

$P$  (polynomial) problem  $\rightarrow$   $P$  problems refer to problems where an algorithm would take a polynomial amount of time to solve, or Big- $O$  is a polynomial like  $O(1)$ ,  $O(n)$ ,  $O(n^2)$  etc.

$NP$  (non-deterministic polynomial) problems  $\rightarrow$  non-deterministic polynomial time solving, problem which can't be solved in polynomial time. But  $NP$  problems are checkable in polynomial time means that given a solution of a problem, we can check that whether the solution is correct or not in polynomial time.

$NP$ -hard is the class of decision problems to which all problems in  $NP$  can be reduce to in polynomial time by a deterministic Turing machine.

$NP$ -complete is the class of decision problems in  $NP$  to which all other problems reduce to in polynomial time by a deterministic Turing machine.

A language  $B$  is  $NP$ -complete if it satisfy two conditions

- $B$  is in  $NP$
- Every  $A$  in  $NP$  is polynomial time reducible to  $B$

If a language satisfies the second property, but not necessarily the first one, the language  $B$  is known as  $NP$ -Hard. A search problem  $B$  is  $NP$ -hard if there exists some  $NP$ -complete problem  $A$  that Turing reduce to  $B$ .

Below is a venn diagram of different classes



Date :

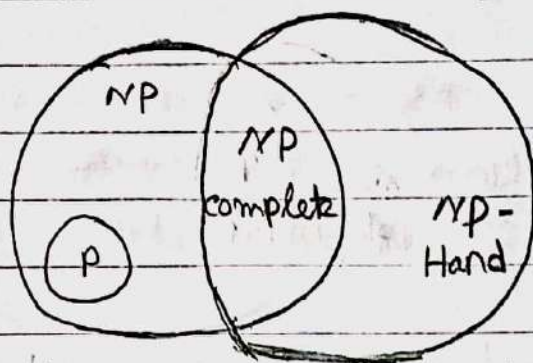


fig → Venn diagram of  $P$ ,  $P$ -hard,  $NP$ ,  $NP$ -complete.

Some examples of  $NP$ -complete and  $NP$ -Hard problems

problems like - Knapsack problem, Hamiltonian path problem.

Travelling salesman problem, The circuit-satisfiability problem, Boolean satisfy problem. etc.

The difference between  $NP$ -Hard and  $NP$ -complete is

$NP$ -Hard	$NP$ -complete
① $NP$ -Hard problems can be solved if and only if there is a $NP$ -complete problem can be reduce into in polynomial time.	① A problem is $NP$ -complete if any problem in $NP$ can be reduce to it in polynomial time and its a $NP$ .
② To solve, $NP$ -Hard, it must be a $NP$ -problems.	② To solve, $NP$ -complete, it must be both $NP$ and $NP$ -complete.
③ $NP$ -problems are → Halting problems, Traveling salesman	③ $NP$ -completes problems are → Knapsack problem, Hamiltonian path problem etc.