

## Before you begin

- These slides are used in presentations at workshops.
- They are best viewed with a **pdf reader** like **Acrobat Reader** (free download).
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  - Navigation buttons are provided at the bottom of each screen if needed (see below).
- Viewing in **web browsers** is not recommended.

## Do not try to print the slides

There are many more pages than the number of slides listed at the bottom right of each screen!

Apologies for any inconvenience.

# Inferential Statistics (testing hypotheses)

(mα+hs)Smart Workshop

Semester 2, 2016

Geoff Coates

*(This workshop is a follow-up to the earlier “Introduction to Statistical Inference” session.) These slides go through the steps used to conduct the one sample t-test and demonstrates how to extract the necessary information from a description of an experiment.*

# What can ( $\alpha+\text{hs}$ )Smart do for you?

## Online Stuff

- presentation slides from workshops on many topics
- practice exercises
- short videos
- and more!

## Drop-in Study Sessions

- Monday, Wednesday, Friday, 10am-12pm, Ground Floor Barry J Marshall Library, *teaching weeks and study breaks.*

## Workshops

- See our current Workshop Calendar for this Semester's topics.

Email: [geoff.coates@uwa.edu.au](mailto:geoff.coates@uwa.edu.au)

- Can't find what you want?
- Got a question?  
Drop us a line!

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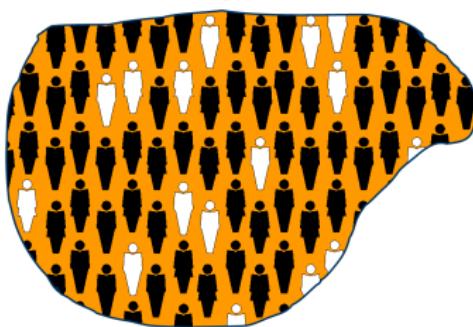
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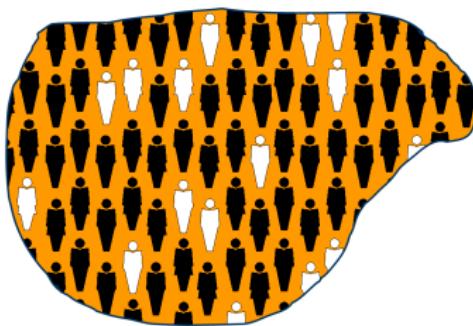
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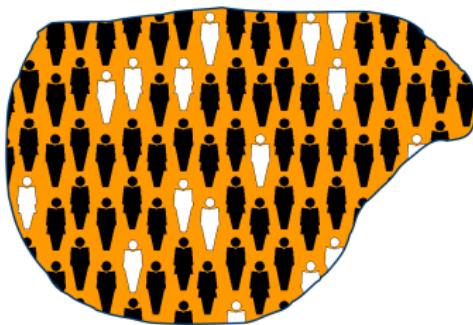
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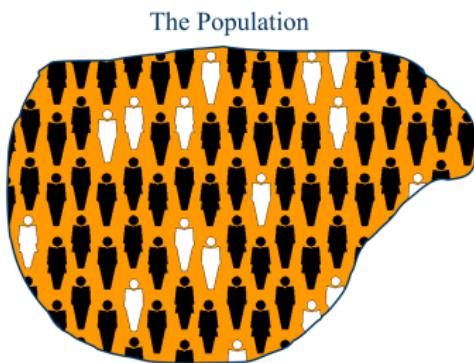
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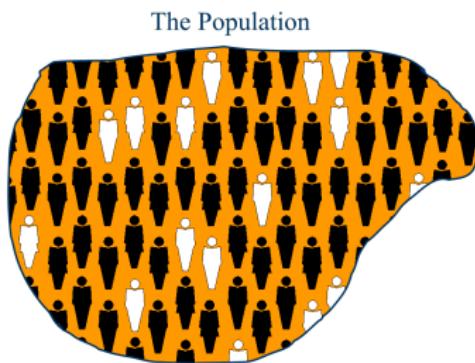


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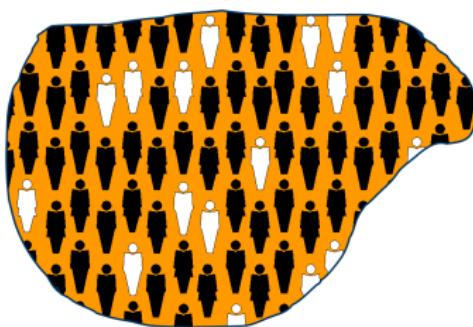
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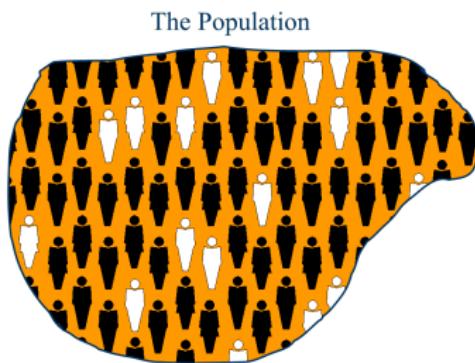
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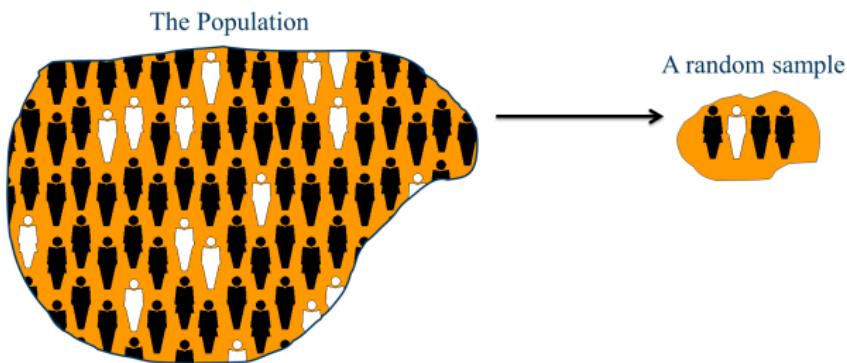
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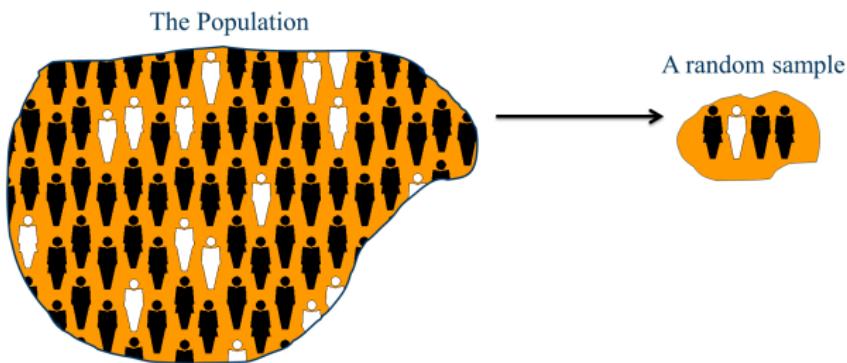
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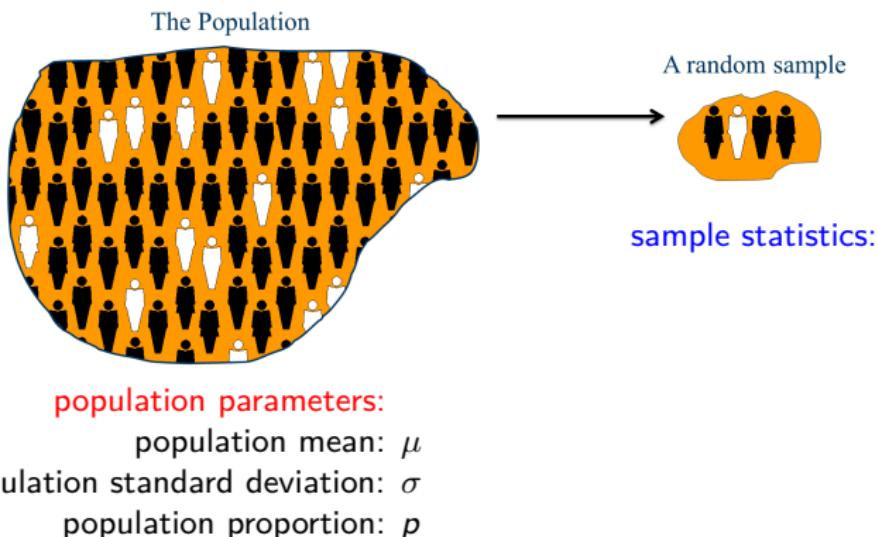
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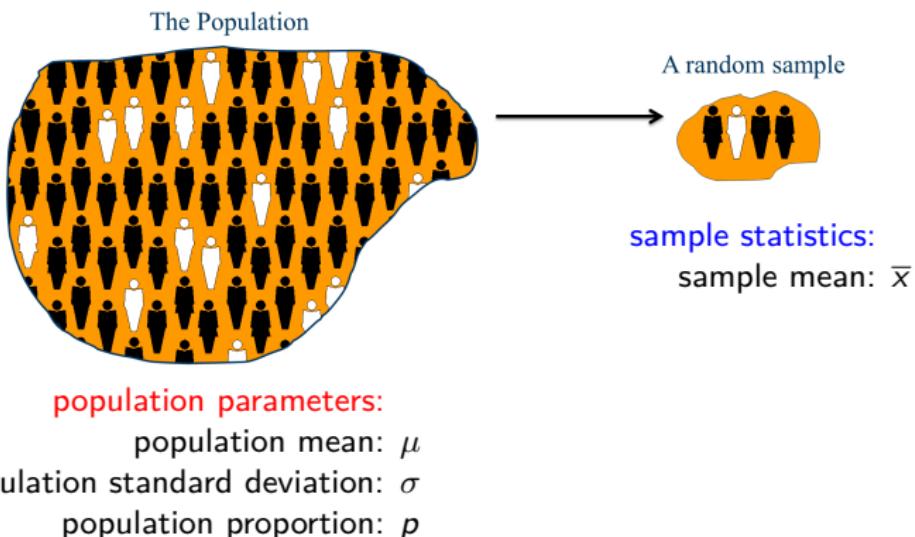
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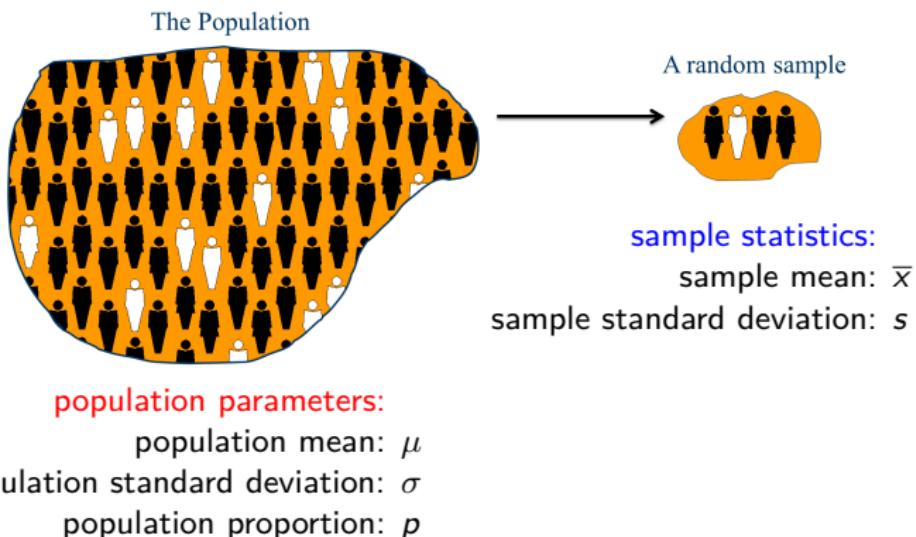
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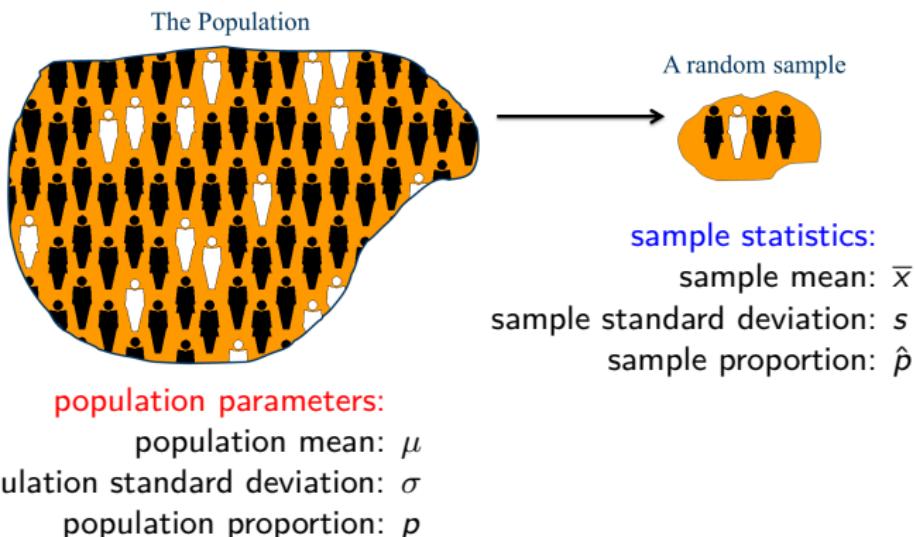
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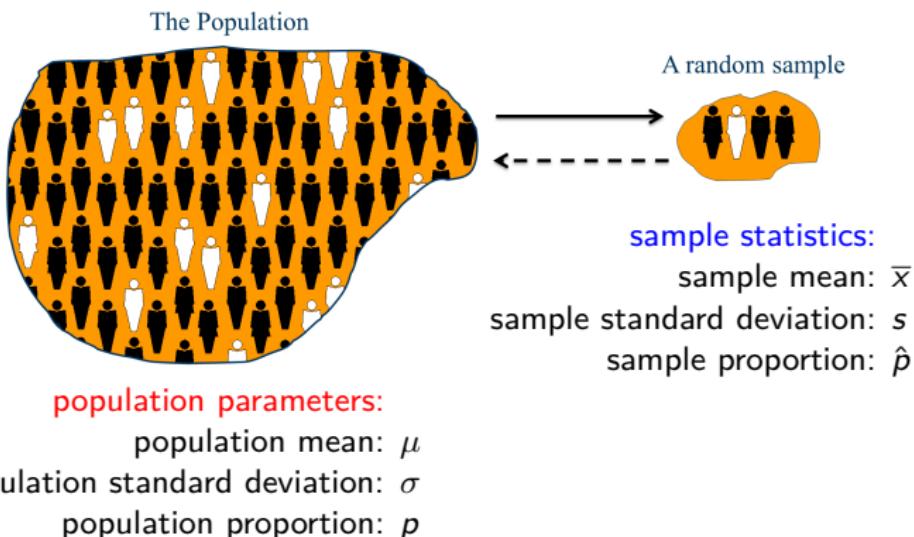
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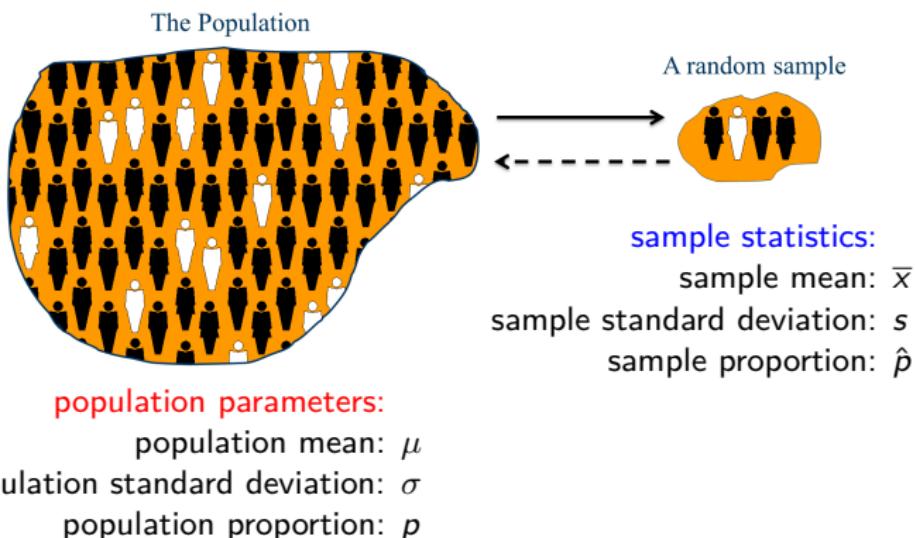
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**Note:** separating **population parameters** and **sample statistics** helps organise all the notation used in statistics.

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**Decision:** if  $p$ -value “too small” (ie.  $<$  significance level  $\alpha$ ), we reject  $H_0$  in favour of  $H_1$  at the  $100\alpha\%$  significance level.

## A typical test/exam-style question

We will now apply this general framework to a commonly used test, the **one-sample *t*-test**, using data from a published paper:

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Some of the analysis in this paper has been re-cast as a typical test/exam-style question.

## A typical test/exam-style question: setting up the hypotheses

A study of the dental status of critically ill children in a Paediatric Intensive Care Unit examined 16 children with permanent teeth and found that the mean number of missing or filled teeth was 1.2 with a standard deviation of 1.9. Extensive analysis has established that the mean number of such teeth in the wider population of children is 1.4. Test whether the mean for critically ill children differs from this.

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$\bar{x} = 1.2$  teeth. In most Intro Stats units the only available testing procedure is the *one-sample t-test*. This uses a "standardized" version of  $\bar{x}$ :

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} =$$

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$$H_0 : \mu = 1.4 \text{ missing/filled teeth} \quad H_1 : \mu \neq 1.4 \text{ missing/filled teeth}$$

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**Test Statistic:** Suitable estimate of the population parameter (or combination of parameters) derived from these data.

$\bar{x} = 1.2$  teeth. In most Intro Stats units the only available testing procedure is the *one-sample t-test*. This uses a "standardized" version of  $\bar{x}$ :

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1.2 - 1.4}{\frac{1.9}{\sqrt{16}}}$$

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$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1.2 - 1.4}{\frac{1.9}{\sqrt{16}}} = -0.421$$

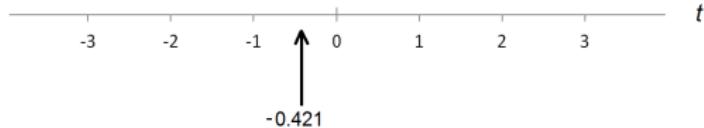
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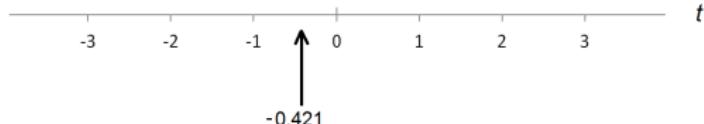
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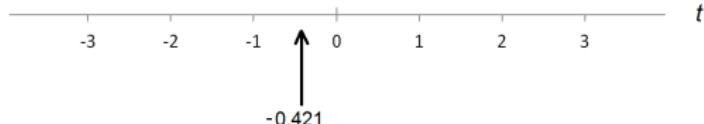


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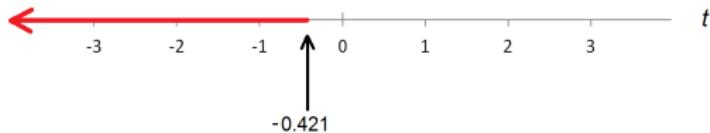


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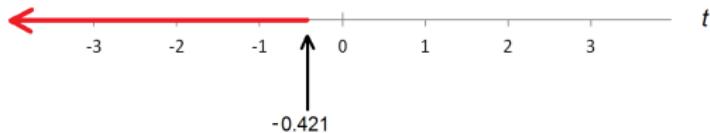
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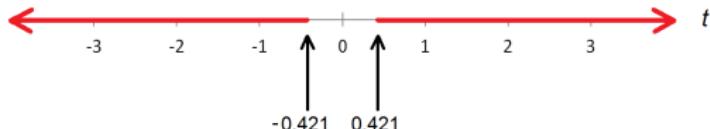
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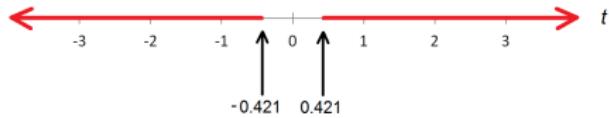
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These values correspond to  $t > 0.421$  (a useful feature of  $t$ ).

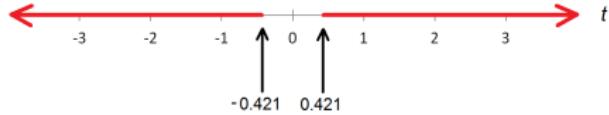
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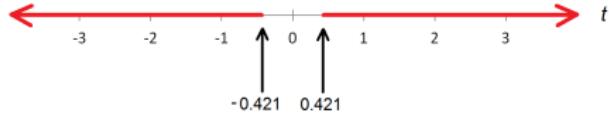
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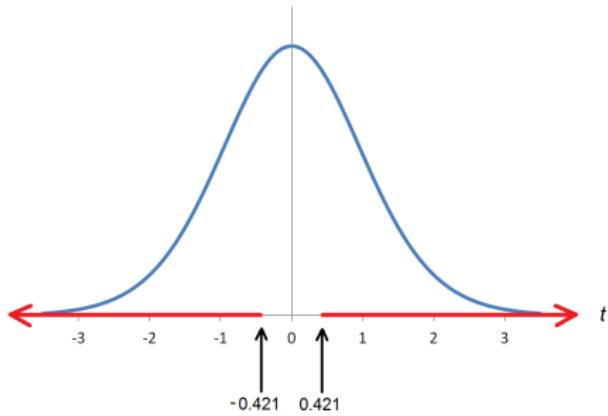
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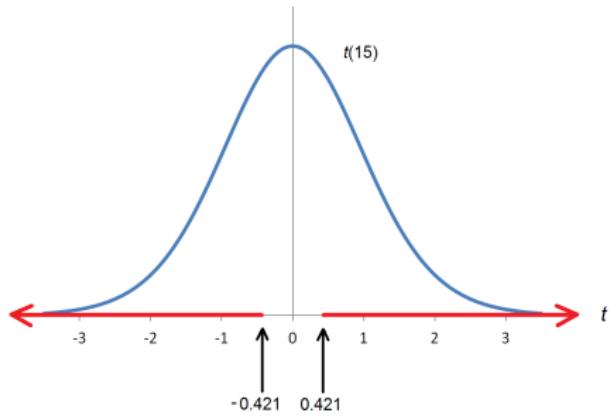
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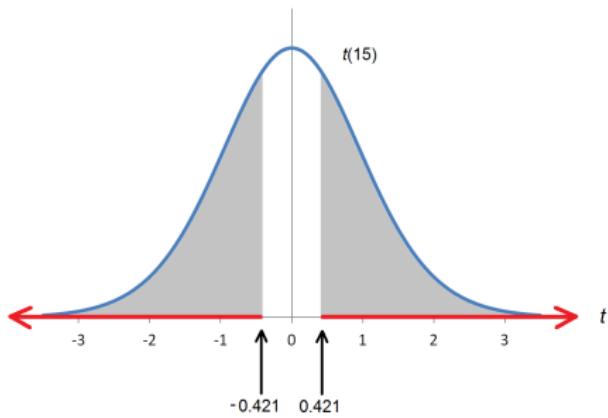
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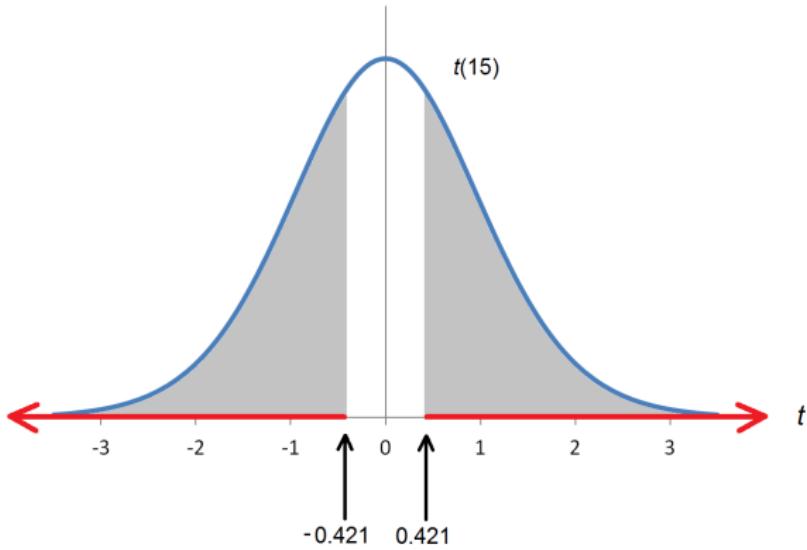
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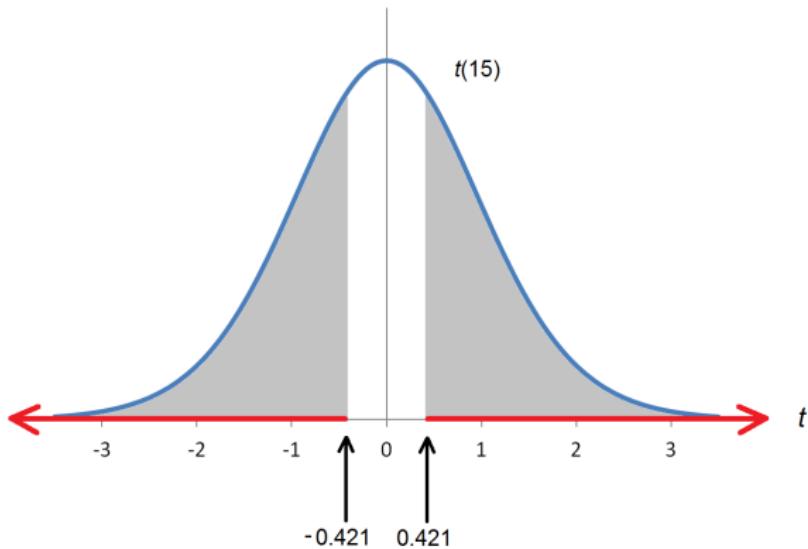
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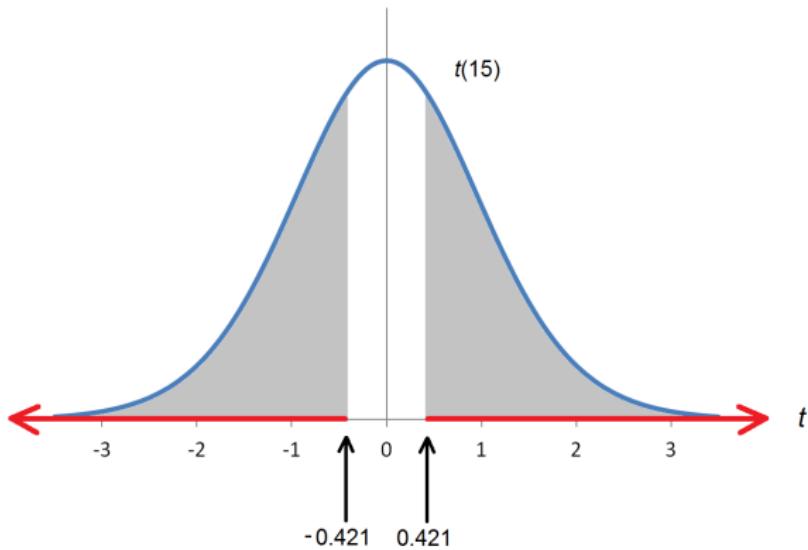
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“=T.DIST.2T(0.421,15)” returns  $p$  – value = 0.680.

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`=T.DIST.2T(0.421,15)` returns  $p$  – value = 0.680. Was your guess close?

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$$p\text{-value} = 0.680$$

**Decision:** if  $p$ -value “too small” (ie.  $<$  significance level  $\alpha$ ), we reject  $H_0$  in favour of  $H_1$  at the  $100\alpha\%$  significance level.

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- Some Intro Stats units use a ***t*-table** to find a suitable approximation for the  $p$ -value. If you don't use this method, you can skip the next section by clicking

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## Using a $t$ -table to find the $p$ -value

- A  $t$ -table provides enough information about a  $p$ -value to make decisions:

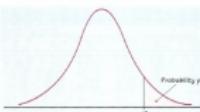
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TABLE D  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.001	.0005	
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.183	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.358	1.782	2.179	2.303	2.681	3.058	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.344	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.894	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.879	3.197	3.611	3.922
19	0.688	0.861	1.064	1.324	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.843	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.811	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.318	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.849	1.045	1.298	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level C



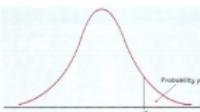
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23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.318	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.849	1.045	1.298	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

Confidence level C



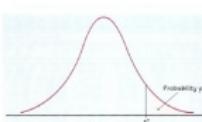
- Each row refers to a different  $t$ -distribution. In this case we need the row for 15 df.

# Using a $t$ -table to find the $p$ -value

- A  $t$ -table provides enough information about a  $p$ -value to make decisions:

TABLE D  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	65.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.183	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.943	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.769	2.179	2.303	2.681	3.058	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.774	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.073	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.777	4.140
15	<b>0.691</b>	<b>0.866</b>	<b>1.074</b>	<b>1.341</b>	<b>1.753</b>	<b>2.131</b>	<b>2.249</b>	<b>2.602</b>	<b>2.947</b>	<b>3.286</b>	<b>3.733</b>	<b>4.073</b>
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.680	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.894	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.064	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.843	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.810	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.316	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.301	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.674	2.937	3.261	3.496
60	0.679	0.848	1.045	1.299	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.633	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.624	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$\infty$	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291



Confidence level  $C$

- Each row refers to a different  $t$ -distribution. In this case we need the row for 15 df.

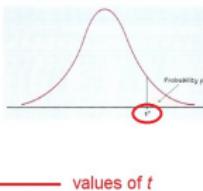
# Using a $t$ -table to find the $p$ -value

- A  $t$ -table provides enough information about a  $p$ -value to make decisions:

TABLE D  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	65.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.183	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.943	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.769	2.179	2.303	2.681	3.058	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.075	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.777	4.140
15	<b>0.691</b>	<b>0.866</b>	<b>1.074</b>	<b>1.341</b>	<b>1.753</b>	<b>2.131</b>	<b>2.249</b>	<b>2.602</b>	<b>2.947</b>	<b>3.286</b>	<b>3.733</b>	<b>4.073</b>
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.894	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.064	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.843	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.810	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.316	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.301	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.298	1.676	2.009	2.109	2.403	2.674	2.937	3.261	3.496
60	0.679	0.848	1.045	1.298	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.633	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.624	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$\infty$	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291

Confidence level  $C$



- Each row refers to a different  $t$ -distribution. In this case we need the row for 15 df.

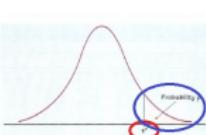
# Using a $t$ -table to find the $p$ -value

- A  $t$ -table provides enough information about a  $p$ -value to make decisions:

TABLE D  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	65.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.183	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.943	1.190	1.553	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.265	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.769	2.179	2.303	2.681	3.058	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.869	1.073	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.777	4.140
15	<b>0.691</b>	<b>0.866</b>	<b>1.074</b>	<b>1.341</b>	<b>1.753</b>	<b>2.131</b>	<b>2.249</b>	<b>2.602</b>	<b>2.947</b>	<b>3.286</b>	<b>3.733</b>	<b>4.073</b>
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.894	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.064	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.843	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.810	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.316	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.301	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.674	2.937	3.261	3.496
60	0.679	0.848	1.045	1.298	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.633	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$\infty$	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291

Confidence level  $C$

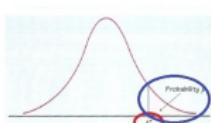


- Each row refers to a different  $t$ -distribution. In this case we need the row for 15 df.

# Using a $t$ -table to find the $p$ -value

TABLE D  $t$  distribution critical values

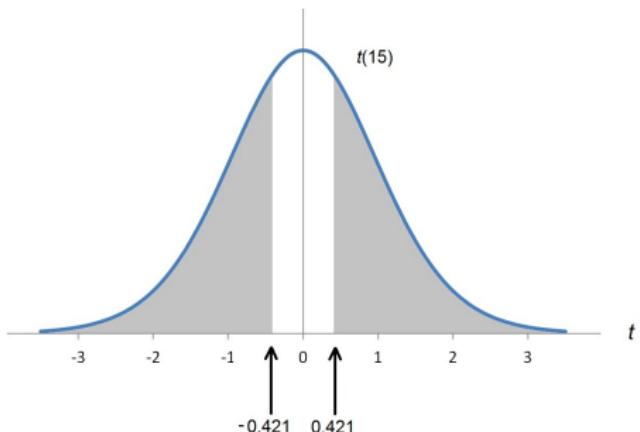
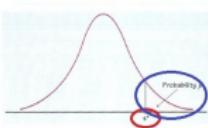
df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	65.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.265	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
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2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.995	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
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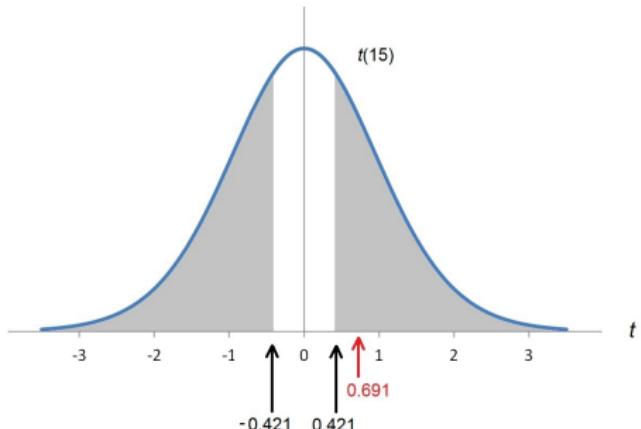
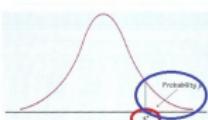


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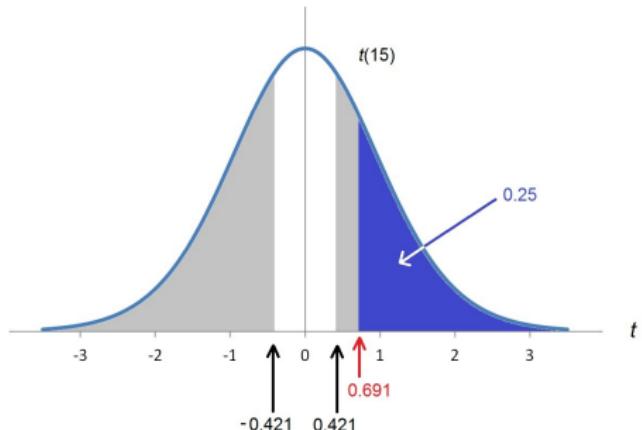
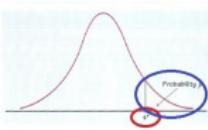


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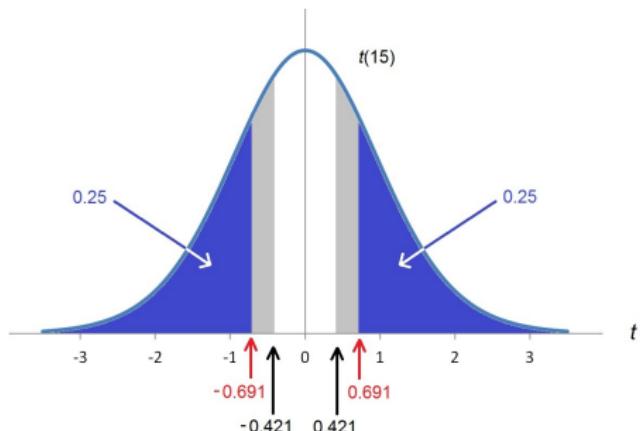


In this case, the positive version of  $t$  is 0.421 and the smallest value of  $t$  in this row is 0.691. We can now say that the blue shaded area is 0.25.

# Using a $t$ -table to find the $p$ -value

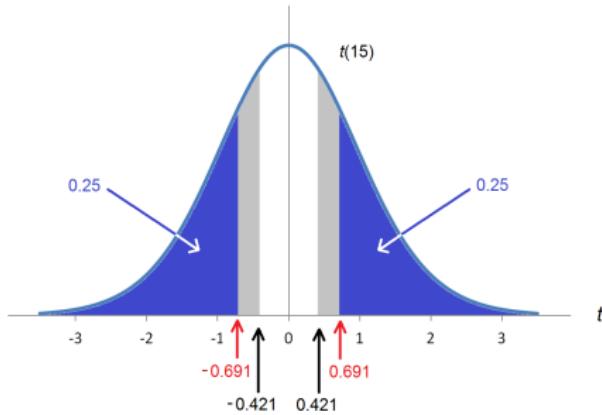
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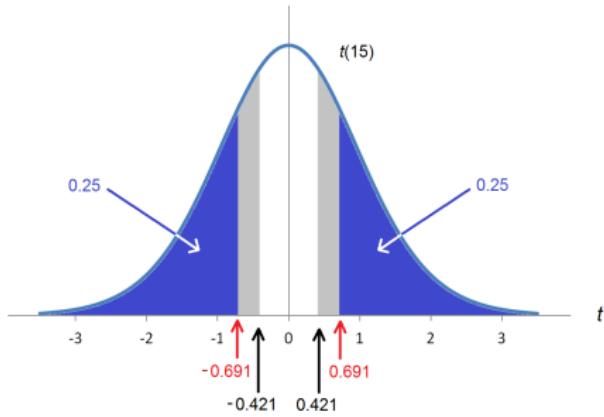


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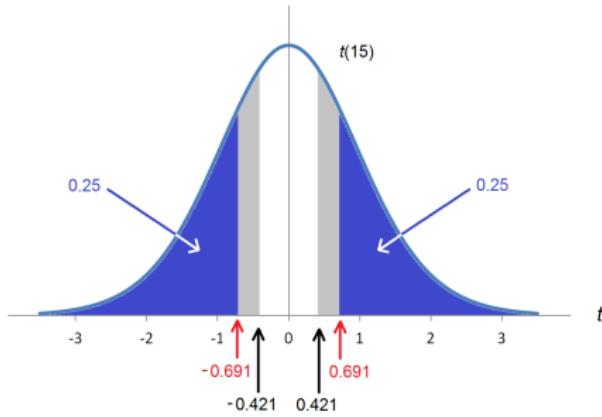


## Using a $t$ -table to find the $p$ -value



So, we can say that the  $p$ -value (grey area partially obscured by the blue area) is greater than the blue area:

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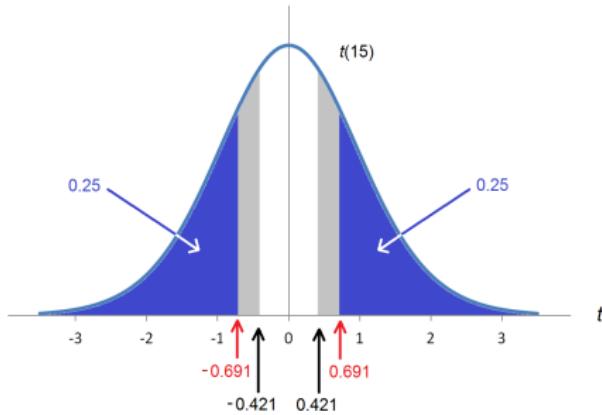


So, we can say that the  $p$ -value (grey area partially obscured by the blue area) is greater than the blue area:

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## Using a $t$ -table to find the $p$ -value



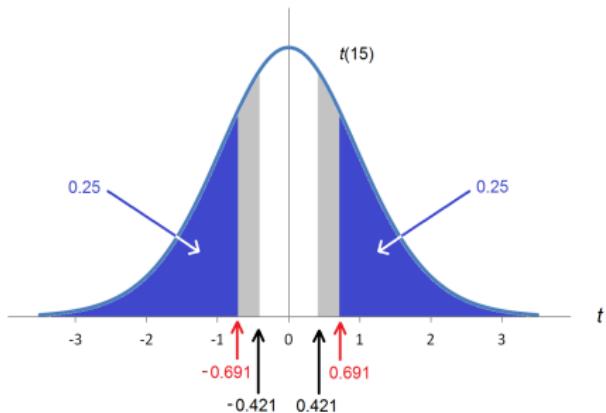
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(Remember that the exact answer is  $p\text{-value} = 0.680$ ).

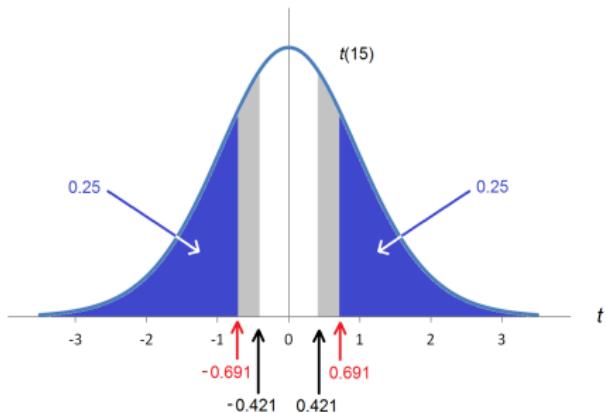
## Using a $t$ -table to find the $p$ -value



This is enough to make our decision:

**Decision:** if  $p$ -value “too small” (ie.  $<$  significance level  $\alpha$ ), we reject  $H_0$  in favour of  $H_1$  at the  $100\alpha\%$  significance level.

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Clearly, since  $p$ -value  $> 0.5$  does not meet the criterion of “too small”, we would “retain  $H_0$  at any sensible significance level” as before.

## Assumptions for the $t$ -test

- You may also be asked to consider whether the **assumptions** required for a hypothesis test to work have been met. If you don't discuss this topic in your unit, you can skip the next section by clicking [here](#).

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We have to assume that the chosen 16 children represent a **random sample** of such children ...

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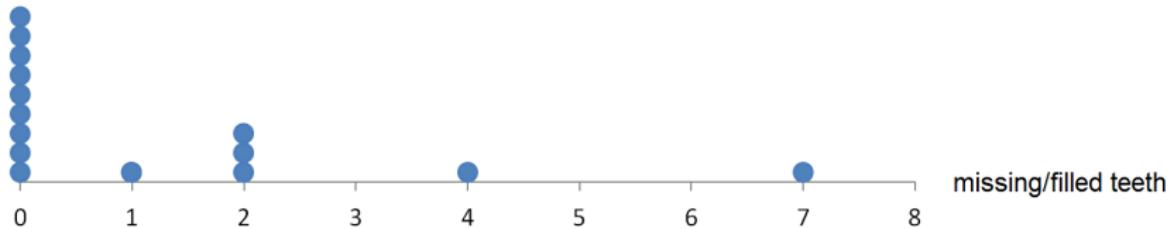
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You can't go backwards from the mean by even one standard deviation! This means that there must be data values much bigger than 1.2 in order to create such a large standard deviation.

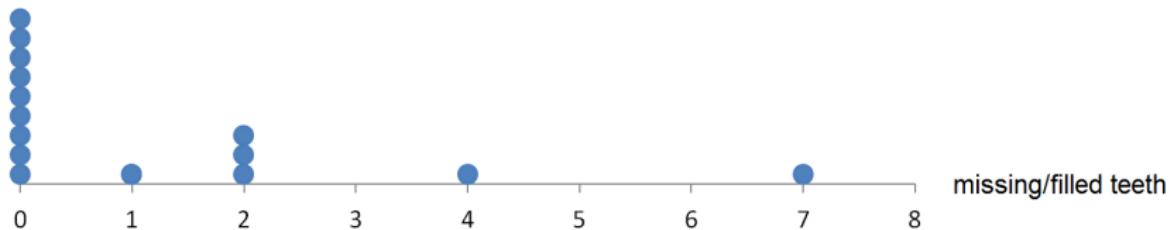
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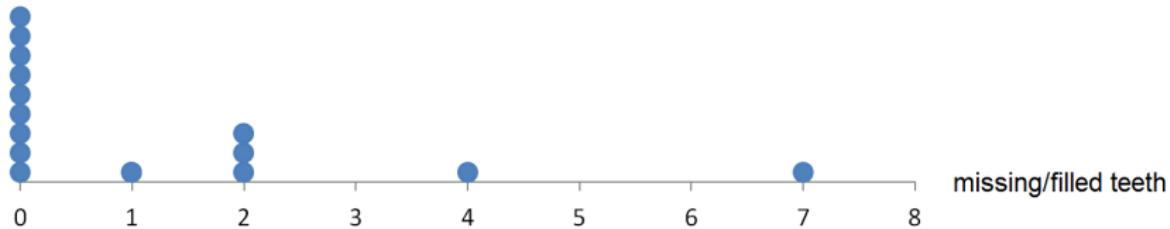
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## Another $t$ -test question: setting up the hypotheses

Plaque develops on teeth in response to the presence of bacteria and can lead to harmful effects. The difference in plaque coverage (% of all teeth surfaces with plaque) between admission and discharge was measured for each of the 16 critically ill children in the previous example. The mean of these differences (discharge – admission) was 4.0% with a standard deviation of 7.4%. Test whether there was a mean change in plaque coverage between admission and discharge.

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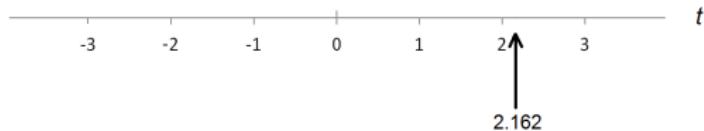
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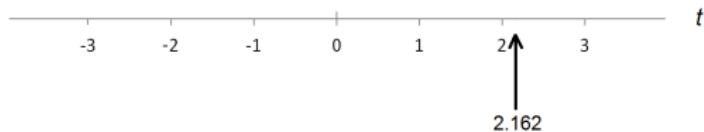
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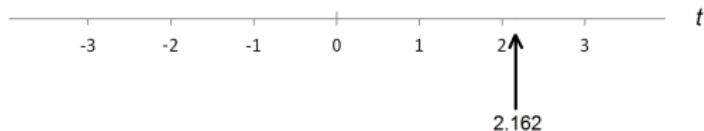


What sort of values of  $\bar{x}$  would have been more favourable to  $H_1$ ?

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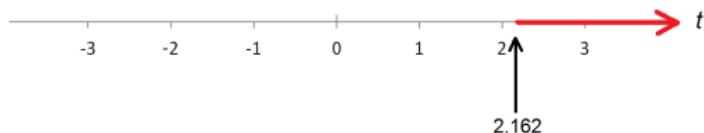


What sort of values of  $\bar{x}$  would have been more favourable to  $H_1$ ? Well, anything further away from 0 than the observed 4.0, such as 4.5 or 5.0, etc (ie. any  $\bar{x} > 4.0$ ).

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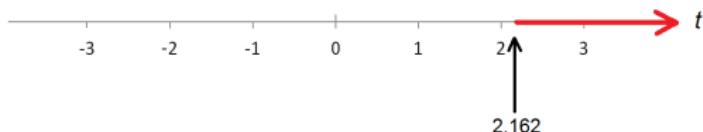
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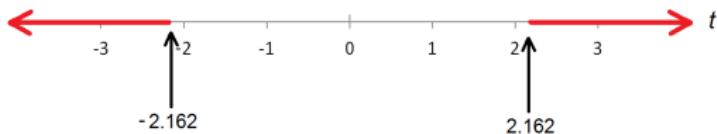
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In fact, since  $H_1$  is two sided, any value further away from 0 on the other side (ie. less than -4.0) would have been more favourable to  $H_1$  as well.

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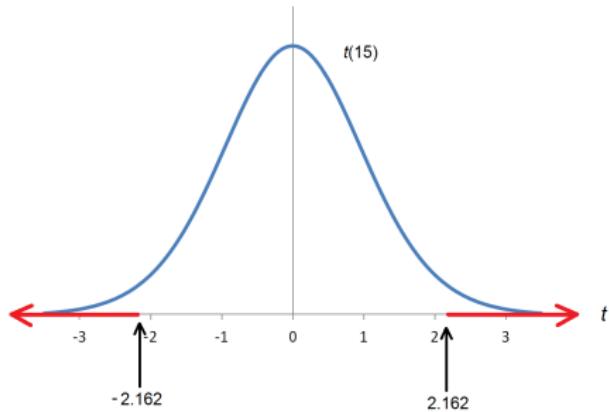
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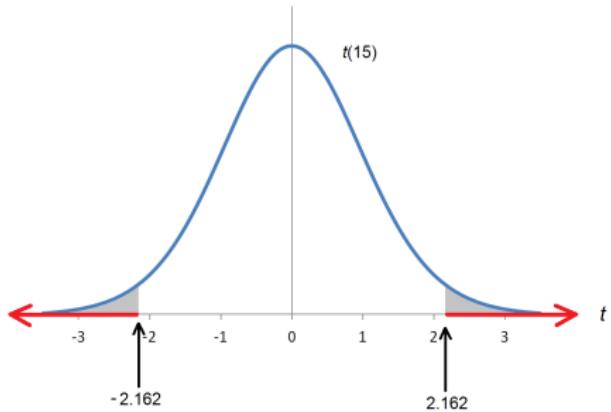
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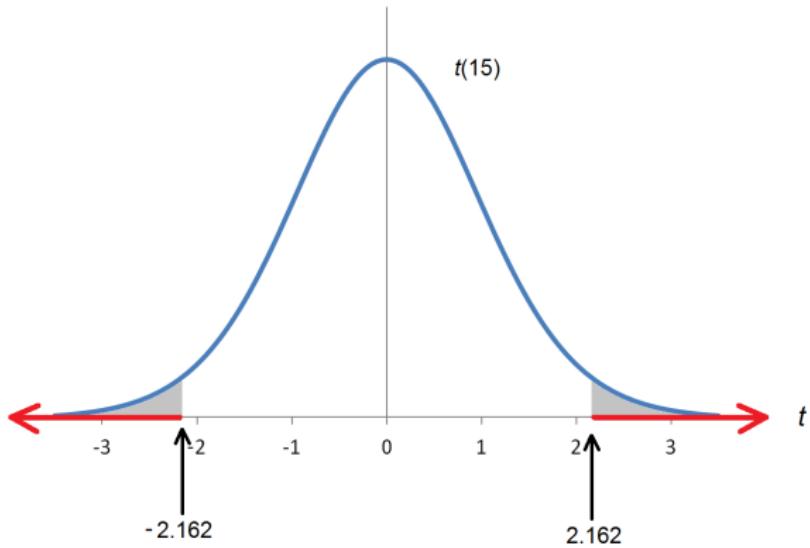
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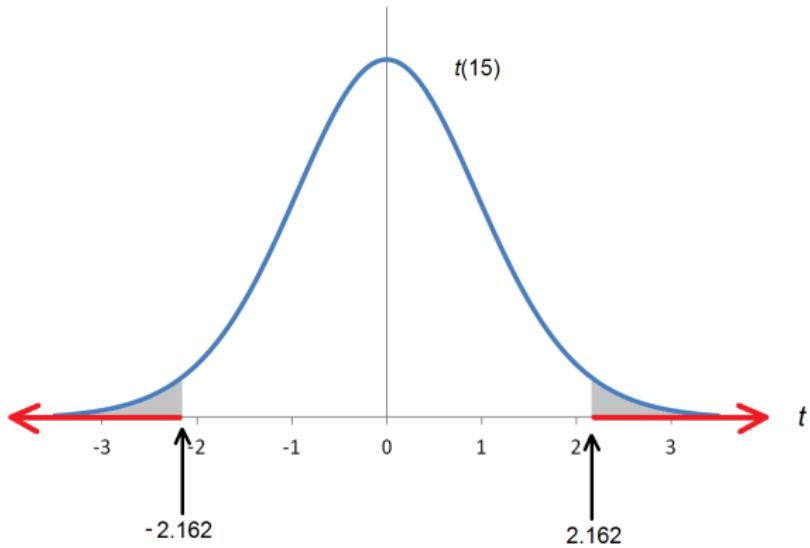
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Before we calculate the  $p$ -value, have a guess at what you think it is. (Hint: the total area under the curve is 1).

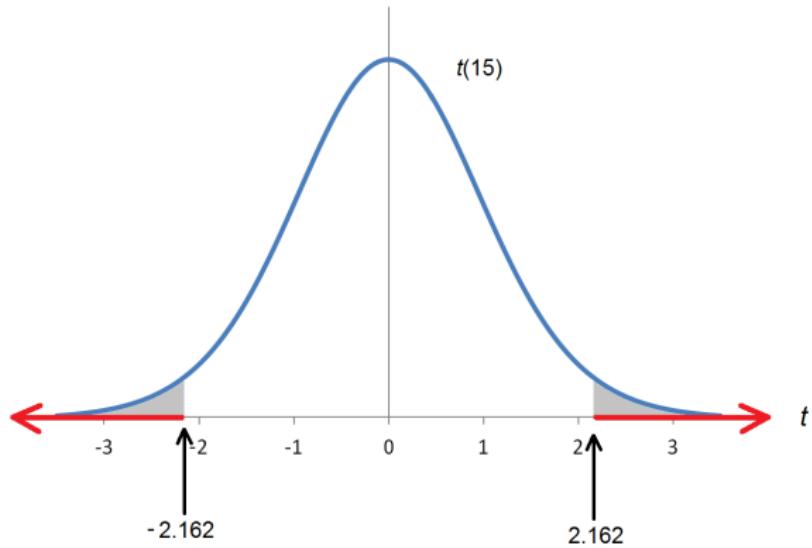
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**Decision:** if  $p$ -value “too small” (ie.  $<$  significance level  $\alpha$ ), we reject  $H_0$  in favour of  $H_1$  at the  $100\alpha\%$  significance level.

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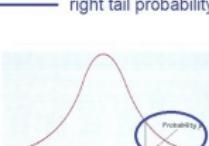
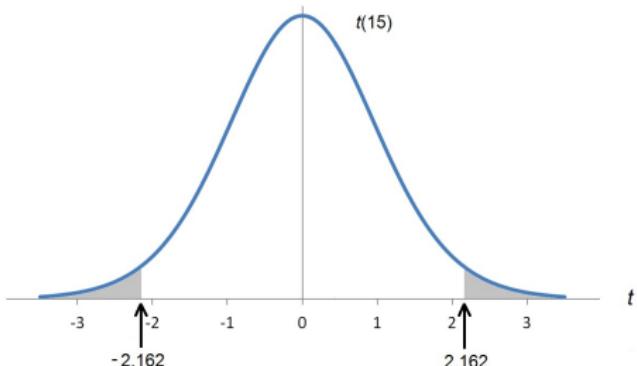
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- If your unit doesn't cover  $t$ -tables, you can skip the next section by clicking [here](#).

# Using a $t$ -table to find the $p$ -value

TABLE D  $t$  distribution critical values

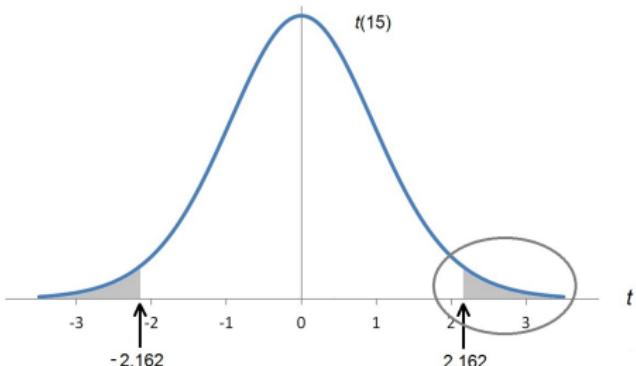
df	Tail probability $p$												
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005	
1	1.660	1.770	1.983	3.078	3.156	12.73	15.39	31.82	63.66	127.3	318.3	636.6	
2	0.816	1.061	1.386	1.880	2.920	4.303	4.849	6.985	9.925	14.09	22.33	31.60	
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92	
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610	
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869	
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959	
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408	
8	0.706	0.889	1.108	1.394	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041	
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781	
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587	
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437	
12	0.695	0.873	1.083	1.355	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318	
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221	
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140	
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073	



# Using a $t$ -table to find the $p$ -value

TABLE D  $t$  distribution critical values

df	Tail probability $p$												
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005	
1	4.600	3.708	3.078	2.918	2.776	2.571	2.326	2.132	1.983	1.863	1.750	1.650	
2	0.816	1.061	1.386	1.880	2.920	4.303	4.849	6.985	9.925	14.09	22.33	31.60	
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92	
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610	
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869	
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959	
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408	
8	0.706	0.889	1.108	1.394	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041	
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right tail probability

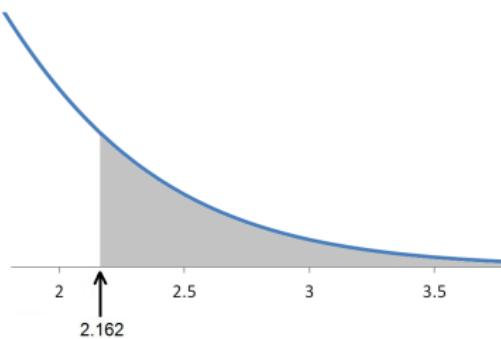
values of  $t$

Let's take a close up look at the grey shaded area.

# Using a $t$ -table to find the $p$ -value

TABLE D  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.370	1.983	3.078	6.316	12.73	15.30	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.880	2.920	4.303	4.849	6.985	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
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right tail probability

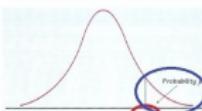
values of  $t$

Let's take a close up look at the grey shaded area.

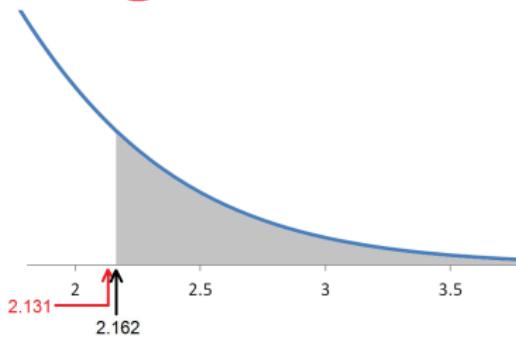
# Using a $t$ -table to find the $p$ -value

TABLE D  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
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2	0.816	1.061	1.386	1.880	2.920	4.303	4.849	6.985	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
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values of  $t$



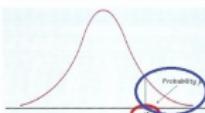
Let's take a close up look at the grey shaded area.

From the  $t$ -table, our  $t = 2.162$  sits between 2.131

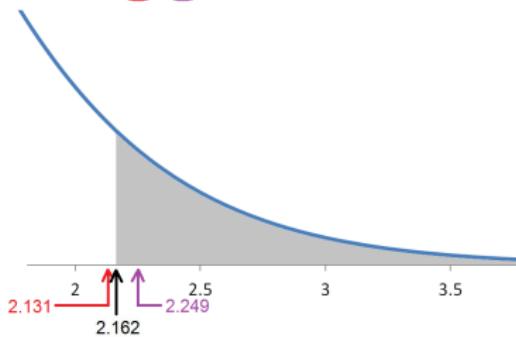
# Using a $t$ -table to find the $p$ -value

TABLE D  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.645	1.770	1.863	3.078	3.178	12.73	15.39	31.82	63.66	127.3	318.3	634.6
2	0.816	1.061	1.386	1.860	2.920	4.303	4.849	6.985	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
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values of  $t$



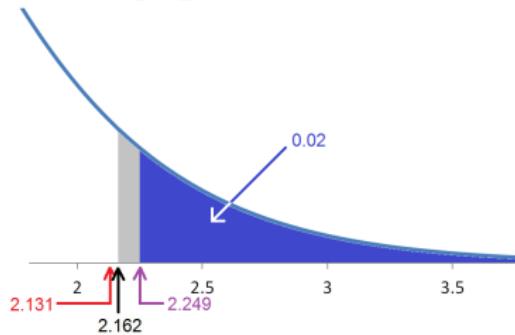
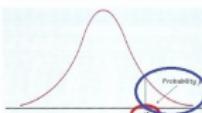
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# Using a $t$ -table to find the $p$ -value

TABLE D  $t$  distribution critical values

df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.660	1.770	1.983	3.078	3.156	12.71	13.32	33.66	127.3	318.3	634.6	
2	0.816	1.061	1.386	1.860	2.920	4.303	4.849	6.985	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
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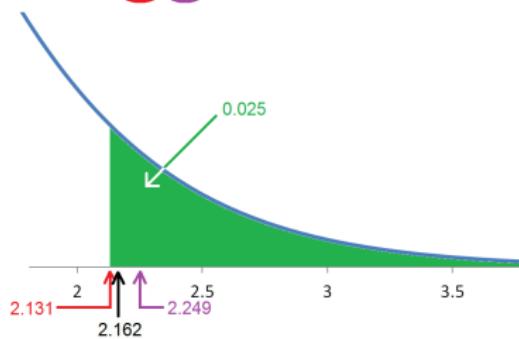
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# Using a $t$ -table to find the $p$ -value

TABLE D  $t$  distribution critical values

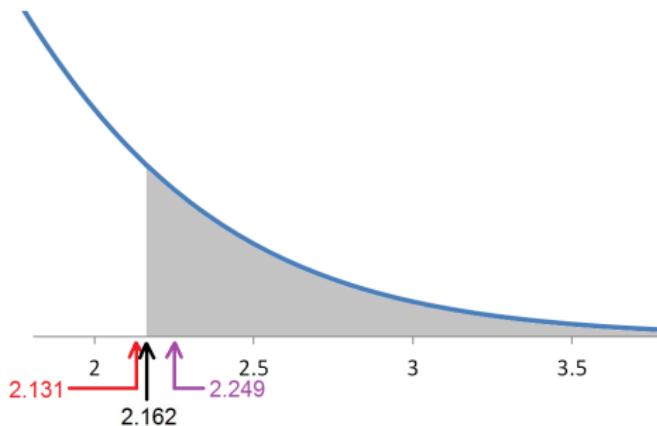
df	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.01	.005	.0025	.001	.0005	
1	1.660	1.770	1.883	2.078	2.318	2.771	3.182	3.346	3.727	3.833	3.846	
2	0.816	1.061	1.386	1.880	2.920	4.303	4.849	6.985	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
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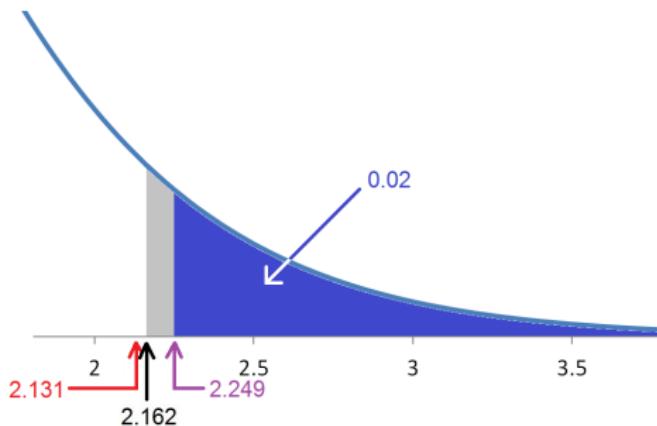
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## Using a $t$ -table to find the $p$ -value



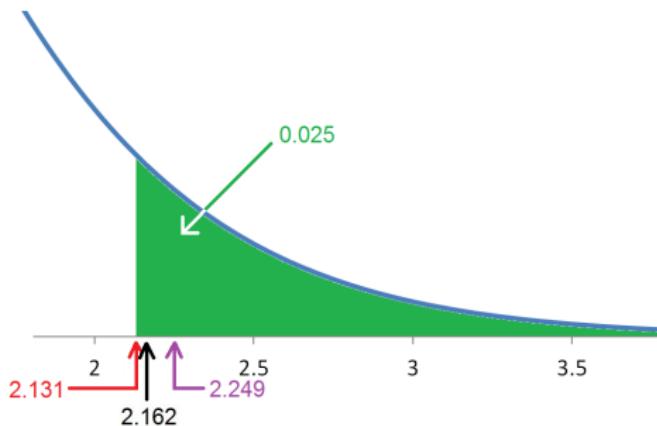
So, we don't know the exact  $p$ -value but we can say that the  $p$ -value (grey area) is

## Using a $t$ -table to find the $p$ -value



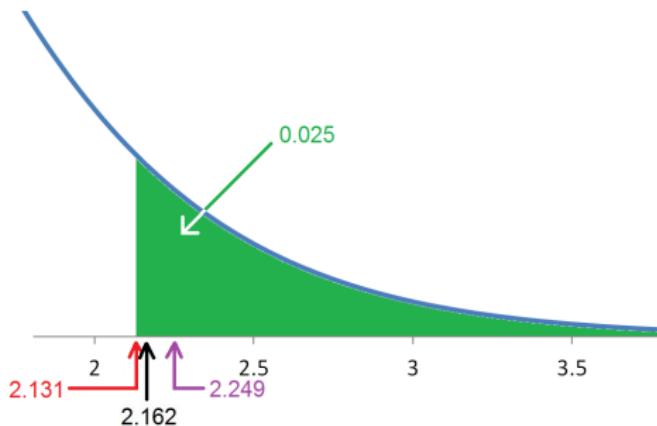
So, we don't know the exact  $p$ -value but we can say that the  $p$ -value (grey area) is larger than the **blue area (0.02)**

## Using a $t$ -table to find the $p$ -value



So, we don't know the exact  $p$ -value but we can say that the  $p$ -value (grey area) is larger than the blue area (0.02) but smaller than the green area (0.025).

## Using a $t$ -table to find the $p$ -value

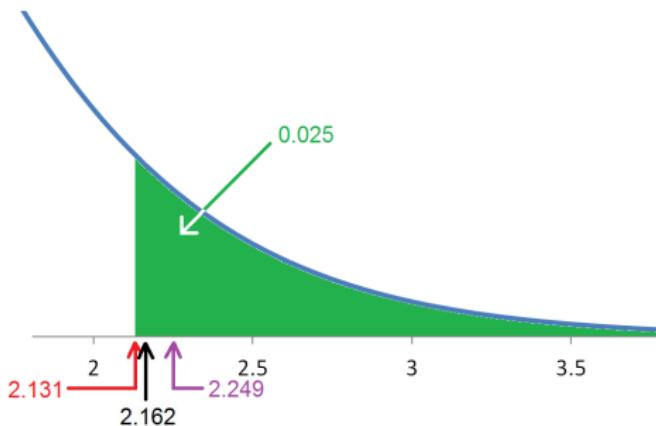


So, we don't know the exact  $p$ -value but we can say that the  $p$ -value (grey area) is larger than the blue area (0.02) but smaller than the green area (0.025). Remembering to double the above values to add in the left-hand tail, we can say:

$$2 \times 0.02 < p\text{-value} < 2 \times 0.025$$

$$0.04 < p\text{-value} < 0.05$$

## Using a $t$ -table to find the $p$ -value



So, we don't know the exact  $p$ -value but we can say that the  $p$ -value (grey area) is larger than the blue area (0.02) but smaller than the green area (0.025). Remembering to double the above values to add in the left-hand tail, we can say:

$$2 \times 0.02 < p\text{-value} < 2 \times 0.025$$

$$0.04 < p\text{-value} < 0.05$$

(Recall that the exact answer is  $p\text{-value} = 0.047$ .)

## Using a $t$ -table to find the $p$ -value

$$0.04 < p\text{-value} < 0.05$$

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This is enough to make our decision:

**Decision:** if  $p$ -value “too small” (ie.  $<$  significance level  $\alpha$ ), we reject  $H_0$  in favour of  $H_1$  at the  $100\alpha\%$  significance level.

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If  $\alpha$  is chosen to be 0.05 (5%), our  $p$ -value does meet the criterion of “too small” (just) and we would “reject  $H_0$  at the 5% significance level” as before.

## Assumptions for the *t*–test

- You may also be asked to consider whether the **assumptions** required for a hypothesis test to work have been met. If you don't discuss this topic in your unit, you can skip the next section by clicking [▶ here](#).

## Checking the assumptions for the $t$ -test

For the one sample  $t$ -test we just performed, a key assumption is

*the population follows a Normal Distribution or the sample size is large.*

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$n = 16$  is not a very large sample so a good statistician would examine the sample itself for evidence of non-Normality and/or outliers.

Luckily, differences in “before-and-after” studies like this are highly likely not to be skewed (there’s no boundary at 0 for a start).

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For the one sample  $t$ -test we just performed, a key assumption is

*the population follows a Normal Distribution or the sample size is large.*

$n = 16$  is not a very large sample so a good statistician would examine the sample itself for evidence of non-Normality and/or outliers.

Luckily, differences in “before-and-after” studies like this are highly likely not to be skewed (there’s no boundary at 0 for a start).

However, outliers are still a potential problem (in this case a child who acquires or loses an unusually large amount of plaque while in hospital).

## Checking the assumptions for the $t$ -test

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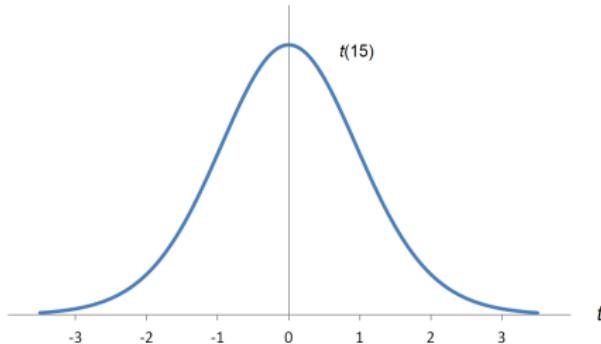
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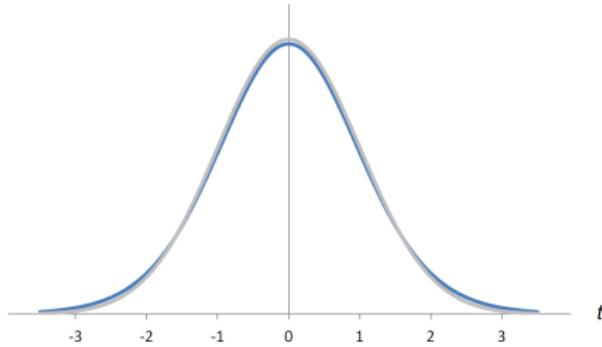
However, outliers are still a potential problem (in this case a child who acquires or loses an unusually large amount of plaque while in hospital). You would need the raw data to check.

## Appendix: A tip for understanding $t$ -tests



There should be something familiar about the  $t$ -distribution.

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## Appendix: A tip for understanding *t*-tests

This means that a *t*-statistic is like a ***z-score*** for the sample mean (when  $\mu = 1.4$ ). That is, it roughly follows the “**68-95-97.5% Rule**”:

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So,  $t = -0.421$  (ie. half a standard deviation below the population mean) is a pretty typical result when  $\mu = 1.4$ . In other words, we have no compelling evidence against  $H_0$ .

On the other hand,  $t = 2.162$  is (roughly) 2 standard deviations above the mean so it's an unusual result when  $\mu = 1.4$ . In other words, we probably do have compelling evidence against  $H_0$ .

## Using **STUDYSmarter** Resources

This resource was developed for UWA students by the **STUDYSmarter** team for the numeracy program. When using our resources, please retain them in their original form with both the **STUDYSmarter** heading and the UWA crest.

