probability that we estimate the error “well enough” for every is:

Now we can calculate the minimal number of samples to guarantee that with probability of at least , training error will be within of generalization error, for any given and for any . This is done by setting .

Solving for we find that for any (logarithmic in ).

It holds that with probability for any . The training set size that a certain method or algorithm requires in order to achieve a certain level of performance is also called the algorithm’s .

# LECTURE 2 – DETERMINISTIC DECISION PROCESSES

**Today we covered:**

1. Deterministic Model
2. Finite Horizon
   1. Shortest paths
   2. Dynamic programming
3. Policies
   1. History
   2. Stochastic
4. Average rewards
   1. Min Mean cost cycle

### Deterministic Decision Processes

Before diving to talk about deterministic decision processes we present a few general definitions that we will use during this course.

**Discrete time dynamic system**

* is the set of possible states at time .
* is the set of possible control at time .
* is the transition function.

**Model**

* Can be finite or infinite.
* Control variable can depend on the state.
* Time invariant system: for all : , , .

**Observations and Rewards**

* Observation
  + Fully observable:
* Costs / Rewards
  + Per state action
    - Costs
    - Rewards

**Linear Dynamics**

* Continuous state and action .
* Linear dynamics:
  + when
* Quadratic cost

**Finite models**

* State rather than .
* Actions rather than

**Deterministic decisions process**

The model of deterministic decisions process represented as a directed graph when:

* Nodes are states
* Edges are actions

**Feasible paths** are a sequence of states and actions.

For example, where and .

The cost of this path is .

the reward of this path is .

We define the optimal path as follow: .

When we defining decisions process, we also need to decide if we are dealing with strategy or policy.

Another different is if the process is deterministic or stochastic.

**What is the different between policy and strategy?**

Strategy is when we map full history to distribution over actions.

Policy is when we map states to distribution over actions, meaning we don’t care about the history and we are looking only on the last state to decided what will be the next state given state and action.

**What is the different between stochastic and deterministic?**

Distribution over actions versus a single action

Full definition to all combinations discussed above: