Домашняя работа

6. $\sqrt{\dfrac{3x^2+2x-1}{4+3x-x^2}} < 1$

ОДЗ: $\begin{cases} 4+3x-x^2 \neq 0 & (1) \\ \dfrac{3x^2+2x-1}{4+3x-x^2} \geq 0 & (2) \end{cases}$

(1) $-x^2+3x+4=0 \quad \Big| \; (x+1)(x-4)$

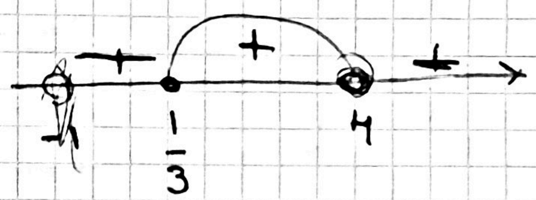
$D = 9+4\cdot 4 = 25$

$\left[\begin{array}{l} x=4 \\ x=-1 \end{array}\right.$

2) $\dfrac{3x^2+2x-1}{4+3x-x^2} \geq 0 \qquad \dfrac{3\cancel{(x+1)}(x-\frac{1}{3})}{-\cancel{(x+1)}(x-4)} \geq 0$

$3x^2+2x-1=0 \quad \Big| \; 3(x+1)(x-\frac{1}{3})$

$\left[\begin{array}{l} x=-1 \\ x=\frac{1}{3} \end{array}\right.$



$\begin{cases} x \in (-\infty; \frac{1}{3}) \cup [4; +\infty) \\ x \neq -1; 4 \end{cases} \qquad x \in [\frac{1}{3}; 4)$ - ОДЗ;

$\sqrt{\dfrac{3x^2+2x-1}{4+3x-x^2}} < 1$

$\sqrt{\dfrac{3\cancel{(x+1)}(x-\frac{1}{3})}{-\cancel{(x+1)}(x-4)}} < 1$

$\sqrt{\dfrac{1-3x}{x-4}} < 1$

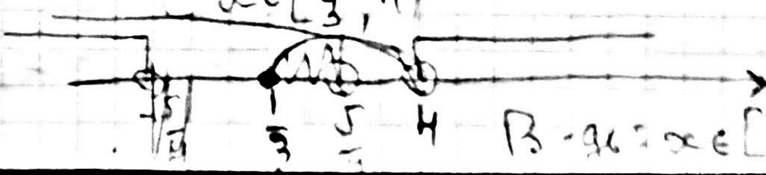
$\dfrac{1-3x}{x-4} - 1 < 0$

$\dfrac{5-4x}{x-4} < 0$

$\dfrac{5-4x}{x-4}$

$\left[\begin{array}{l} \begin{cases} 5-4x<0 \\ x-4>0 \end{cases} \\ \begin{cases} 5-4x>0 \\ x-4<0 \end{cases} \end{array}\right. \qquad \left[\begin{array}{l} \begin{cases} x>\frac{5}{4} \\ x>4 \end{cases} \\ \begin{cases} x<\frac{5}{4} \\ x<4 \end{cases} \end{array}\right.$

$\begin{cases} \left[\begin{array}{l} x \in (4; +\infty) \\ x \in (-\infty; +\frac{5}{4}) \end{array}\right. \\ x \in [\frac{1}{3}; 4) \end{cases}$



В-вн: $x \in [\frac{1}{3}; \frac{5}{\ldots}$

7. $\sqrt{2x-1}+\sqrt{x+15}\le 5$

ОДЗ: $\begin{cases} 2x-1\ge 0 \\ x+15\ge 0 \end{cases}$ $\begin{cases} x\ge \frac{1}{2} \\ x\ge -15 \end{cases}$ $x\in\left[\frac{1}{2};+\infty\right)$

$2x-1+2\sqrt{(2x-1)(x+15)}+x+15\le 25$

$3x+14+2\sqrt{2x^2+29x-15}\le 25$

$2\sqrt{2x^2+29x-15}\le 11-3x$

$\begin{cases} 2\sqrt{2x^2+29x-15}\ge 0 \quad (1) \\ 11-3x\ge 0 \\ 8x^2+116x-60\le 9x^2-66x+121 \quad (2) \end{cases}$

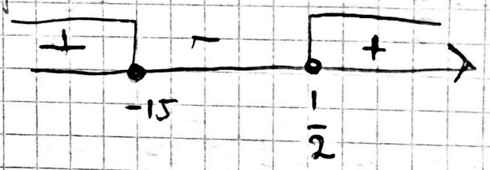
(1) $2\sqrt{2x^2+29x-15}\ge 0$

$2x^2+29x-15\ge 0$

$D=841+4\cdot 2\cdot 15=961$ | $(x-\frac{1}{2})(x+15)$

$x=\frac{-29+31}{4}=\frac{1}{2}$

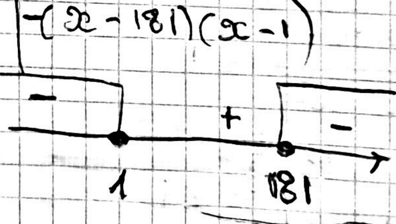
$x=\frac{-29-31}{4}=-15$



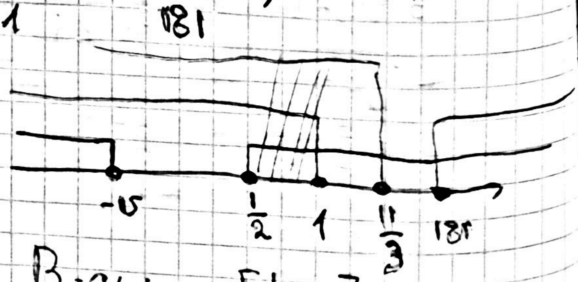
(2) $8x^2+116x-60\le 9x^2-66x+121$

$-x^2+182x-181\le 0$ | $-(x-181)(x-1)$

$\left[\begin{array}{l} x=181 \\ x=1 \end{array}\right.$



$\begin{cases} x\in(-\infty;-15]\cup\left[\frac{1}{2};+\infty\right) \\ x\le\frac{11}{3} \\ x\in(-\infty;1]\cup[181;+\infty) \end{cases}$



В-дь: $x\in\left[\frac{1}{2};1\right]$

8. $\sqrt{\dfrac{x^2-8x-20}{x-5}} \geq x-2$

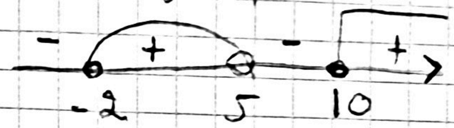
ОДЗ: $\begin{cases} \dfrac{x^2-8x-20}{x-5} \geq 0 \quad (1) \\ x \neq 5 \end{cases}$

(1) $x^2-8x-20=0$ | $(x-10)(x+2)$

$D = 64+4\cdot 20 = 144$

$\left[\begin{array}{l} x = 10 \\ x = -2 \end{array}\right.$

$$\frac{(x-10)(x+2)}{(x-5)} \geq 0$$



$x \in [-2;5) \cup [10;+\infty)$

$$\left[\begin{array}{l} \begin{cases} \dfrac{x^2-8x-20}{x-5} \geq 0 \\ x-2 < 0 \end{cases} \\ \begin{cases} \dfrac{x^2-8x-20}{x-5} \geq 0 \\ x-2 \geq 0 \\ \dfrac{x^2-8x-20}{x-5} \geq x^2-4x+4 \quad (2) \end{cases} \end{array}\right.$$

$$\left[\begin{array}{l} \begin{cases} x \in [-2;5) \cup [10;+\infty) \\ x < 2 \end{cases} \\ \begin{cases} x \in [-2;5) \cup [10;+\infty) \\ x \geq 2 \\ \dfrac{x^2-8x-20}{x-5} \geq x^2-4x+4 \end{cases} \end{array}\right.$$

(2) $\dfrac{x^2-8x-20}{x-5} - x^2+4x-4 \geq 0$

$\dfrac{x^2-8x-20 - x^2(x-5)+4x(x-5) - 4(x-5)}{x-5} \geq 0$

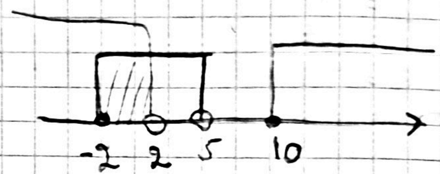
$\dfrac{10x^2-32x-x^3}{x-5} \geq 0$

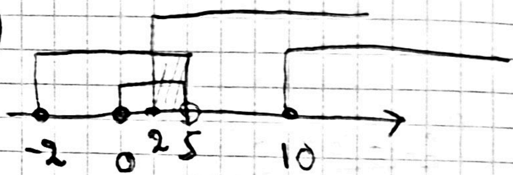
$\left[\begin{array}{l} \begin{cases} 10x^2-32x-x^3 \geq 0 \\ x-5 > 0 \end{cases} \\ \begin{cases} 10x^2-32x-x^3 \leq 0 \\ x-5 < 0 \end{cases} \end{array}\right.$

$$\left[\begin{array}{l}\begin{cases} x(10x-32-x^2)\ge 0\\ x>5\end{cases}\\ \begin{cases} x(10x-32-x^2)\le 0\\ x<5\end{cases}\end{array}\right.$$

$$\left[\begin{array}{l}\begin{cases} x\in(-\infty;0]\\ x>5\end{cases} \quad \varnothing\\ \begin{cases} x\in[0;+\infty)\\ x<5\end{cases} \quad x\in[0;5)\end{array}\right.$$

$$\left[\begin{array}{l}\begin{cases} x\in[-2;5)\cup[10;+\infty)\\ x<2\end{cases}\\ \begin{cases} x\in[-2;5)\cup[10;+\infty)\\ x\ge 2\\ x\in[0;5)\end{cases}\end{array}\right.$$



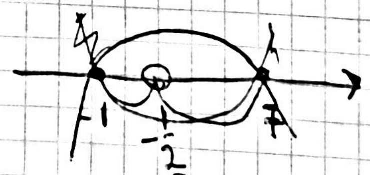


$$\left[\begin{array}{l} x\in[-2;2)\\ x\in[2;5)\end{array}\right.$$

В-дь: $x\in[-2;5)$

9. $$\frac{\sqrt{7+6x-x^2}}{x+7}\ge\frac{\sqrt{7+6x-x^2}}{2x+1}$$

ОДЗ: $\begin{cases} 7+6x-x^2\ge 0\\ x\ne -7\\ x\ne -\frac{1}{2}\end{cases}$



$x\in[-1;-\frac{1}{2})\cup(-\frac{1}{2};7]$

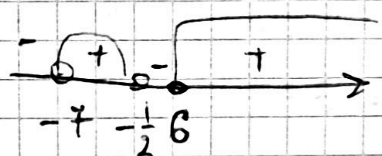
$$\frac{\sqrt{7+6x-x^2}}{x+7}-\frac{\sqrt{7+6x-x^2}}{2x+1}\ge 0$$

$$\frac{(2x+1)\sqrt{7+6x-x^2}-(x+7)\sqrt{7+6x-x^2}}{(2x+1)(x+7)}\ge 0$$

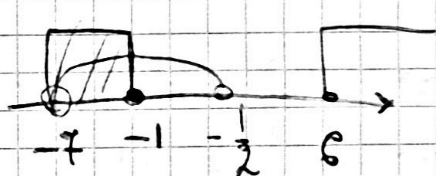
$$\sqrt{7+6x-x^2}\left(-\frac{1}{2x+1}+\frac{1}{x+7}\right)\geq 0$$

$$\left[\begin{array}{l}\begin{cases}\sqrt{7+6x-x^2}\geq 0\\ \frac{1}{x+7}-\frac{1}{2x+1}\geq 0\end{cases} \quad (1)\\ \begin{cases}\sqrt{7+6x-x^2}\leq 0\\ \frac{1}{x+7}-\frac{1}{2x+1}\leq 0\end{cases} \quad (2)\end{array}\right.$$

(1) $\frac{(x-6)}{2(x+7)(x+\frac{1}{2})}\geq 0$



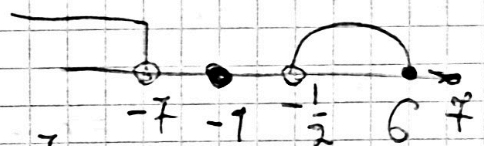
$$\begin{cases}x\in[-1;7]\\ x\in(-7;-\frac{1}{2})\cup[6;+\infty)\end{cases}$$



(2) $x=-1$;

$x=7$;

$x\in(-\infty;-7)\cup(-\frac{1}{2};6]$



$x\in\varnothing$

$$\left[\begin{array}{l}x\in(-7;-1)\\ x\in\varnothing\end{array}\right.$$

В-дь: $x\in(-7;-1)$

ДЗ. Похідна.

№6.16

1) $\begin{cases}f(x)=x^2-5x\\ f(x)\text{дот}=-x\end{cases}$

$f'(x)=2x-5$; $k_{дот}=k_{прям}$ - умова паралел.

$\left.\begin{array}{l}k_{дот}=f'(x_0)\\ k_{прям}=-1\end{array}\right\}\Rightarrow f'(x_0)=-1$ $\quad$ $\underline{y_д=-1x-4}$

$f'(x_0)$: $2x-5=-1$; $x=2$ $\Rightarrow f(x)=-6$;

2) $\begin{cases} f(x) = x - \frac{1}{x^2} \\ f(x)_{\text{дот}} \parallel 3x \end{cases}$

$k_{\text{дот}} = k_{\text{прямої}}$ → умова парал.

$\left.\begin{array}{l} k_{\text{дот}} = f'(x_0) \\ k_{\text{прямої}} = 3 \end{array}\right\} \Rightarrow k_{\text{дот}} = 3 = f'(x_0)$

$f'(x) = 1 - (-2) \cdot x^{-3} = 1 + \frac{2}{x^3}$

$f'(x_0) = 3; \Rightarrow 1 + \frac{2}{x^3} = 3 \Rightarrow x_0 = 1$

$f(x_0) = 0$

$y_{\text{дот}} = f'(x_0) \cdot (x - x_0) + f(x_0)$

$y_{\text{дот}} = 3 \cdot (x - 1) + 0 = 3x - 3$

3) $\begin{cases} f(x) = 2x^3 + 3x^2 - 10x - 1 \\ f(x)_{\text{дот}} \parallel 2x + 1 \end{cases}$

$k_{\text{дот}} = k_{\text{прямої}}$ → умова паралел.

$\begin{cases} k_{\text{дот}} = f'(x_0) \\ k_{\text{прямої}} = 2 \end{cases} \Rightarrow k_{\text{дот}} = 2 = f'(x_0)$

$f'(x) = 6x^2 + 6x - 10$

$f'(x_0) = 2 \Rightarrow 6x^2 + 6x - 10 = 2$

$x^2 + x - 2 = 0$; $x = x_0$

$\left[\begin{array}{l} x_1 = -1 \\ x = 2 \end{array}\right.$

~~Удот~~

$x_0 = -1 \Rightarrow f(x_0) = 10$;

$x_0 = 2 \Rightarrow f(x_0) = 7$;

$\left[\begin{array}{l} y_{\text{дот}} = 2(x+1) + 10 = 2x + 12 \\ y_{\text{дот}} = 2(x-2) + 7 = 2x + 3 \end{array}\right.$

46.15

$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 4$

Пряма $y = kx + b$ паралельна віcі Ox при $k = 0 \Rightarrow$

$f'(x_0) = 0$

$f'(x) = \frac{1}{3} \cdot 3x^2 - 2x - 3 = x^2 - 2x - 3$

$x^2 - 2x - 3 = 0$

$\left[\begin{array}{l} x = -1 \\ x = 3 \end{array}\right. \longrightarrow \left[\begin{array}{l} x_0 = -1 \\ x_0 = 3 \end{array}\right.$

$y_{дот} = \underbrace{f'(x_0)}_{k=0}(x - x_0) + f(x_0)$

1) $x_0 = -1 \Rightarrow f(x_0) = f(-1) = -\frac{1}{3} - 1 + 3 + 4 \Rightarrow y_{дот} = 5\frac{2}{3}$

2) $x_0 = 3 \Rightarrow f(x_0) = f(3) = \frac{1}{3} \cdot 27 - 9 - 9 + 4 \Rightarrow y_{дот} = -5$

46.18.

$y_{дот} = 12x - 10$; $f(x) = 4x^3$

$k_{дот} = 12 = f'(x)$; $f'(x) = 12x^2$

$12 = 12x^2$; $x^2 = \pm 1$

$x = -1 \Rightarrow f(x) = -4$; $y_{дот} = 12(x+1) - 4 = 12x - 8$ - не зад

$x = 1 \Rightarrow f(x) = 4$; $y_{дот} = 12(x-1) + 4 = 12x + 8$ - не зад.

46.26.

$f(x) = -\sqrt{2x+1}$

$y_{дот} \perp y - 2x + 1 = 0$; $y = 2x - 1$

$k_{дот} \cdot k_{прям} = -1 \rightarrow$ умова перп.

$k_{прямої} = 2 \Rightarrow k_{дот} = -\frac{1}{2} = f'(x_0)$

$f'(x) = -\frac{2}{2\sqrt{2x+1}} = -\frac{1}{1\sqrt{2x+1}}$; $-\frac{1}{\sqrt{2x+1}} = -\frac{1}{2}$

$\sqrt{2x+1} = 2 \Rightarrow x = \frac{3}{2} = x_0 \Rightarrow f(x_0) = -2$

В-дь: $\left(\frac{3}{2}; -2\right)$